The Beam Matrix Method to perform the Near-Far Extrapolation in MINOS

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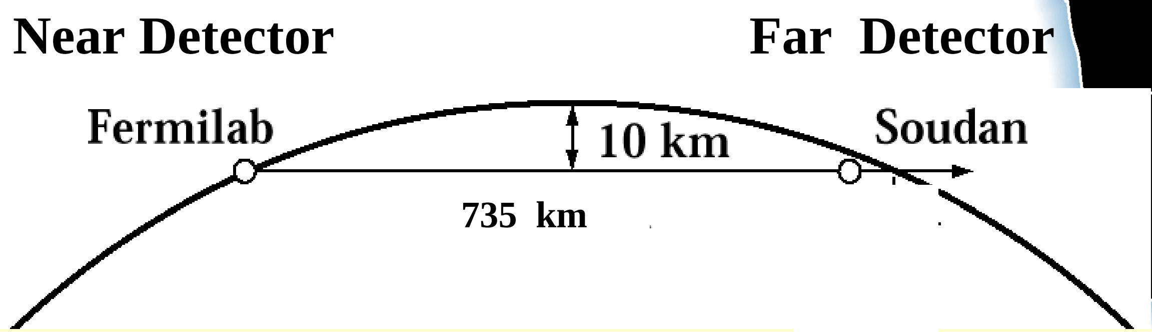
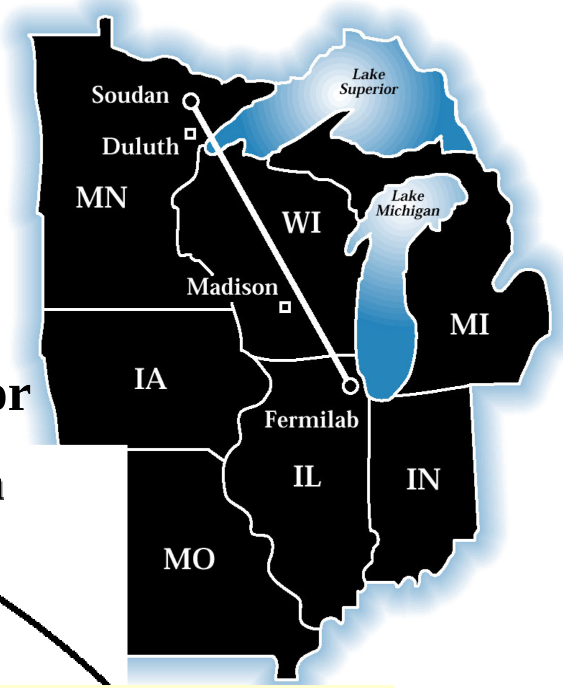
Outline

- Brief MINOS Experiment Overview.
- Main Systematic Uncertainties in the oscillation measurement.
- Near Detector Data : What did we learn
- Near – Far Extrapolation using the "Beam Matrix Method"
  - Basic Idea & Description of the Method
  - Cancellation of Beam & Cross Section uncertainties
  - Performance on "Mock Data Challenge"
  - Far Detector Oscillation Analysis Results
- Summary

MINOS Experiment

MINOS (**M**ain **I**njector **N**eutrino **O**scillation **S**earch) is a two detector long baseline neutrino oscillation experiment.

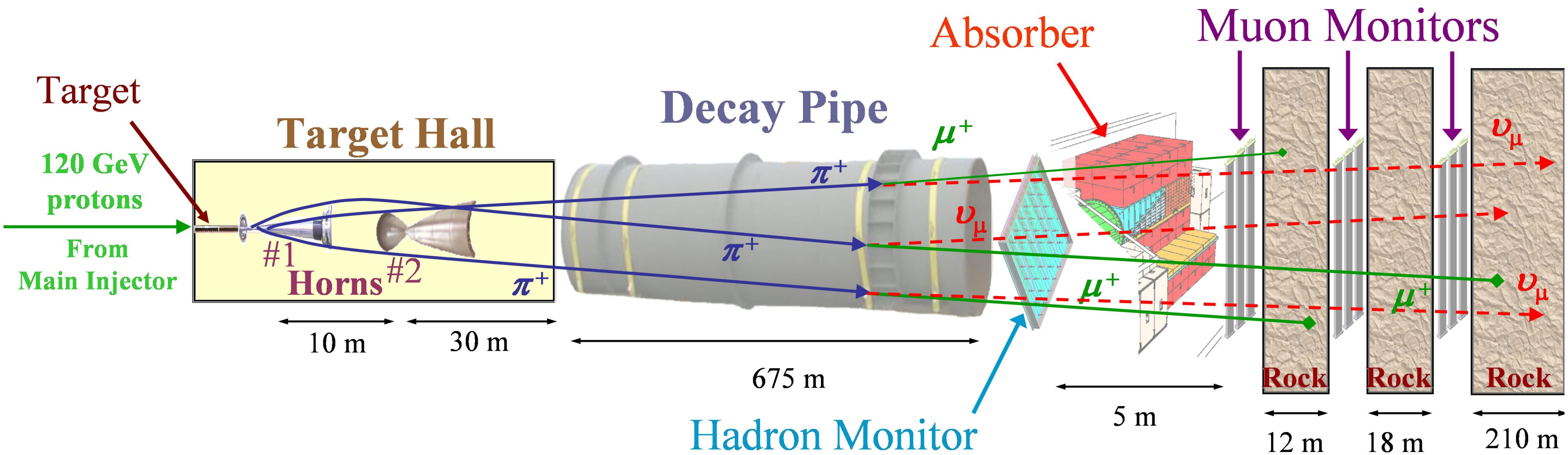




Comparison between Near/Far measurements will establish the oscillation signal and characteristics

Basic Idea : Two detectors "identical" in all their important features.

Producing the neutrino beam



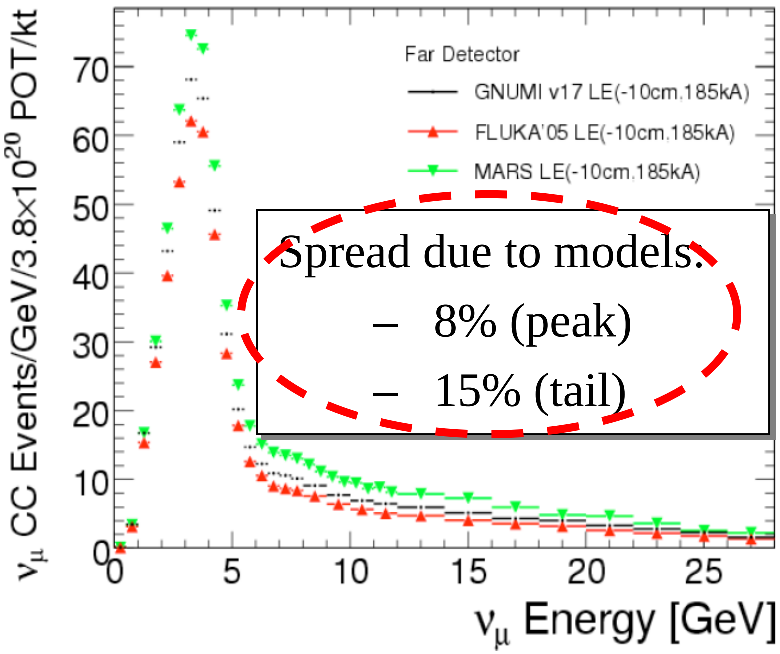
The Near and Far Detector neutrino energy spectra predicted by our MC have uncertainties due to :

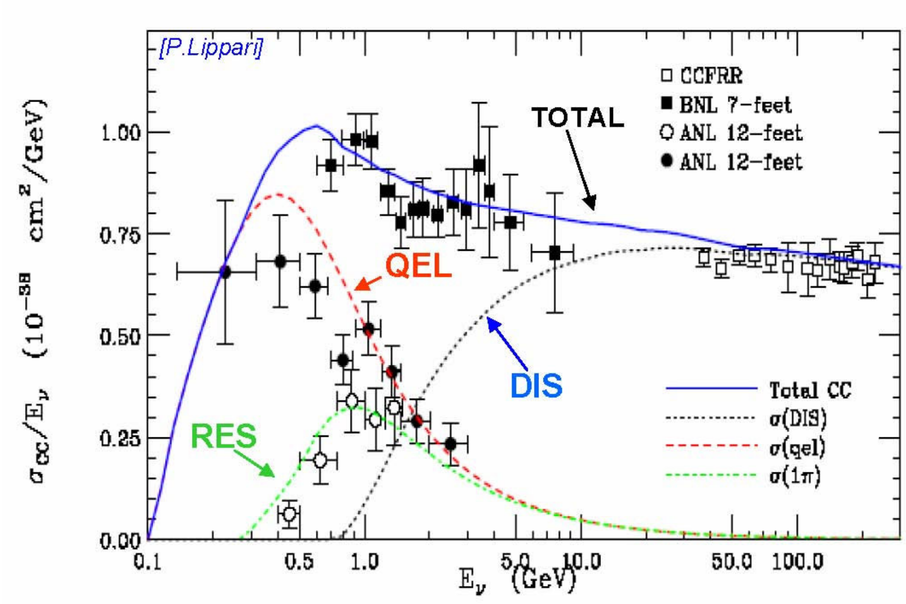
- Hadron production model uncertainties

- Neutrino cross section uncertainties

Goal is to minimize their effect on the final oscillation measurement using the Near Detector Data.

Systematic Uncertainties: Hadron Production & Cross Sections





Model	$\langle p_T \rangle$ (GeV/c)
Geant/Fluka[16]	0.37
Fluka 2005[20]	0.43
Sanford-Wang[17]	0.42
CKP[18]	0.44
Malensek[19]	0.50
MARS-v.15[14]	0.38

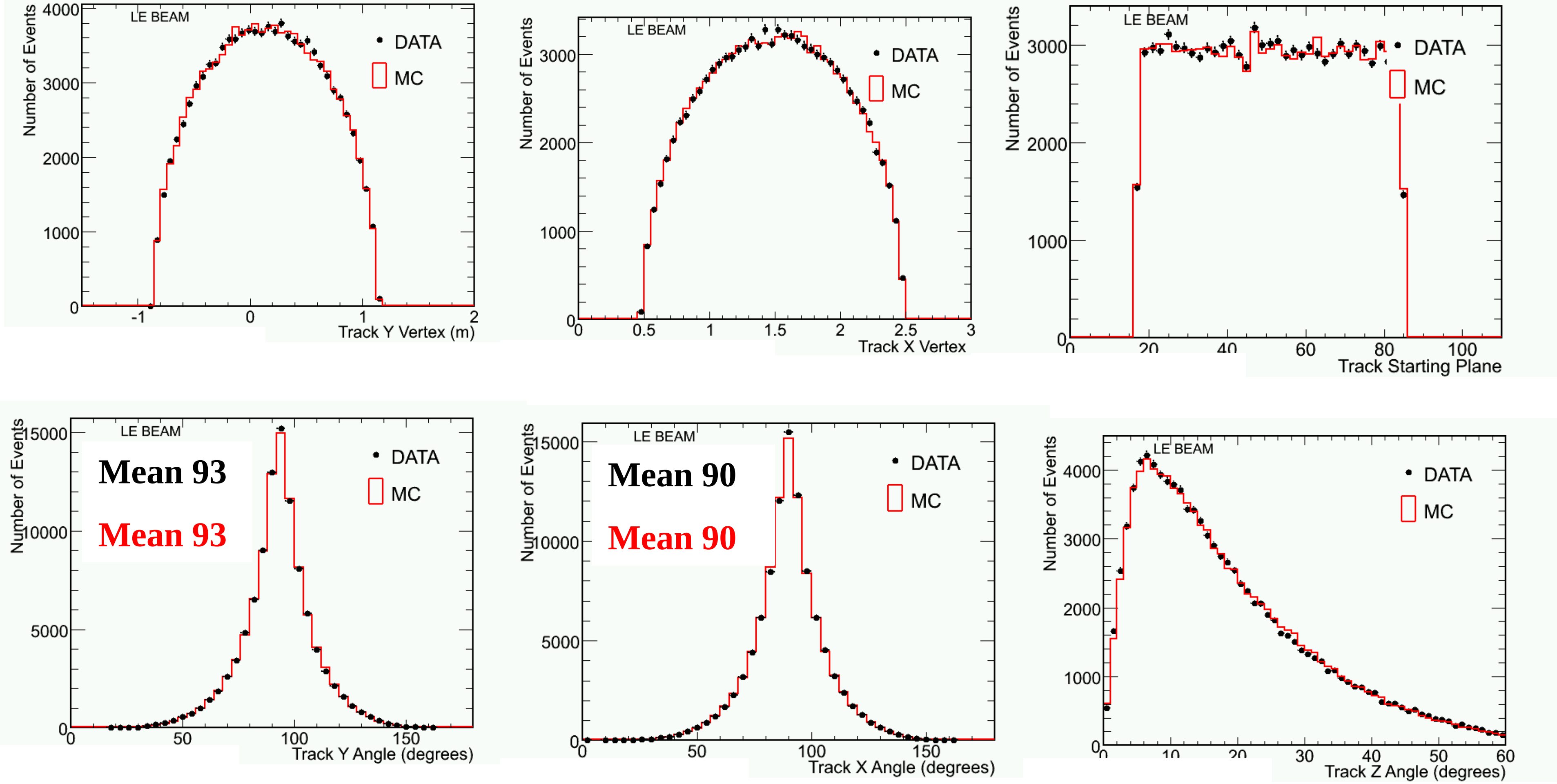
Neutrino Cross Sections at Low Energies not very well measured (uncertainties of the order of 10%)

MINOS ND CC SPECTRUM Composition (up to 6 GeV):

- **~ 19% QE**
- **~ 25% RES**
- **~ 56% DIS**

Near Detector : Data/MC

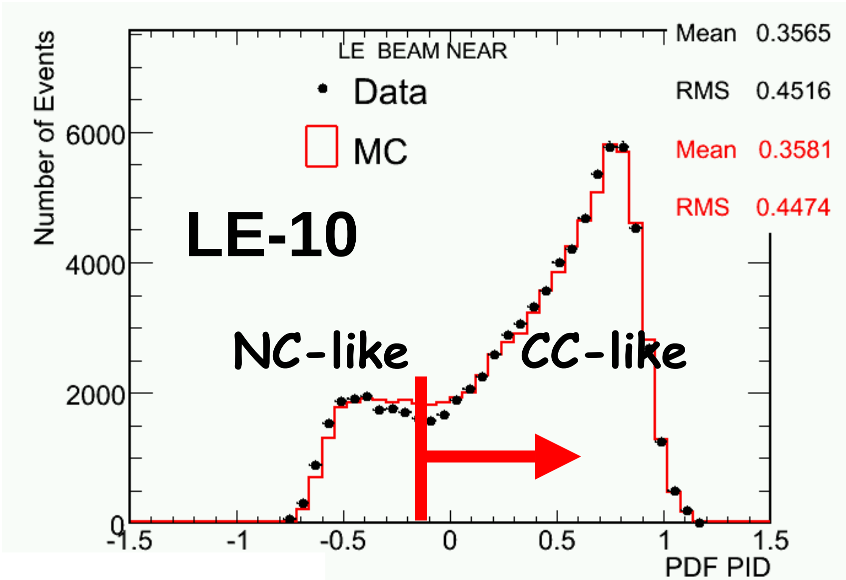
Plots normalized to area

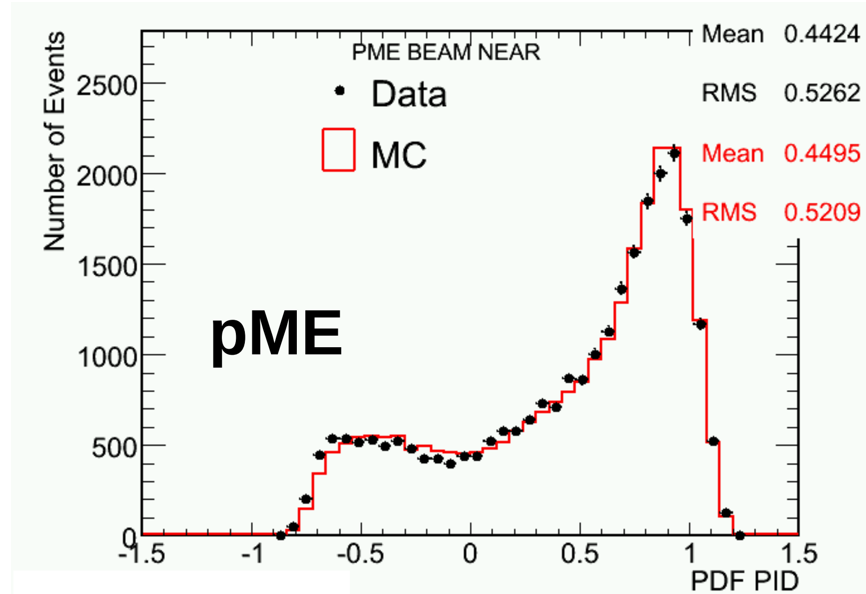


Low Level ND Quantities agree quite well.

Near Detector : Data/MC

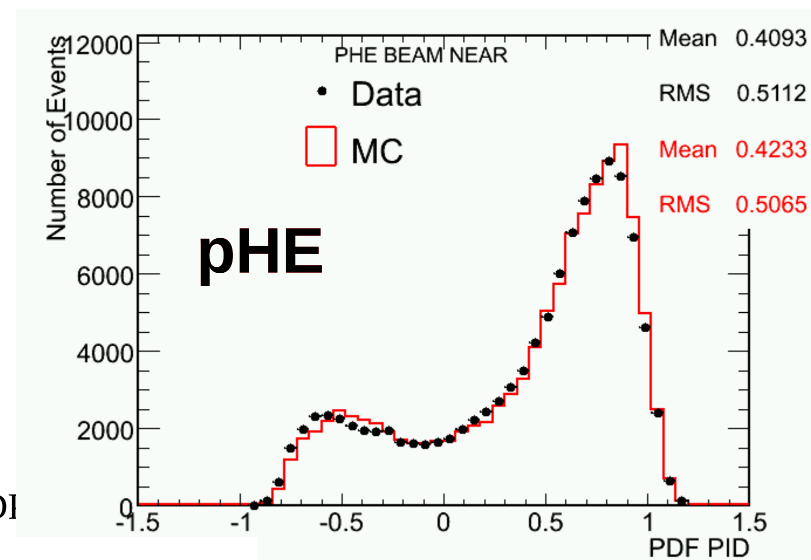
Particle IDentification Distributions





Cut to select CC-like events

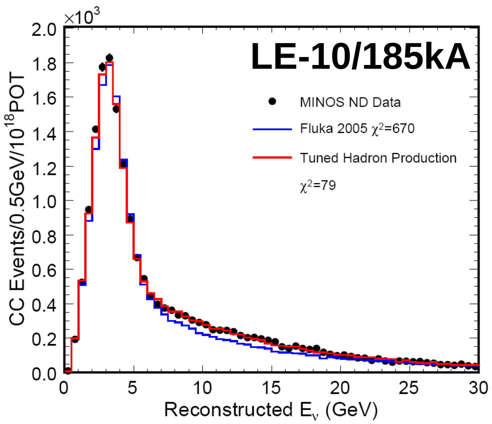
Agreement between Data and MC very good.

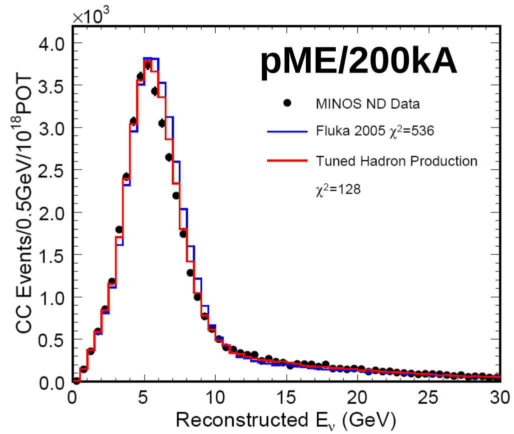


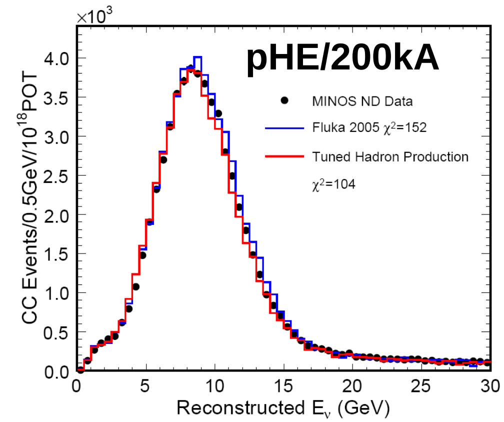
Near Detector : Data/MC Energy Spectra

By tuning the MC improved agreement between data and MC can be obtained.

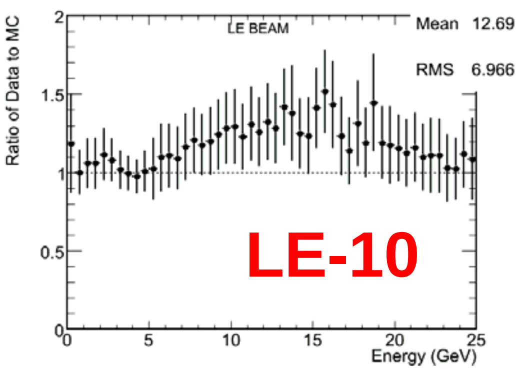
Reconstructed Energy (GeV)

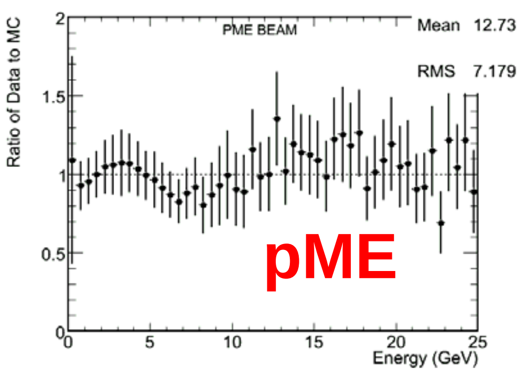


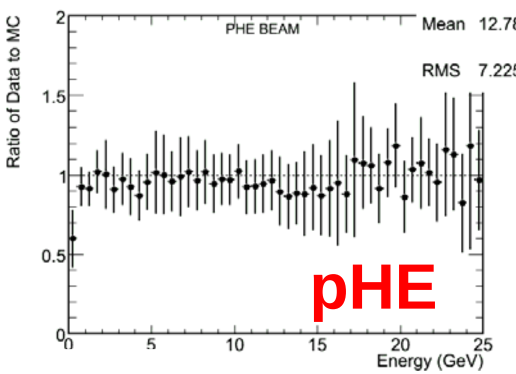




Ratios of Data/MC







"Dip" moves with energy, discrepancy between data and MC due to hadron production modeling uncertainties.

Near Detector Data : What did we learn

- The agreement between Data/MC of low level quantities indicates that there are no major detector/reconstruction effects not modeled by our MC.

- The disagreement between Data/MC of the reconstructed neutrino energy spectrum is related with the main uncertainties that we mentioned earlier (hadron production and cross sections modeling).

- We would like to use a Near-Far extrapolation technique as insensitive to these systematics uncertainties as possible.

Near – Far Extrapolation Techniques

- There are two main Near - Far extrapolation techniques:

(A) Fitting techniques (Indirect Use of the ND Data)

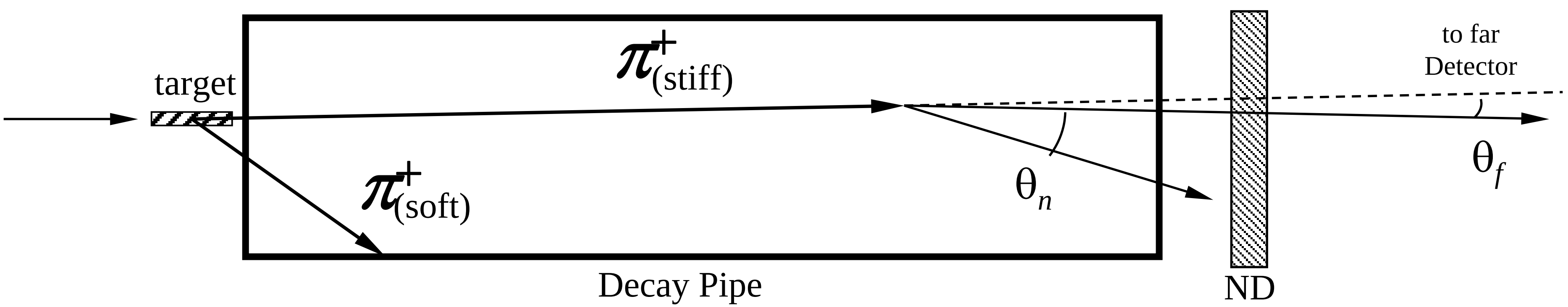
- These use the Near Detector Data in order to tune the Nominal MC.
- The fits use the **specific hadron production and cross section models used by the MC** and try to improve them.

(B) Techniques that make Direct Use of the ND Data

- These **use the Near Detector Data directly to extrapolate without attempting to tune the Nominal MC.**
- The Nominal MC is used to provide corrections due to energy smearing and acceptance.

Predicting Unoscillated FD Spectrum

- Use the "Beam Matrix" method with which Beam modeling & Cross sections uncertainties cancel out between the two detectors.
- The "Beam Matrix" method uses :
  - The ND Reconstructed Energy Distribution (Data),
  - The knowledge of pion 2 body decay kinematics,
  - The geometry of our beamline,
  - Our Monte Carlo to provide necessary corrections due to energy smearing and acceptance.



Schematic Description of the Method

A) $$E_{Near}^{\text{Reconstructed}} CC-like \Rightarrow E_{Near}^{True} CC$$

Correction for purity, Reconstructed => True, Correction for efficiency

B) $$E_{Near}^{True} CC \Rightarrow E_{Far}^{True} CC$$

BEAM MATRIX

C) $$E_{Far}^{True} CC \Rightarrow E_{Far}^{\text{Reconstructed}} CC-like$$

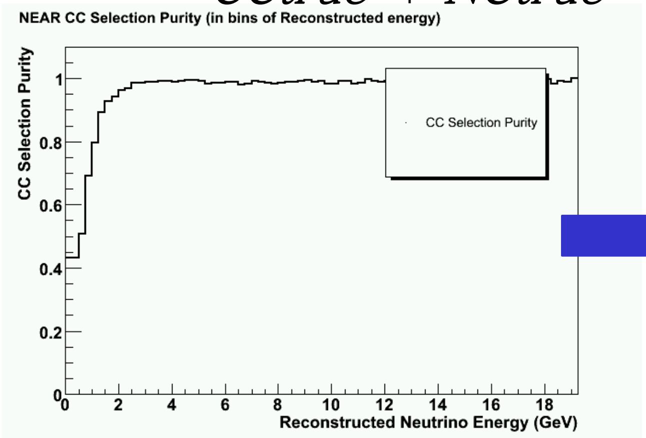
i) Oscillation, True => Reconstructed, Correction for efficiency to obtain CC oscillated spectrum

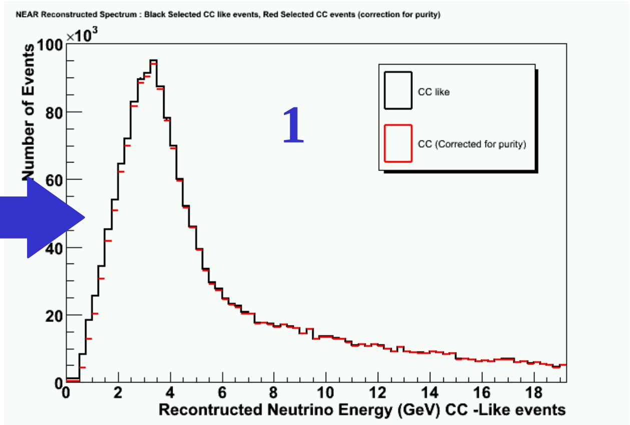
ii) Unoscillated True => Reconstructed, Use purity to obtain NC background

Beam Matrix Method : Step A

$$Purity\,(bin) = \frac{CCtrue}{CCtrue + NCtrue}(bin)$$

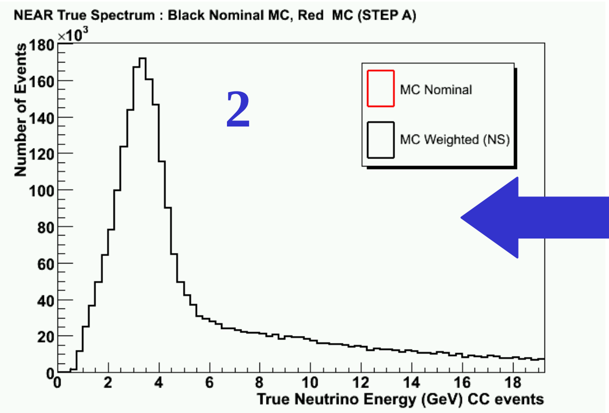
Correction for purity

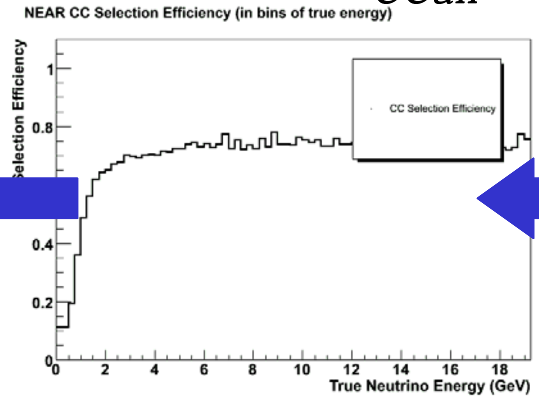


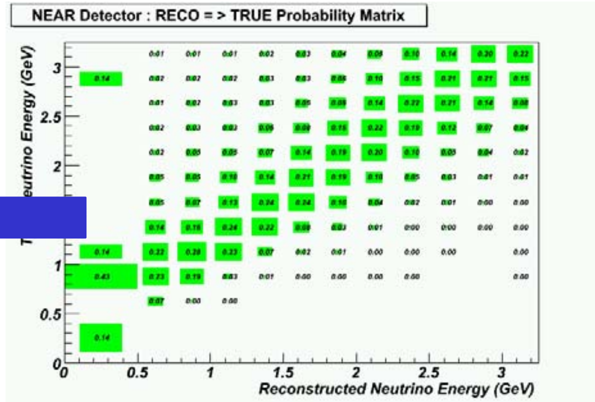


Reco=>True and Correction for efficiency

$$Efficiency\;(bin) = \frac{CCtrue}{CCall}(bin)$$

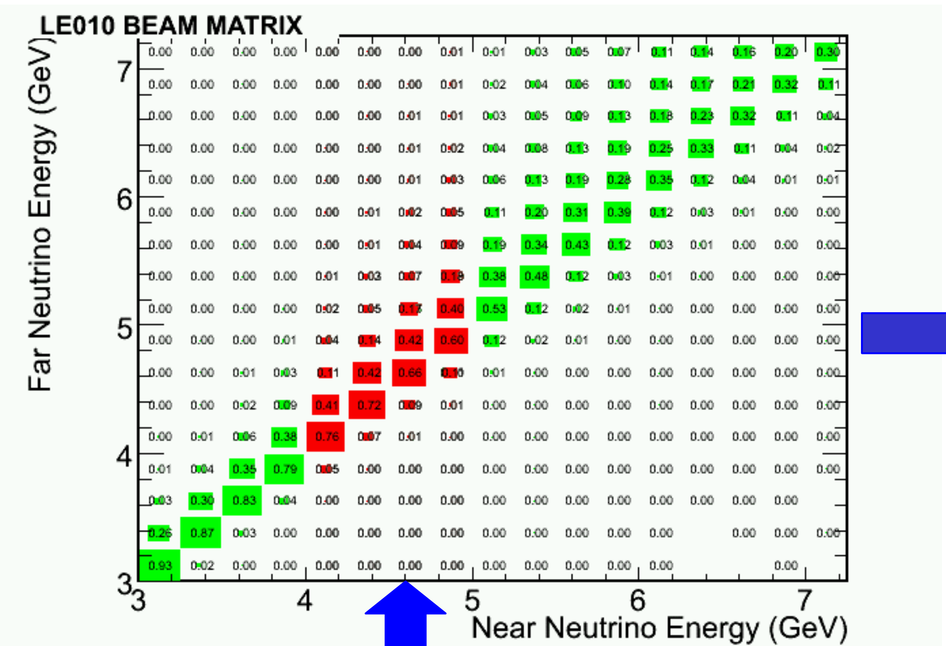


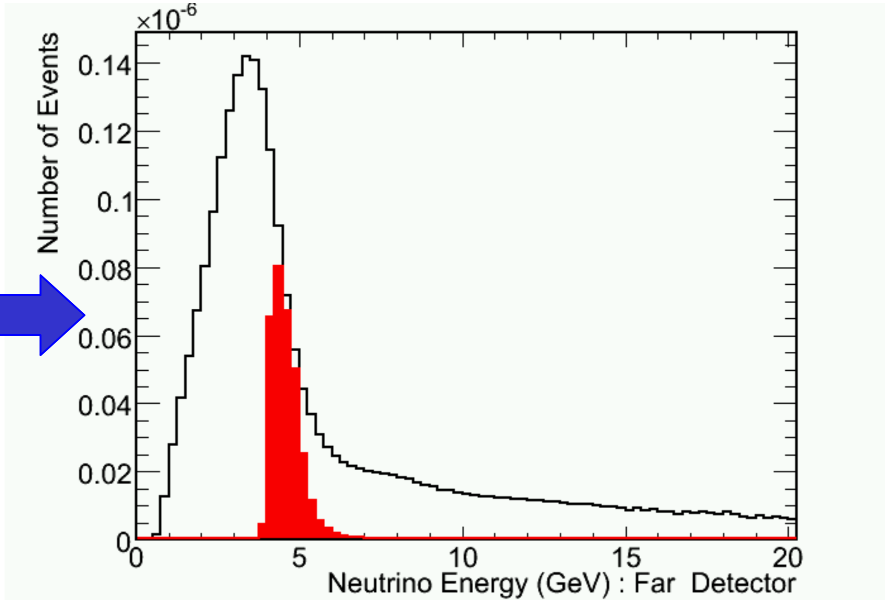


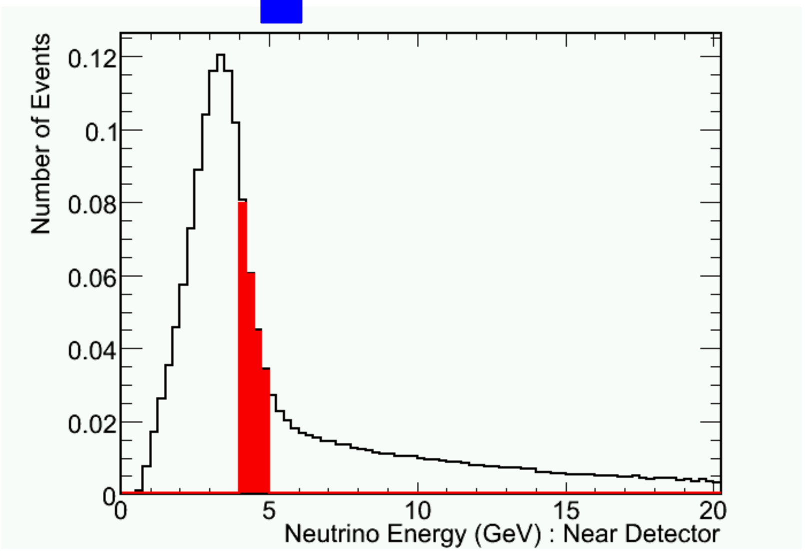


Each column is normalized to one (probability matrix)

Beam Matrix Method : Step B



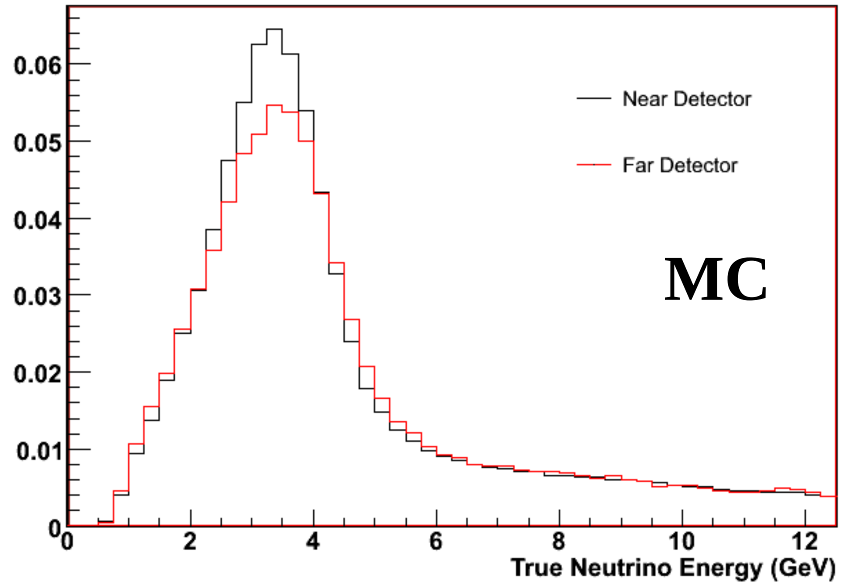




- The Beam Matrix is constructed with basically the knowledge of pion 2 body decay kinematics & geometry of the beamline.

- The Beam Matrix provides a very good representation of how the far detector spectrum relates to the near one.

Why does the "Beam Matrix" Method work?



- *The neutrino beam is common in both detectors, therefore knowing the neutrino flux in the Near determines what the expected neutrino flux is in the Far. Hadron production uncertainties are expected to cancel out (next slides)*
- *The neutrino spectra are very similar in the two detectors, therefore neutrino cross section uncertainties are expected to cancel out (next slides).*
- *Near Detector Data/MC differences do not arise from detector/reconstruction effects.*

Beam Matrix Method: Systematics

BEAM & Cross Section Uncertainties Cancel out (Do They?)

Method to check the validity of the hypothesis:

- Use "Fake Data" for which different hadron production model and different cross sections, than nominal MC, are used.

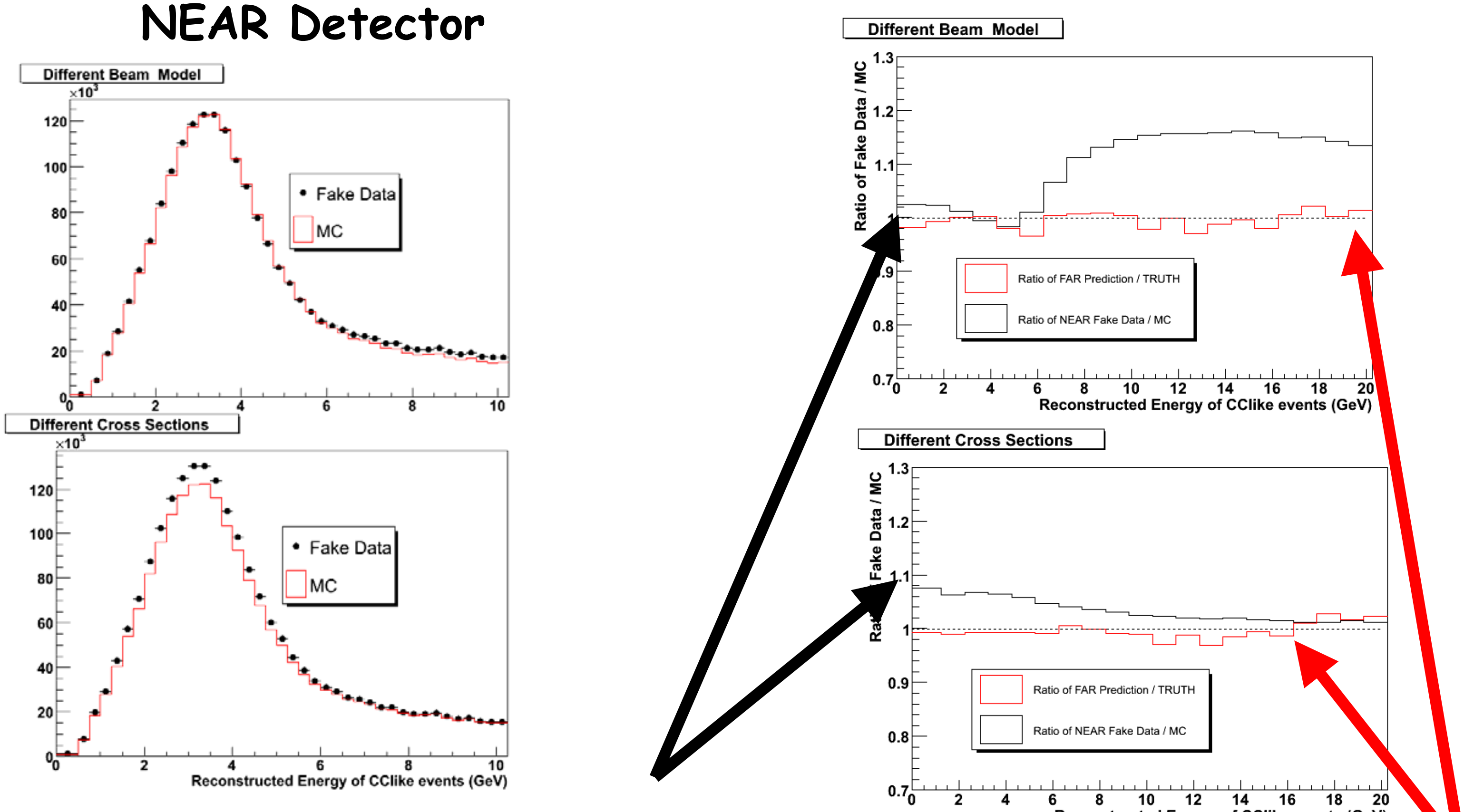
(How dramatic the difference is can be seen in Ratio of Near Detector "Fake Data"/Nominal MC)

- **Examine how different Predicted Far spectrum is from True one:**
  - -> The **magnitude of the difference** is a **direct measurement** of the **Cancellation** of the introduced **Uncertainty**.
  - -> The **magnitude** of the **difference** is a **direct measurement** of the **systematic shift** that will be introduced to the **oscillation measurement** from this particular source of systematics

Beam Matrix Method :Beam Systematics

BEAM & Cross Section Uncertainties Cancel out (Do They?)

NEAR Detector



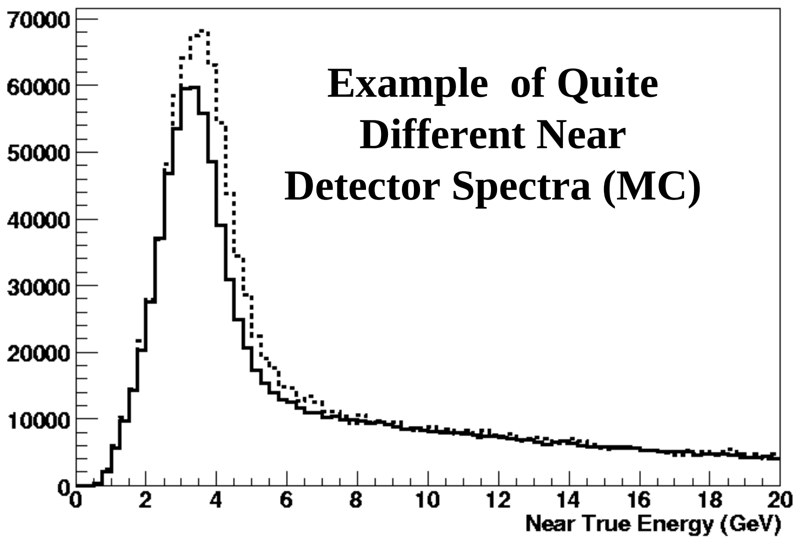
Ratio of "Fake Data" to Nominal MC in the Near Detector

Ratio of Beam Matrix Far unoscillated prediction (using the "Fake Data") to True Far spectrum.

Beam Related uncertainties Cancel

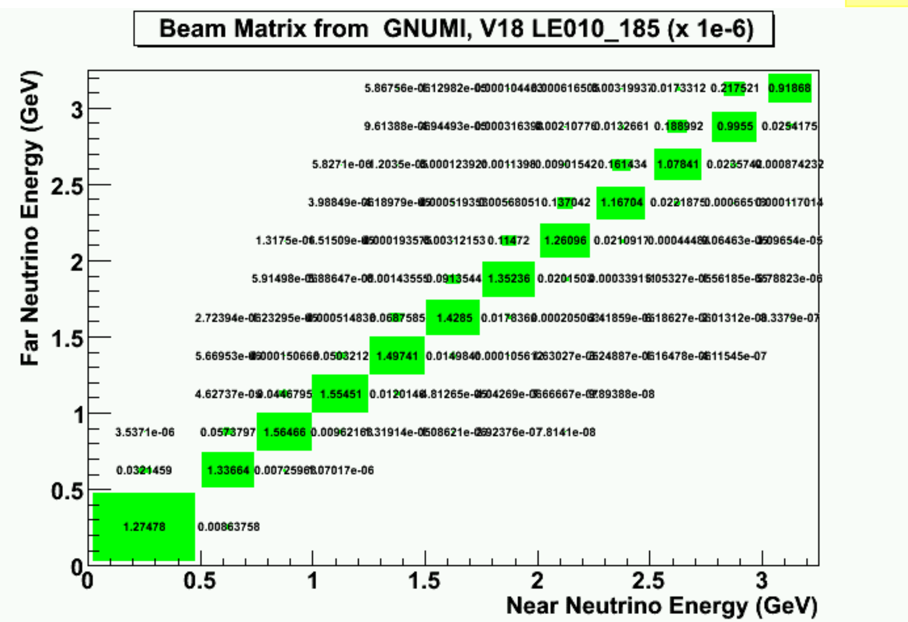
Cross Section uncertainties Cancel

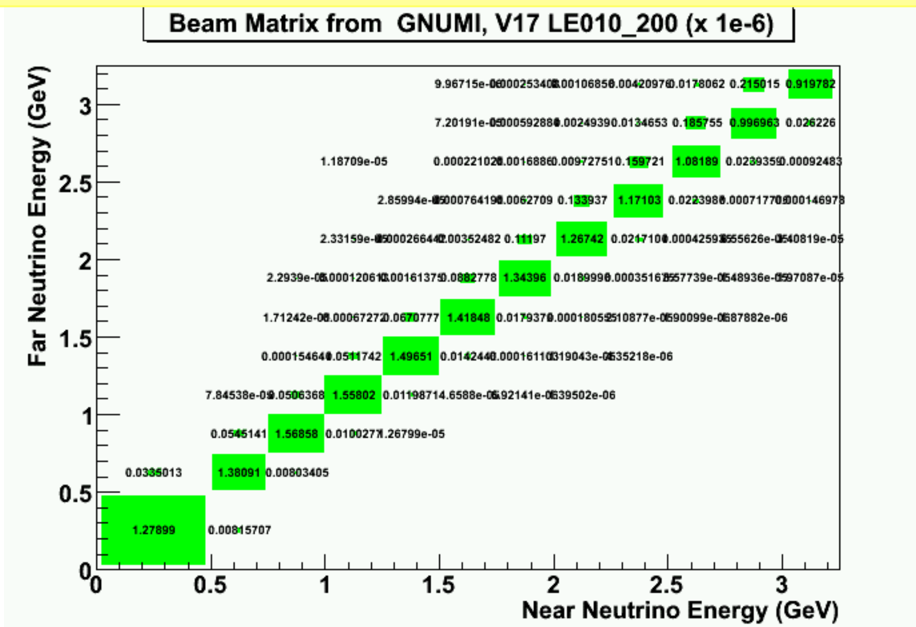
Why Beam Modeling uncertainties Cancel



- Elements of Beam Matrices that correspond to quite different near detector spectra are very similar (spread in each column determined primarily by the geometry of the beamline)

- This means that even if our Beam Matrix is not the one corresponding to the actual Data Spectrum (beam model of Nature and not beam model of our MC) the Far Prediction will be accurate





Why Cross Section Uncertainties Cancel

Beam Matrix

$$\text{ND Spectrum} \times \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_b & 0 \\ 0 & 0 & \sigma_c \end{pmatrix}^{-1} \begin{pmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{pmatrix} \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_b & 0 \\ 0 & 0 & \sigma_c \end{pmatrix} \text{FD Spectrum} \Rightarrow$$

ND Flux —X→ **FD Flux** —X→

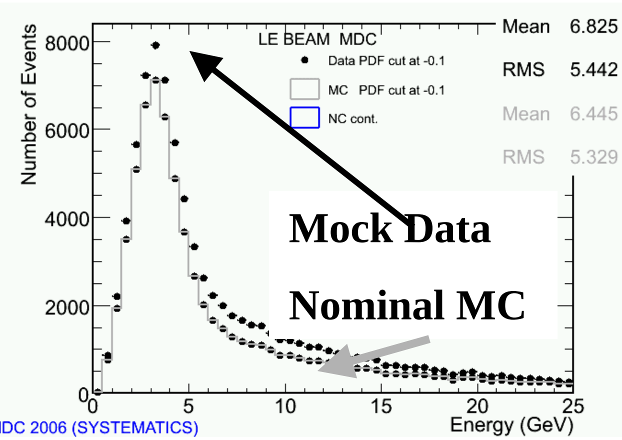
Cross Section matrices diagonal, Beam Matrix almost diagonal => They Commute!

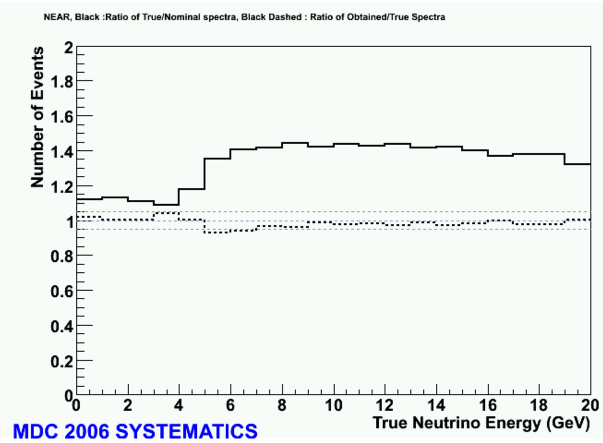
$$\Rightarrow \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_b & 0 \\ 0 & 0 & \sigma_c \end{pmatrix}^{-1} \begin{pmatrix} \sigma_a & 0 & 0 \\ 0 & \sigma_b & 0 \\ 0 & 0 & \sigma_c \end{pmatrix} \begin{pmatrix} b_1 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & b_3 \end{pmatrix}$$

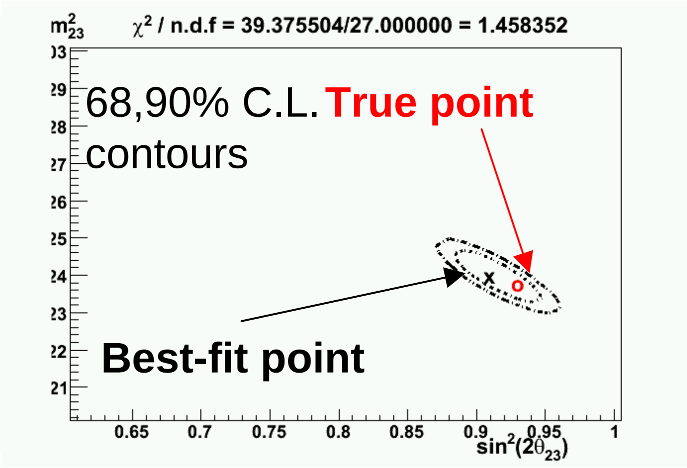
Their Product is I regardless of their values!

Test on 1E22 POT Mock Data Challenge Set

In order to test the robustness of the oscillation analysis that uses the Beam Matrix Method to extrapolate, "fake datasets" were generated with tweaked beam/generator parameters and unknown oscillation parameters.







Ratio of True/ Nominal (Black line)

Ratio of Predicted /True (Black dotted Line)

Oscillation analysis using the Beam Matrix Method to extrapolate yields to an accurate estimation of the oscillation parameters despite the large differences between "Mock Data" and Monte Carlo (even for 1E22 protons on target!)

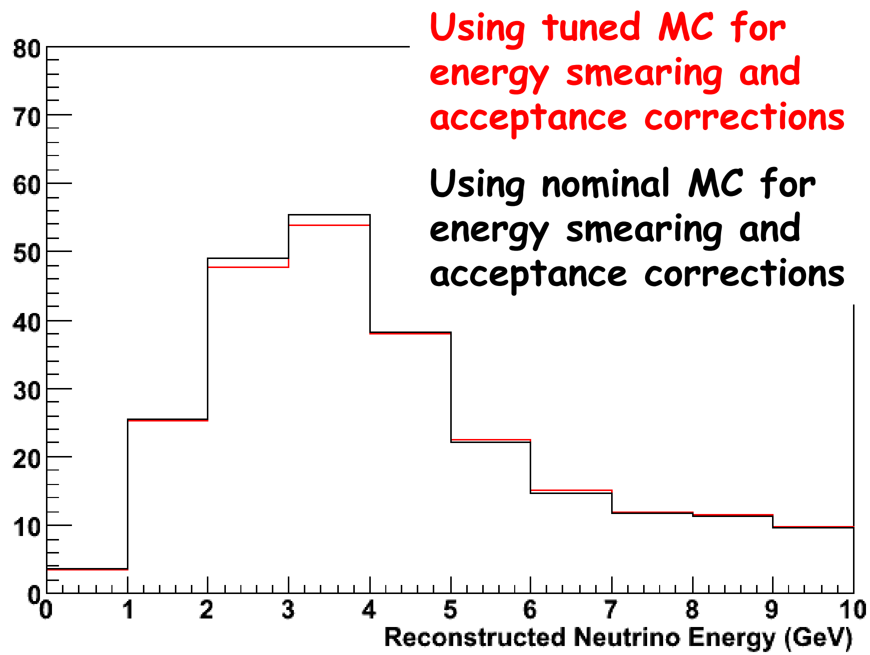
Remaining systematic uncertainties

Beam and Cross Section Uncertainties using the Beam Matrix Method Cancel. The main remaining systematic uncertainties are Near/Far Normalization, Absolute hadronic energy scale and NC contamination.

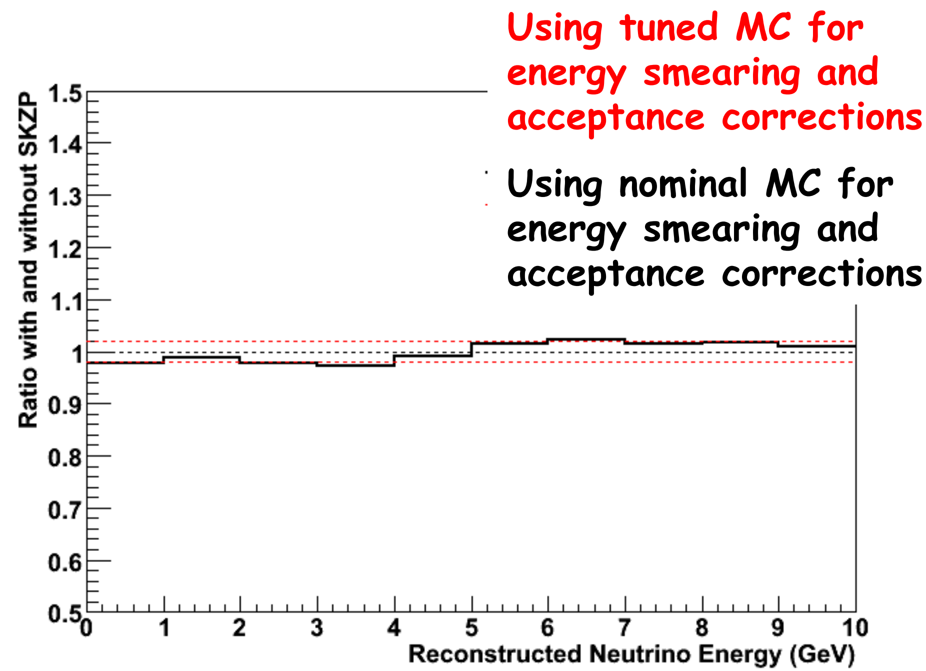
Uncertainty	Shift in Δm^2 (10^{-3} eV^2)	Shift in $\sin^2(2\theta)$
Near/Far normalization ±4%	0.050	0.005
Absolute hadronic energy scale ±11%	0.060	0.048
NC contamination ±50%	0.090	0.050
All other systematic uncertainties	0.044	0.011
Total systematic (summed in quadrature)	0.13	0.07
Statistical error (data)	0.36	0.12

Effect of MC tuning on the measurement

Far Predicted Spectra using the Beam Matrix and with/without hadron production tuning

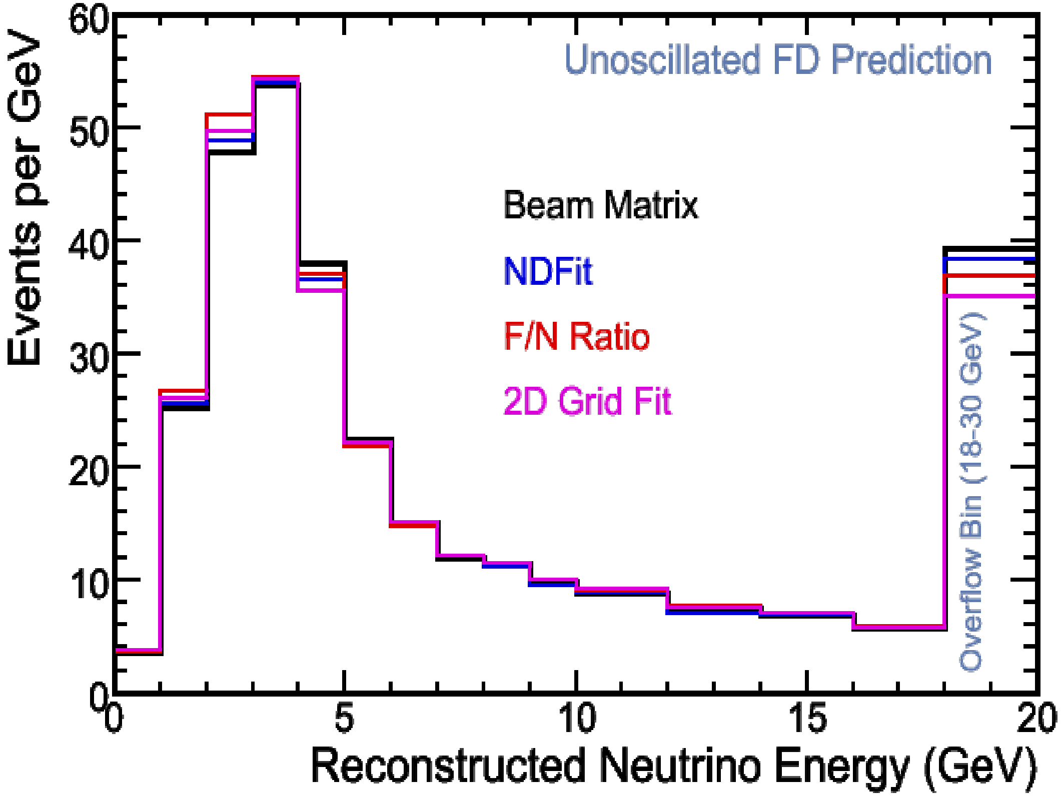


Ratio of Far Prediction using the Beam Matrix and with/without hadron production tuning



- Using Beam Matrix Method, hadron production tuning does not affect the Unoscillated prediction (obtained from the ND data) by more than 1-2%.
- However, its use improves the MC (make it more similar to the data) and therefore uncertainties due to energy smearing-unsmearing and acceptance become smaller.

How does the Beam Matrix Method compares to other Near-Far extrapolation techniques

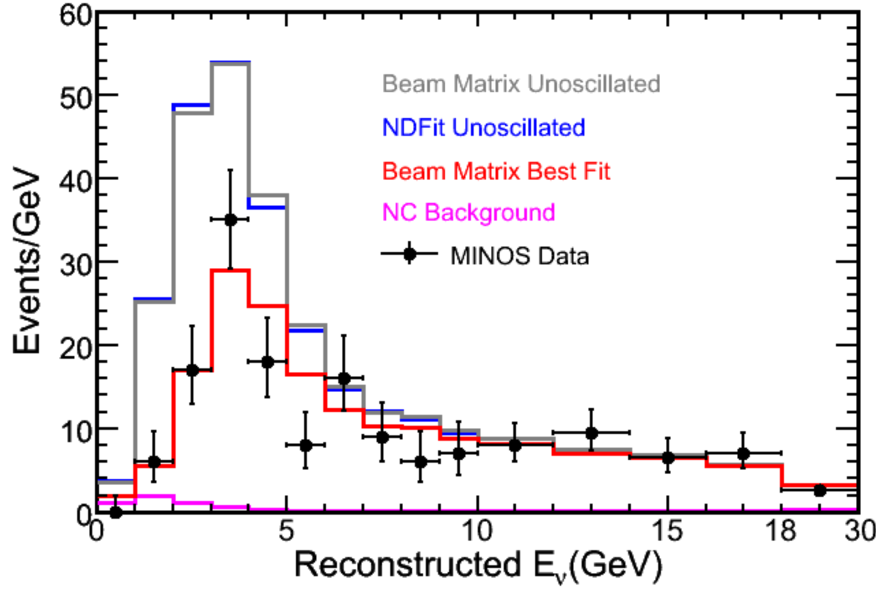


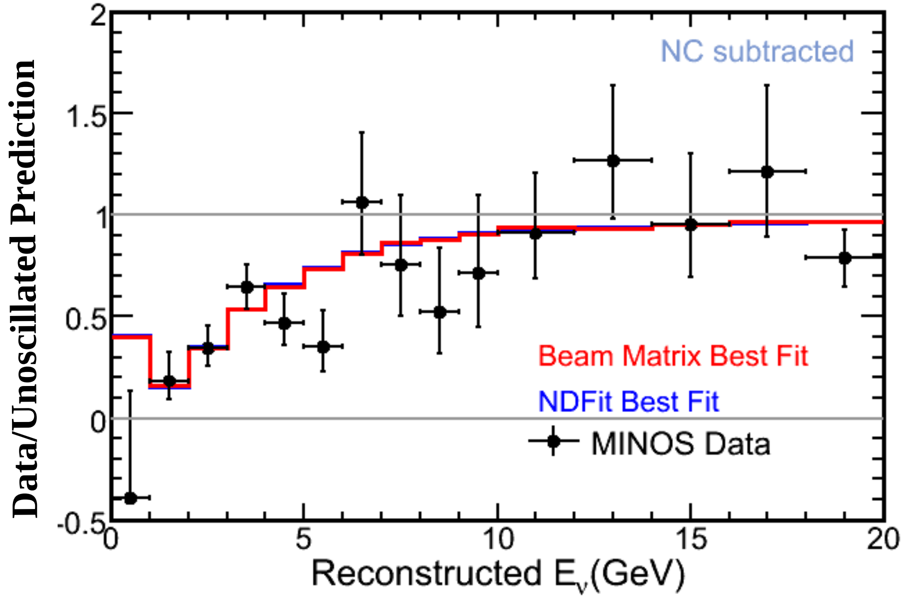
- In parallel to the Beam Matrix method, 3 other extrapolation methods were applied to the data.

- The 4 extrapolation methods investigated give consistent predictions

FD CC_{like} Events: Best Fit Spectrum

$$\chi^2(\Delta m^2, \sin^2 2\theta, s_1, \ldots, s_{nsys}) = \sum_{i=1}^{nbins} 2(e_i - o_i) + 2o_i \ln(o_i / e_i) + \sum_{j=1}^{nsys} \Delta s_j^2 / \sigma_{s_j}^2$$





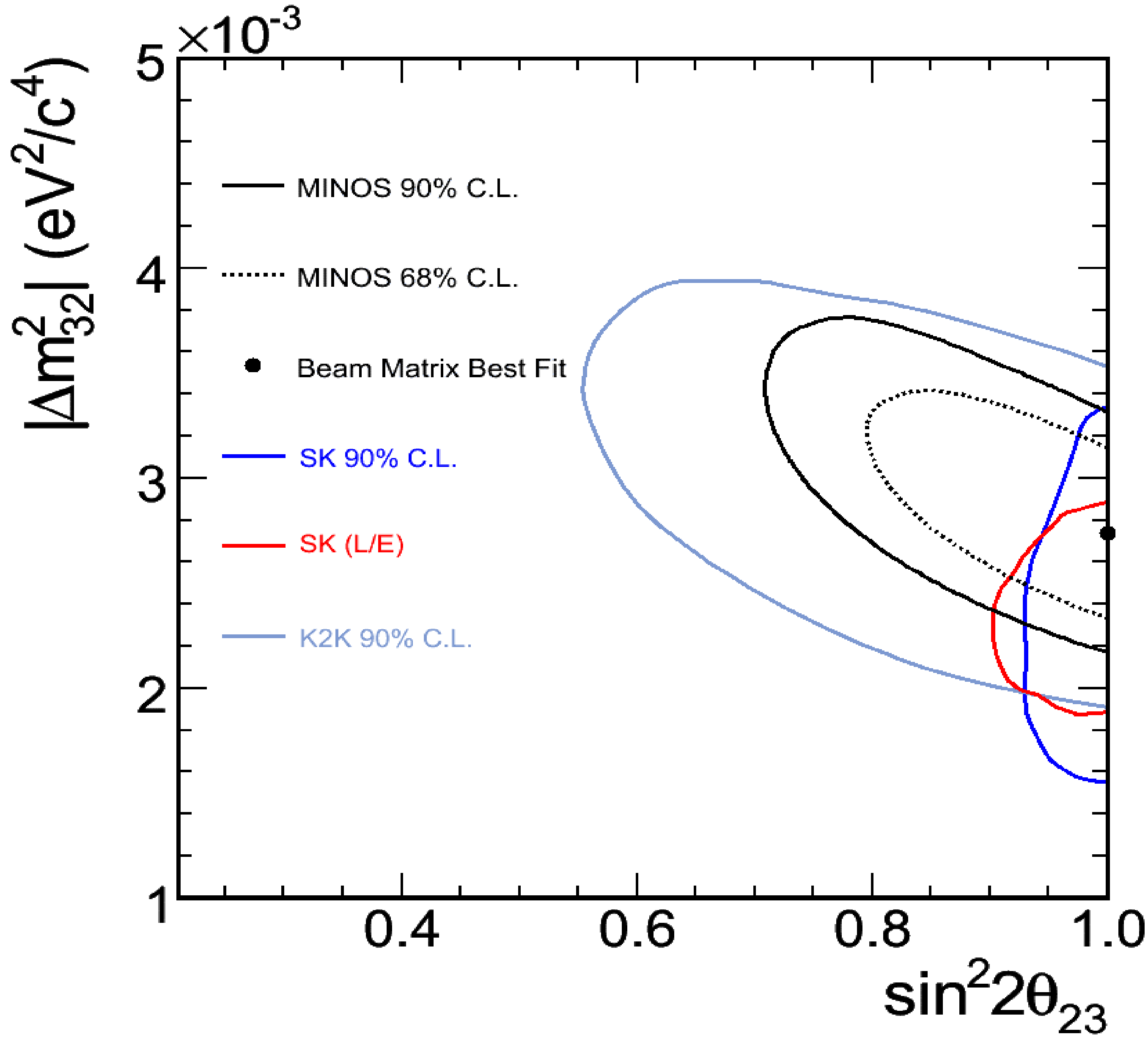
$$\left|\Delta m^2_{32}\right| = 2.74^{+0.44}_{-0.26} \text{ (stat + syst)} \times 10^{-3} \text{eV}^2$$

$$\sin^2(2\theta_{23}) = 1.00_{-0.13} \text{ (stat + syst)}$$

$$\text{Normalization} = 0.98$$

$$\text{Constrained to } \sin^2(2\theta_{23}) \le 1$$

FD CC_{like} Events: MINOS allowed region



SUMMARY

- The Beam Matrix Method fully utilizes the fact that the MINOS experiment has two "identical" detectors.

- It uses the Near Detector Data directly to obtain the Far Unoscillated Spectrum, without attempting to tune the MC.

- It is a quite powerful technique, very robust against Beam Modeling and Cross Section uncertainties, which practically cancel out.

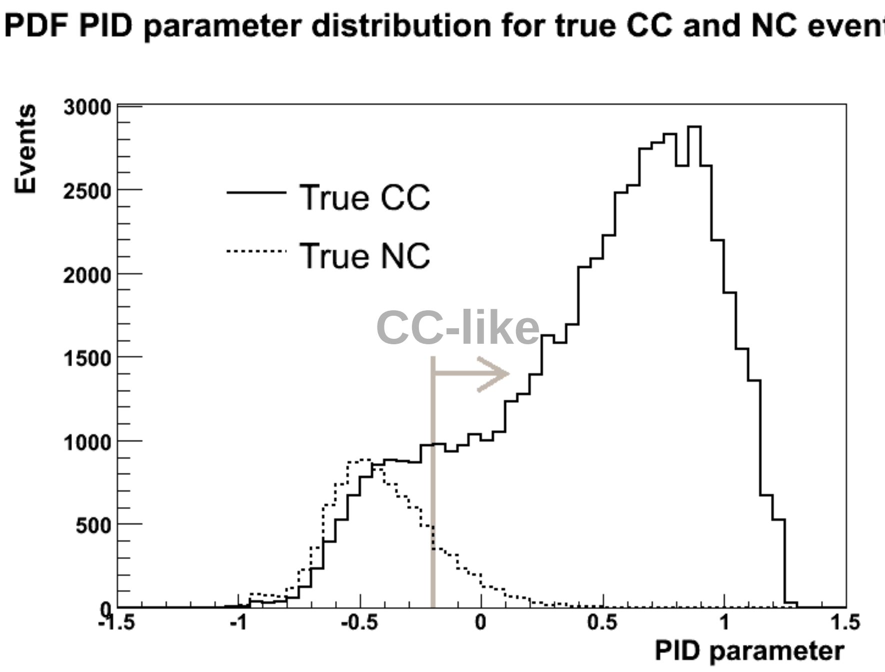
BACKUP SLIDES

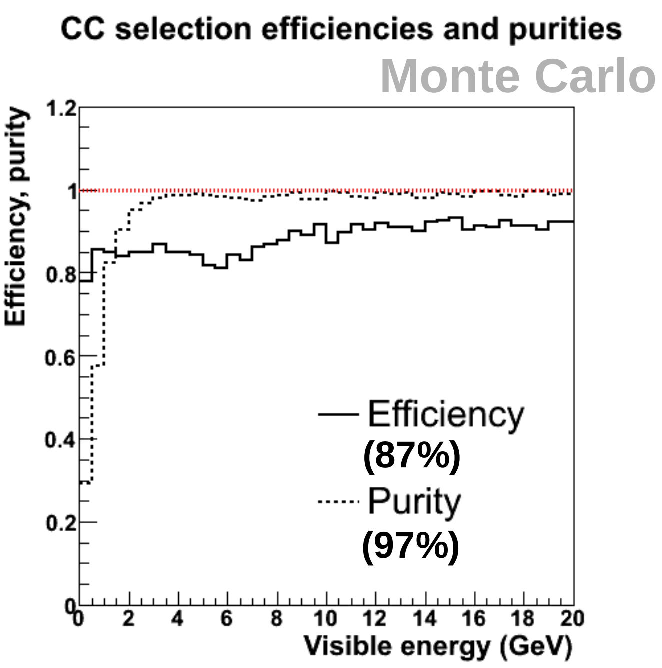
CC selection efficiencies

- The Particle ID (PID) parameter is defined thus:

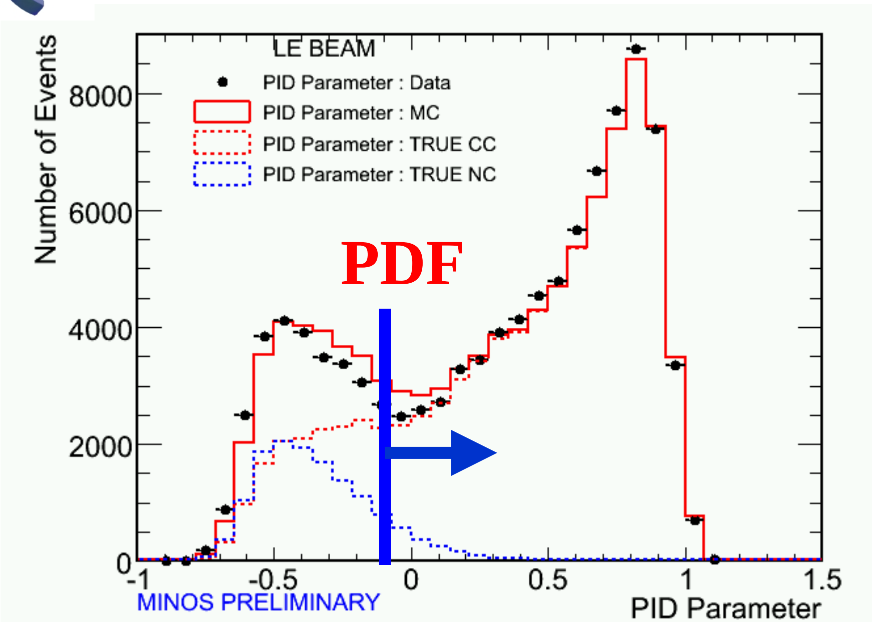
$$PID = -\left(\sqrt{-\log(P_{\mu})} - \sqrt{-\log(P_{NC})}\right)$$

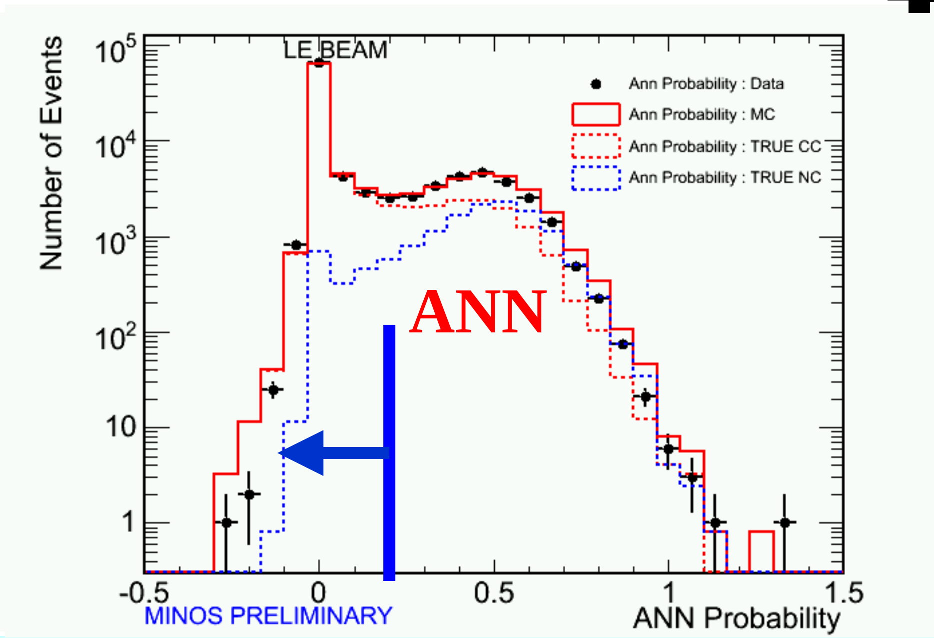
- CC-like events are defined by the cut PID>-0.2 in the FD (>-0.1 in the ND)
  - NC contamination is limited to the lowest visible energy bins (below 1.5 GeV)
  - Selection efficiency is quite flat as a function of visible energy





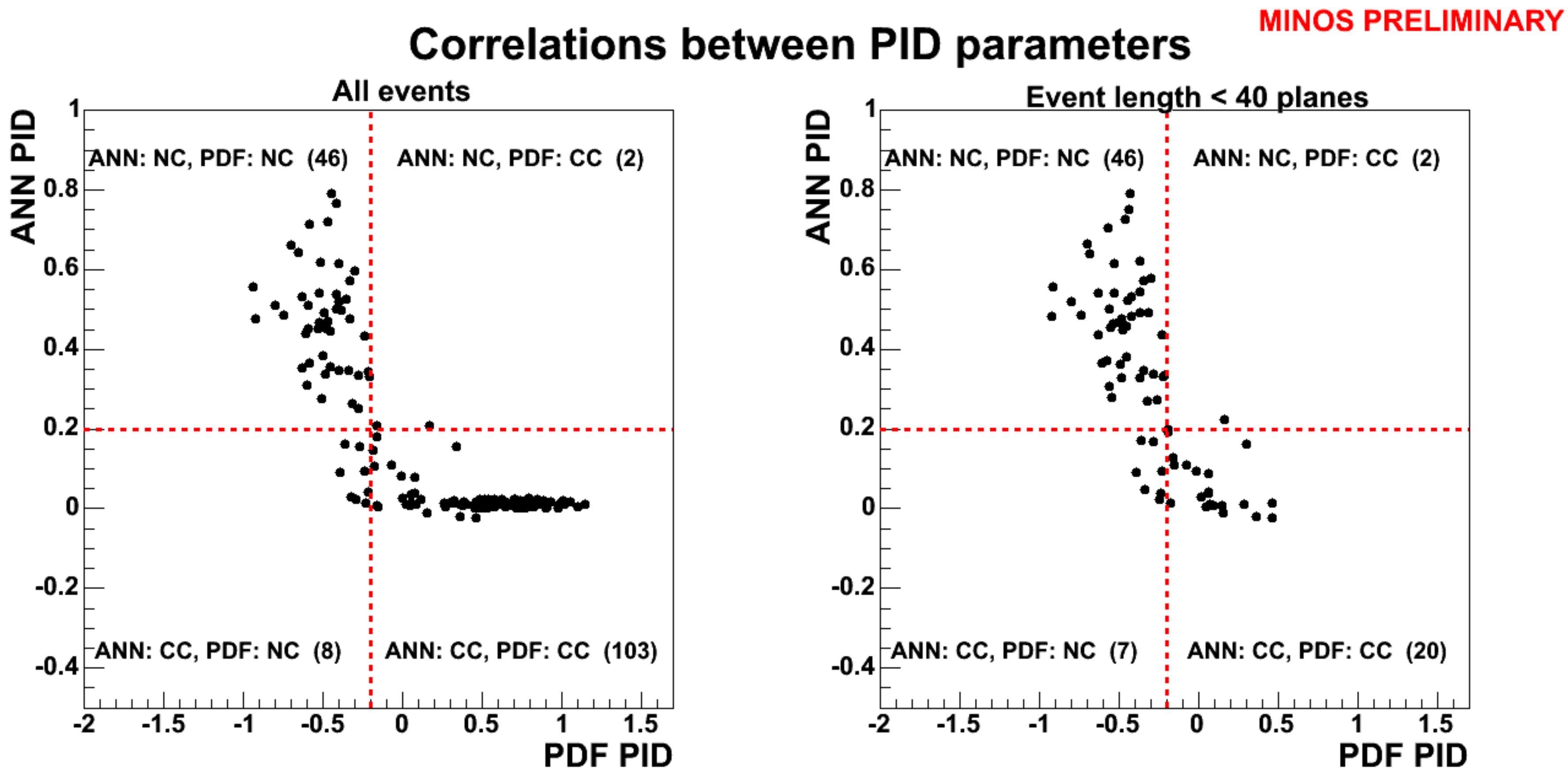
Beam Data : CC – like Selection





- **In order to select CC-like candidates we have :**
  - Simple method based on existence or not of a track (quite robust but with limited sensitivity).
  - PDF based method.
  - ANN based method.
- **PDF and ANN selection methods tested on high statistics neutrino sample from our Near Detector and performance is quite good.**

Far Detector Beam Data : CC – like Selection



PDF and ANN selection methods give very consistent results on Far Detector Beam neutrino events.

- CC-like Selection : Importance of ANN Variables MC -

Relative weight (%)	ANN Variable
10.565750	Total Pulse Height
10.446102	Total # of Strips
9.2708178	Event Length
8.7206430	Number of Tracks
8.6607571	Track Pulse Height per Plane
8.5564222	Pulse height per Plane
8.1546698	Shower Pulse Height per Digit
7.4450355	Pulse height per Strip
6.6567850	Difference of Track-Shower Length (V view)
6.4418235	Pulse height Fraction in first 3 planes
5.7088947	Pulse height Fraction in planes 3-6
5.1340508	Difference of Track-Shower Length (V view)
4.2382522	Pulse height Fraction in planes 6-last.

- **All ANN variables are important.**

ANN vs PDF Selection : Systematics

- In order to compare how sensitive the ANN and PDF selections are to the three major systematics we generated fake samples in which are altered :

  a) NC Background : +/- 50%

  b) Shower Energy : +/- 10%

  c) Normalization : +/- 4 %

  Then using the exact same code and the exact same pre-selection cuts as we did for the CC measurement using the Beam Matrix we performed the fits and compared the resulting shifts in Δm^2 and $\sin^2(2\theta)$:

	PDF Selection		ANN Selection (cut@0.2)	
	Δm^2	$\sin^2(2\theta)$	Δm^2	$\sin^2(2\theta)$
NC +/- 50%	**0.970**	**0.0525**	**0.325**	**0.0100**
E_{shower} +/- 10%	**0.500**	**0.0125**	**0.650**	**0.0075**
Normalization +/- 4%	**0.375**	**0.0050**	**0.400**	**0.0002**
TOTAL	**1.154**	**0.0542**	**0.829**	**0.0125**
TOTAL (% of PDF)	**100%**	**100%**	**71.8%**	**23.1%**

ANN vs PDF Selection : Efficiency, Purity & Sensitivity

- The best sensitivity with the ANN selection is obtained when placing the ANN cut at ~ 0.3-0.35. We repeated the sensitivity and systematics studies for 6 different ANN cuts 0.2, 0.25, 0.3, 0.35, 0.4, 0.45 and the results are summarized in the following table:
- The conclusion is that the ANN selection (for any of the above cuts) always gives better sensitivity (by ~ 5% in dm2 and ~ 7% in sin2theta) than the PDF selection.

ANN cut	Ratio of ANN /PDF selected CC events	Ratio of ANN /PDF selected CC events	Ratio of 1 sigma errors : ANN error / PDF error (%)	
			Δm^2	$\sin^2(2\theta)$
0.20	0.99	0.49	97%	93%
0.25	1.00	0.64	96%	93%
0.30	1.02	0.84	95%	93%
0.35	1.04	1.12	95%	93%
0.40	1.05	1.50	95%	93%
0.45	1.07	2.06	97%	93%

ANN vs PDF Selection : Systematics (Δm^2)

- The ANN Selection reduces the error on Δm^2 coming from the NC systematic by ~ 60%
- The ANN Selection reduces the total error on Δm^2 by ~ 25% for any of the six examined ANN cuts.

	PDF	ANN (cut@0.2)	cut@0.25	cut@0.3	cut@0.35	cut@0.4	cut@0.45
	Δm^2 (1E-4)	Δm^2 (1E-4)	Δm^2 (1E-4)	Δm^2 (1E-4)	Δm^2 (1E-4)	Δm^2 (1E-4)	Δm^2 (1E-4)
NC 50%	0.970	0.325	0.425	0.425	0.425	0.300	0.425
$E_{sh.}$ 10%	0.500	0.650	0.625	0.575	0.650	0.600	0.600
Norm. 4%	0.375	0.400	0.425	0.425	0.425	0.375	0.400
TOTAL	1.154	0.829	0.867	0.832	0.885	0.769	0.837
TOTAL (% of PDF)	100%	71.8%	75.0%	72.0%	77.0%	66.6%	72.5%

ANN vs PDF Selection : Systematics ($sin^2(2\theta)$)

- The ANN Selection reduces the error on $\sin^2(2\theta)$ coming from the NC systematic from maximum ~ 80% to ~ 35% depending on the ANN cut.

  (Optimum cut based on sensitivity studies is ~ 0.35)
- The ANN Selection reduces the total error on $\sin^2(2\theta)$ from maximum ~ 80% to ~ 30% depending on the ANN cut.

	PDF	ANN (cut@0.2)	cut@0.25	cut@0.3	cut@0.35	cut@0.4	cut@0.45
	$\sin^2 2\theta$	$\sin^2 2\theta$	$\sin^2 2\theta$	$\sin^2 2\theta$	$\sin^2 2\theta$	$\sin^2 2\theta$	$\sin^2 2\theta$
NC 50%	0.0525	0.0100	0.0150	0.0200	0.0275	0.0275	0.0350
$E_{sh.}$ 10%	0.0125	0.0075	0.010	0.0125	0.0125	0.0175	0.0175
Norm. 4%	0.0050	0.0002	0.0002	0.0002	0.0002	0.0050	0.0002
TOTAL	0.0542	0.0125	0.0180	0.0236	0.0303	0.0330	0.0391
TOTAL (% of PDF)	**100%**	**23.1%**	**34.3%**	**43.5%**	**56.0%**	**60.9%**	**72.1%**