Supersymmetry as a part of Higher dimensional gauge symmetry

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Introduction

The hierarchy problem can be solved by

- **Geometry of extra dimensions**

  N.Arkani-Hamed, S.Dimopoulos, G.R.Dvali, PLB429(1998)263

  L.Randall and R.Sundrum, PRL83(1999)3370

- **Higher dimensional gauge symmetry**

  H.Hatanaka, T.Inami, C.S.Lim, MPLA13(1998)2601

- **Supersymmetry**

  boson ⟷ fermion

Introduction

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- ◆Supersymmetry

  boson ⟷ fermion

Supersymmetry always exist in higher dimensional gauge theory

A setup

- Space-time

$$x^M = (x^\mu, y)$$

4d Minkowski space-time   an extra space

- Metric

$$ds^2 = \Delta(y)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu + dy^2\right)$$

$$\Delta(y) = \frac{1}{ky}$$: Randall-Sundrum metric

A setup

- A pure abelian gauge theory

$$S = \int d^4x dy \sqrt{-g} \left(-\frac{1}{4} g^{MM'} g^{NN'} F_{MM'} F_{NN'} \right)$$

$$F_{MN}(x,y) = \partial_M A_N(x,y) - \partial_N A_M(x,y)$$

$$g_{MN} = \Delta(y)^2 \eta_{MN}$$

$$\eta_{MN} = \begin{pmatrix} -1 & & & & 0 \\ & 1 & & & \\ & & 1 & & \\ & 0 & & 1 & \\ & & & & 1 \end{pmatrix}$$

Mode expansions 1

$$A_M(x,y) = (A_\mu(x,y), A_y(x,y))$$

- **Mode expansions**

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) f_n(y)$$

$$A_y(x,y) = \sum_n A_y^{(n)}(x) g_n(y)$$

Mode expansions 1

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regard as 4d gauge fields

Mode expansions 1

$$A_M(x,y) = (A_\mu(x,y), A_y(x,y))$$

complete sets

- **Mode expansions**

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) f_n(y)$$

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Mode expansions 1

$$A_M(x,y) = (A_\mu(x,y), A_y(x,y))$$

complete sets

- Mode expansions

$$A_\mu(x,y) = \sum_n A_\mu^{(n)}(x) f_n(y)$$

$$A_y(x,y) = \sum_n A_y^{(n)}(x) g_n(y)$$

question

Which complete sets?

Consistent with gauge theory？

Mode expansions 2

- **Diagonalization of mass term of** $A_\mu^{(n)}$

$$S = \int d^4x dy \sqrt{-g} \left(-\frac{1}{4} g^{MM'} g^{NN'} F_{MM'} F_{NN'} \right) \quad \Rightarrow \quad \int dy \Delta \left(\partial_y A_\mu\right) \left(\partial_y A^\mu\right)$$

mass term of $A_\mu^{(n)}$

Diagonalized

$$\left[-\frac{1}{\Delta(y)} \partial_y \Delta(y) \partial_y \right] f_n(y) = m_n^2 f_n(y)$$

Mode expansions 2

- Diagonalization of mass term of $A_\mu^{(n)}$

$$S = \int d^4x dy \sqrt{-g} \left(-\frac{1}{4} g^{MM'} g^{NN'} F_{MM'} F_{NN'} \right) \rightarrow \int dy \Delta \left(\partial_y A_\mu \right) \left(\partial_y A^\mu \right)$$

mass term of $A_\mu^{(n)}$

Diagonalized

$$\left[-\frac{1}{\Delta(y)} \partial_y \Delta(y) \partial_y \right] f_n(y) = m_n^2 f_n(y)$$

◆ Inner product

$$\langle f_n | f_m \rangle = \int dy \; \Delta(y) \; f_n(y) f_m(y)$$

Mode expansions 2

- Diagonalization of mass term of $A_\mu^{(n)}$

$$S = \int d^4x dy \sqrt{-g} \left(-\frac{1}{4} g^{MM'} g^{NN'} F_{MM'} F_{NN'} \right) \Rightarrow \int dy \Delta \left(\partial_y A_\mu \right) \left(\partial_y A^\mu \right)$$

mass term of $A_\mu^{(n)}$

Diagonalized

$$\left[-\frac{1}{\Delta(y)} \partial_y \Delta(y) \partial_y \right] f_n(y) = m_n^2 f_n(y)$$

Hermitian operator

◆ Inner product

$$\langle f_n | f_m \rangle = \int dy \ \Delta(y) \ f_n(y) f_m(y)$$

Mode expansions 3

- Relation between f_n & g_n
  - Gauge transformation

$$\delta A_M = \partial_M \epsilon(x,y) \;\Rightarrow\; \begin{array}{l} \delta A_\mu = \partial_\mu \epsilon(x,y) \\ \delta A_y = \partial_y \epsilon(x,y) \end{array}$$

$$\Downarrow \quad \epsilon(x,y) = \sum_n \epsilon^{(n)}(x) k_n(y)$$

$$\begin{array}{c} f_n(y) \sim k_n(y) \\ g_n(y) \sim \partial_y k_n(y) \sim \partial_y f_n(y) \end{array} \;\Rightarrow\; g_n(y) \sim \partial_y f_n(y)$$

Mode expansions 4

$$\left[-\frac{1}{\Delta(y)}\partial_y \Delta(y) \partial_y\right] f_n(y) = m_n^2 f_n(y)$$

$\partial_y \times$

$\partial_y f_n(y) \sim g_n(y)$

$$\left[-\partial_y \frac{1}{\Delta(y)}\partial_y \Delta(y)\right] g_n(y) = m_n^2 g_n(y)$$

Hermitian operator

- **Inner product**

$$\langle g_n | g_m \rangle = \int dy \; \Delta(y) \; g_n(y) g_m(y)$$

Supersymmetric structure

$$\begin{pmatrix} -\frac{1}{\Delta}\partial_y \Delta \partial_y & 0 \\ 0 & -\partial_y \frac{1}{\Delta}\partial_y \Delta \end{pmatrix} \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix} = m_n^2 \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix}$$

$$H \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix} = m_n^2 \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix}$$

$$\{Q_i, Q_j\} = 2\delta_{ij} H$$

Supersymmetry algebra (in quantum mechanics)

$$Q_1 = \begin{pmatrix} 0 & -\frac{1}{\Delta}\partial_y \Delta \\ \partial_y & 0 \end{pmatrix}$$

$$H = \begin{pmatrix} -\frac{1}{\Delta}\partial_y \Delta \partial_y & 0 \\ 0 & -\partial_y \frac{1}{\Delta}\partial_y \Delta \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 0 & i\frac{1}{\Delta}\partial_y \Delta \\ i\partial_y & 0 \end{pmatrix}$$

Schrodinger like equation

$$\begin{pmatrix} -\frac{1}{\Delta}\partial_y \Delta \partial_y & 0 \\ 0 & -\partial_y \frac{1}{\Delta}\partial_y \Delta \end{pmatrix} \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix} = m_n^2 \begin{pmatrix} f_n(y) \\ g_n(y) \end{pmatrix}$$

$$F_n(y) = \Delta^{-\frac{1}{2}} f_n(y)$$

$$G_n(y) = \Delta^{-\frac{1}{2}} g_n(y)$$

$$W = \frac{1}{2\Delta}\frac{d\Delta}{dy}$$

$$\begin{pmatrix} -\frac{d^2}{dy^2} + W(y)^2 + \frac{dW(y)}{dy} & 0 \\ 0 & -\frac{d^2}{dy^2} + W(y)^2 - \frac{dW(y)}{dy} \end{pmatrix} \begin{pmatrix} F_n(y) \\ G_n(y) \end{pmatrix} = m_n^2 \begin{pmatrix} F_n(y) \\ G_n(y) \end{pmatrix}$$

N=2 Supersymmetric quantum mechanics

E. Witten, NPB188(1981)513

Supersymmetry 1

Supersymmetric strucutre

→ Supersymmetry in the lagrangian?

- Lagrangian with gauge fixing term

$$L = \sqrt{-g}\left(-\frac{1}{4}g^{MM'}g^{NN'}F_{MM'}F_{NN'}\right)$$

$$L_{GF} = -\frac{1}{2}\left(\partial_\mu A^\mu + \frac{1}{\Delta}\partial_y \Delta A^y\right)^2$$

$$L + L_{GF} = \frac{\Delta}{2}\begin{pmatrix} A^y & A^\mu \end{pmatrix}\begin{pmatrix} \partial_\mu\partial^\mu & 0 \\ 0 & \partial_\mu\partial^\mu \end{pmatrix}\begin{pmatrix} A_y \\ A_\mu \end{pmatrix}$$

$$-\frac{\Delta}{2}\begin{pmatrix} A^y & A^\mu \end{pmatrix}\begin{pmatrix} -\partial_y \frac{1}{\Delta}\partial_y \Delta & 0 \\ 0 & -\frac{1}{\Delta}\partial_y \Delta \partial_y \end{pmatrix}\begin{pmatrix} A_y \\ A_\mu \end{pmatrix}$$

$$H = Q^2$$

Supersymmetry 2

$$L + L_{GF} = \frac{\Delta}{2}\begin{pmatrix} A^y & A^\mu \end{pmatrix}\begin{pmatrix} \partial_\mu\partial^\mu & 0 \\ 0 & \partial_\mu\partial^\mu \end{pmatrix}\begin{pmatrix} A_y \\ A_\mu \end{pmatrix}$$

$$-\frac{\Delta}{2}\begin{pmatrix} A^y & A^\mu \end{pmatrix}\begin{pmatrix} -\partial_y\frac{1}{\Delta}\partial_y\Delta & 0 \\ 0 & -\frac{1}{\Delta}\partial_y\Delta\partial_y \end{pmatrix}\begin{pmatrix} A_y \\ A_\mu \end{pmatrix}$$

$$H = Q^2$$

$$[H, Q] = 0$$

Supersymmetry?

$$\begin{pmatrix} A_y \\ A_\mu \end{pmatrix} \to e^{i\theta Q}\begin{pmatrix} A_y \\ A_\mu \end{pmatrix} \qquad Q = \begin{pmatrix} 0 & i\frac{1}{\Delta}\partial_y\Delta \\ i\partial_y & 0 \end{pmatrix}$$

mismatch !

Supersymmetry 3

$$A_\mu = A_\mu^T(x,y) + A_\mu^L(x,y) \qquad \textbf{with} \qquad \partial^\mu A_\mu^T = 0$$

$$A_\mu^T = \frac{1}{\partial^2}\left(\partial^2 \eta_{\mu\nu} - \partial_\mu \partial_\nu\right) A^\nu$$

$$A_\mu^L = \frac{1}{\partial^2}\partial_\mu \partial_\nu A^\nu$$

Supersymmetry 3

$$A_\mu = A^T_\mu(x,y) + A^L_\mu(x,y) \qquad \textbf{with} \qquad \partial^\mu A^T_\mu = 0$$

$$A^T_\mu = \frac{1}{\partial^2}\left(\partial^2 \eta_{\mu\nu} - \partial_\mu \partial_\nu\right) A^\nu$$

$$A^L_\mu = \frac{1}{\partial^2}\partial_\mu \partial_\nu A^\nu = \frac{1}{\sqrt{-\partial^2}}\partial_\mu \rho$$

$$\rho \equiv -\frac{1}{\sqrt{-\partial^2}}\partial_\nu A^\nu$$

$$\Downarrow$$

$$A_\mu \to (A^T_\mu, \rho)$$

Supersymmetry 4

$$L + L_G = \frac{\Delta}{2}(A^y \quad \rho)\begin{pmatrix} \partial_\mu\partial^\mu & 0 \\ 0 & \partial_\mu\partial^\mu \end{pmatrix}\begin{pmatrix} A_y \\ \rho \end{pmatrix}$$

$$-\frac{\Delta}{2}(A^y \quad \rho)\begin{pmatrix} -\partial_y \frac{1}{\Delta}\partial_y\Delta & 0 \\ 0 & -\frac{1}{\Delta}\partial_y\Delta\partial_y \end{pmatrix}\begin{pmatrix} A_y \\ \rho \end{pmatrix}$$

$$+\frac{\Delta}{2}A^T_\mu(\partial_\mu\partial^\mu + \frac{1}{\Delta}\partial_y\Delta\partial_y)A^{\mu T}$$

- **SUSY transformation**

$$\begin{pmatrix} A_y \\ \rho \end{pmatrix} \to \begin{pmatrix} A'_y \\ \rho' \end{pmatrix} = e^{i\theta Q}\begin{pmatrix} A_y \\ \rho \end{pmatrix}$$

$$Q = \begin{pmatrix} 0 & i\partial_y \\ i\frac{1}{\Delta}\partial_y\Delta & 0 \end{pmatrix}$$

SUSY as a gauge symmetry

$$\begin{pmatrix} \delta A_y \\ \delta\rho \end{pmatrix} = e^{i\theta Q} \begin{pmatrix} A_y \\ \rho \end{pmatrix} = \theta \begin{pmatrix} 0 & -\partial_y \\ -\frac{1}{\Delta}\partial_y\Delta & 0 \end{pmatrix} \begin{pmatrix} A_y \\ \rho \end{pmatrix}$$

$$\delta A_y = -\theta\partial_y\rho$$
$$\delta\rho = -\theta\frac{1}{\Delta}\partial_y\Delta A_y$$

$$\delta A_y = -\theta\partial_y\rho$$
$$\delta A^L_\mu = -\theta\partial_\mu\rho$$

using gauge fixing condition

$$\delta A_M = -\theta\partial_M\rho$$

gauge transformation

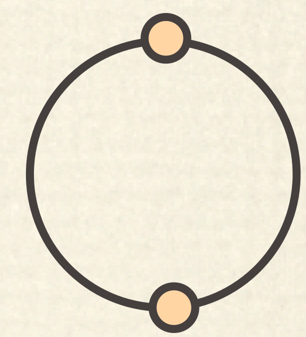
Application

- Extra dimensions with boundaries

interval

fixed point

Orbifold



fixed point

We require the boundary conditions to preserve the supersymmetry

There are only two types of BC's at each boundary

very restrictive!

T. Nagasawa, M. Sakamoto, K. Takenaga, PLB562(2003)358, PLB583(2004)357 J. Phys. A38(2005)8053

Generalization

- **R-ξ gauge**

$$L_{GF} = -\frac{1}{2}\left(\partial_\mu A^\mu + \frac{1}{\Delta}\partial_y \Delta A^y\right)^2 \quad \Rightarrow \quad L_{GF} = -\frac{1}{2\xi}\left(\partial_\mu A^\mu + \xi\frac{1}{\Delta}\partial_y \Delta A^y\right)^2$$

Supersymmetry exist

- **(4+1)→(4+n)**

$$x^M = (x^\mu, y^i) \quad (i = 1, 2, \cdots, n) \qquad ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + g_{ij}dy^i dy^j$$

$$S = \int dx^4 d^n y \sqrt{-g}\left(-\frac{1}{4}g^{MM'}g^{NN'}F_{MN}F_{M'N'}\right) \qquad A_M = (A_\mu, A_i)$$

Supersymmetry exist

$$A^L_\mu \leftrightarrow A^{(1)} \qquad A^{(1)} \equiv A_i dy^i$$

SUSY in Gravity

- **Gravity with Randall-Sundrum background**

$$ds^2 = g_{MN} dx^M dx^N$$
$$= \frac{1}{(ky)^2} (\eta_{MN} + h_{MN}) dx^M dx^N$$

$$h_{\mu\nu} = \sum_n h_{\mu\nu}^{(n)} f_n(y)$$
$$h_{\mu y} = \sum_n h_{\mu y}^{(n)} g_n(y)$$
$$h_{yy} = \sum_n h_{yy}^{(n)} k_n(y)$$

SUSY

$$f_n(y) \leftrightarrow g_n(y)$$
$$g_n(y) \leftrightarrow k_n(y)$$

C.S. Lim, T. Nagasawa, K. Sakamoto, M. Sakamoto, S.Ohya, in preparation

Summary

- Summary
  - The higher dimensional gauge theories possess the supersymmetry as a part of gauge symmetry
  - We can obtain the allowed boundary conditions from a susy point of view
- Outlook
  - SUSY of matter field?
  - Application