Gauge Messenger Models

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Outline

1. Little Hierarchy Problem

2. Radiatively Generated Maximal Stop Mixing Scenario for Higgs Mass

3. Gauge Messenger Model

4. Neutralino Dark Matter in Gauge Messenger Model
   - Neutralino mass matrix
There are several good motivations for SUSY.

- Stabilize weak scale (Higgs mass)
- Gauge Coupling Unification with $M_{\text{GUT}} \sim 2 \times 10^{16}$ GeV.
- Compatible with EW precision data.
- Dark Matter with weak scale SUSY (with R-parity)
SUSY suffers from \textit{Little Hierarchy Problem}

- We haven’t found any experimental discovery of SUSY yet. Especially, LEP2 did not see Higgs. Higgs mass is logarithmically sensitive to the SUSY breaking scale.

\[ m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2} \]

\[ \rightarrow \text{Current observational bound } m_h \geq 114\text{GeV} \text{. generically pushes up superpartner masses } m_{\tilde{t}}, m_{H_u}, \ldots \text{ to } \mathcal{O}(\text{TeV}) \]

\( M_Z \) is determined by soft SUSY breaking parameters.

\[ \frac{M_Z^2}{2} \approx -\mu^2(M_Z) - m_{H_u}^2(M_Z) \]

\[ \rightarrow 0.5 \% \text{ level fine-tuning} \rightarrow \text{Little Hierarchy Problem} \]
Radiative correction to Higgs mass

- Effective Potential has radiative corrections of the form

\[ V^1(Q) = V^0(Q) + \Delta V^1(Q) \]
\[ \Delta V^1(Q) = \frac{1}{64\pi^2} \text{Str} M^4(h) \left[ \log \frac{M^2(h)}{Q^2} - \frac{3}{2} \right] \]

w/o mixing between \( \tilde{t}_L \) and \( \tilde{t}_R \),

\[ m_h^2 \approx M_Z^2 \cos^2 2\beta + \frac{3G_F m_t^4}{\sqrt{2}\pi^2} \log \frac{m_{\tilde{t}}^2}{m_t^2} \]

→ logarithmically sensitive to \( m_{\tilde{t}}^2 \).

To have \( m_h > 114 \) GeV, \( m_{\tilde{t}} \sim \mathcal{O}(\text{TeV}) \).
Electroweak Symmetry Breaking is triggered mainly by two parameters $\mu$ and $m_{Hu}^2$ in MSSM:

$$\frac{M_Z^2}{2} \approx -\mu^2(M_Z) - m_{Hu}^2(M_Z)$$

$\mu$ term does not change much from GUT scale value.

$$\frac{d\mu}{d \log Q} = \frac{\mu}{16\pi^2}(3y_t^2 + \ldots) \propto \mu \quad \text{(no big change 5 \sim 10\%)}$$

However, $m_{Hu}^2$ has a big radiative correction $\propto m_t^2$ through RG evolution.

$$\frac{dm_{Hu}^2}{d \log Q} = \frac{3y_t^2}{8\pi^2}(m_{Q_3}^2 + m_{\tilde{\tau}}^2 + m_{Hu}^2 + |A_t|^2)$$

$$\delta m_{Hu}^2 \approx -\frac{3y_t^2}{4\pi^2} m_t^2 \log \frac{M_{\text{GUT}}}{m_t} \approx -m_t^2 \sim O(\text{TeV})^2$$
We need fine-tuning of parameters to get $M_Z = 90$ GeV.

$$\frac{M_Z^2}{2} \approx -\mu^2(M_{\text{GUT}}) - m_{H_u}^2(M_{\text{GUT}}) + m_{\tilde{t}}^2$$

- For $m_{\tilde{t}} \sim 1$ TeV,
  
  $m_{H_u}^2(M_{\text{GUT}}) \sim (1 \text{ TeV})^2$
  
  and $\mu^2(M_{\text{GUT}}) \sim (1 \text{ TeV})^2$

$$\Delta = \frac{\delta M_Z^2}{M_Z^2} = \frac{2m_{H_u}^2(M_{\text{GUT}})}{M_Z^2} \sim 240$$

- 0.5% fine tuning.
Radiatively Generated Maximal Stop Mixing Scenario


- Large Mixing between $\tilde{t}_L$ and $\tilde{t}_R$ helps higgs mass lift-up.

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{Q}_3}^2 + m_t^2 + \ldots & -m_{\tilde{u}_c^3} (A_t^* + \mu \cot \beta) \\ -(A_t + \mu^* \cot \beta) m_{\tilde{u}_c^3}^* & m_{\tilde{u}_c^3}^2 + m_t^2 + \ldots \end{pmatrix}$$

$$m_h^2 \sim M_Z^2 + \frac{3G_F m_t^4}{\sqrt{2} \pi^2} \left\{ \log \frac{M_S^2}{m_t^2} + \frac{A_t^2}{M_S^2} \left( 1 - \frac{A_t^2}{12M_S^2} \right) \right\}$$

$\rightarrow$ Maximum at $A_t = \pm \sqrt{6} M_S$. 

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Gauge Messenger
Physical Higgs Mass vs Stop Mixing (Carena et al)

Leading $m_t^4$ approximation at $O(\alpha \alpha_\tau)$

$M_s^{\text{MS}} = 1000 \text{ GeV}$, $m_A = 1000 \text{ GeV}$, $\tan\beta = 30$
To satisfy LEP2 bound $m_h > 114\text{GeV}$ (using FeynHiggs) for $m_\tilde{t}(M_Z) \approx 300$ GeV

$$|A_t(M_Z)| \approx 450\text{GeV, } \tan \beta \gtrsim 50$$

$$|A_t(M_Z)| \approx 500\text{GeV, } \tan \beta \gtrsim 8$$

$$|A_t(M_Z)| \approx 600\text{GeV, } \tan \beta \gtrsim 6$$

Therefore, $\left| \frac{A_t(M_Z)}{m_\tilde{t}(M_Z)} \right| \gtrsim 1.5$ is crucial.

Unfortunately, the maximal mixing is not easily achieved due to the RG running.

However, stop can be light.

$$\log \frac{m_\tilde{t}^2}{m_t^2} = 3$$

$$\rightarrow m_\tilde{t}_{(L,R)} \approx e^{-\frac{3}{2}} 1 \text{ TeV} \approx 250 \text{ or } 300 \text{ GeV.}$$

$$\rightarrow m_{\tilde{t}_1} \sim m_\tilde{t} - m_t \sim 130 \text{ GeV. } (\frac{A_t}{m_\tilde{t}} = 2)$$
- D0 squark gluino search (1st and 2nd generation)
  - $m_{\tilde{g}} \geq 500$ GeV, $m_{\tilde{q}} \geq 200$ GeV
  - $m_{\tilde{g}} \geq 230$ GeV, $m_{\tilde{q}} \geq 380$ GeV
  - $m_{\tilde{g}} \geq 330$ GeV, $m_{\tilde{q}} \geq 330$ GeV
Expressions for Weak scale parameters in terms of UV parameters
( for $\tan \beta = 10$ )

\[
m_t^2(M_Z) \approx 5.0M_3^2 + 0.6m_t^2 + 0.2A_tM_3
\]
\[
M_3(M_Z) \approx 3.0M_3
\]
\[
A_t(M_Z) \approx -2.3M_3 + 0.2A_t
\]

\[
\left| \frac{A_t(M_Z)}{M_\tilde{t}(M_Z)} \right| = \frac{|-2.3M_3 + 0.2A_t|}{\sqrt{5.0M_3^2 + 0.6m_t^2 + 0.2A_tM_3}} \lesssim 1 \text{ for positive } m_t^2.
\]

To achieve large stop mixing, we need negative stop mass squared at $M_{\text{GUT}}$. 
- \( A_t \) at \( M_{\text{GUT}} \) does not help.

\[
A_t(M_Z) \approx -2.3M_3 + 0.2A_t
\]

\[
\frac{dA_t}{d\log Q} = \frac{A_t}{16\pi^2}(18y_t^2 + \ldots) \quad \text{(exponential suppression)}
\]

- Maximal mixing is not possible unless \( A_t \geq 10M_3 \) with opposite sign at the GUT scale.
Negative stop mass squared also reduces fine-tuning.

- From the RG running of $m_{H_u}^2$,

$$\frac{dm_{H_u}^2}{d \log Q} \approx \frac{3y_t^2}{4\pi^2} m_t^2,$$

$m_{H_u}^2$ is lifted up by Yukawa loop if $m_t^2 < 0$. This enables $m_{H_u}^2$ to stay around $M_Z^2$.

- In terms of fine-tuning to obtain $M_Z^2$,

$$M_Z^2 \approx -1.9\mu^2 - 1.2m_{H_u}^2 + 5.9M_3^2 + 1.5m_t^2 + \ldots$$
For $m^2_t \approx -4M_3^2$, stop contribution almost cancels gluino contribution so that $\mu$ and $m_{H_u}$ can remain to be small.

Near $m^2_t \approx -4M_3^2$, 

$$\left| \frac{A_t(M_Z)}{M_t(M_Z)} \right| = \frac{| -2.3M_3 + 0.2A_t |}{\sqrt{5.0M_3^2 + 0.6m^2_t + 0.2A_tM_3}} \sim 1.5 - 0.2 \frac{A_t}{M_3}$$

$A_t \sim -2.5M_3$ at the GUT scale gives $\frac{A_t}{m^2_t} = 2$ at the weak scale
FIG. 1: Renormalization group running of relevant SSBs for \( \tan \beta = 10 \) and GUT scale boundary conditions: \( -A_t = M_3 = 200 \text{ GeV} \), \( m_{\tilde{t}}^2 = -(400 \text{ GeV})^2 \) and \( m_{H_u}^2 = 0 \text{ GeV}^2 \). In order to have both mass dimension one and two parameters on the same plot and keep information about signs, we define \( m_{H_u} \equiv m_{H_u}^2 / \sqrt{|m_{H_u}^2|} \) and \( m_{\tilde{t}} \equiv m_{\tilde{t}}^2 / \sqrt{|m_{\tilde{t}}^2|} \).
Cosmologically Viable?

- Large VEV Color/Charge Breaking Vacuum
  - $m^2 < 0$ at high energy.
  - Along the D-flat direction, there is no quartic coupling.
  - Example

\[
W = \frac{1}{M_{Pl}^3} (u^c d^c d^c)^2 \\
V(\phi) = m^2 |\phi|^2 + \frac{1}{M_{Pl}^6} |\phi|^{10}.
\]

→ Then $\langle \phi \rangle \sim (mM_{Pl}^3)^{1/4} \sim 10^{14}$ GeV: Large VEV CCB minimum.
  - The large VEV CCB minimum is always deeper than the EW vacuum (It is the AdS vacuum).
**Large VEV CCB Vacuum**

- Effective potential is lifted up at finite temperature and there is no CCB vacuum. Universe will settle down to the EW vacuum after inflation. The tunneling rate to the deeper minimum is negligible.
- We can not reach AdS vacuum from expanding universe. It just collapses instantaneously.
- Coleman-De Luccia suppression
**EW Scale CCB Vacuum: Kusenko, Langacker, Segre**

- Large $A_t$ can generate dangerous CCB vacuum at around the weak scale.
- Assuming finite temperature after inflation, $T_R \geq 100$ GeV, the field is at the origin.
- Phase transition to the EW vacuum is the second order while phase transition to $A_t$ CCB vacuum is the first order.
- The field just rolls to the EW vacuum unless tunneling rate to $A_t$ CCB vacuum is very large.
- Empirical bound:

$$|A_t|^2 + 3\mu^2 \leq 7.5(m_{t_L}^2 + m_{t_R}^2)$$

- Maximal stop mixing is compatible with the cosmological CCB bound.
(Middle) Conclusion

- Negative stop mass squared can make small $\mu$ and can make supersymmetry natural with maximal stop mixing. (other manifest solution: lower the cutoff $\Lambda$)
- Large $A_t$ can be a solution of the little hierarchy problem.
- Either stop should be different from sleptons or gauginos should be non-universal (bino heavier than gluino).
- Naturalness points out the meta-stable vacuum.
- There are new regions of parameter space that should be seriously explored. (possibly better than CMSSM)
- But how?
In SUSY GUT, $X, Y$ gauge bosons $\in SU(5)/G_{321}$ become massive at $M_{\text{GUT}}$ by adjoint chiral superfield $\Sigma$. We consider the case where $F$-term of $\Sigma$ is also induced.

$SU(5) \xrightarrow{\langle \Sigma \rangle} G_{321} = SU(3) \times SU(2) \times U(1)$

$\Sigma = M_{\text{GUT}} \text{ diag } (2, 2, 2, -3, -3) + \theta^2 F \text{ diag } (2, 2, 2, -3, -3)$

$X, Y$ and $\lambda_X, Y$ are split in mass.

$M_{\text{SUSY}} = \frac{\alpha_{\text{GUT}}}{4\pi} \left| \frac{F}{M} \right|$, $b_G$ is $\beta$-func coeff. in $SU(5)$.

$b_G = 3$ in the minimal case with a single adjoint.
Gauge Messenger Model

- Soft supersymmetry breaking parameters at $M_{\text{GUT}}$:

Gaugino

\[ M_3 = -4M_{\text{SUSY}}, \quad M_2 = -6M_{\text{SUSY}}, \quad M_1 = -10M_{\text{SUSY}} \]

Scalar

\[ m_{\tilde{Q}}^2 = (-20 + 3b_G)M_{\text{SUSY}}^2, \quad m_{\tilde{t}}^2 = (-16 + 4b_G)M_{\text{SUSY}}^2, \quad m_{\tilde{u}}^2 = (-16 + 4b_G)M_{\text{SUSY}}^2, \quad m_{\tilde{d}}^2 = (-12 + 2b_G)M_{\text{SUSY}}^2, \quad m_{\tilde{L}}^2 = (-12 + 3b_G)M_{\text{SUSY}}^2, \quad m_{\tilde{e}}^2 = (-12 + 3b_G)M_{\text{SUSY}}^2 \]

Tri-linear

\[ A_t = 10M_{\text{SUSY}} \]
Gaugino Masses are not universal and have opposite sign to conventional GMSB.

\[ b X \frac{\alpha}{4\pi} \frac{F}{M} \]

→ Bino is the heaviest at \( M_{\text{GUT}} \) scale.

\[ M_{\text{GUT}} \rightarrow |M_1| : |M_2| : |M_3| = 2.5 : 1.5 : 1 \]

\[ M_{Z} \rightarrow |M_1| : |M_2| : |M_3| \sim 1 : 1 : 2 \]

- Negative soft scalar masses squared are generated and squark masses squared are most negative.
- \( M_{\text{GUT}} : A_t \) is sizable (\( A_t = -2.5M_3 \)).
- \( M_{Z} : \frac{A_t(M_{Z})}{m_{\tilde{t}}(M_{Z})} \) close to maximal (\( \sim 2 \)).
(Minimal) Gauge Messenger (using SoftSUSY)

- $M_{\text{SUSY}} = 56$ GeV, $\tan \beta = 22$
Gauge Messenger
Geometric Hierarchy

- Dimopoulos, Raby

\[ W = \text{Tr} \Sigma \tilde{\Sigma}^2 + \chi (\text{Tr} \tilde{\Sigma}^2 - M^2) \]

- \( F_\Sigma \) and \( F_\chi \) can not be zero at the same time.
- \( M \) is the intermediate scale \( \sim \sqrt{M_{Pl} M_Z} \).
- \( F_\Sigma \sim F_\chi \sim M^2 \).
- \( \langle \Sigma \rangle \sim M_{\text{GUT}} \) becomes messenger scale.
Gravity mediation is not entirely negligible.

\[ \frac{\alpha}{4\pi} \frac{F}{M_{\text{mess}}} \sim \frac{F}{M_{\text{Pl}}} \]

Group theoretic numbers make gauge mediation dominates.
Gauge Messenger + Higgs
- $M_{\text{SUSY}} = 51 \text{GeV}$, $\tan \beta = 32$, $c_{H_u} = 40$, $c_{H_d} = 34$,
- $m_{H_u}^2 = (-3 + c_{H_u})M_{\text{SUSY}}^2$
Pure Gauge Messenger: Parameter space
Gauge Messenger + Higgs : Parameter space

$$c_{Hd} = 50$$  $$\tan \beta = 25$$

$$C_{Hu}$$  $$M_{SUSY} \text{ (GeV)}$$

Tachyon

$$\tilde{\tau}$$  $$\tilde{\tau} \text{ NLSP}$$  $$\tilde{\tau} \text{ NLSP}$$
Additional effect to the soft parameters

- Adding a contribution from gravity mediation opens a possibility of generating the $\mu$ term using the Giudice-Masiero mechanism. Comparable in size soft masses squared for $H_u$ and $H_d$ are also generated. We parametrize additional contribution to the Higgs soft masses squared by $c_{H_u} M_{\text{SUSY}}^2$ and $c_{H_d} M_{\text{SUSY}}^2$.

- We can add a universal contribution to all scalar masses from gravity mediation. We parametrize this contribution as $c_{\text{SUGRA}} M_{\text{SUSY}}^2$. 
Free parameters of gauge messenger model

- In the minimal ($b_G = 3$ and $N_{mess} = 0$) case, all the soft parameters at the GUT scale are determined by only one parameter $M_{SUSY}$. And at the EW scale, we need additional two parameters, $\mu$ and $\tan \beta$.

- In addition, we can include gravity contribution to Higgs sector: $c_{H_u}$ and $c_{H_d}$.

- We can include universal gravity contribution to scalar masses: $c_{mSUGRA}$.

- Including these, every mass spectrum is determined by 6 parameters $M_{SUSY}$, $\mu$, $\tan \beta$, $c_{H_u}$, $c_{H_d}$ and $c_{mSUGRA}$. 
Mass spectra for various parameter regions

- If \( c_{\text{mSUGRA}} = 0 \), NLSP is sneutrino (for low \( c_{H_u} \)) or stop (for high \( c_{H_u} \)) for low \( \tan \beta \). And as \( \tan \beta \) increases, down-type Yukawa couplings become large so that stau mixing becomes large and stau replaces sneutrino.

- But as \( c_{\text{mSUGRA}} \) increases (\( \gtrsim 5 \)), every scalar masses become larger than gaugino masses, and the LSP is neutralino.
Mass spectra for various parameter regions

In this figure, we can see that (N)LSP changes as \( \tan \beta \) and \( c_{H_u} \) vary when \( c_{mSUGRA} = 5 \).

- Gravitino is LSP and \( m_{3/2} \sim 100 \text{ GeV} \)
- Stau, sneutrino or stop is NSLP (Stau is a mixture of LR)
- In most of the parameter space, neutralino is LSP when \( c_{mSUGRA} \geq 5 \).
Neutralino masses in MSSM Lagrangian

where

\[ N_1 = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix} \]

and

\[ M_N^1 = \begin{pmatrix}
M_1 & 0 & -M_Z s_{\beta + \frac{\pi}{4}} s_{\theta_W} & M_Z s_{\beta} - \frac{\pi}{4} c_{\theta_W} \\
0 & M_2 & M_Z s_{\beta + \frac{\pi}{4}} c_{\theta_W} & -M_Z s_{\beta} - \frac{\pi}{4} c_{\theta_W} \\
-M_Z s_{\beta + \frac{\pi}{4}} s_{\theta_W} & M_Z s_{\beta + \frac{\pi}{4}} c_{\theta_W} & \mu & 0 \\
M_Z s_{\beta - \frac{\pi}{4}} s_{\theta_W} & -M_Z s_{\beta - \frac{\pi}{4}} c_{\theta_W} & 0 & -\mu
\end{pmatrix} \]
Neutralino masses in MSSM Lagrangian

Since $M_1$, $M_2$ and $\mu$ are sufficiently larger than $M_Z$, we can take off-diagonal terms as perturbation like quantum mechanics problems.

- When the difference between $M_1$ and $M_2$ is not extremely small, $(M_1 - M_2)\mu^2 \geq M_1 M_Z^2$, non degenerate perturbation formalism can be applied. First order result is given by,

$$
\alpha^{(1)} = 1, \quad \beta^{(1)} = 0, \quad \gamma^{(1)} = \frac{-M_Z s_\beta + \frac{\pi}{4} s_{\theta_W}}{M_1 - \mu}, \quad \delta^{(1)} = \frac{M_Z s_\beta - \frac{\pi}{4} s_{\theta_W}}{M_1 + \mu},
$$

where $\alpha$, $\beta$, $\gamma$ and $\delta$ are defined by lightest neutralino vector,

$$
\chi_1 = \alpha \tilde{B} + \beta \tilde{W}^0 + \gamma \tilde{H}_1^0 + \delta \tilde{H}_2^0.
$$

And superscript (1) means first order perturbation.
Neutralino masses in MSSM lagrangian

As $\beta^{(1)} = 0$ at the first order, it is necessary to calculate the next order. Next order correction is small in general but can significantly alter the result when $M_1 \sim M_2$. The second order perturbation gives

$$
\beta^{(2)} = \frac{M_Z^2 s_{\beta+}^2/4}{4\Delta M(\mu - M_1)} - \frac{M_Z^2 s_{\beta-}^2/4}{4\Delta M(\mu + M_1)}
\approx \frac{M_Z^2 M_1 s_{2\theta_W}}{2\Delta M(\mu^2 - M_1^2)} = \frac{M_Z^2 s_{2\theta_W}}{2\epsilon(\mu^2 - M_1^2)},
$$

where $\Delta = M_2 - M_1$ and $\epsilon = (M_2 - M_1)/M_1$. The second line is for $\tan \beta \geq 10$. 

Neutralino in mSUGRA

- Since gaugino masses are universal at GUT scale in mSUGRA model, we know $|M_1|:|M_2|:|M_3| \sim 1:2:7$ at EW scale from the relation $M_1/g_1^2 = M_2/g_2^2 = M_3/g_3^2$ at any scale.

- In mSUGRA case, gaugino masses are a few hundred GeV, so $M_Z \ll |\mu \pm M_1|$. And since $\epsilon = 1$, second order correction eq.(15) is negligible. Thus the lightest neutralino is mostly Bino(from eq.(13)).

- It is well-known that mostly Bino neutralino is overproduced and has too large relic density. So we need another suppression mechanism like co-annihilation or resonance cross section.

- Or if neutralino has sufficient wino composition, we can reduce the relic density to fit the observed value.
Main features in neutralino sector

- Starting from $|M_1| : |M_2| : |M_3| = 2.5:1.5:1$ at the GUT scale, we get $|M_1| : |M_2| : |M_3| \sim 1:1:2$ at EW scale.
- Remind the second order term in $\chi$ eq.(15),

$$\beta^{(2)} \simeq \frac{M_Z^2 s_{2\theta_W}}{2\epsilon (\mu^2 - M_1^2)}.$$

Since $\epsilon \simeq 0.1$ in our case, the second order contribution becomes important and neutralino has sizable Wino contribution.
- Such a Wino-mixed neutralino can reduce the relic density. So we can find easily the parameter region which gives correct relic density.
Neutralino relic density for Bino/Wino, Higgsino mixture

- In most parameter region of mSUGRA, neutralino is usually pure Bino. And its relic density is much larger than observed value. If neutralino is a mixture of Bino and Wino, since the cross section of Wino is much larger than other contributions, from the eq.(44) relic density is approximately

\[ \Omega \chi h^2 \approx \frac{1}{|\beta|^2} \times 0.13 \left( \frac{M_2}{2.5\text{TeV}} \right). \]

- If \(|\beta|^2 = 0.04\), i.e. 4% Wino, we get the Wino mass to fit the observed value,

\[ 0.09 \leq 25 \times 0.13 \left( \frac{M_2}{2.5\text{TeV}} \right) \leq 0.13, \]

\[ 416\text{GeV} \leq M_2 \leq 500\text{GeV} \]
Bench mark point for relic density

- We choose the bench mark point as $M_{\text{SUSY}} = 90 \text{GeV}$, $c_{\text{mSUGRA}} = 20$, $c_{H_u} = 10$, $c_{H_d} = -40$ and $\tan \beta = 10$.
- At EW scale, $M_1 = 379 \text{GeV}$, $M_2 = 423 \text{GeV}$, $\mu = -604 \text{GeV}$, and

  $$|\alpha|^2 = 0.948, \quad |\beta|^2 = 0.034, \quad |\gamma|^2 = 0, \quad |\delta|^2, = 0.018$$

  $$\chi = \alpha \tilde{B} + \beta \tilde{W}^0 + \gamma \tilde{H}^0_1 + \delta \tilde{H}^0_2.$$ 

- Using the formulae for pure Wino and pure Higgsino cases, we can get the relic density of neutralino,

  $$\Omega_\chi h^2 \sim \left( \frac{|\beta|^2}{\Omega_{\tilde{W}} h^2} + \frac{|\gamma|^2 + |\delta|^2}{\Omega_{\tilde{H}} h^2} \right)^{-1}$$

  $$= \left[ \frac{|\beta|^2}{0.13} \left( \frac{2.5 \text{TeV}}{M_2} \right)^2 + \frac{|\gamma|^2 + |\delta|^2}{0.10} \left( \frac{1.0 \text{TeV}}{\mu} \right)^2 \right]^{-1}$$

  $$= 0.108$$
Bench mark point for relic density

- It satisfies the observed value, $0.9 \leq \Omega \chi h^2 \leq 0.13$.
- In this parameter region, the numerical result is 0.088 for DARKSUSY.
Numerical results for the relic density

- Left figure is for simple gauge messenger case, i.e. no gravity contribution to Higgs. For high $\tan \beta$ region is co-annihilation with stau and for low $\tan \beta$ region is pseudoscalar Higgs resonance region.

- Right figure is for large gravity contribution to scalar and Higgs. There is bulk region to fit the observed value. There is no co-annihilation and no resonance.
Direct detection of neutralino

- Nucleon-neutralino scattering is described by effective four-fermi Lagrangian which is given by

\[ \mathcal{L} = \bar{\chi} \gamma^{\mu} \gamma^{5} \chi \bar{q}_{i} \gamma_{\mu} \left( \alpha_{1i} + \alpha_{2i} \gamma^{5} \right) q_{i} + \alpha_{3i} \bar{\chi} \chi \bar{q}_{i} q_{i} + \alpha_{4i} \bar{\chi} \gamma^{5} \chi \bar{q}_{i} q_{i} + \alpha_{5i} \bar{\chi} \chi \bar{q}_{i} \gamma^{5} q_{i} + \alpha_{6i} \bar{\chi} \gamma^{5} \chi \bar{q}_{i} q_{i}, \]

where \( i = 1 \) is for up-quark and \( i = 2 \) is for down-quark.

- For spin-independent case, \( \alpha_{3i} \) term contributes. Since t-channel Higgs exchange scattering is dominant, cross section is approximately given by,

\[
\sigma_{\chi p, n}^{SI} = \frac{g'^4 m_N^4}{4\pi} \left( \frac{\mu^2}{(\mu^2 - M_1^2)^2} \right) \left[ \frac{X_d \tan \beta}{m_h^2} \left( \frac{M_1}{\mu} \right) + \frac{X_u}{m_h^2} \right]^2.
\]

where \( X_u \) and \( X_d \) is determined by form factor of target material.
Direct detection of neutralino

Blue: spin-dependent, Red: spin-independent

- Our numerical result of nucleon-neutralino spin-independent cross section is below the current CDMS bound. But it can be detected by CDMSII near future.
- Spin-dependent cross section bound of CDMS is about $10^{-37}$ cm$^2$. It may be impossible to detect neutralino by spin-dependent scattering.
Conclusion

- **Negative Stop Mass Squared → Meta-stable Vacuum**
  → Large $A_t/M_\tilde{t}$ → lightest Higgs mass bound even with light stop.

- **Non-universal Gaugino Masses**
  Gauge messenger model → Non-universal gaugino mass, negative squark mass at GUT scale and large $A$ term → smaller fine-tuning

- **Degenerate Spectrum at the EW scale**
  Gluino and Bino/Wino ($< 500$ GeV) and squarks and sleptons also have similar masses ($< 500$ GeV).

- **Fine Tuning is improved up to 10 %**
  Almost factor 5 or 10 improvement compared to CMSSM.
Conclusion

- **Gravitino LSP and Stop NLSP**
  Detection of NSLP decay using stopper might be possible.

- **Light Gluino, Light Stop**
  Collider signal would be very interesting at the early year of LHC. (even at the Tevatron)
  R-hadron (stop-light quark) is produced and is stable at the collider time scale

- **Neutralino Dark Matter**
  Bino/Wino/Higgsino mixture with almost fixed ratio of mixing explains the dark matter relic density.
  Direct detection might take longer than a few years.

- **mSUGRA vs Gauge Messenger Models**
  Gauge messenger model is as good as mSUGRA or is better from phenomenological considerations.