Infrared features of the ghost propagators in quenched and unquenched lattice

Landau gauge QCD

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I. Introduction

- Unquenched lattice Landau gauge simulation using MILC Asqtad Kogut-Susskind fermions.

- Color confinement
  - The Kugo-Ojima confinement parameter $c$.
  - $A^2$ condensate in running coupling, gluon propagator and quark propagator.
  - Ghost condensate parameter $\nu$ and Binder cumulant of the color anti-symmetric ghost propagator.

- Dynamical chiral symmetry breaking
II. Color Confinement

- Kugo-Ojima theory based on the Lagrangian satisfying BRST symmetry yields a confinement criterion (A two-point function at $q = 0$). (Kugo-Ojima 1979)

- Gribov-Zwanziger theory gives a sufficient condition of the confinement for infrared exponents of gluon propagator and ghost propagator. (Gribov 1978, Zwanziger 1991)

- The lattice simulation of running coupling $\alpha_s(q)$ in $\text{MOM}$ scheme suggests presence of mass-dimension 2 ($A^2$) condensates. (Boucaud et al. 2000)
• The mass-dimension 2 condensates can be related to Zwanziger’s horizon condition generated by restriction of the gauge field in fundamental modular region. (Dudal et al. 2005)

• The $A^2$ is not BRST invariant. A mixed condensate with $\bar{c}c$ becomes on-shell BRST invariant. (Kondo 2003)

• Local Composite Operator (LCO) approach suggests that $\bar{c}c$ condensate manifest itself in the color anti-symmetric ghost propagator. (Dudal et al. 2005)

• Investigation of the ghost condensate in SU(2) lattice Landau gauge was performed. (Cucchieri et al. 2005)
• The Kugo-Ojima confinement criterion:

\[(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2})u^{ab}(q^2)\]

\[= \frac{1}{V} \sum_{x,y} e^{-ip(x-y)} \langle \text{tr} \left( \Lambda^{a\dagger} D_\mu \frac{1}{-\partial D} [A_\nu, \Lambda^b] \right) \rangle_{xy}.\]

The fact that the parameter \(c\) defined as \(u^{ab}(0) = -\delta^{ab} c\) becomes 1 is the confinement criterion.

• The parameter \(c\) is related to the renormalization factor as

\[1 - c = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_1^\psi}{Z_2}\]

• If the finiteness of \(\tilde{Z}_1\) is proved, divergence of \(\tilde{Z}_3\) is a sufficient condition. If \(Z_3\) vanishes in the infrared, \(Z_1\) should have higher order 0. If \(Z_2\) is finite \(Z_1^\psi\) should vanish.
• Zwanziger’s horizon condition

\[ \sum_{x,y} e^{-ip(x-y)} \left\langle \text{tr} \left( \Lambda^a \left( \frac{1}{-\partial D} (-D_\nu) \Lambda^b \right) \right)_{xy} \right\rangle \]

\[ = G_{\mu\nu}(p) \delta^{ab} = \left( \frac{e}{d} \right) \frac{p_\mu p_\nu}{p^2} \delta^{ab} - \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u^{ab}, \]

where, with use of the covariant derivativative \( D_\mu(U) \)

\[ D_\mu(U_{x,\mu}) \phi = S(U_{x,\mu}) \partial_\mu \phi + [A_{x,\mu}, \phi], \]

\( \partial_\mu \phi = \phi(x + \mu) - \phi(x), \) and \( \bar{\phi} = \frac{\phi(x + \mu) + \phi(x)}{2} \)

\[ e = \left\langle \sum_{x,\mu} \text{tr}(\Lambda^a \Lambda^b) \right\rangle \left/ \{(N_c^2 - 1)V\} \right. \]
The horizon condition reads \( \lim_{p \to 0} G_{\mu\mu}(p) - e = 0 \), and the l.h.s. of the condition is \( \left( \frac{e}{d} \right) + (d - 1)c - e = (d - 1)h \) where \( h = c - \frac{e}{d} \) and dimension \( d = 4 \), and it follows that \( h = 0 \) → horizon condition, and thus the horizon condition coincides with Kugo-Ojima criterion provided the covariant derivative approaches the naive continuum limit, i.e., \( e/d = 1 \).

The renormalization group flow of the ghost propagator is assumed to follow perturbative renormalization-group flow equation.

Suppression of the infrared modes of the gauge field corresponds to the vanishing of the gluon propagator.
III. The lattice Landau gauge

- Two types of the gauge field definitions:
  1. log $U$ type: $U_{x,\mu} = e^{A_{x,\mu}}$, $A_{x,\mu}^\dagger = -A_{x,\mu}$,

  2. $U$ linear type: $A_{x,\mu} = \frac{1}{2} (U_{x,\mu} - U_{x,\mu}^\dagger)|_{trlp}$,

     ( $A_{\mu}(x) = i \sum_a A_{\mu}^a(x) \frac{\Lambda^a}{\sqrt{2}}$, $\text{tr}\Lambda^a \Lambda^b = \delta^{ab}$)

- The optimizing function
  1. $F_U(g) = ||A^g||^2 = \sum_{x,\mu} \text{tr} \left( A_{x,\mu}^g A_{x,\mu}^{g\dagger} \right)$,

  2. $F_U(g) = \sum_{x,\mu} \text{tr} \left( 2 - (U_{x,\mu}^g + U_{x,\mu}^{g\dagger}) \right)$,
• Under infinitesimal gauge transformation $g^{-1}\delta g = \epsilon$, its variation reads for either definition as

$$\Delta F_U(g) = -2\langle \partial A^g|\epsilon \rangle + \langle \epsilon| - \partial D(U^g)|\epsilon \rangle + \cdots,$$

• Stationality (Landau gauge), Local minimum (Gribov Region), Global minimum (Fundamental modular (FM) region)
Kugo-Ojima parameter $c$ of quenched SU(3)

$U-$linear(left) and log $U$(right). $\beta = 6.0$ and 6.4.

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<th>$L$</th>
<th>$c_1$</th>
<th>$e_1/d$</th>
<th>$h_1$</th>
<th>$c_2$</th>
<th>$e_2/d$</th>
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<td>0.576(79)</td>
<td>0.860(1)</td>
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<td>0.628(94)</td>
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<tr>
<td>6.0</td>
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<td>0.695(63)</td>
<td>0.861(1)</td>
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<td>0.774(76)</td>
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<td>6.0</td>
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<td>6.4</td>
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<td>0.883(1)</td>
<td>-0.23</td>
<td>0.700(42)</td>
<td>0.953(1)</td>
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<td>0.793(61)</td>
<td>0.954(1)</td>
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<td></td>
<td></td>
<td></td>
<td>0.814(89)</td>
<td>0.954(1)</td>
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MILC configurations used in our simulation

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<th>$\beta_{imp}$</th>
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<th>$N_f$</th>
<th>$1/a$(GeV)</th>
<th>$L_s$</th>
<th>$L_t$</th>
<th>$aL_s$(fm)</th>
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<td>6.83</td>
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<td>1.64</td>
<td>20</td>
<td>64</td>
<td>2.41</td>
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<td>6.76</td>
<td>0.007/0.050</td>
<td>2+1</td>
<td>1.64</td>
<td>20</td>
<td>64</td>
<td>2.41</td>
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<td>MILC$_f$</td>
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<td>0.0124/0.031</td>
<td>2+1</td>
<td>2.19</td>
<td>28</td>
<td>96</td>
<td>2.52</td>
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<tr>
<td></td>
<td>7.09</td>
<td>0.0062/0.031</td>
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<td>2.19</td>
<td>28</td>
<td>96</td>
<td>2.52</td>
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<tr>
<td>MILC$_{ft}$</td>
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<td>2.244</td>
<td>24</td>
<td>12</td>
<td>2.11</td>
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The Kugo-Ojima parameter of unquenched SU(3)

Table 1: The Kugo-Ojima parameter for the polarization along the spacial directions $c_x$ and that along the time direction $c_t$ and the average $c$, trace divided by the dimension $e/d$, horizon function deviation $h$ of the unquenched KS fermion (MILC$_c$, MILC$_f$, MILC$_{ft}$) with use of log $U$ definition.

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<th>$c_t$</th>
<th>$c$</th>
<th>$e/d$</th>
<th>$h$</th>
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<td>6.76</td>
<td>1.04(11)</td>
<td>0.74(3)</td>
<td>0.97(16)</td>
<td>0.9325(1)</td>
<td>0.03(16)</td>
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<td></td>
<td>6.83</td>
<td>0.99(14)</td>
<td>0.75(3)</td>
<td>0.93(16)</td>
<td>0.9339(1)</td>
<td>-0.00(16)</td>
</tr>
<tr>
<td>MILC$_f$</td>
<td>7.09</td>
<td>1.06(13)</td>
<td>0.76(3)</td>
<td>0.99(17)</td>
<td>0.9409(1)</td>
<td>0.04(17)</td>
</tr>
<tr>
<td></td>
<td>7.11</td>
<td>1.05(13)</td>
<td>0.76(3)</td>
<td>0.98(17)</td>
<td>0.9412(1)</td>
<td>0.04(17)</td>
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<tr>
<td>MILC$_{ft}$</td>
<td>5.65</td>
<td>0.72(13)</td>
<td>1.04(23)</td>
<td>0.80(21)</td>
<td>0.9400(7)</td>
<td>-0.14</td>
</tr>
<tr>
<td></td>
<td>5.725</td>
<td>0.68(15)</td>
<td>0.77(16)</td>
<td>0.70(15)</td>
<td>0.9430(2)</td>
<td>-0.24</td>
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<tr>
<td></td>
<td>5.85</td>
<td>0.63(19)</td>
<td>0.60(12)</td>
<td>0.62(17)</td>
<td>0.9465(2)</td>
<td>-0.33</td>
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Fig. 1: Kugo-Ojima parameter $u(0)$ of MILC$_f$ $N_f = 2 + 1$ KS fermion unquenched configurations of $\beta_{imp} = 7.11$(green diamonds), $\beta_{imp} = 7.09$(red stars).

Fig. 2: Kugo-Ojima parameter $u(0)$ of MILC finite temperature configurations of $\beta = 5.65$(blue diamonds), $\beta = 5.725$(red stars) and $\beta = 5.85$(green triangles).
IV. The Ghost propagator

\[
FT[D_G^{ab}(x, y)] = FT(\text{tr}(\Lambda^a (\mathcal{M}[U])^{-1})_{xy} \Lambda^b),
\]

\[
= \delta^{ab} D_G(q^2),
\]

\[
\mathcal{M} = -\partial_\mu D_\mu.
\]

- Ghost dressing function \( G(q^2) = q^2 D_G(q^2) \).

- Solve the equation with plane wave sources.

\[
-\partial_\mu D_\mu f_s^b(x) = \frac{1}{\sqrt{V}} \Lambda^b \sin \mathbf{q} \cdot \mathbf{x} \tag{1}
\]

\[
-\partial_\mu D_\mu f_c^b(x) = \frac{1}{\sqrt{V}} \Lambda^b \cos \mathbf{q} \cdot \mathbf{x}. \tag{2}
\]
• Color diagonal ghost propagator

\[ D_G(q) = \frac{1}{N_c^2 - 1V} \]

\[ \times \delta^{ab}(\langle \Lambda^a \cos q \cdot x|f_c^b(x)\rangle + \langle \Lambda^a \sin q \cdot x|f_s^b(x)\rangle) \]

(3)

• Color anti-symmetric ghost propagator

\[ \phi^c(q) = \frac{1}{\mathcal{N} V} \]

\[ \times f^{abc}(\langle \Lambda^a \cos q \cdot x|f_s^b(x)\rangle - \langle \Lambda^a \sin q \cdot x|f_c^b(x)\rangle) \]

(4)

where \( \mathcal{N} = 2 \) for SU(2) and 6 for SU(3).
Fig. 3: Log of the ghost dressing function $\log_{10} G(q)$ as a function of $\log_{10} q$(GeV) of MILC$_f$ $\beta_{imp} = 7.09$ (diamonds) and that of quenched $\beta = 6.45$ 56$^4$ (stars).

Fig. 4: The ghost dressing function of MILC$_f$ $\beta_{imp} = 7.09$ (stars) Dashed line is the 4-loop $N_f = 2$ pQCD result ($\lambda_G = 3.01, y = 0.0246100$)
Fig. 5: Log of the color diagonal ghost propagator of $\log_{10}[D_G(q)]$ as a function of $q$(GeV) MILC$_c$.

Fig. 6: Log of the color antisymmetric ghost propagator squared $\log_{10}[\phi(q)^2]$ as a function of $q$(GeV). 20$^3 \times 64$ MILC$_c$. 
The ghost condensate

- The ghost condensate $\langle f^{abc} c^{b} c^{c} \rangle$ in the color anti-symmetric ghost propagator is parametrized by $v$, $r$ and $z$.

- Expression of the finite size effect in the $L^3_t L_t$ lattice for $q_\mu$ under cylinder cut is difficult. Our choice is,

\[
\frac{1}{N_c^2 - 1} \sum_a \frac{\sqrt{L^3_x L_t}}{\cos(\pi \tilde{q}/8 \sqrt{L_x L_t})} \langle |\phi^a(q)| \rangle = \frac{r}{q^z}, \quad (5)
\]

\[
\tilde{q}^2 = \sum_{i=1}^{3} \left(2 \sin \frac{\pi \tilde{q}_i}{L_x} \right)^2 + \left(2 \sin \frac{\pi \tilde{q}_4}{L_t} \right)^2 \quad (6)
\]

\[
\frac{1}{N_c^2 - 1} \sum_a \langle |\phi^a(q)| \rangle = \frac{r/\sqrt{L^3_x L_t} + v}{q^4 + v^2} \quad (7)
\]
Fig. 7: The logarithm of the color antisymmetric ghost propagator of MILC$_f$ as the function of $\log_{10}(q(\text{GeV}))$ and the fit using $r = 134$ and $v = 0.026\text{GeV}^2$.

Fig. 8: The logarithm of the color antisymmetric ghost propagator of MILC$_{ft}$ of $T = 143\text{MeV}$ (blue diamonds), $T = 159.5\text{MeV}$ (red stars) and $T = 187\text{MeV}$ (green triangles) and their fits.
The Binder cumulant

- Binder cumulant of the color antisymmetric ghost propagator

\[ U(q) = 1 - \frac{\langle \phi(q)^4 \rangle}{3\langle \phi(q)^2 \rangle^2}. \]

- Suppression away from the d-dimensional gaussian distribution

\[ \frac{\langle \phi^4 \rangle}{\langle \phi^2 \rangle^2} = \frac{d + 2}{d} \] may imply presence of ghost condensate.
• Quenched SU(2) is compatible with gaussian. MILC$_c$ shows a larger randomness than the Gaussian.

Fig. 9: The momentum dependence of Binder cumulant $U(q)$ of SU(2), $\beta = 2.2$, $a = 1.07\text{GeV}^{-1}$ of PT samples (blue triangles) and first copy samples (green diamonds).

Fig. 10: The momentum dependence of Binder cumulant $U(q)$ of unquenched SU(3), $a = 1.64\text{GeV}^{-1}$ MILC$_c$. 
Exceptional samples

- In $\beta = 6.4, 56^4$ quenched SU(3) configurations, we found a copy whose $\alpha_G = 0.272$ v.s. $\alpha_G$(average)$=0.223$, and whose gluon propagator has an axis along which the reflection positivity is manifestly violated.

- In the $\beta_{imp} = 6.76, 20^3 \times 64$ MILC$c$ configurations (21 samples), we find a similar exceptional sample.

- The exceptional sample makes the sample average of the Binder cumulant of color anti-symmetric ghost propagator in the infrared small, and the standard deviation large.
Fig. 11: The Binder cumulant of the color antisymmetric ghost propagator of MILC_{ft} \( N_f = 2 \) configurations of \( \beta = 5.725 \) (red stars).

Fig. 12: Averages over momenta excluding the lowest momentum point of the Binder cumulants of MILC finite temperature configurations. \( \beta = 5.65 \) (blue diamonds), \( \beta = 5.725 \) (red stars) and \( \beta = 5.85 \) (green triangles).
The fitted parameters $r, z, v$ of $|\phi(q)|$, $\bar{r}, \bar{z}, \bar{v}$ of $\phi(q)^2$ and $U$ of MILC$_c$ and MILC$_f$ samples. ($U$ of MILC$_f$ corresponds to the average below $q = 1$GeV and the average above 1GeV, respectively)

<table>
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<tr>
<th>$\beta_{imp}$</th>
<th>$m_0$(MeV)</th>
<th>$r$</th>
<th>$z$</th>
<th>$v$</th>
<th>$\bar{r}$</th>
<th>$\bar{z}$</th>
<th>$\bar{v}$</th>
<th>$U$</th>
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<td>6.76</td>
<td>11.5/82.2</td>
<td>37.5</td>
<td>3.90</td>
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<td>0.045</td>
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</tr>
<tr>
<td>6.83</td>
<td>65.7/82.2</td>
<td>38.7</td>
<td>3.85</td>
<td>0.007</td>
<td>33.5</td>
<td>7.6</td>
<td>0.048</td>
<td>0.57(4)</td>
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<tr>
<td>7.09</td>
<td>13.6/68.0</td>
<td>134</td>
<td>3.83</td>
<td>0.026</td>
<td>251</td>
<td>7.35</td>
<td>0.044</td>
<td>0.57(4)/0.56(1)</td>
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<tr>
<td>7.11</td>
<td>27.2/68.0</td>
<td>112</td>
<td>3.81</td>
<td>0.028</td>
<td>164</td>
<td>7.34</td>
<td>0.002</td>
<td>0.58(2)/0.52(1)</td>
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The data of MILC$_f$ $\beta_{imp} = 7.11$ are rather noisy. $U(q)$ below 1GeV and above 1GeV are different. The anomaly is correlated with that of the dynamical quark mass function.
The infrared exponents of ghost and gluon

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<td>0.23</td>
<td>-0.65</td>
<td>-0.19</td>
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VII. The quark propagator

- The statistical average over Landau-gauge-fixed samples

\[ S_{\alpha\beta}(p) = \left\langle (\chi_{p,\alpha} | \frac{1}{i\not{\partial}(U) + m} | \chi_{p,\beta}) \right\rangle \]

The inversion, \( \frac{1}{i\not{\partial}(U) + m} \), is performed via conjugate gradient method after preconditioning.

\[ S_{\alpha\beta}(q) = Z_2(q) \frac{-i\gamma q + M(q)}{q^2 + M(q)^2} \]
• The mass function in large $q$.

\[
M(q) = -\frac{4\pi^2 d_M \langle \bar{\psi} \psi \rangle_{\mu} [\log(q^2/\Lambda_{QCD}^2)]^{d_M-1}}{3q^2 [\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}} + \frac{m(\mu^2) [\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}}{[\log(q^2/\Lambda_{QCD}^2)]^{d_M}},
\]

\[
d_M = 12/(33 - 2N_f)
\]

• The mass function in the infrared region.

\[
M(q) = \frac{\bar{c} \Lambda^3}{q^2 + \Lambda^2} + m_0
\]
The mass function

The parameters \( \tilde{c} \) and \( \Lambda \) of the Staple+Naik action (left) and the Asqtad action.

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<th>( \beta_{imp} )</th>
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<th>( \tilde{c} )</th>
<th>( \Lambda(\text{GeV}) )</th>
<th>( \tilde{c}\Lambda(\text{GeV}) )</th>
<th>( \tilde{c} )</th>
<th>( \Lambda(\text{GeV}) )</th>
<th>( \tilde{c}\Lambda(\text{GeV}) )</th>
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Fig. 13: The mass function $M(q)$ of the Asqtad action of MILC$_f$ with the bare quark mass $m_0 = 13.6\text{MeV}$ (green stars) and with the bare quark mass $m_0 = 68\text{MeV}$ (red diamonds).

Fig. 14: The chiral symmetry breaking mass $\bar{c}\Lambda$ as a function of bare mass and its chiral limit. Dotted line is an extrapolation of MILC$_f$ and the dash-dotted line is that of MILC$_c$, Asqtad action.
VIII. Summary and Discussion

- The Kugo-Ojima parameter $c$ saturated at about 0.8 in the quenched $56^4$ lattice, but it is consistent with 1 in the zero temperature MILC configurations.
- The quark has the effect of quenching randomness.
- The condensate parameter $v$ in the color anti-symmetric ghost propagator in quenched SU(2) is consistent with 0 and also small in MILC$_c$ and MILC$_f$.
- The Binder cumulants of zero temperature MILC are close to those of Gaussian distributions except the lowest momentum point.
- The Binder cumulants of MILC finite temperature ($T > T_c$) show larger randomness than those of Gaussian distributions.
• The gluon propagator is infrared finite?

• Kugo-Ojima confinement criterion

\[ \frac{Z_1}{Z_3} = \frac{o((q^2)^{-\alpha_D+s})}{O((q^2)^{-\alpha_D})} \quad [s > 0]? \]

\[ \frac{Z_1}{Z_3} = \frac{1}{\infty} \]

\[ \frac{Z_1^\psi}{Z_2} = \frac{o((q^2)^s)}{O(1)} \quad [s > 0]? \]

• How does the true vacuum manifest itself in the \( V \rightarrow \) limit of the simulation?
References


