A

HETEROTIC

STANDARD MODEL

AND THE

COSMOLOGICAL CONSTANT

VOLKER BRAUN, YANG-HUI HE

BURT OVRUT, TONY PANTEV
Observable Sector:

\[ W, \text{hol}(W) = \mathbb{Z}_3 \times \mathbb{Z}_3 \]

Specimen:

\[ N = 1 \]

\[ V, G = SU(4) \]

\[ \mathbb{R}^4 \]

\[ SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \]

Spectrum:

1. 3 families of quark/leptons. Each family is

\[ Q = (3, 2, 1, 1), \quad u = (3, 1, -4, -1), \quad d = (\bar{3}, 1, 2, 1) \]

\[ L = (1, 2, -3, 3), \quad e = (1, 1, 6, -3), \quad \nu = (1, 1, 0, 3) \]
2. A pair of Higgs–Higgs fields

\[ H = (1, 2, 3, 0), \quad \bar{H} = (1, 2, -3, 0) \]

NO

EXOTIC MATTER / VECTOR-LIKE FIELDS

EXACT MSSM SPECTRUM

3. 6 geometric and 13 vector bundle moduli

Physical Properties:

a. Higgs \( \mu \)-terms

Leading spectral "quantum number" forbid all

\[ \langle \phi \rangle H \bar{H} \]

interactions.
However, can have

\[ \frac{<\Phi_c^p H H^\dagger \quad \rho \geq 2>}{M_c^{\rho-1}} \]

\[ \Rightarrow \text{ naturally small } \mu - \text{terms} \]

6. Yukawa couplings

"Every special "quantum number" forbids all Yukawa couplings except"

\[ \lambda^{(u)}_{ij} \frac{Q_j}{(H/H)} (\nu_{ij}) + \lambda^{(u)}_{ij} \frac{Q_j}{(H/H)} (\nu^\dagger_{ij}) \]

\[ \lambda^{(d)}_{ij} \frac{E_j}{(H/H)} (\nu_{ij}) + \lambda^{(d)}_{ij} \frac{E_j}{(H/H)} (\nu^\dagger_{ij}) \]

where \( j = 2,3 \) \[ \Rightarrow \text{ a "texture" of quark/lepton masses.} \]
example: up-quark mass matrix. $<H> \neq 0 \Rightarrow$

\[
\begin{pmatrix}
0 & \lambda_{1,2} <H> & \lambda_{1,3} <H> \\
\lambda_{2,1} <H> & 0 & 0 \\
\lambda_{3,1} <H> & 0 & 0
\end{pmatrix}
\rightarrow
\begin{pmatrix}
0 & 0 & 0 \\
0 & \lambda <H> & 0 \\
0 & 0 & \lambda' <H>
\end{pmatrix}
\]

similar result for down-quark and lepton mass matrices. However, have unrestricted terms

\[
\lambda <\phi_c > \frac{p Q (H) / M_c \omega}{P}\left(\frac{H}{H}\right) (\nu) + \lambda' <\phi_c > \frac{p L (H) / M_c}{P-1} (\nu)
\]

for $p \geq 2. \Rightarrow$ naturally small first family masses

c. Proton decay

a) $M_c \nu O(10^{16} \text{GeV}) \Rightarrow$

dim 6 decay suppressed
6) Leaky spatial "quantum number" + $\mathcal{U}(1)_{B-L}$ forbid the $\Delta B = 1$, $\Delta L = 1$ cubic term

\[ \lambda Q d \bar{d} + \beta L \bar{L} + \gamma u d \bar{d} \]

\[ \Rightarrow \]

$\text{dim} \frac{4}{5}$ decay forbidden

remains sufficiently small if $\mathcal{U}(1)_{B-L}$ is broken at $\mathcal{O}(10^3 - 10^4 \text{GeV})$.

c) Natural doublet-triplet splitting projects out color triplet Higgs $c, \bar{c}$. However, these quantum states appear in the Kaluza-Klein tower

\[ \Rightarrow \text{dim} \frac{5}{4} \] operators such as

\[ \alpha \frac{\lambda^2}{M_c} u d s v \Rightarrow p \rightarrow K^+ + \nu_{\mu, \tau} \]
Vector Bundle Stability

Slope-stable bundle $V \Rightarrow$ connection solved $
\frac{\theta}{\phi} F_{\theta} = 0$

The Kahler cone of $X$ is found to be

\begin{align*}
\text{Vol}(\tau_1^2) &= \frac{a^2}{3} \to 0 \\
\text{Vol}(\tau_2^2) &= \frac{a^2}{3} \to 0 \\
\text{Vol}(\tau_1 \tau_2 - \tau_1^2 - \tau_2^2) &= a^3 \to 0
\end{align*}

Figure 1: Kahler Cone. The observable sector vector bundle is slope-stable in the region $K^S$.

$V$ is slope-stable with respect to each Kahler

modulus in

$K^S \subset H^2(X, \mathbb{R}) \times \mathbb{R}^3$
What is the hidden sector?

To motivate this, we must discuss:

Moduli Stability:

A) 5-brane only

moduli:

\[ S, T^2, Z_2, \phi, Y \]

\[ h^{i_1}, h^{i_2}, \text{5-brane} \]

dilaton \quad \text{vector bundle}

simplification - assume 1 vector bundle moduli

Can construct

\[ K = K_{S,T} + K_Z + K_S + K\phi \]

and

\[ W = W_f + W_g + W_{np} + W^{(1)}_S + W^{(2)}_S \]
For example

\[ k_z = -M^2_{pl} \ln (-i \int_x \alpha \, d\bar{\alpha}) \]

where \( \alpha \) is the holomorphic 3-form and

\[ W_4 = \frac{1}{v_f^{1/2}} \int_x H \wedge \alpha \]

\( H \) is the B-flux.

Result:

• Can always solve

\[ D_{\xi} W = 0 \]

for all \( F = S, T^I, \bar{Z}_A, \phi, Y \).

• These equations fit all

\[ \langle S \rangle, \langle T^I \rangle, \langle \bar{Z}_A \rangle, \langle \phi \rangle, \langle Y \rangle \]
\( \langle F \rangle \) have phenomenologically acceptable values such as

\[ \text{Re} \langle s \rangle \simeq 1, \quad R \simeq 1, \quad 0 < \text{Re} Y < R \]

\[ \langle D_F W \rangle = 0 \implies \text{SUSY unbroken} \implies \]

\[ V_{\text{min}} = \frac{3\langle W \rangle^2}{\lambda^2 M_{\text{pl}}^2} \]

\[ V_{\text{min}} - (10^{-4} M_{\text{pl}})^4 v = 10^{60} \text{ GeV}^4 \]

\( \implies \) a deep, negative cosmological constant
8) \(5\)-brane + anti \(5\)-brane \((+ V' v' O_x)\)

The anti \(5\)-brane adds the term

\[
\Delta U_5 = \frac{4 T_5}{(R \varepsilon S)^{4/3}} \int \omega \wedge J
\]

\(J = c_2(v) - c_2(TX) + [W] + [\bar{W}]\),

\(w\) is the Kaluza form

\[
w = a^I \omega_I
\]

and \(T_5\) is the \(5\)-brane tension. The anomaly cancellation condition \(\Rightarrow\)

\[
c_2(v) - c_2(TX) + [W] - [\bar{W}] = 0
\]

\(\Rightarrow\)

\[
J = 2[\bar{W}]
\]
Therefore

\[ \Delta U \frac{3}{5} = + \frac{8 \frac{T_5}{\langle \text{Res} \rangle^{4/3}}}{R^2} \]

where

\[ V \frac{3}{5} = \frac{1}{2^{2/3}} \int \omega \wedge [W^3] = \int \omega \]

Add to the Lagrangian a rescale the equations of motion \( \Rightarrow \)

**Result:**

- Meta-stable minimum with
  1. \( <s>, <T^I>, <Z_2>, <\phi>, <Y> \) all fixed
  2. \( <Q_F^W > \neq 0 \Rightarrow \text{SUSY broken} \)
  3. \( <F> \) have phenomenologically acceptable values
The cosmological constant can be made small as long as one chooses

\[ \frac{V_5}{V_{1/3}} \sim 10^{-16} M_{PL}^4 \]

or, equivalently

\[ \frac{V_5}{V_{1/3}} \sim 10^{-7} \]

Some <F> have acceptable values \( \Rightarrow \)

\[ \frac{V_5}{V_{1/3}} \sim 1 \]

where

\[ V_5 = \frac{1}{3} \int_{x}^{z_5} \omega^{1/3} \text{d}x = \int_{z_5}^{\omega} \]
Question: For a realistic vacuum can and be solved such that the observable sector vector bundle is slope-stable with respect to \( \omega \)?

Answer: Yes?

Consider the MSSM realistic standard model.

On the hidden sector choose

\[ V' \sim O_x \]

which is trivially slope-stable.
Hidden Sector:

\[ \mathbb{C} \]

\[ V, G = SU(4) \]

\[ W' = 1 \]

\[ V \approx O_x \]

\[ E_8' \]

\[ \mathbb{T}^4 \]

Anomaly Cancellation

\[ \mathbb{C} \] = \( c_2(TX) - c_2(V) \)

\[ \omega = a' \gamma_1 + a^2 \gamma_2 + a^3 \phi \in \mathbb{K} \]

Kähler Cone

We find that

\[ c_2(V) = \gamma_1^2 + 4\gamma_2^2 + 4\gamma_3 \gamma_2 \]

\[ c_2(TX) = 12(\gamma_1^2 + \gamma_2^2) \]
\[ \mathcal{C} = [W] - [\overline{W}] \]

where

\[
[W] = 7 \gamma_1^2 + 4 \gamma_2^2, \quad [\overline{W}] = 4(\gamma_1 \gamma_2 - \gamma_1^2 - \gamma_2^2)
\]

Then using the \( \gamma_1, \gamma_2, \phi \) intersection numbers

\[
\frac{V_5}{V_y^{1/3}} = \frac{1}{V_y} \int \omega \wedge [\overline{W}] = 4a^3
\]

and

\[
\frac{V_5}{V_y^{1/3}} = \frac{1}{V_y} \int \omega \wedge [W] = \frac{4a^1 + 7a^2}{3}
\]

Consider the region

\[ K_5 \subset K \]

As one approaches the bottom of \( K_5 \Rightarrow a^3 \rightarrow 0 \)
one can choose
\[ \frac{V_5}{V_{cy}^{1/3}} \approx 10^{-7} \]

Note that

\[ \text{Re } S = \frac{1}{6} \left( (a')^2 a^2 + a' (a')^2 + 6a'a^3 \right) \]

and

\[ \text{Re } S \approx 1 \]

\( S \) is bounded on the left by the vertical line where

\[ a' = a^2 \]

\[ \Rightarrow \text{ on vertical line at } a^3 \rightarrow 0 \]

The modulus

\[ a' = (3)^{1/3} - \frac{\varepsilon}{2}, \quad a^2 = (3)^{1/3} + \frac{\varepsilon}{2} \]

\[ \Rightarrow \text{Re } S \approx 1 \text{ and are in } S_5. \text{ For this region} \]

\[ \frac{V_5}{V_{cy}^{1/3}} \approx 1 \]
Figure 1: Kähler Cone. The observable sector vector bundle is slope-stable in the region $\mathcal{K}$. 

In the region

\[
\frac{V_{5}}{V_{CY}^{1/3}} \approx 10^{-7}, \quad \frac{V_{5}}{V_{CY}^{1/3}} \approx 1
\]

\[
0 < \Lambda / \mu_{\text{pl}} < 1
\]

and

Observable $V$ is slope-stable.
Conclusion:

For the MSSM heterotic standard model

- Take $V \propto \phi^4$. Anomaly cancellation $\Rightarrow$ both 5-brane and anti 5-brane on $S^1/\mathbb{Z}_2$ interval and fixed their cohomology classes.

- Neglecting the anti 5-brane, all moduli are stabilized, but at $N = 1$ preserving minimum with $V_{\text{min}} \approx 10^{-16} \text{M}_{\text{Pl}}^4$.

- Add anti 5-brane lifts the minimum to a meta-stable vacuum with a positive cosmological constant. The moduli are fixed on their vacuum and have phenomenologically acceptable values.
There is a region of the Kähler cone for which the cosmological constant has its observed value and for which the observable sector vector bundle is slope-stable.

One expects the Kähler moduli can be fine-tuned to lie in this region.