A

HETEROTIC

STANDARD MODEL

AND THE

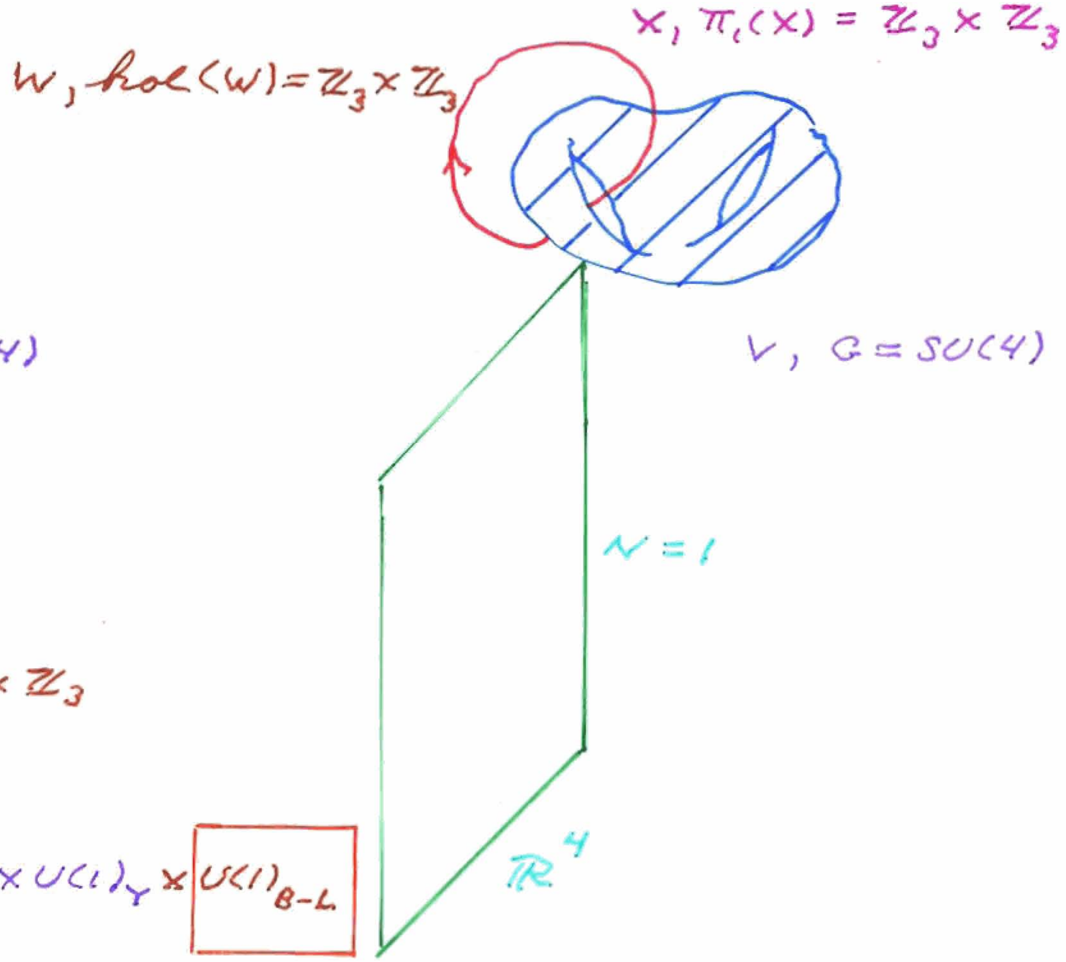
COSMOLOGICAL CONSTANT

VOLKER BRAUN, YANG-HUI HE

BURT OVRUT, TONY PANTEV

Observable Sector:

$$E_8 \xrightarrow{SU(4)} Spin(10) \xrightarrow{\mathbb{Z}_3 \times \mathbb{Z}_3} SU(3)_C \times SU(2)_L \times U(1)_Y \times \boxed{U(1)_{B-L}}$$



Spectrum:

1. 3 families of quark/leptons. Each family is

$Q = (3, 2, 1, 1)$, $u = (3, 1, -4, -1)$, $d = (\bar{3}, 1, 2, -1)$

$L = (1, 2, -3, -3)$, $e = (1, 1, 6, -3)$, $\boxed{\nu = (1, 1, 0, 3)}$

2. 1 pair of Higgs - $\overline{\text{Higgs}}$ fields

$$H = (1, 2, 3, 0), \quad \overline{H} = (1, \overline{2}, -3, 0)$$

NO

EXOTIC MATTER / VECTOR-LIKE FIELDS!

$\Downarrow$

$$\boxed{\text{EXACT MSSM SPECTRUM}}$$

3. 6 geometric and 13 vector bundle moduli

Physical Properties:

a. Higgs μ-Terms

Leray spectral "quantum numbers" forbid all

$$\langle \phi_i \rangle H \overline{H}$$

interactions.

However, can have

$$\frac{\langle\phi_i\rangle^{p} H \bar{H}}{M_c^{\,p-1}} \quad , \quad p \geq 2$$

$\Longrightarrow$ naturally small μ-terms

6. Yukawa couplings

Many special "quantum numbers" forbid all Yukawa couplings except

$$\lambda_{\binom{u}{d}1,j}\, Q_1 \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} u_j \\ d_j \end{pmatrix} + \lambda_{\binom{u}{d}j,1}\, Q_j \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} u_1 \\ d_1 \end{pmatrix}$$

$$+$$

$$\lambda_{\binom{\nu}{e}1,j}\, L_1 \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} \nu_j \\ e_j \end{pmatrix} + \lambda_{\binom{\nu}{e}j,1}\, L_j \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} \nu_1 \\ e_1 \end{pmatrix}$$

where $j = 2,3$. $\Longrightarrow$ a "texture" of quark/lepton masses.

example: up-quark mass matrix. $\langle H\rangle \neq 0 \Rightarrow$

$$\begin{pmatrix} 0 & \lambda_{u1,2}\langle H\rangle & \lambda_{u1,3}\langle H\rangle \\ \lambda_{u2,1}\langle H\rangle & 0 & 0 \\ \lambda_{u3,1}\langle H\rangle & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} \boxed{0} & 0 & 0 \\ 0 & \lambda\langle H\rangle & 0 \\ 0 & 0 & \lambda'\langle H\rangle \end{pmatrix}$$

similar result for down-quark and lepton mass matrices. However, have unrestricted terms

$$\lambda \frac{\langle\phi_i\rangle^p}{M_c^{p-1}} Q \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix} + \lambda' \frac{\langle\phi_i\rangle^p}{M_c^{p-1}} L \begin{pmatrix} H \\ \bar{H} \end{pmatrix} \begin{pmatrix} \nu \\ e \end{pmatrix}$$

for $p \geq 2$. $\Rightarrow$ naturally small first family masses

c. Proton decay

a) $M_c \sim \mathcal{O}(10^{16}\,\mathrm{GeV}) \Rightarrow$

dim $\underline{6}$ decay suppressed

b) Lecay spectral "quantum numbers" + $U(1)_{B-L}$ forbid the $\Delta B = 1$, $\Delta L = 1$ cubic terms

$$\alpha\, Q L d + \beta\, L L e + \gamma\, u d d$$

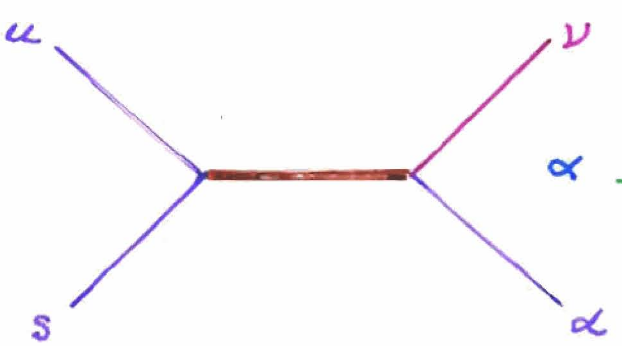
$\Rightarrow$

dim $\underline{\underline{4}}$ decay <u>forbidden</u>

remains sufficiently small if $U(1)_{B-L}$ is broken at $\mathcal{O}(10^3\text{-}10^4\,\text{GeV})$.

c) Natural doublet-triplet splitting <u>projects</u> <u>out</u> color triplet Higgs $c, \bar{c}$. However, these quantum states appear on the Kaluza-Klein tower

$\Rightarrow$ dim $\underline{\underline{5}}$ operators such as



$$\propto \frac{\lambda^2}{M_c}\, u d s \nu \Rightarrow p \rightarrow K^+ + \nu_{\mu,\tau}$$

Vector Bundle Stability:

Slope-stable bundle V $\Rightarrow$ connection solves $\boxed{g^{a\bar{b}} F_{a\bar{b}} = 0}$

The Kahler cone of X is found to be

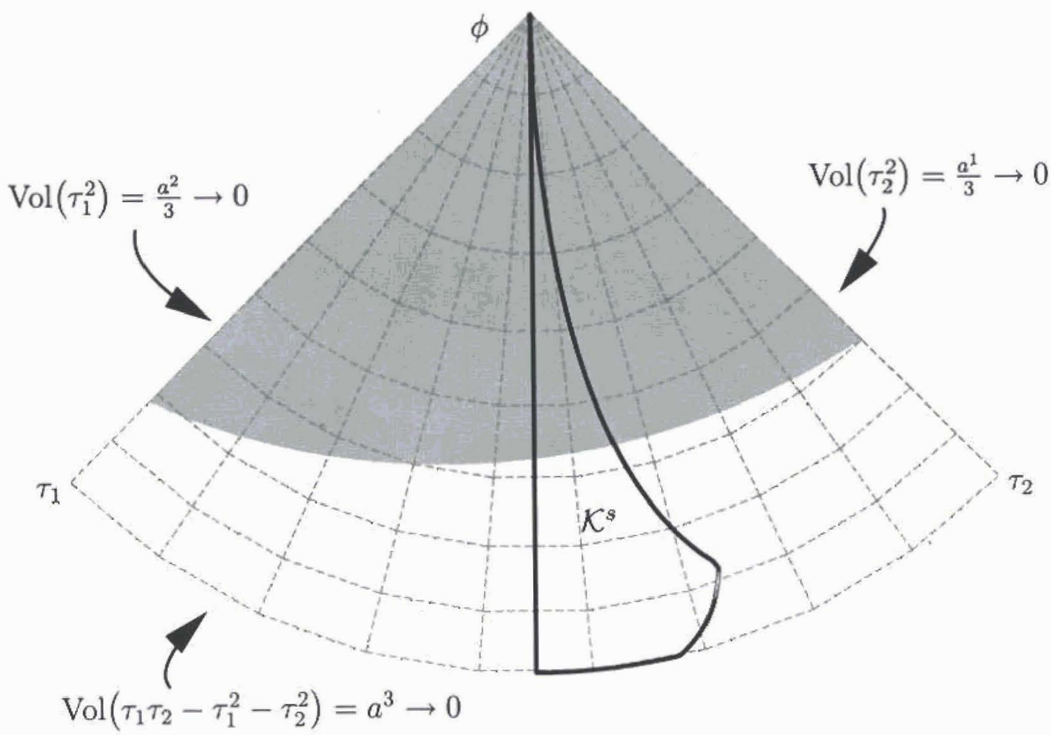


Figure 1: Kähler Cone. The observable sector vector bundle is slope-stable in the region $\mathcal{K}^s$.

V is slope-stable with respect to each Kahler moduli in

$$\mathcal{K}^s \subset H^2(X, \mathbb{R}) \simeq \mathbb{R}^3$$

What is the hidden sector?

To motivate this must discuss

Moduli Stability:

A) 5-brane only

moduli:

$$\underbrace{S}_{\text{dilaton}},\ \underbrace{T^I}_{h^{1,1}},\ \underbrace{Z_\alpha}_{h^{1,2}},\ \underbrace{\phi}_{\text{vector bundle}},\ \underbrace{Y}_{\text{5-brane}}$$

simplification - assume 1 vector bundle modulus

Can construct

$$K = K_{S,T} + K_Z + K_5 + K_\phi$$

and

$$W = W_f + W_g + W_{np} + W_5^{(1)} + W_5^{(2)}$$

For example

$$K_Z = -M_{Pl}^2 \ln\left(-i\int_X \Omega\wedge\bar{\Omega}\right)$$

where Ω is the holomorphic 3-form and

$$W_f = \frac{1}{\mathcal{V}_{CY}^{1/2}}\int_X H\wedge\Omega$$

H is the B-flux.

Result:

- Can always solve

$$D_F W = 0$$

for all $F = S, T^I, Z_\alpha, \phi, Y$.

- These equations fix all

$$\langle S\rangle, \langle T^I\rangle, \langle Z_\alpha\rangle, \langle\phi\rangle, \langle Y\rangle$$

- $\langle F \rangle$ have phenomenologically acceptable values such as

$$Re\langle S\rangle \sim 1 \qquad R \sim 1\ ,\ 0 < Re\,Y < R$$

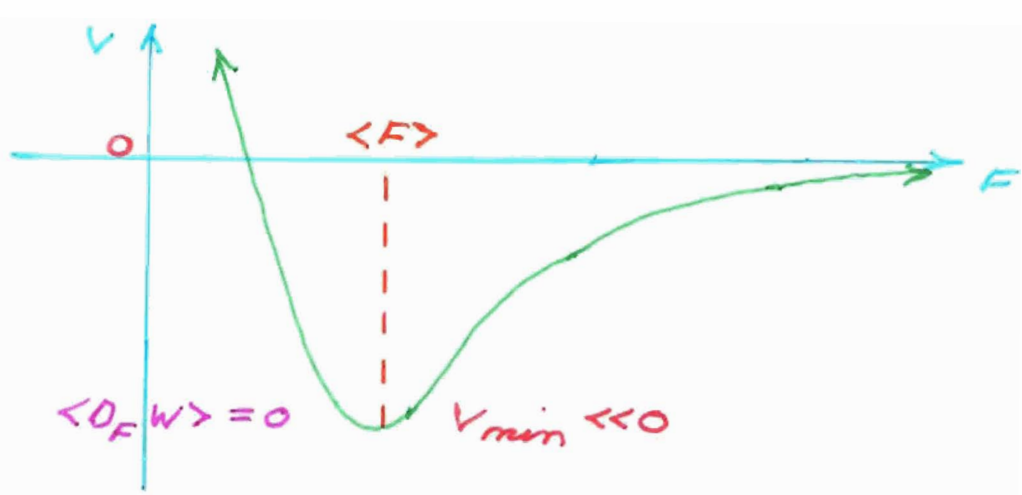
- $\langle \partial_F W \rangle = 0 \Rightarrow$ SUSY unbroken $\Rightarrow$

$$V_{min} \propto -\frac{3|\langle W\rangle|^2}{M_{Pl}^2}$$

- For these values of $\langle F \rangle$

$$V_{min} \sim -(10^{-4} M_{Pl})^4 \sim -10^{60}\ GeV^4$$

$\Rightarrow$ a deep, negative cosmological constant



B) 5-brane + anti 5-brane $(+ V' \sim \mathcal{O}_X)$

The anti 5-brane adds the term

$$\Delta U_{\bar{5}} = \frac{4 T_5}{(\mathrm{Re}S)^{4/3} R^2} \frac{1}{\mathcal{V}_{CY}^{2/3}} \int_X \omega \wedge J$$

to the $N=1$ supersymmetric Lagrangian, where

$$J = c_2(V) - c_2(TX) + \underset{\overset{\uparrow}{5}}{[W]} + \underset{\overset{\uparrow}{\bar{5}}}{[\bar{W}]} \, ,$$

ω is the Kahler form

$$\omega = a^I \omega_I$$

and T_5 is the 5-brane tension. The anomaly cancellation condition $\Rightarrow$

$$c_2(V) - c_2(TX) + [W] - [\bar{W}] = 0$$

$\Rightarrow$

$$J = 2[\bar{W}]$$

Therefore

$$\Delta U_{\bar{5}} = + \frac{8 \, T_5 \, \nu_{\bar{5}}}{(\mathrm{Re}S)^{4/3} R^2}$$

where

$$\nu_{\bar{5}} = \frac{1}{\upsilon_{CY}^{2/3}} \int_X \omega \wedge [\bar{W}] = \int_{z_{\bar{5}}} \omega$$

Add to the Lagrangian & resolve the equations of motion $\Longrightarrow$

Result:

- Meta-stable minimum with

1. $\langle S \rangle, \langle T^I \rangle, \langle Z_\alpha \rangle, \langle \phi \rangle, \langle Y \rangle$ all fixed
2. $\langle D_F W \rangle \neq 0 \Longrightarrow$ SUSY broken
3. $\langle F \rangle$ have phenomenologically acceptable values

- The cosmological constant can be made small as long as one chooses

$$T_5 \, v_{\bar{5}} \sim 10^{-16} M_{Pl}^4$$

or, equivalently

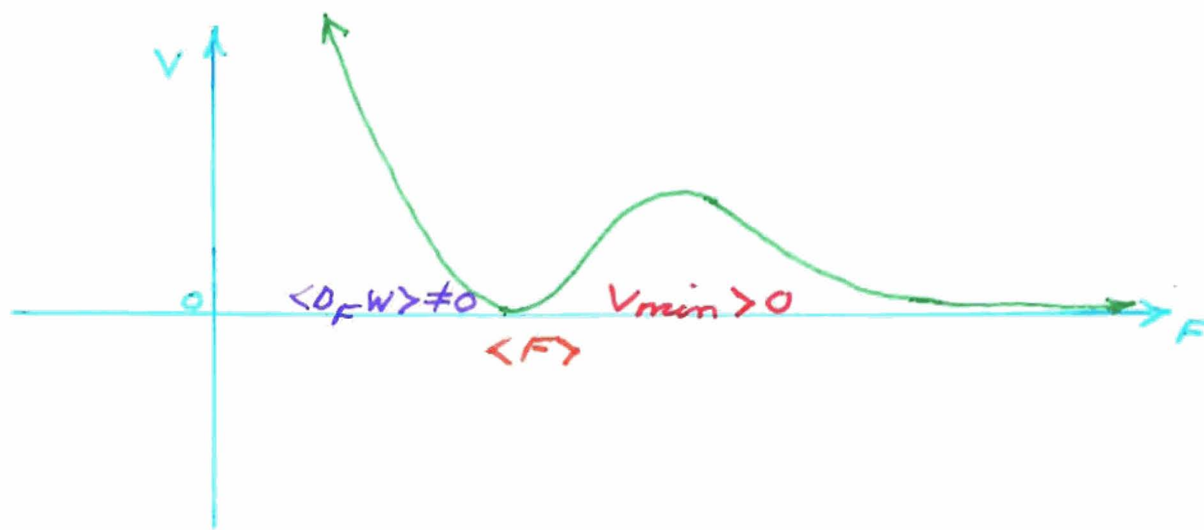
$$\boxed{\frac{v_{\bar{5}}}{v_{CY}^{1/3}} \sim 10^{-7}} \quad *$$

- Since $\langle F \rangle$ have acceptable values $\Rightarrow$

$$\boxed{\frac{v_5}{v_{CY}^{1/3}} \sim 1} \quad **$$

where

$$v_5 = \frac{1}{v_{CY}^{2/3}} \int_X \omega \wedge [W] = \int_{z_5} \omega$$



Question: For a realistic vacuum can * and ** be solved such that the observable sector vector bundle is slope-stable with respect to ω ?

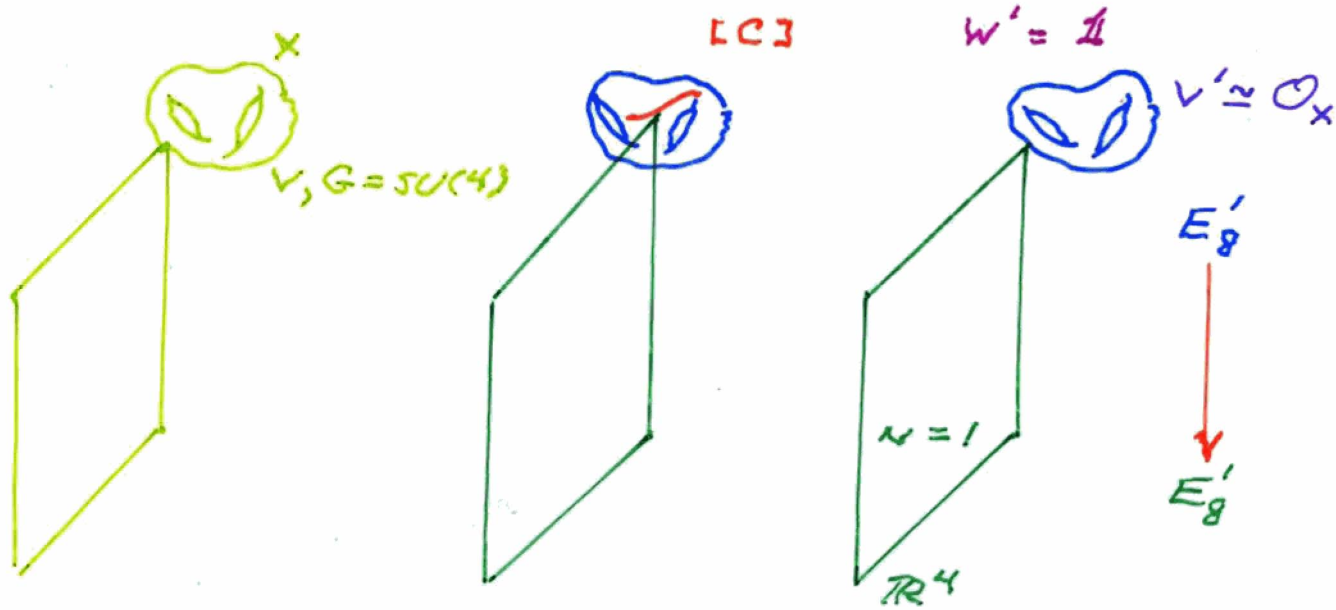
Answer. Yes!

Consider the MSSM heterotic standard model. In the hidden sector choose

$$V' \sim \mathcal{O}_X$$

which is trivially slope-stable.

Hidden Sector:



Anomaly Cancellation

$$[C] = c_2(TX) - c_2(V)$$

Kahler Cone

$$\omega = a^1\tau_1 + a^2\tau_2 + a^3\phi \in \mathcal{K}$$

We find that

$$c_2(V) = \tau_1^2 + 4\tau_2^2 + 4\tau_1\tau_2 \;, \quad c_2(TX) = 12(\tau_1^2 + \tau_2^2)$$

$\Longrightarrow$

$$[C] = [W] - [\bar{W}]$$

where

$$[W] = 7\tau_1^2 + 4\tau_2^2 , \quad [\bar{W}] = 4(\tau_1\tau_2 - \tau_1^2 - \tau_2^2)$$

Then using the τ_1, τ_2, & intersection numbers $\Longrightarrow$

$$\frac{v_{\bar{5}}}{v_{CY}^{1/3}} = \frac{1}{v_{CY}} \int_X \omega \wedge [\bar{W}] = 4a^3$$

and

$$\frac{v_5}{v_{CY}^{1/3}} = \frac{1}{v_{CY}} \int_X \omega \wedge [W] = \frac{4}{3} a^1 + \frac{7}{3} a^2$$

Consider the region

$$K_5 \subset K$$

As one approaches the bottom of $K_5 \Longrightarrow a^3 \rightarrow 0$

$\Longrightarrow$

one can choose

$$\frac{\mathcal{V}_5}{\mathcal{V}_{CY}^{1/3}} \sim 10^{-7} \checkmark$$

Note that

$$\mathrm{Re}\, S = \frac{1}{6}\left((a^1)^2 a^2 + a^1 (a^2)^2 + 6 a^1 a^2 a^3 \right)$$

and

$$\mathrm{Re}\, S \sim 1$$

K_S is bounded on the left by the vertical line where

$$a^1 = a^2$$

$\Rightarrow$ on vertical line as $a^3 \to 0$

$$a^1 = a^2 \longrightarrow (3)^{1/3}$$

The moduli

$$a^1 = (3)^{1/3} - \frac{\epsilon}{2}, \quad a^2 = (3)^{1/3} + \frac{\epsilon}{2}$$

$\Rightarrow$ $\mathrm{Re}\, S \sim 1$ and are in K_S. For this region

$$\frac{\mathcal{V}_5}{\mathcal{V}_{CY}^{1/3}} \sim 1 \checkmark$$

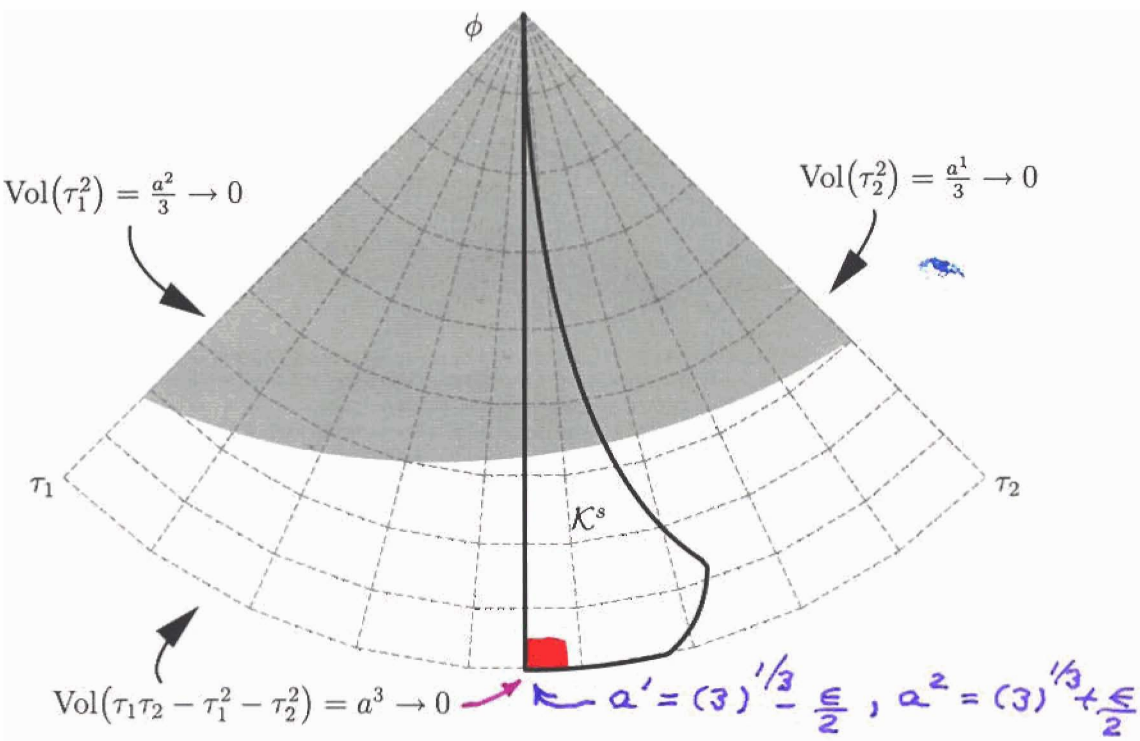


Figure 1: Kähler Cone. The observable sector vector bundle is slope-stable in the region $\mathcal{K}^s$.

In the ■ region

$$\frac{\mathcal{V}_{\bar{5}}}{\mathcal{V}_{CY}^{1/3}} \sim 10^{-7}, \qquad \frac{\mathcal{V}_5}{\mathcal{V}_{CY}^{1/3}} \sim 1$$

$$\Downarrow \qquad\qquad\qquad \Downarrow$$

$$0 < \Lambda / M_{P\ell}^4 \ll 1 \qquad\qquad \mathrm{Re}\, S \sim 1$$

and

observable V is <u>slope-stable</u>

Conclusion:

For the MSSM heterotic standard model

- Take $V' \simeq \mathcal{O}_X$. Anomaly cancellation $\Rightarrow$ both 5-brane and anti 5-brane on $S^1/\mathbb{Z}_2$ interval and fixes their cohomology classes.

- Neglecting the anti 5-brane, all moduli are stabilized, but at $N=1$ preserving minimum with $V_{min} \sim -10^{-16} M_{Pl}^4$.

- Add anti 5-brane lifts the minimum to a meta-stable vacuum with a positive cosmological constant. The moduli are fixed on this vacuum and have phenomenologically acceptable values.

- There is a region of the Kahler cone for which the cosmological constant has its observed value and for which the observable sector vector bundle is slope-stable.

- One expects the Kahler moduli can be fine-tuned to lie in this region.