

B_s Lifetime and $\Delta\Gamma_s$ Measurements at DØ

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For the DØ Collaboration

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http://www-d0.fnal.gov/~dstrom/DPF06.pdf_

Outline

Introduction

B_s Physics @ DØ

B_s Lifetime

October 30, 2006

$\Delta\Gamma_s: \text{Br}(B_s \rightarrow D_s^* D_s^*)$

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$\Delta\Gamma_s: B_s \rightarrow J/\psi\phi$

- B_s Physics Program at DØ
- The DØ Detector
- B_s Lifetime Measurement
- Measurement of $\text{Br}(B_s \rightarrow D_s^* D_s^*)$ and $\Delta\Gamma_s$
- Direct Measurement of $\Delta\Gamma_s$ and $\delta\Phi_s$ CPV phase
- Summary

B_s Program @ DØ

- DØ has a rich B physics program.
 - First double-sided bound on B_s mixing parameter, Δm_s .
 - Precision lifetime measurements of B_s, Λ_b , B_c, B⁰, B⁻.
 - Large boost at Tevatron is good for lifetime studies.
 - $\Delta\Gamma_s$ measurement provides tests SM predictions.
 - $\delta\Phi_s$ measurement tests for CPV and new physics.

DØ Detector

Introduction

B_s Physics @ DØ

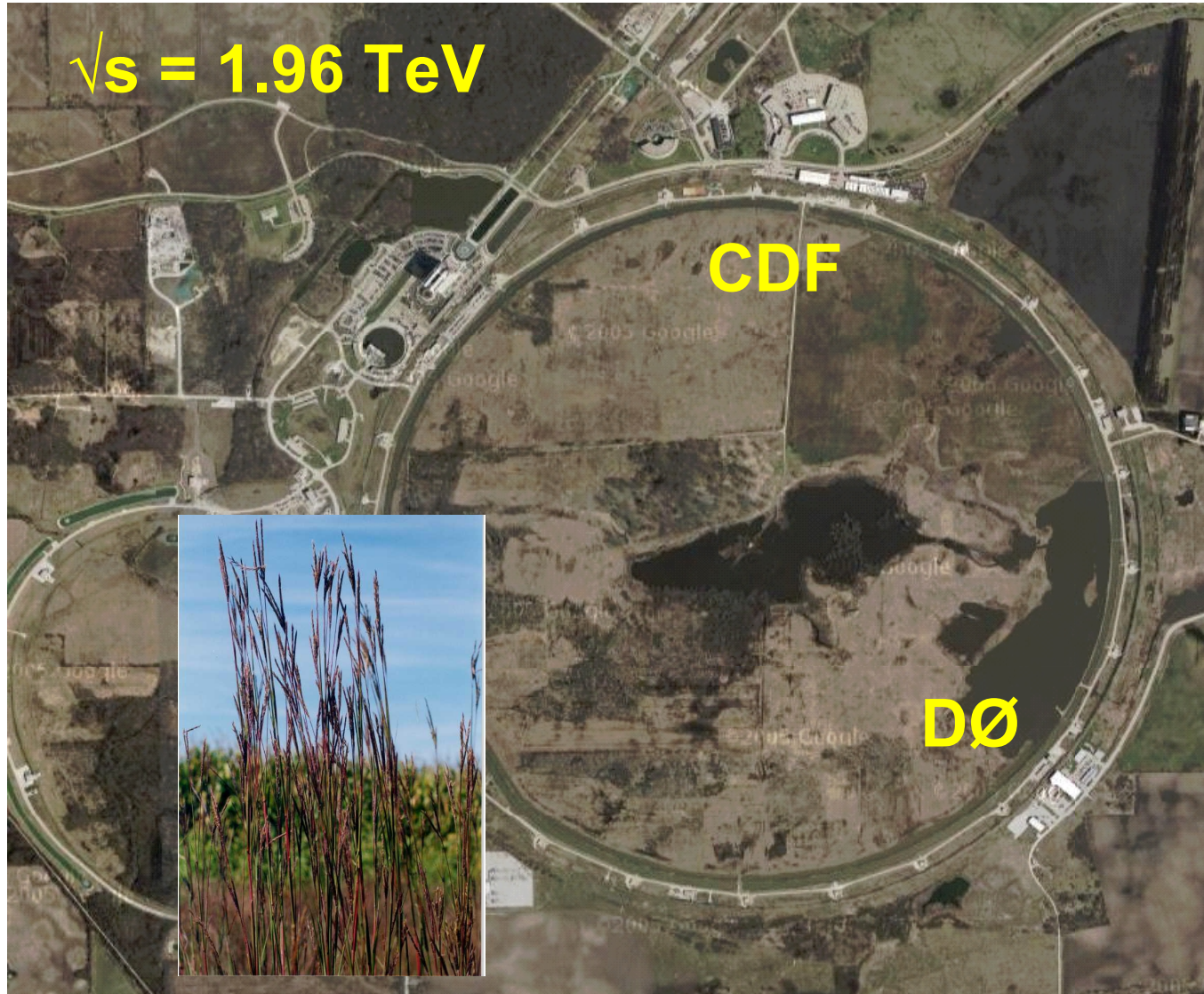
B_s Lifetime

October 30, 2006

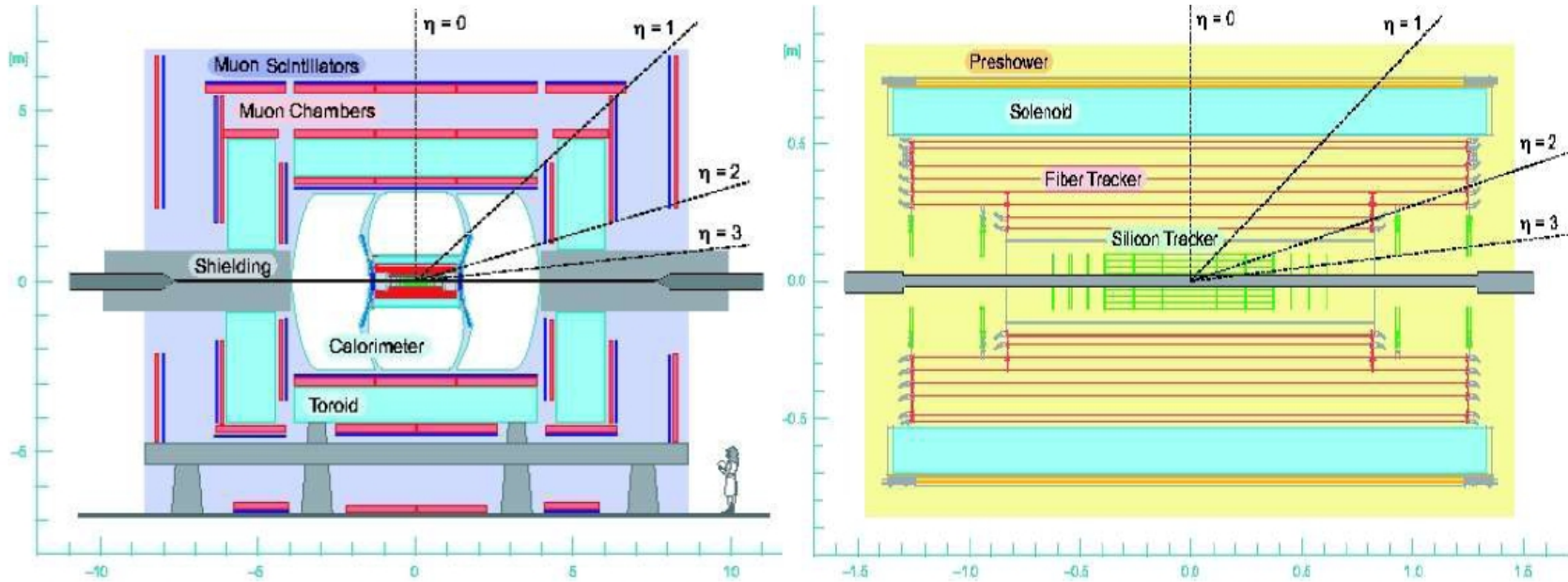
$\Delta\Gamma_s: \text{Br}(B_s \rightarrow D_s^+ D_s^-)$

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$\Delta\Gamma_s: B_s \rightarrow J/\psi\phi$



DØ Detector



Tracking ($|\eta| < 3$) – Silicon Microstrip Tracker (SMT), Layer0 (RunIIb Upgrade) Silicon Detector, and Central Fiber Tracker (CFT) inside a 2T magnetic field.

Muon ($|\eta| < 2$) – One layer tracking and one layer scintillation trigger counters.

1.8T toroid followed by two similar tracking and trigger layers.

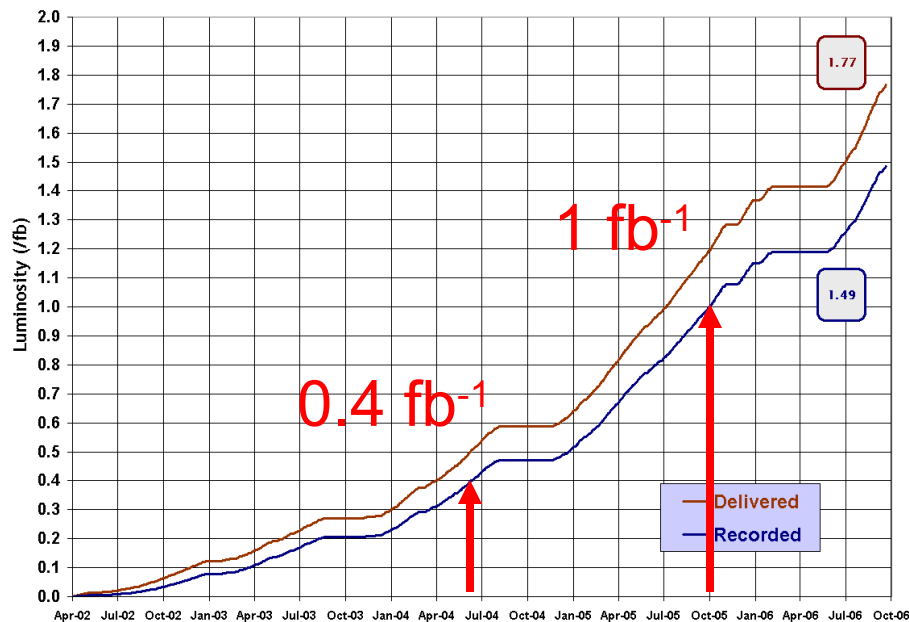
Good muon triggers produce high yield.

→ Large B samples.



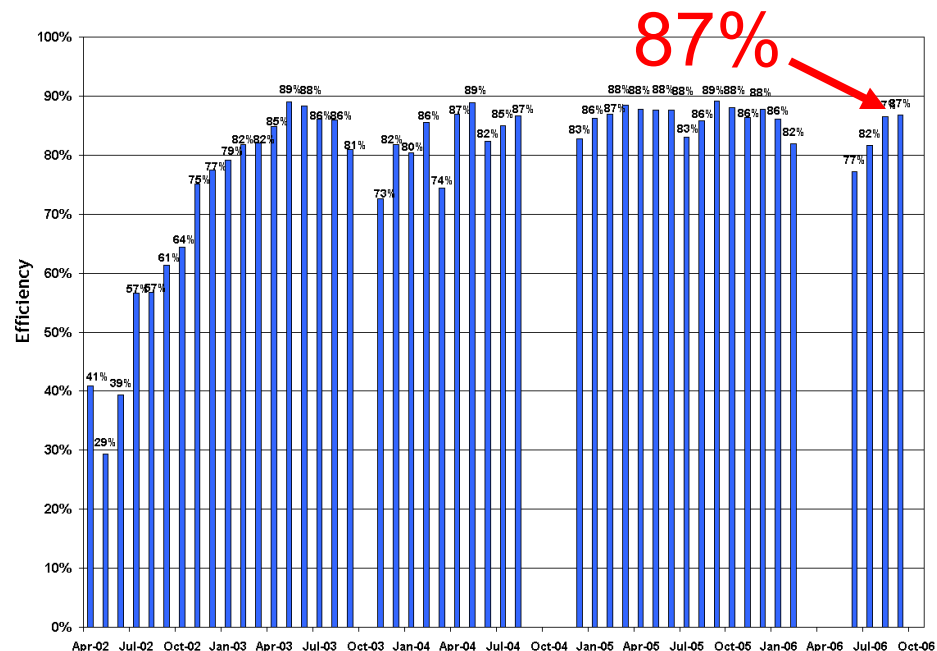
Run II Integrated Luminosity

19 April 2002 - 8 October 2006



Monthly Data Taking Efficiency

19 April 2002 - 24 September 2006



Data used in these analyses:

B_s Lifetime: 0.4 fb⁻¹

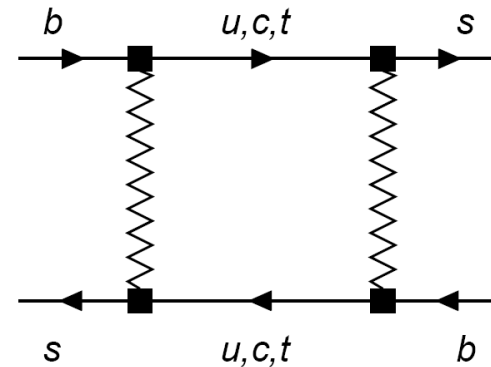
$\Delta\Gamma_s: \text{Br}(B_s \rightarrow D_s^* D_s^*): 1 \text{ fb}^{-1}$

$\Delta\Gamma_s: B_s \rightarrow J/\Psi \Phi: 1 \text{ fb}^{-1}$

Schrodinger Equation

$$i \frac{d}{dt} \begin{pmatrix} |B_s(t)\rangle \\ |\bar{B}_s(t)\rangle \end{pmatrix} = \left(M - i \frac{\Gamma}{2} \right) \begin{pmatrix} |B_s(t)\rangle \\ |\bar{B}_s(t)\rangle \end{pmatrix}$$

B_s- \bar{B}_s mixing



Two physical B_s eigenstates, B_H and B_L

B_H and B_L expected to have different masses and lifetimes.

$$B_L = p |B_s\rangle + q |\bar{B}_s\rangle \approx \text{cp odd}$$

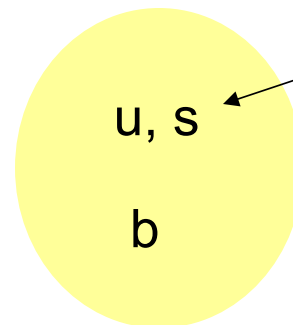
$$B_H = p |B_s\rangle - q |\bar{B}_s\rangle \approx \text{cp even}, \quad p^2 + q^2 = 1$$

$$\Delta m = M_H - M_L \approx 2 |M_{12}|$$

$$\Delta\Gamma = \Gamma_L - \Gamma_H \approx 2 |\Gamma_{12}| \cos \phi$$

B_s Lifetime

- **Spectator model** of heavy hadron decays predicts all hadrons with the same heavy flavor content have identical lifetimes.
- Observed charmed hadron lifetimes suggest non-spectator effects are not negligible in such decays.
- **Heavy Quark Expansion (HQE)** theory accounts for non-spectator effects in decays and predicts lifetime differences among different b hadrons.
- B-meson lifetime measurements tests the predictions of HQE theory.



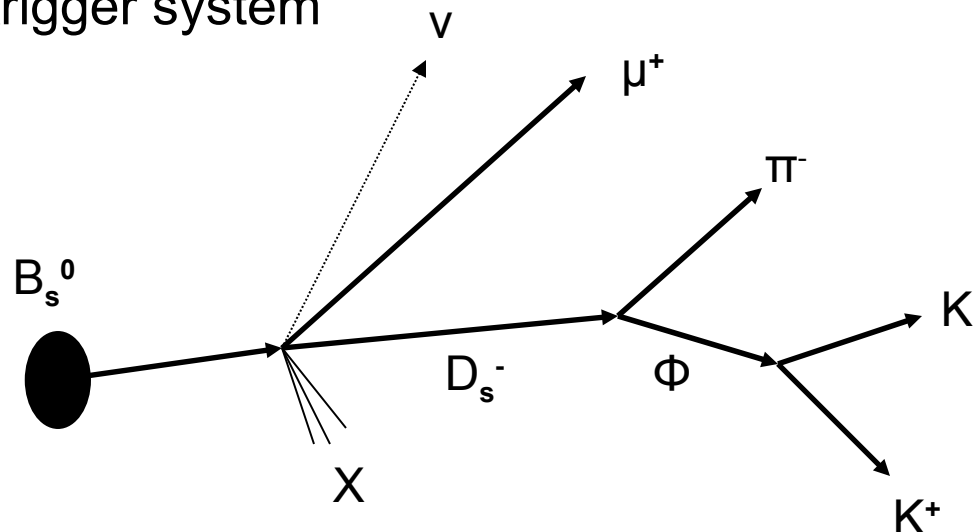
Spectator quark not negligible in b hadron lifetimes.

Hierarchy of B lifetimes

Test HQE prediction

Channel

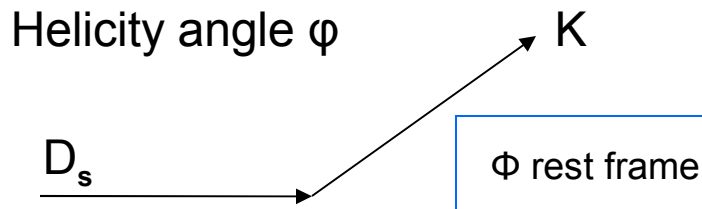
- B_s identified through the semileptonic decay channel
 - $B_s \rightarrow D_s \mu \nu X$
 - $D_s \rightarrow \Phi \pi$
 - $\Phi \rightarrow K K$
- Events selected with inclusive single-muon triggers.
 - 3 level trigger system



Selection

- Muon penetrates toroid
- All tracks within same jet
- χ^2 prob (D_s) > 0.1%
- χ^2 prob (B_s) > 0.01%
- Helicity(D_s) > 0.4
- Tracks with nSMT and nCFT ≥ 1
- No lifetime cuts!
- D_s decay vertex displaced from primary vertex in direction of $P(D_s)$

- $p_T(\mu) > 2.0$ GeV
- $p(\mu) > 3.0$ GeV
- $p_T(K) > 1.0$ GeV
- $p_T(\pi) > 0.7$ GeV
- $p_T(D_s) > 3.5$ GeV
- $p_T(\mu \text{ w.r.t } D_s) > 2.0$ GeV
- $1.008 < M(\Phi) < 1.032$ GeV
- $1.6 < M(D_s) < 2.3$ GeV
- $3.4 < M(B_s) < 5.0$ GeV



Reconstruction

$$D_s \rightarrow \Phi \pi$$

- Tracks with $p_T > 1.0$ GeV assigned M(K)
- Oppositely charged pairs combined to form a Φ candidate.
- $1.008 < M(KK) < 1.032$ GeV.
- Φ Combined with a track with $p_T > 0.7$ GeV
 - Opposite charge from muon gives “right sign” combination.
 - Track assigned M(π)
- Three tracks (KK π) used to form the D_s vertex.
- Additional track requirements:
 - At least one hit in SMT and CFT.
 - $p_T(D_s) > 3.5$ GeV

$$B_s \rightarrow D_s \mu X$$

- B_s decay vertex found by intersecting the μ track with the flight direction of the D_s candidate.
 - Required to be displaced from the primary vertex in the direction of the D_s momentum.
- $3.4 < \text{Mass}(B_s) < 5.0$ GeV/ c^2
- B_s not fully reconstructed because neutrino is not detected.

Analysis

- B_s lifetime, τ , is related to the decay length, L_{xy} , in the transverse plane.

$$c\tau = m L_{xy} / p_T$$

τ = lifetime
 p_T = transverse momentum of B_s
 m = invariant B_s mass

- Missing neutrino prevents a full reconstruction of the B_s.
- Correction factor K introduced to estimate $p_T(B_s)$

$$K = p_T(D_s \mu) / p_T(B_s) \quad \text{Determined with MC}$$

- Pseudo-proper decay length (PPDL), λ , used to determine the B_s lifetime.

$$\lambda = L_{xy} m / p_T(D_s \mu) = c\tau / K$$

Signal

Signal: Gaussian fit to “right-sign” D_sμ candidates.

Background: second-order polynomial fit.

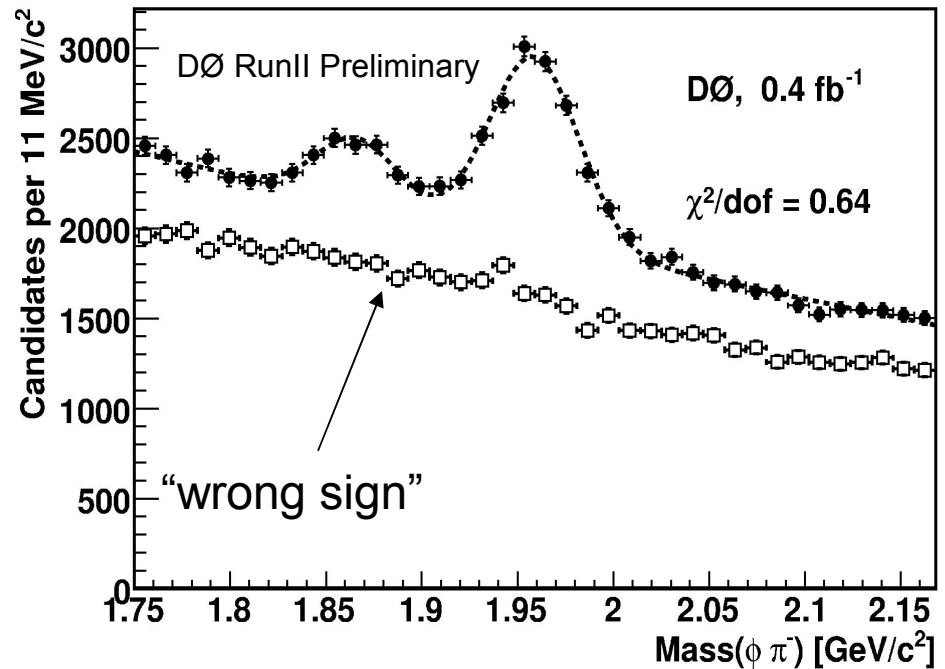
D → Φπ (Cabbibo suppressed):
Gaussian fit.

Reconstructed D_s candidates

400 pb⁻¹

5176 ± 242 (stat) ± 314 (syst)

$M(D_s) = 1958.8 \pm 0.9$ MeV



Background

- “Prompt”
 - μ produced at primary vertex + reconstructed D_s
- “Physics Background”
 - $\mu + D_s$ where neither come from semileptonic decay of B_s
 - Included in signal sample
 - Prompt D_s from cc production + μ
 - Short lifetimes
 - Can bias lifetime measurement
- “Non-B_s Background”
 - $B \rightarrow \mu D_s$
 - Long lifetimes
 - Smaller effects
 - Softer $p_T(\mu)$ from decay of secondary c-hadron
 - Reduced by kinematic cuts
- Background contribution of each process evaluated with MC.

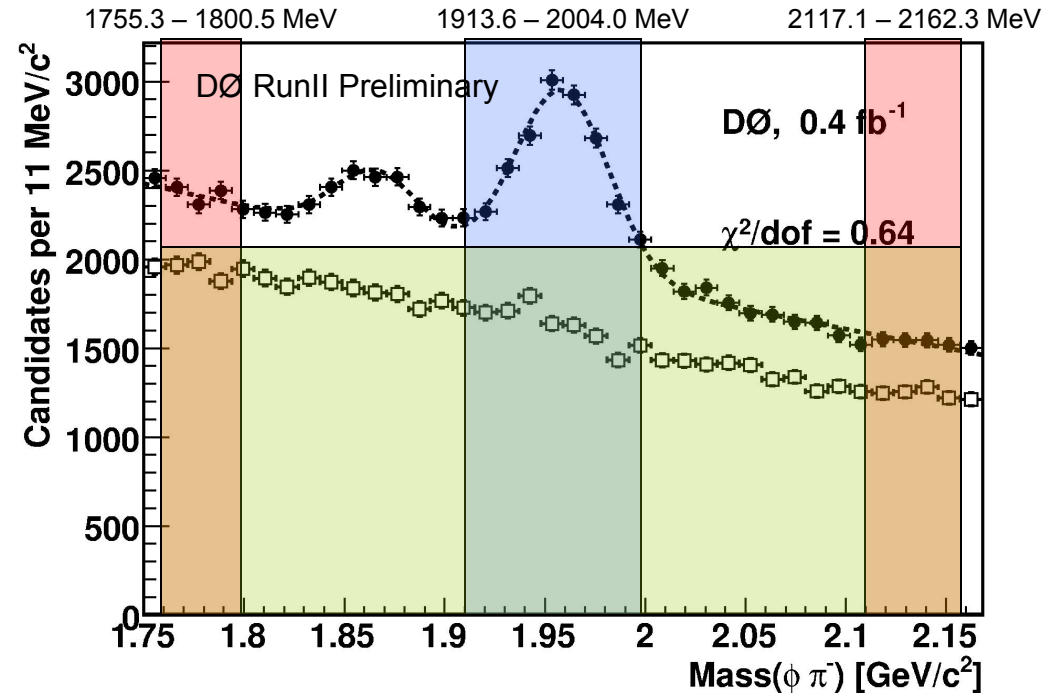
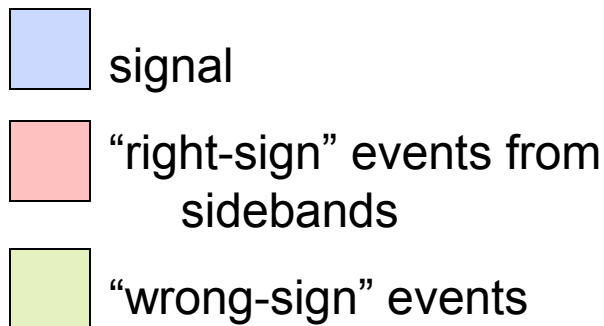
$$B^0 \rightarrow D_s^{*-} D^{*+} X$$

$$B^- \rightarrow D_s^{*-} D^{*0} X$$

$$B_s \rightarrow D_s^{*-} D^* X$$

Background

- Combinatorial background events contained in the signal sample parameterized using “right-sign” events from sidebands and “wrong-sign” events



Fit

- PDDL distribution from signal region fitted with an unbinned maximum log-likelihood method.
- Both B_s lifetime and background shape determined in a simultaneous fit to the signal and background samples.

$$\mathcal{L} = \mathcal{C}_{sig} \prod_i^{N_S} [f_{sig} \mathcal{F}_{sig}^i + (1 - f_{sig}) \mathcal{F}_{bck}^i] \prod_j^{N_B} \mathcal{F}_{bck}^j,$$

N_S – Number of events in signal sample

N_B – Number of events in background sample

f_{sig} – ratio of D_s signal events from mass distribution to total number of events in the signal sample

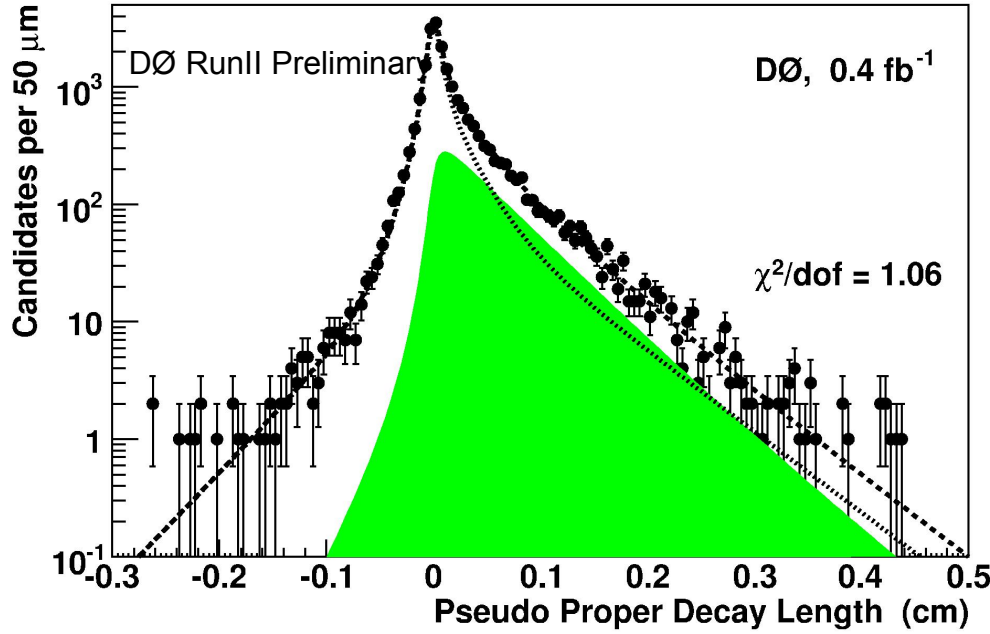
\mathcal{F}_{sig} – signal probability distribution function (normalized exponential decay, K factor, and a gaussian resolution function).

\mathcal{F}_{bkg} – combinatorial background sample

Results

- $\tau(B_s) = 1.398 \pm 0.044$ (stat) $_{-0.025}^{+0.028}$ (syst) ps
- Most precise result to date!
- $W_{\text{PDG}}: \tau(B_s) = 1.461 \pm 0.057$ ps
- $\tau(B^0) = 1.536 \pm 0.014$ (stat) ps
- B_s lifetime is different from the B⁰ lifetime by more than 1%, consistent with HQE

Source	Syst. Uncertainty (ps)
Detector alignment	± 0.007
Combinatorial background	± 0.014
Selection criteria	+0.01 -0.0001
K factor determination	+0.012 -0.007
Decay length resolution	± 0.012
non-B _s background	+0.010 -0.014
cchar background	+0.008 -0.003
Total	+0.028 -0.025



Comparison to other results.

Experiment	dataset	$\tau(B_s^0)$ (ps)
World Average(PDG) [11]		1.461 ± 0.057
ALEPH	91-95	$1.54^{+0.14}_{-0.13} \pm 0.04$
CDF	92-96	$1.36 \pm 0.09^{+0.06}_{-0.05}$
DELPHI	91-95	$1.42^{+0.14}_{-0.13} \pm 0.03$
OPAL	90-95	$1.50^{+0.16}_{-0.15} \pm 0.04$
Average of $D_s l$ measurements [16]		1.442 ± 0.066
CDF [17]	02-04	$1.381 \pm 0.055^{+0.052}_{-0.046}$
This Measurement	02-04	$1.398 \pm 0.044^{+0.028}_{-0.025}$

[hep-ex/0604046]

$$\Delta\Gamma_s: \text{Br}(B_s \rightarrow D_s^* D_s^*)$$

- How to measure $\Delta\Gamma_s$?

- Directly from $B_s \rightarrow J/\Psi \Phi$ decays
 - Disentangle CP even and odd final states.

$$\Delta\Gamma_s = \Gamma_L - \Gamma_H$$

$$|B_L\rangle = |B_s^{\text{EVEN}}\rangle$$

$$|B_H\rangle = |B_s^{\text{ODD}}\rangle$$

- Estimate from $\text{Br}(B_s \rightarrow D_s^* D_s^*)$
 - CP even dominated final state
 - Gives largest contribution in the lifetime difference.

$$\Delta\Gamma_{\text{CP}} = \Gamma(B_s^{\text{EVEN}}) - \Gamma(B_s^{\text{ODD}})$$

Theory suggest:

$$2 \text{Br}(B_s \rightarrow D_s^* D_s^*) \approx \Delta\Gamma_{\text{CP}} / \Gamma (1 + O(\Delta\Gamma/\Gamma))$$

- Relate measurement to CPV phase, Φ_s

- $\Delta\Gamma_{\text{CP}} = \Delta\Gamma / \cos\Phi_s$

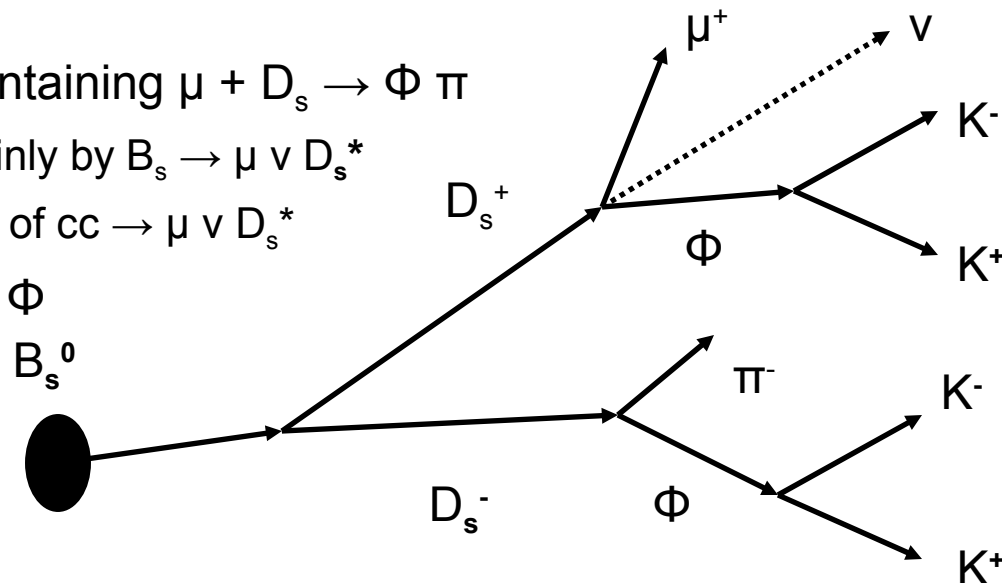
- Φ_s is related to CP violation in B_s mixing and is expected to be small in the SM.

Channel

- B_s identified through decay to two D_s^{*}
 - B_s⁰ → D_s^{*} D_s^{*}
 - D_s^{*} → μ⁺ ν Φ, Φ → K⁺ K⁻ (semileptonic)
 - D_s^{*} → Φ π⁻, Φ → K⁺ K⁻ (hadronic)

- Define 2 Samples

- (μD_s): events containing μ + D_s → Φ π
 - Produced mainly by B_s → μ ν D_s^{*}
 - Small sample of cc → μ ν D_s^{*}
- (μΦD_s): (μD_s) + Φ



Analysis

- Estimate the following;
 - $N(\mu D_s)$ – number of signal events in (μD_s) sample
 - $N(\mu\Phi D_s)$ – number of signal events in $(\mu\Phi D_s)$ sample
 - $f(B_s \rightarrow \mu\nu D_s^*)$ – fraction of $B_s \rightarrow \mu\nu D_s^*$ decays in (μD_s) sample
 - $N_{\text{bkg}}(\mu\Phi D_s)$ – Number of background events in $(\mu\Phi D_s)$ sample
 - $f(B_s \rightarrow \mu\nu D_s^*)$ and $N_{\text{bkg}}(\mu\Phi D_s)$ determine by subtracting contributions from all other sources from each sample
- Measure R (detector uncertainties cancel) and extract the branching ratio.
 - $R \propto N(\mu\Phi D_s) - N_{\text{bkg}}(\mu\Phi D_s) / N(\mu D_s) f(B_s \rightarrow \mu\nu D_s)$
 - $R = \text{Br}(B_s \rightarrow D_s^* D_s^*) \text{Br}(D_s \rightarrow \mu\nu\Phi) / \text{Br}(B_s \rightarrow \mu\nu D_s^*)$
- Compute $\text{Br}(B_s \rightarrow D_s^* D_s^*)$ using;
 - Measured value of R
 - PDG value for $\text{Br}(D_s \rightarrow \mu\nu\Phi)$
 - PDG value for $\text{Br}(B_s \rightarrow \mu\nu D_s^*)$
 - BaBar measurement of $\text{Br}(D_s \rightarrow \Phi\pi)$ – combined average with PDG

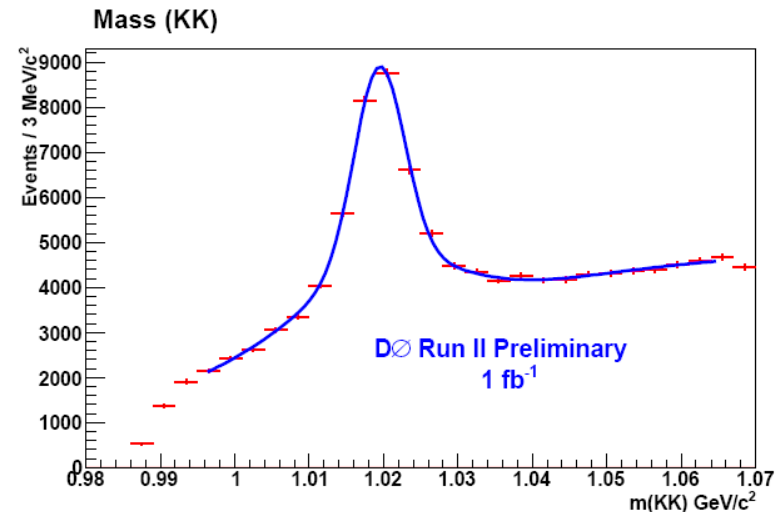
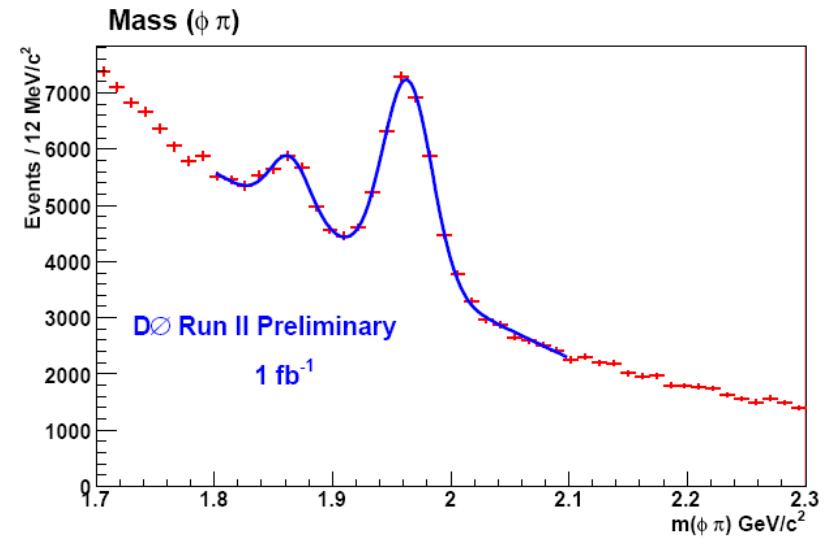
Selection

- Number of muon hits ≥ 2
- All tracks:
 - nSMT hits ≥ 2
 - nCFT hits ≥ 2
- $P_T(\mu) > 2 \text{ GeV}$
- $P(\mu) > 3 \text{ GeV}$
- $P_T(\pi) > 1.0 \text{ GeV}$
 - Opposite charge from μ
- $P_T(K) > 0.8 \text{ GeV}$
 - Opposite charge combinations
- $\Phi(D_s \rightarrow \Phi\pi): 1.01 < m(KK) < 1.03 \text{ GeV}$
- $\Phi(D_s \rightarrow \Phi\mu): 0.998 < m(KK) < 1.07 \text{ GeV}$
- $D_s \rightarrow \Phi\pi$
 - $1.7 < m(\Phi\pi) < 2.3 \text{ GeV}$
 - $\chi^2(\text{vertex}) < 16$
 - $|\cos(\theta)| > 0.35$, Helicity between D_s and K
- $D_s \rightarrow \Phi\mu\nu$
 - $1.2 < m(\Phi\mu) < 1.85 \text{ GeV}$
 - $\chi^2(\text{vertex}) < 16$
- $B_s \rightarrow \mu D_s$
 - $m(\mu D_s) < 5.2 \text{ GeV}$
 - $\chi^2(\text{B vertex}) < 16$
 - $L(\mu D_s) > 150 \mu\text{m}$
- $D_s \rightarrow \mu\Phi D_s$
 - $4.3 < m(\mu\Phi D_s) < 5.2 \text{ GeV}$
 - $\chi^2(\text{vertex}) < 16$
 - $L(\mu\Phi D_s) > 150 \mu\text{m}$

N(μD_s) Sample

- Number of (μD_s) events estimated from a binned fit to $M(\Phi\pi)$ distribution.
- Gaussian fit to both D and D_s decays
- Second-order polynomial fit to background

$$N(\mu D_s) = 15225 \pm 310$$



N($\mu\Phi D_s$) Sample

• Number of ($\mu\Phi D_s$) events estimated from an unbinned Log-Likelihood fit.

Used all events in range

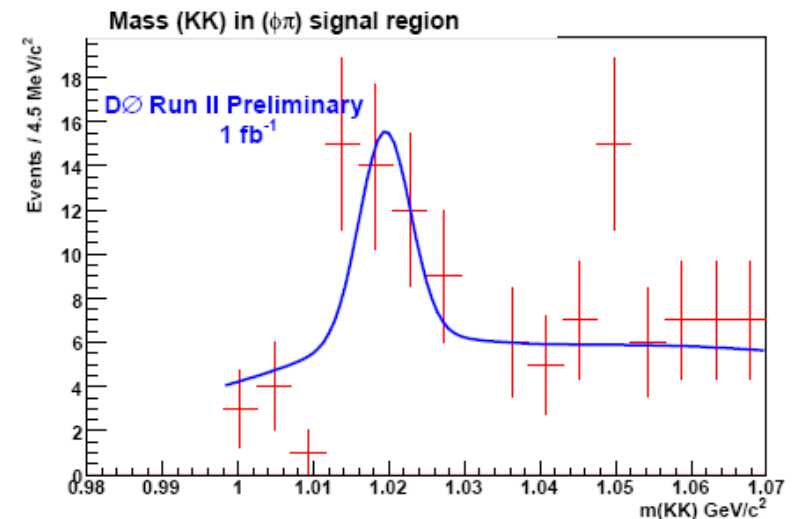
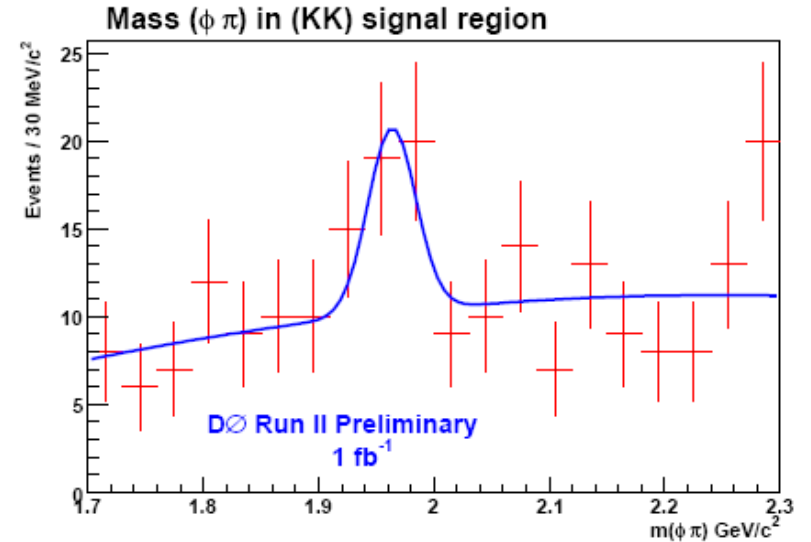
$$1.7 < M(D) < 2.3 \text{ GeV}$$

$$0.998 < M(\Phi) < 1.07 \text{ GeV}$$

Single Gaussian used to describe the D_s signal

Double Gaussian used to describe the Φ peak.

$$N(\mu\Phi D_s) = 19.34 \pm 7.85$$



Sample Composition

Process	Br(%)
$\text{Br}(B_s \rightarrow \mu\nu D_s)$	2.10
$\text{Br}(B_s \rightarrow \mu\nu D_s^*)$	5.60
$\text{Br}(B_s \rightarrow \mu\nu D_{s0}^*)$	0.20
$\text{Br}(B_s \rightarrow \mu\nu D'_{s1})$	0.37
$\text{Br}(B_s \rightarrow \tau\nu D_s^*) \text{ Br}(\tau \rightarrow \mu\nu)$	0.51

Process	Br(%)
$B^0 \rightarrow D_s D^* X$	10.5 ± 2.6
$B^\pm \rightarrow D_s D^* X$	10.5 ± 2.6
$B_s \rightarrow D_s^* D_s^*$	12_{-7}^{+11}
$B_s \rightarrow D_s D X$	15.4 ± 15.4

Fraction of events in (μD_s) sample coming from $B_s \rightarrow \mu\nu D_s X$

$$f(B_s \rightarrow \mu\nu D_s^*) = 0.79 \pm 0.05$$

Background Composition

Processes Contributing to $N_{\text{bkg}} (\mu\Phi D_s)$	Number of Events
$B_s \rightarrow D_s^* D_s^*$	
$B \rightarrow D_s^* D_s^* K X$	0.44 ± 0.30
$B_s \rightarrow D_s^* D_s^* X$	~ 0
$B_s \rightarrow \mu\nu D_s^* \Phi$	1.27 ± 1.14
$cc \rightarrow \mu\Phi D_s^*$	~ 0
$B_s \rightarrow \mu\nu D_s^*$ and Φ from fragmentation.	~ 0

$$N_{\text{bkg}} (\mu\Phi D_s) = 1.7 \pm 1.2$$

Results

N (μD_s)	15225 ± 310
N ($\mu\Phi D_s$)	19.34 ± 7.85
$f(B_s \rightarrow \mu\nu D_s)$	0.79 ± 0.05
N _{bkg} ($\mu\Phi D_s$)	1.7 ± 1.2
$\text{Br}(D_s \rightarrow \Phi\pi)_{\text{BaBar}}$	4.81 ± 0.52 ± 0.38

Source	Uncertainty in $\text{Br}(B_s \rightarrow D_s^* D_s^*)$
$\text{Br}(D_s \rightarrow \Phi\pi) = 0.0440 \pm 0.0520$	+0.020 -0.014
$\text{Br}(B_s \rightarrow \mu\nu D_s^*) \text{Br}(D_s \rightarrow \Phi\pi)$	±0.012
$\text{Br}(D_s \rightarrow \Phi\mu\nu) / \text{Br}(D_s \rightarrow \Phi\pi)$	±0.005
$f(B_s \rightarrow \mu\nu D_s^*) = 0.79 \pm 0.05$	±0.005
Background contribution in N($\mu\Phi D_s$)	±0.007
Ratio of efficiencies	±0.010
Reweighting of MC	±0.011

$$\text{Br}(B_s \rightarrow D_s^* D_s^*) = 0.071 \pm 0.032 \text{ (stat)}_{-0.025}^{+0.029} \text{ (syst)}$$

$$\Delta\Gamma_{\text{CP}} / \Gamma = 0.142 \pm 0.064 \text{ (stat)}_{-0.050}^{+0.058} \text{ (syst)}$$

This estimate is in good agreement with the SM predictions

$$\Delta\Gamma_{\text{CP}} / \Gamma_{\text{SM}} = 0.12 \pm 0.06$$

[DØ Conference Note 5068]

$\Delta\Gamma_s: B_s \rightarrow J/\Psi \Phi$

- To a good approximation, mass eigenstates in B_s system are expected to be CP eigenstates.
- New phenomena may introduce a non-vanishing mixing phase $\delta\Phi_s$, leading to a reduction in the observed $\Delta\Gamma$ compared to SM predictions $\Delta\Gamma_{SM}$:
 - $\Delta\Gamma = \Delta\Gamma_{SM} * |\cos(\delta\Phi_s)|$
- B_s → J/Ψ Φ gives rise to a mixture of CP-even and CP-odd final states.
- Possible to separate the two CP components of the decay and measure the lifetime differences.
 - Simultaneous fit to the mass, proper decay length, and three angles of the decay products J/Ψ and Φ.

Selection

- B_s → J/Ψ Φ
 - J/Ψ → μμ
 - Φ → KK

$$2.9 < M(J/\Psi) < 3.3 \text{ GeV}$$

$$1.01 < M(\Phi) < 1.03 \text{ GeV}$$

- 2 Reconstructed μ's

$$5.0 < M(B_s) < 5.8 \text{ GeV}$$

- Number muon hits ≥ 1

J/Ψ and Φ have common vertex

- P_T(μ) > 1.5 GeV

$$\sigma(\text{ct}) < 60 \text{ } \mu\text{m}$$

- P_T(K) > 0.7 GeV

$$\text{ct} = L_{xy} M_B / p_T$$

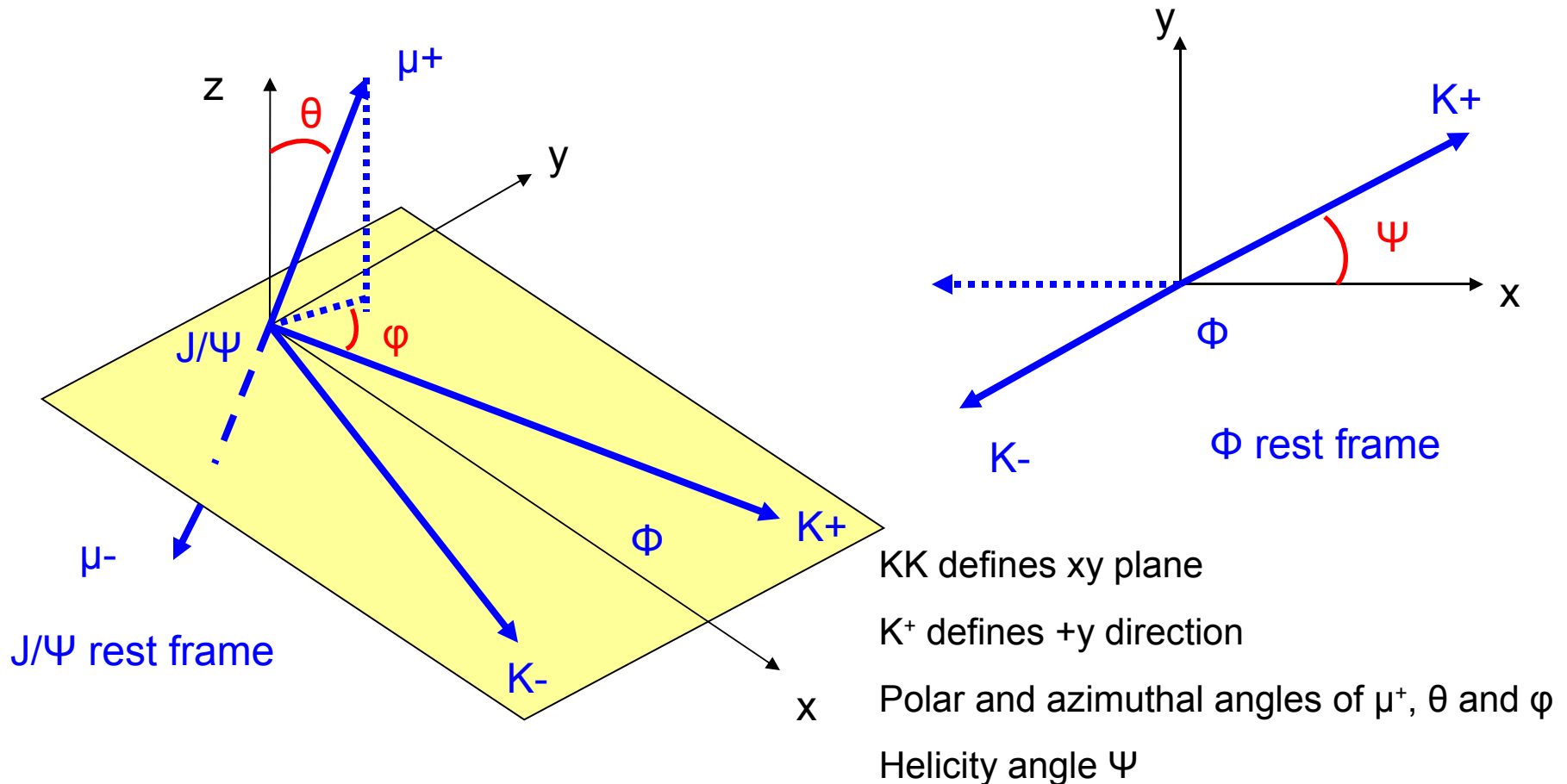
- P_T(Φ) > 1.5 GeV

- P_T(B_s) > 6.0 GeV

23343 candidates

3 Angle Analysis

- The CP content $B_{s \rightarrow J/\psi \Phi}$ can be analyzed by studying the 3 angular distributions (ϕ , θ , Ψ) of the decay products.



Background

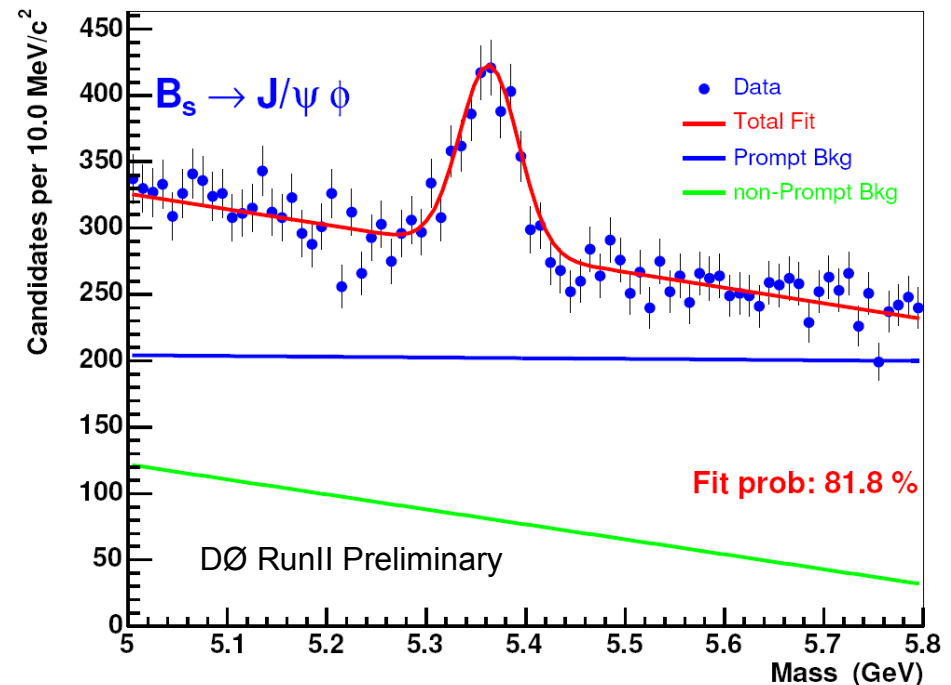
- Background is divided into two categories:
 - “Prompt” – directly produced J/ψ mesons formed together with random tracks.
 - Fitted with a gaussian function
 - “Non-prompt” – J/ψ product of B decay and Φ tracks come from same B hadron
 - Fitted with two exponentials: negative and positive cτ regions.

Fit

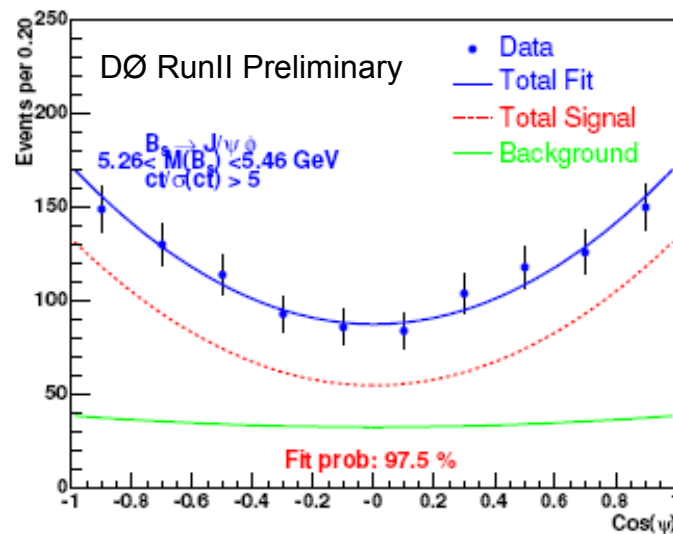
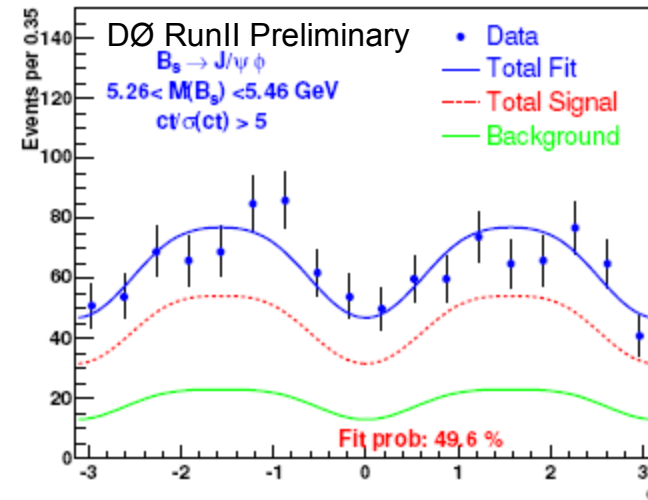
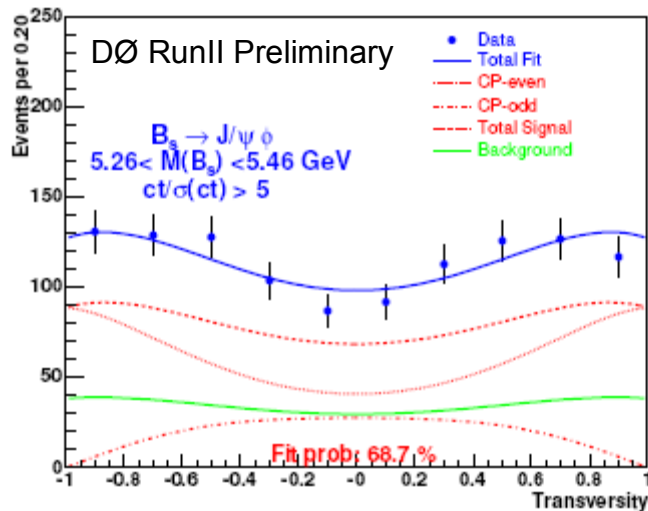
- Simultaneous unbinned maximum likelihood fit to the proper decay length, three angles, and the B_s mass.

$$\mathcal{L} = \prod_{i=1}^N \left[f_{sig} F_{sig}^i + (1 - f_{sig}) F_{bkd}^i \right]$$

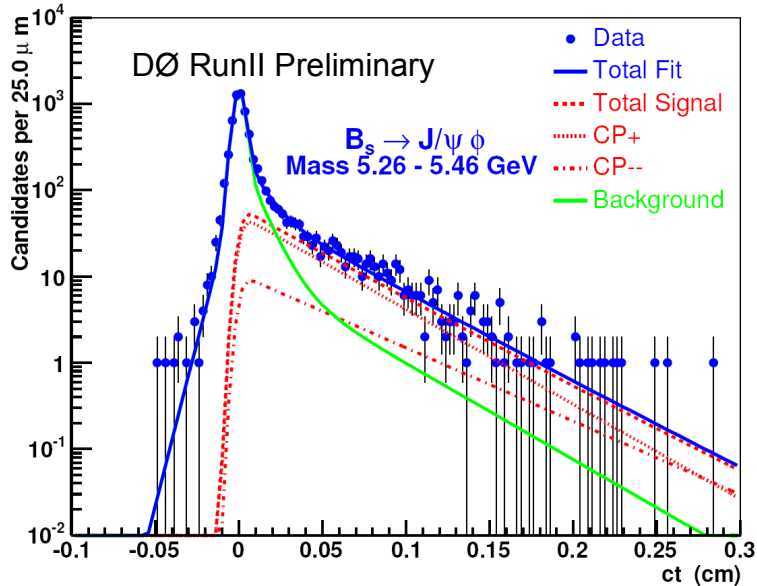
- N – total number of events
- f_{sig} – signal fraction
- F_{sig} – function of the signal mass, proper decay length, and the decay angles (φ, θ, Ψ).
- F_{bkg} – product of the background mass, proper decay length, and angular density functions.



Angular Fit Results



Results



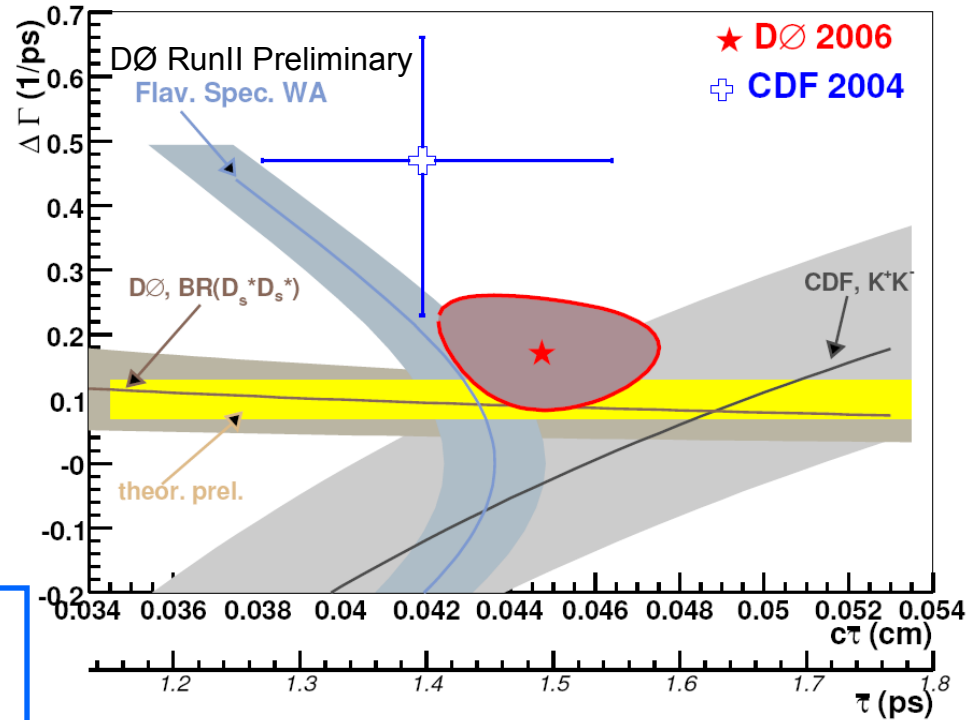
No CP violation hypothesis ($\delta\Phi = 0$)
 $\overline{\tau}(B_s) = 1.52 \pm 0.08$ (stat) $_{-0.03}^{+0.01}$ (syst) ps
 $\Delta\Gamma = 0.12 \pm 0.08 \pm 0.03$ ps⁻¹

CPV ($\delta\Phi$ allowed to vary)

$\overline{\tau}(B_s) = 1.49 \pm 0.08$ (stat) $_{-0.03}^{+0.01}$ (syst) ps

$\Delta\Gamma = 0.17 \pm 0.09 \pm 0.03$ ps⁻¹

$\delta\Phi = -0.79 \pm 0.56 \pm 0.01$, $\delta\Phi_{\text{SM}} = -0.03$



[DØ Conference Note 5144]

Summary

- DØ continues to produce interesting and competitive results in B physics
- Most precise results to date on B_s lifetime!
 - $\tau(B_s) = 1.398 \pm 0.044$ (stat) $_{-0.025}^{+0.028}$ (syst) ps
 - Stringent test of HQE theory
- Tests of Standard Model
 - $\text{Br}(B_s \rightarrow D_s^* D_s^*) = 0.071 \pm 0.032$ (stat) $_{-0.025}^{+0.029}$ (syst)
 - $\Delta\Gamma_{\text{CP}}/\Gamma = 0.142 \pm 0.064$ (stat) $_{-0.050}^{+0.058}$ (syst)
 - $B_s \rightarrow J/\psi \phi$
 - $\tau(B_s) = 1.49 \pm 0.08$ (stat) $_{-0.03}^{+0.01}$ (syst) ps
 - $\Delta\Gamma = 0.17 \pm 0.09 \pm 0.03$ ps⁻¹

Backup Slides

$\Delta\Gamma_s$: Fit

- The time evolution of the three-angle distribution of the products of the decay of untagged B_s mesons, expressed in terms of the linear polarization amplitudes $|A_x(t)|$

$$\begin{aligned} \frac{d^3\Gamma(t)}{d\cos\theta d\varphi d\cos\psi} &\propto 2|A_0(0)|^2 \mathcal{T}_+ \cos^2\psi(1 - \sin^2\theta \cos^2\varphi) \\ &+ \sin^2\psi\{|A_{\parallel}(0)|^2 \mathcal{T}_+ (1 - \sin^2\theta \sin^2\varphi) + |A_{\perp}(0)|^2 \mathcal{T}_- \sin^2\theta\} \\ &+ \frac{1}{\sqrt{2}} \sin 2\psi |A_0(0)| |A_{\parallel}(0)| \cos(\delta_2 - \delta_1) \mathcal{T}_+ \sin^2\theta \sin 2\varphi \\ &+ \left\{ \frac{1}{\sqrt{2}} |A_0(0)| |A_{\perp}(0)| \cos \delta_2 \sin 2\psi \sin 2\theta \cos \varphi \right. \\ &\left. - |A_{\parallel}(0)| |A_{\perp}(0)| \cos \delta_1 \sin^2\psi \sin 2\theta \sin \varphi \right\} \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \sin(\delta\phi) . \end{aligned}$$

where, $\mathcal{T}_+ = \frac{1}{2} ((1 + \cos\delta\phi)e^{-\Gamma_L t} + (1 - \cos\delta\phi)e^{-\Gamma_H t})$ and $\mathcal{T}_- = \frac{1}{2} ((1 - \cos\delta\phi)e^{-\Gamma_L t} + (1 + \cos\delta\phi)e^{-\Gamma_H t})$.