Hermiticity and Majorana condition for two-dimensional super Yang-Mills on a lattice with Dirac-Kähler twist

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Ref. PLB633,645 (2006)[`hep-lat/0507029`] and work in progress

1 Introduction

Several approaches toward lattice SUSY use the Dirac-Kähler(or topological) twist: S.Catterall et.al, D.B.Kaplan et.al, F.Sugino, A.d'Adda et.al, J.Giedt,...

Dirac-Kähler(D-K) twist

- recombination of supercharges
- useful to make nilpotent supercharge

Using D-K twist, we proposed the following approach for 2-dim. $N = 2$ and 3-dim. $N = 4$ cases: [DKKN]

- assign supercharges ∇_A on links
- keep the algebra — all charges are exactly kept

Questions

In the gauge theory, the gauge link variables satisfy

$$U_\mu(x, x+\mu)^\dagger = U_\mu(x+\mu, x)$$: real cond. for gauge filed

What is the corresponding relations for supercharges on link?

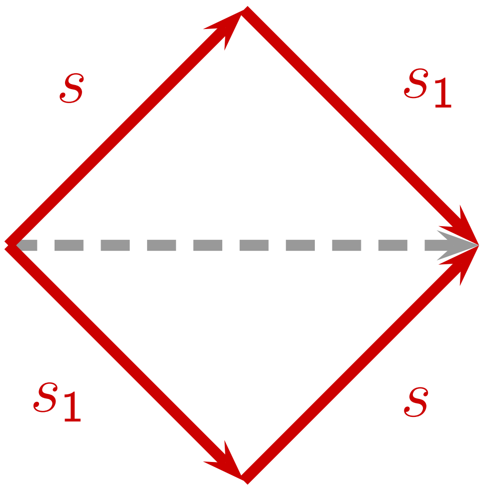
What is the Majorana condition?

Plan of this talk

2 Model: two-dimensional $N = 2$ SYM

A.D'Adda, I.K. N.Kawamoto and K.Nagata, PLB633,645 (2006)

Supertransformation$(s) \sim \sqrt{\partial}$: $\{s, s_1\} = \partial$



SUSY trans.(s) on link

It gives shift of the field

$$s(fg)_{x,x}$$
$$= (sf)_{x+a,x}g_{x,x} + f_{x+a,x+a}(sg)_{x+a,x}$$

Constraints for SYM

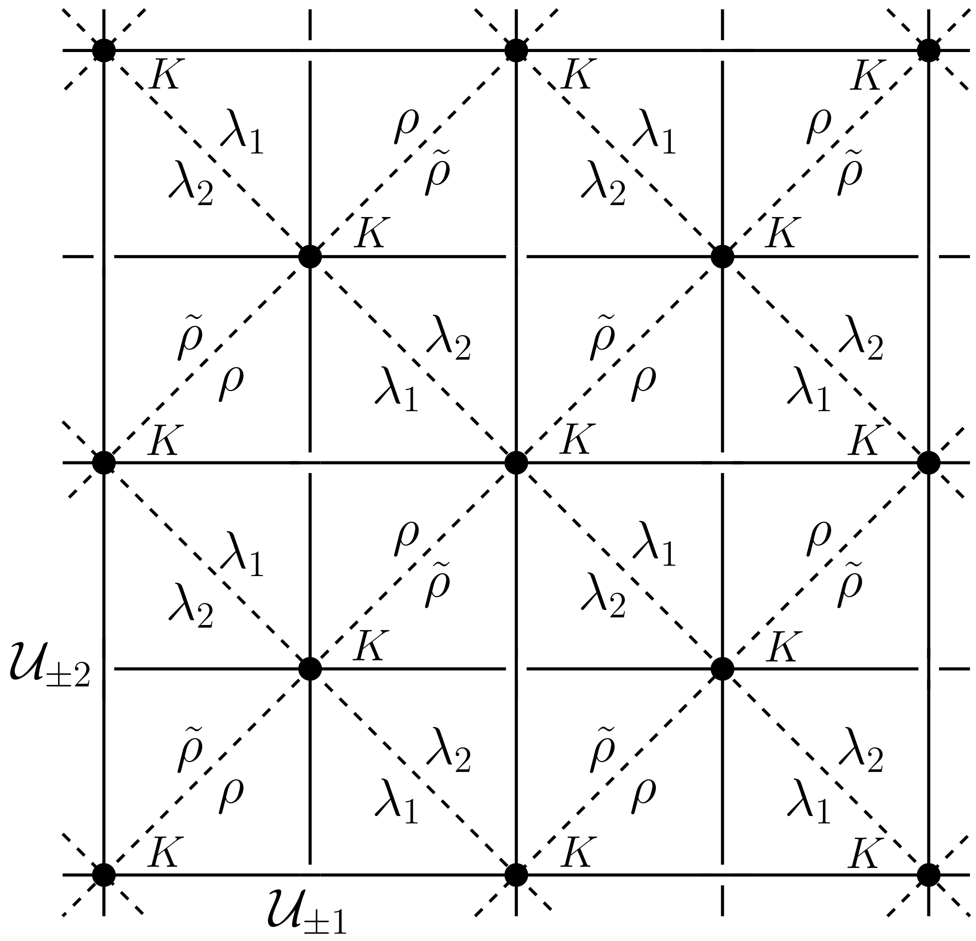
$$\{s, s_\mu\} = i\mathcal{U}_{+\mu}, \quad \{\tilde{s}, s_\mu\} = i\epsilon_{\mu\nu}\mathcal{U}_{-\nu}, \quad \{\text{others}\} = 0. \quad \left(s_A^2 = 0\right)$$

$\mathcal{U}_{\pm\mu}$: Covariant difference

Action(s-exact)

$$S = \frac{1}{4}\sum_x \mathrm{Tr}\, s\tilde{s}s_1 s_2 \left(\mathcal{U}_{+\mu}\mathcal{U}_{-\mu} + \mathcal{U}_{-\mu}\mathcal{U}_{+\mu}\right)$$

Confi guration on the Lattice



Gauge field and scalar

$$(\mathcal{U}_{\pm\mu})_{x\pm\mu,x}$$
$$\sim \mp(\partial_\mu - iA_\mu \pm \phi^{(\mu)})$$

Dirac-Kähler fermion

$$\psi_{\alpha i} = \frac{1}{2}\left(\mathbb{1}\rho + \gamma_\mu\lambda^\mu + \gamma_5\tilde{\rho}\right)_{\alpha i}$$

Aux. field K

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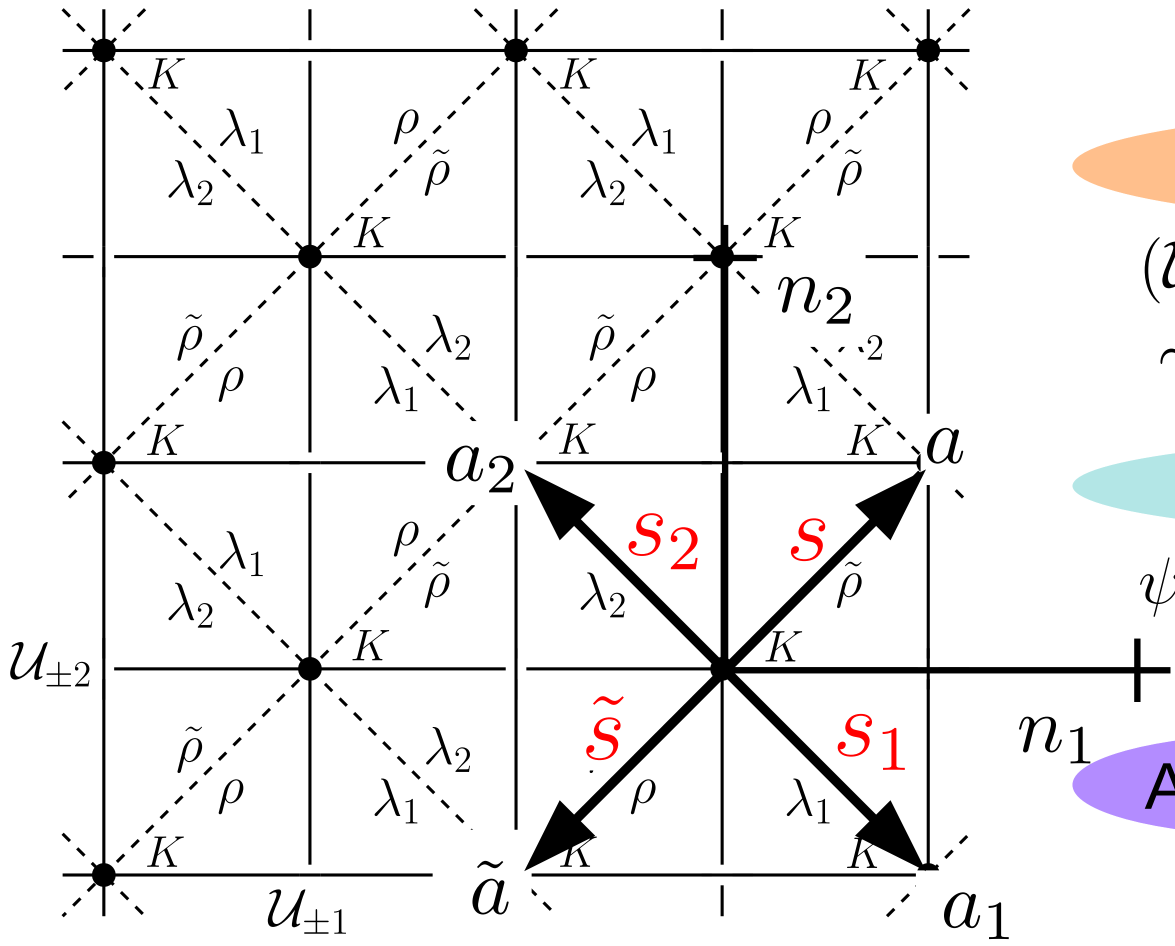

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s_A should be expressed by a *super covariant derivative* ∇_A:

$$s_A\phi = \{\nabla_A, \phi] \qquad \text{with } \{\nabla, \nabla_\mu\} = i\mathcal{U}_{+\mu}, \ldots$$

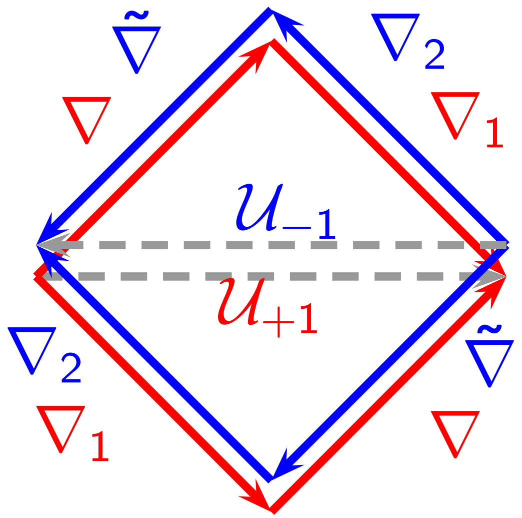
3 Hermiticity and Majorana condition

$$U_\mu(x, x+\mu)^\dagger = U_\mu(x+\mu, x) \qquad \left(= U_\mu(x, x+\mu)^{-1}\right):$$

real condition for the gauge field

$\nabla_A^\dagger = ?$: Majorana condition for the supercharge

($\nabla_A^\dagger \neq \nabla_A$ since it lives on link)



$$\{\tilde{\nabla}, \nabla_2\} = i\mathcal{U}_{-1}$$
$$\{\nabla, \nabla_1\} = i\mathcal{U}_{+1}$$

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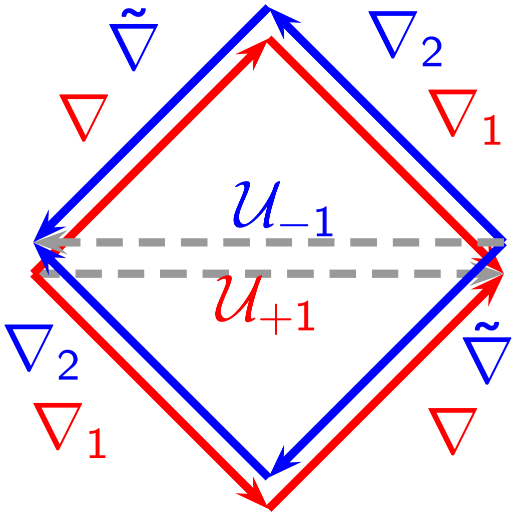

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A naive guess:

$(i\mathcal{U}_{\pm\mu})^\dagger = (i\mathcal{U}_{\mp\mu})$, $\nabla^\dagger \sim \tilde{\nabla}$ and $\nabla_1^\dagger \sim \nabla_2$

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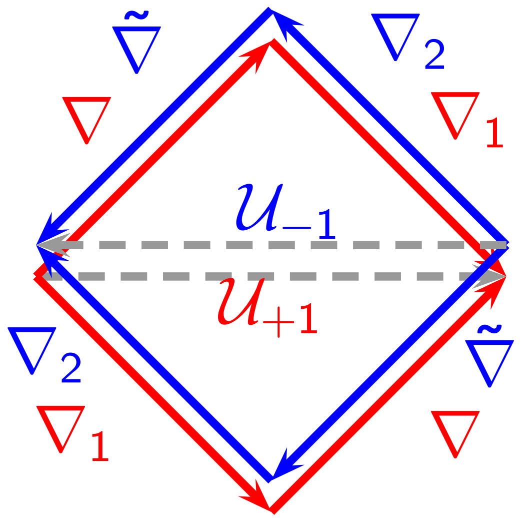

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A naive guess:

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But it does not work because it gives $(\nabla_A^\dagger)^\dagger = -\nabla_A$.

Original algebra(Euclidean): $\{\nabla, \nabla_\mu\} = i\mathcal{U}_\mu, \{\tilde{\nabla}, \nabla_\mu\} = i\epsilon_{\mu\nu}\mathcal{U}_{-\nu}$

$$\{\nabla, \nabla_1\} = i\mathcal{U}_{+1} \qquad \{\nabla, \nabla_2\} = i\mathcal{U}_{+2}$$

$$\{\tilde{\nabla}, \nabla_2\} = i\mathcal{U}_{-1} \qquad \{\tilde{\nabla}, \nabla_1\} = \boxed{-i\mathcal{U}_{-2}}$$

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$\Downarrow$ (counter-)Wick rotation

New algebra(Lorentzian): $\{\nabla, \nabla_\mu\} = i\mathcal{U}_\mu, \{\tilde{\nabla}, \nabla_\mu\} = i\varepsilon_{\mu\nu}\mathcal{U}^{-\nu}$

with $\eta^{\mu\nu} = \eta_{\mu\nu} = (+,-), \varepsilon^{01} = -\varepsilon_{01} = 1$

$$\{\nabla, \nabla_1\} = i\mathcal{U}_{+1} \qquad \{\nabla, \nabla_0\} = i\mathcal{U}_{+0}$$

$$\{\tilde{\nabla}, \nabla_0\} = -i\mathcal{U}^{-1} = i\mathcal{U}_{-1} \qquad \{\tilde{\nabla}, \nabla_1\} = i\mathcal{U}^{-0} = i\mathcal{U}_{-0}$$

$$\Rightarrow \boxed{\nabla^\dagger = \tilde{\nabla}, \quad \nabla_0^\dagger = \nabla_1, \quad (i\mathcal{U}_{\pm\mu})^\dagger = (i\mathcal{U}_{\mp\mu})}$$ Majorana cond.

Spinors

∇ in terms of spinor(in continuum)

$$\nabla_{i\alpha} = \left(\nabla C^{-1} + i\nabla_\mu \gamma^\mu C^{-1} + \tilde{\nabla}\gamma^5 C^{-1}\right)_{i\alpha}$$

α: SO(1,1) spinor of spacetime

i: SO(1,1) spinor of R-symmetry

C: charge conjugation matrix

Dirac conjugation: $\overline{\nabla}^{j\beta} = (\nabla_{i\alpha})^* (\gamma^0)_i{}^j (\gamma^0)_\alpha{}^\beta$

Majorana condition for twisted charge: $\nabla^\dagger = \tilde{\nabla}, \quad \nabla_0^\dagger = \nabla_1$

$$\Downarrow$$

Majorana cond. $\boxed{\nabla_{i\alpha} = (\gamma^5 C^{-1})_{ij} C^{-1}_{\alpha\beta} \overline{\nabla}^{j\beta}}$

Algebra

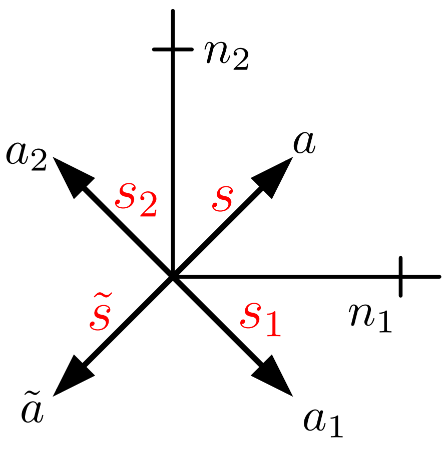
$$\{\nabla_{i\alpha}, \overline{\nabla}^{j\beta}\} = -2(\gamma^5)_i{}^j (\gamma^\mu)_\alpha{}^\beta \nabla_{\underline{\mu}} + 2(\gamma^5\gamma^a)_i{}^j \delta_\alpha{}^\beta \phi_a$$

$\nabla_{\underline{\mu}}$: covariant derivative

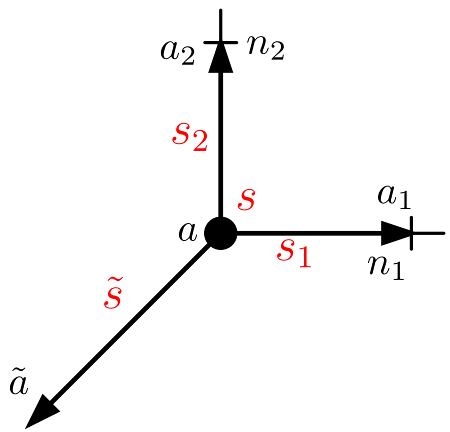
ϕ_a: scalar $\qquad \mathcal{U}_{\pm\mu} \to \nabla_{\underline{\mu}} \pm \phi_a$

4 Discussion

- Majorana cond. for supercharges $\Rightarrow$ Lorentzian
- R-symmetry is SO(1,1): noncompact
- $(i\mathcal{U}_{\pm\mu})^\dagger = i\mathcal{U}_{\mp\mu} \Rightarrow$ gauge group is unitary because
$$i\mathcal{U}_{-\mu} \to G(i\mathcal{U}_{-\mu})G^{-1} = G^{-1\dagger}(i\mathcal{U}_{+\mu})^\dagger G^\dagger$$
- Other choice of the lattice structure?
- Another way of treatment:
introduce $\nabla_A^\dagger$ as new d.o.f$\Rightarrow$charges are doubled, $N=4$?
($\sim$ dim. reduction from $D=3, N=4$)



the only choice w.r.t Hermiticity?



different structure: s lives on site

$N = 4$ algebra

$$\{\nabla, \nabla_\mu\} = i\mathcal{U}_{+\mu} \qquad \{\tilde{\nabla}, \nabla_\mu\} = i\epsilon_{\mu\nu}\mathcal{U}_{-\nu}$$

$$\{\overline{\nabla}, \overline{\nabla}_\mu\} = i\mathcal{U}_{-\mu} \qquad \{\overline{\tilde{\nabla}}, \overline{\nabla}_\mu\} = i\epsilon_{\mu\nu}\mathcal{U}_{+\nu} \quad \leftarrow \overline{\nabla}_A \sim \nabla_A^\dagger$$

$$\{\nabla, \overline{\nabla}\} = \{\tilde{\nabla}, \overline{\tilde{\nabla}}\} = -iW \quad \{\nabla_\mu, \overline{\nabla}_\nu\} = -i\delta_{\mu\nu}F \quad \leftarrow \text{new scalars}$$

$$\sim \mathcal{U}_{\pm 3}$$

Action

$$S = \mathrm{tr}\sum_x \overline{s}_1 s_1 \overline{s}_2 s_2 W W = \mathrm{tr}\sum_x \overline{s} s \overline{\tilde{s}} \tilde{s} F F$$

Transformation($N=2$)

	s	$\tilde{s}$	s_μ
$\mathcal{U}_{+\nu}$	0	$+\epsilon_{\nu\rho}\lambda_\rho$	$-\epsilon_{\mu\nu}\tilde{\rho}$
$\mathcal{U}_{-\nu}$	$-\lambda_\nu$	0	$-\delta_{\mu\nu}\rho$
λ_ν	0	0	$-i[\mathcal{U}_{+\mu},\mathcal{U}_{-\nu}]$ $+\delta_{\mu\nu}(K+\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}])$
ρ	$-\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}]-K$	$+\frac{i}{2}\epsilon_{\rho\sigma}[\mathcal{U}_{-\rho},\mathcal{U}_{-\sigma}]$	0
$\tilde{\rho}$	$-\frac{i}{2}\epsilon_{\rho\sigma}[\mathcal{U}_{+\rho},\mathcal{U}_{+\sigma}]$	$+\frac{i}{2}[\mathcal{U}_{+\rho},\mathcal{U}_{-\rho}]-K$	0
K	$+\frac{i}{2}[\mathcal{U}_{+\rho},\lambda_\rho]$	$-\frac{i}{2}\epsilon_{\rho\sigma}[\mathcal{U}_{-\rho},\lambda_\sigma]$	$-\frac{i}{2}[\mathcal{U}_{+\mu},\rho]-\frac{i}{2}\epsilon_{\mu\nu}[\mathcal{U}_{-\nu},\tilde{\rho}]$

Dirac-Kähler fermion $\psi_{\alpha i}=\frac{1}{2}\left(\mathbb{1}\rho+\gamma_\mu\lambda^\mu+\gamma_5\tilde{\rho}\right)_{\alpha i}$

Gauge field and scalar

Aux. field

Action: 2-dim $N = 2$ Super Yang-Mills

SUSY is kept exactly: ($s_A^2 = 0$, s_A-exact form)

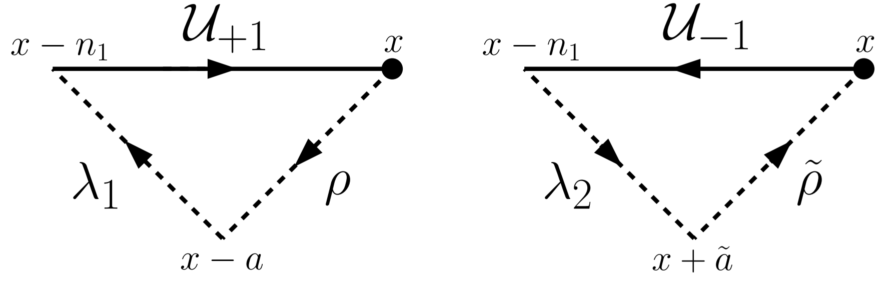
$$S = \frac{1}{4}\sum_x \mathrm{Tr}\, s\tilde{s}s_1 s_2\left(\mathcal{U}_{+\mu}\mathcal{U}_{-\mu} + \mathcal{U}_{-\mu}\mathcal{U}_{+\mu}\right)$$

$$= \sum_x \mathrm{Tr}\Bigg[\frac{1}{4}[\mathcal{U}_{+\mu},\mathcal{U}_{-\mu}]_{x,x}[\mathcal{U}_{+\nu},\mathcal{U}_{-\nu}]_{x,x} + K^2_{x,x}$$

$$- \frac{1}{4}\epsilon_{\mu\nu}\epsilon_{\rho\sigma}[\mathcal{U}_{+\mu},\mathcal{U}_{+\nu}]_{x,x-n_\mu-n_\nu}[\mathcal{U}_{-\rho},\mathcal{U}_{-\sigma}]_{x-n_\rho-n_\sigma,x}$$

$$- \frac{i}{2}[\mathcal{U}_{+\mu},\lambda_\mu]_{x,x-a}(\rho)_{x-a,x} - \frac{i}{2}(\tilde{\rho})_{x,x+\tilde{a}}\epsilon_{\mu\nu}[\mathcal{U}_{-\mu},\lambda_\nu]_{x+\tilde{a},x}$$

$$+ \frac{i}{2}(\rho)_{x,x+a}[\mathcal{U}_{+\mu},\lambda_\mu]_{x+a,x} + \frac{i}{2}\epsilon_{\mu\nu}[\mathcal{U}_{-\mu},\lambda_\nu]_{x,x-\tilde{a}}(\tilde{\rho})_{x-\tilde{a},x}\Bigg]$$



closed loop $\Rightarrow$ gauge inv.