Planck Scale Cosmic Rays in Resummed Quantum Gravity

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Introduction

- New approach to quantum gravity
- Based on resummation of Feynman graphs
- See papers by BFLW, S. Jadach et al.: hep-ph/0503189; JCAP 0402(2004)011; MPL A19(2004)143; A17 (2002)2371; CPC 130(2000)260; and references therein

Topics of Discussion

- Preliminaries
- Review of Feynman's Formulation of Einstein's Theory
- Resummed Quantum Gravity
- Hawking Radiation to Planck Scale Remnants
- Conclusions

Preliminaries

- Newton's Law: Most Basic – Taught To All Beginning Students
- Albert Einstein: Special Case of Classical General Relativity

$g_{00}=1+2\varphi \Rightarrow \nabla^2\varphi = 4\pi G_N \rho$

from

$R^{\alpha\gamma} - \frac{1}{2} g^{\alpha\gamma} R = -8\pi G_N T^{\alpha\gamma}$

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Quantum Mechanics

- Heisenberg & Schrödinger, following Bohr
- Tremendous progress: Quantum Field Theory, Superstrings, Loop Quantum Gravity, etc.
- NO SATISFACTORY QUANTUM TREATMENT OF NEWTON'S LAW IS KNOWN TO BE PHENOMENOLOGICALLY CORRECT

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Today's Talk

- New Approach: Building on work by Feynman (Acta Phys. Pol.**24**(1963)697; *Feynman Lectures on Gravitation*, eds. Moringo and Wagner, 1971)
- BASIC IDEA: QG is a point particle field theory – Bad UV due to our NAIVETE.
- Union of Bohr & Einstein Possible

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Review of Feynman's Formulation of Einstein's Theory

- Given the infancy of our approach, we replace the SM Lagrangian with its scalar sector coupled to Einstein's theory:

1. This already contains all UV divergence issues.

2. Including spinning particles adds inessential complications whose solutions are known.

$$L^G_{SM} \Rightarrow L^G_{SM}(\text{scalar}), \ \text{show relevant dynamics.}$$

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$$
\begin{aligned}
\mathcal{L}(x) &= -\frac{1}{2\kappa^2} R\sqrt{-g} + \frac{1}{2}\left(g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - m_o^2\varphi^2\right)\sqrt{-g} \\
&= \quad \frac{1}{2}\left\{h^{\mu\nu,\lambda}\bar{h}_{\mu\nu,\lambda} - 2\eta^{\mu\mu'}\eta^{\lambda\lambda'}\bar{h}_{\mu\lambda,\lambda'}\eta^{\sigma\sigma'}\bar{h}_{\mu'\sigma,\sigma'}\right\} \\
&\quad + \frac{1}{2}\left\{\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right\} - \kappa h^{\mu\nu}\left[\overline{\varphi_{,\mu}\varphi_{,\nu}} + \frac{1}{2}m_o^2\varphi^2\eta_{\mu\nu}\right] \\
&\quad - \kappa^2\left[\frac{1}{2}h_{\lambda\rho}\bar{h}^{\rho\lambda}\left(\varphi_{,\mu}\varphi^{,\mu} - m_o^2\varphi^2\right) - 2\eta_{\rho\rho'}h^{\mu\rho}\bar{h}^{\rho'\nu}\varphi_{,\mu}\varphi_{,\nu}\right] + \cdots
\end{aligned}
\tag{2}
$$

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where $\varphi_{,\mu} \equiv \partial_\mu \varphi$ and we have

- $g_{\mu\nu}(x) = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}(x)$,
  $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$
- $\bar{y}_{\mu\nu} \equiv \frac{1}{2}\left(y_{\mu\nu} + y_{\nu\mu} - \eta_{\mu\nu} {y_\rho}^\rho\right)$ for any tensor $y_{\mu\nu}$
- Feynman rules already worked-out by Feynman (*op. cit.*), where we use his gauge, $\partial^\mu \bar{h}_{\nu\mu} = 0$

⇔ Quantum Gravity is just another quantum field theory where the metric now has quantum fluctuations as well.

For example, the one-loop corrections to the graviton propagator due to matter loops is just given by the diagrams in Fig. 1.

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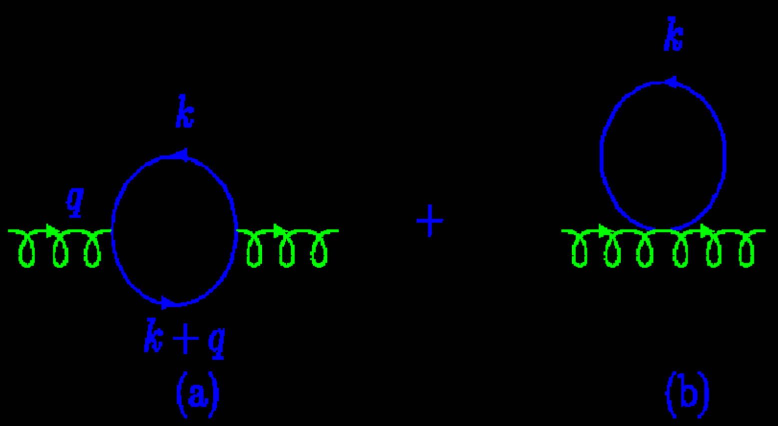

Figure 1: The scalar one-loop contribution to the graviton propagator. q is the 4-momentum of the graviton.

These graphs already illustrate the QG's BAD UV behavior.

Resummed Quantum Gravity

- RESUM PROPAGATORS FOLLOWING YFS IDEA:

$$i\Delta'_F(k)|_{\text{Resummed}} = \frac{ie^{B''_g}}{k^2 - m^2 - \Sigma'_F(k) + i\varepsilon} \ ,$$

$$\Sigma'_F(k) = \sum_{n=0}^{\infty} \Sigma'_{Fn}(k),$$

Σ'_F STARTS IN O(κ^2); DROP IN ONE-LOOP EFF

- EXPLICIT EVALUATION GIVES,

$$B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{m^2}{m^2+|k^2|}\right), \qquad (11)$$

$\Rightarrow$ THE RESUMMED PROPAGATOR FALLS FASTER THAN ANY POWER OF $|k^2|$!

- IF m VANISHES, USING THE USUAL $-\mu^2$ NORMALIZATION POINT WE GET $B_g''(k) = \frac{\kappa^2|k^2|}{8\pi^2}\ln\left(\frac{\mu^2}{|k^2|}\right)$ WHICH AGAIN VANISHES FASTER THAN ANY POWER OF $|k^2|$!

THIS MEANS THAT ONE-LOOP CORRECTIONS ARE FINITE!

INDEED, ALL QUANTUM GRAVITY LOOPS ARE UV FINITE!

All Orders Proof in MPLA**17**, 2371 (2002)

Newton's Law

- Consider the one-loop corrections to Newton's law from Fig. 1 – they directly impact our Hawking radiation analysis. Our resummed propagators ⇒ (k → $(ik^0, \vec{k})$)

$$\Sigma^{1a}_{\bar{\mu}\bar{\nu};\mu\nu} = \kappa^2 \frac{\int d^4k}{2\,(2\pi)^4} \frac{(k'_{\bar{\mu}} k_{\bar{\nu}} + k'_{\bar{\nu}} k_{\bar{\mu}}) e^{\frac{\kappa^2 |k'^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k'^2|}\right)}}{k'^2 - m^2 + i\varepsilon}$$

$$\frac{(k'_{\mu} k_{\nu} + k'_{\nu} k_{\mu}) e^{\frac{\kappa^2 |k^2|}{8\pi^2} \ln\left(\frac{m^2}{m^2 + |k^2|}\right)}}{k^2 - m^2 + i\varepsilon}$$

⇔ CONVERGENT; SO IS FIG. 1b.

In transv.-traceless space, graviton propagator denominator is

$$q^2 + \frac{1}{2} q^4 \Sigma^{T(2)} + i\varepsilon$$

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with the result

$$-\frac{1}{2}\Sigma^{T(2)} \cong \frac{c_2}{360\pi M_{Pl}^2} \text{ for}$$

$$c_2 = \int_0^\infty dx\, x^3 (1+x)^{-4-\lambda_c x} \cong 72.1,$$

$$\lambda_c = \frac{2m^2}{\pi M_{Pl}^2}.$$

$$\Rightarrow$$

$$\Phi_{Newton}(r) = -\frac{G_N M_1 M_2}{r}(1-e^{-ar}),$$

$$a = \frac{1}{\sqrt{-\frac{1}{2}\Sigma^{T(2)}}} \cong 3.96 M_{Pl}, \quad \text{when}$$

$$m \cong 120 \text{ GeV}.$$

Sum over SM particles $\Rightarrow a_{eff} = 0.210 M_{Pl}$

Many consequences, hep-ph/0602025 and refs. therein

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CONTACT WITH ASYMPTOTIC SAFETY APPROACH

- OUR RESULTS IMPLY

$$G_N(k) = G_N/(1+k^2/a_{eff}^2)$$

$\Rightarrow$ FIXED PT. BEHAVIOR FOR $k^2 \rightarrow \infty$

-- AGREES WITH PHENOMENOLOGICAL ASYMPTOTIC SAFETY APPROACH OF BONNANNO & REUTER, PRD62(2000)043008.

- WE FIND MASSIVE ELEMENTARY PARTICLES HAVE NO HORIZON $\Leftrightarrow$ BR FIND BH WITH MASS LESS THAN $M_{cr} \sim M_{Pl}$ HAS NO HORIZON.

BASIC PHYSICS: $G_N(k)$ VANISHES FOR $k^2 \rightarrow \infty$

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QG04

- A FURTHER AGREEMENT: FINAL STATE OF HAWKING RADIATIO
  ORIGINALLY VERY MASSIVE BLACKHOLE
  BECAUSE OUR VALUE OF THE COEFFICIENT,
  $$\frac{1}{a^2_{eff}},$$
  OF k^2 IN THE DENOMINATOR OF $G_N(k)$
  AGREES WITH THAT FOUND BY BONNANNO & REUTER,
  IF WE USE THEIR PRESCRIPTION FOR THE
  RELATIONSHIP BETWEEN k AND r
  IN THE REGIME WHERE THE LAPSE FUNCTION VANISHES,
  WE GET THE SAME HAWKING RADIATION PHENOMEMNOLOGY
  THE BLACK HOLE EVAPORATES UNTIL IT REACHES A MASS
  $$M_{cr} \sim M_{Pl}$$
  AT WHICH THE BEKENSTEIN-HAWKING TEMPERATURE VANISH
  LEAVING A PLANCK SCALE REMNANT.
- FATE OF REMNANT? IN hep-ph/0503189 $\Rightarrow$ OUR QUANTUM LOO
  COMBINED WITH THE $G_N(r)$ OF B-R ACTUALLY THE HORIZON
  SCALE REMNANTIS OBVIATED SO THAT THERE ARE THEN NO
  REMNANTS AT ALL! – CONSISTENT WITH RECENT RESULTS OF

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TO WIT, IN THE METRIC CLASS

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2 d\Omega^2 \quad (26)$$

THE LAPSE FUNCTION IS, FROM B-R,

$$\begin{aligned} f(r) &= 1 - \frac{2G_N(r)M}{r} \\ &= \frac{B(x)}{B(x)+2x^2}\Big|_{x=\frac{r}{G_N M}}, \end{aligned} \quad (27)$$

WHERE

$$B(x) = x^3 - 2x^2 + \Omega x + \gamma\Omega \quad (28)$$

FOR

$$\Omega = \frac{\tilde{\omega}}{G_N M^2} = \frac{\tilde{\omega} M_{Pl}^2}{M^2}. \quad (29)$$

AFTER H-RADIATING TO $M_{cr} \sim M_{Pl}$, QUANTUM LOOPS CHANGE THE $-2x^2$ IN $B(x)$ TO $-2\xi x^2$ WITH $\xi = \xi(x) = 1 - e^{-aG_N M_{cr} x} < 1$, REMOVING THE DOUBLE ZERO AT x_{cr}. MONTONICITY $\Rightarrow$ HORIZON OBVIATED.

NOTE: M. BOJOWALD *et al.*, gr-qc/0503041, – LOOP QG CONCURS.

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B.

- Continuous Transition:

  $1\text{-}2G(r)M/r = 1\text{-}2G_N(1\text{-}e^{-ar})M/r$

Outermost solution: Ω=.2, $\gamma = 0$,

$\Rightarrow r_> = 27.1/M_{Pl}$, $x_+ = 1.15$, $x_- < 0$

$\Rightarrow x_+ \rightarrow 0$ for $M \rightarrow 2.4\ M_{Pl} = M'_{cr}$

with $T_{BH} > 0$, as above.

- Remnant M'_{cr} :

  Decays: 2-body,…,n-body

  $\Rightarrow$ Planck Scale Cosmic Rays, etc.

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Conclusions

- RESUMMATION RENDERS QGR UV FINITE
- SUB-PLANCK SCALE PHYSICS ACCESSIBLE TO QFT (TUT)
- MINIMAL UNION OF BOHR & EINSTEIN
- BLACK HOLES WITH $M<M_{cr} \sim M_{Pl}$ HAVE NO HORIZON
- FINAL STATE OF HAWKING RADIATION ⇒ PLANCK SCALE REMNANT ⇒ PLANCK SCALE COSMIC RAYS, …

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