Infrared features of the ghost propagators in quenched and unquenched lattice Landau gauge QCD

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I. Introduction

- Unquenched lattice Landau gauge simulation using MILC Asqtad Kogut-Susskind fermions.

- Color confinement
  - The Kugo-Ojima confinement parameter c.
  - A^2 condensate in running coupling, gluon propagator and quark propagator.
  - Ghost condensate parameter v and Binder cumulant of the color anti-symmetric ghost propagator.

- Dynamical chiral symmetry breaking

II. Color Confinement

- Kugo-Ojima theory based on the Lagrangian satisfying BRST symmetry yields a confinement criterion (A two-point function at $q = 0$). (Kugo-Ojima 1979)

- Gribov-Zwanziger theory gives a sufficient condition of the confinement for infrared exponents of gluon propagator and ghost propagator.(Gribov 1978, Zwanziger 1991)

- The lattice simulation of running coupling $\alpha_s(q)$ in $\widetilde{MOM}$ scheme suggests presence of mass-dimension 2 (A^2) condensates.(Boucaud et al. 2000)

- The mass-dimension 2 condensates can be related to Zwanziger's horizon condition generated by restriction of the gauge field in fundamental modular region.(Dudal et al. 2005)

- The A^2 is not BRST invariant. A mixed condensate with $\bar{c}c$ becomes on-shell BRST invariant.(Kondo 2003)

- Local Composite Operator(LCO) approach suggests that $\bar{c}c$ condensate manifest itself in the color anti-symmetric ghost propagator. (Dudal et al. 2005)

- Investigation of the ghost condensate in SU(2) lattice Landau gauge was performed.(Cucchieri et al. 2005)

- The Kugo-Ojima confinement criterion:

$$(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) u^{ab}(q^2)$$
$$= \frac{1}{V}\sum_{x,y} e^{-ip(x-y)} \langle \mathrm{tr}\left(\Lambda^{a\dagger} D_\mu \frac{1}{-\partial D}[A_\nu, \Lambda^b]\right)_{xy}\rangle .$$

The fact that the parameter c defined as $u^{ab}(0) = -\delta^{ab}c$ becomes 1 is the confinement criterion.

- The parameter c is related to the renormalization factor as

$$1 - c = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_1^\psi}{Z_2}$$

- If the finiteness of $\tilde{Z}_1$ is proved, divergence of $\tilde{Z}_3$ is a sufficient condition. If Z_3 vanishes in the infrared, Z_1 should have higher order 0. If Z_2 is finite Z_1^ψ should vanish.

- Zwanziger's horizon condition

$$\sum_{x,y} e^{-ip(x-y)} \left\langle \mathrm{tr} \left(\Lambda^{a\dagger} D_\mu \frac{1}{-\partial D} (-D_\nu) \Lambda^b \right)_{xy} \right\rangle$$

$$= G_{\mu\nu}(p)\delta^{ab} = \left(\frac{e}{d}\right) \frac{p_\mu p_\nu}{p^2} \delta^{ab} - \left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) u^{ab},$$

where, with use of the covariant derivatative $D_\mu(U)$

$$D_\mu(U_{x,\mu})\phi = S(U_{x,\mu})\partial_\mu \phi + [A_{x,\mu}, \bar{\phi}],$$

$\partial_\mu \phi = \phi(x+\mu) - \phi(x)$, and $\bar{\phi} = \dfrac{\phi(x+\mu)+\phi(x)}{2}$

$e = \left\langle \sum_{x,\mu} \mathrm{tr}(\Lambda^{a\dagger} S(U_{x,\mu}) \Lambda^a) \right\rangle / \{(N_c^2-1)V\}.$

- The horizon condition reads $\lim_{p \to 0} G_{\mu\mu}(p) - e = 0$, and the l.h.s. of the condition is $\left(\frac{e}{d}\right) + (d-1)c - e = (d-1)h$ where $h = c - \frac{e}{d}$ and dimension $d = 4$, and it follows that $h = 0 \rightarrow$ horizon condition, and thus the horizon condition coincides with Kugo-Ojima criterion provided the covariant derivative approaches the naive continuum limit, i.e., $e/d = 1$.

- The renormalization group flow of the ghost propagator is assumed to follow perturbative renormalization-group flow equation.

- Suppression of the infrared modes of the gauge field corresponds to the vanishing of the gluon propagator.

III. The lattice Landau gauge

- Two types of the gauge field definitions:

  1. $\log U$ type: $U_{x,\mu} = e^{A_{x,\mu}}$, $A^{\dagger}_{x,\mu} = -A_{x,\mu}$,

  2. U linear type: $A_{x,\mu} = \frac{1}{2}(U_{x,\mu} - U_{x,\mu}{}^{\dagger})|_{trlp.}$,

  $$(A_\mu(x) = i \sum_a A_\mu{}^a(x) \frac{\Lambda^a}{\sqrt{2}}, \quad \mathrm{tr}\Lambda^a\Lambda^b = \delta^{ab})$$

- The optimizing function

  1. $F_U(g) = ||A^g||^2 = \sum_{x,\mu} \mathrm{tr}\left(A^g{}_{x,\mu}{}^{\dagger} A^g_{x,\mu}\right)$,

  2. $F_U(g) = \sum_{x,\mu} \mathrm{tr}\left(2 - (\, U^g_{x,\mu} + U^g_{x,\mu}{}^{\dagger})\right)$,

- Under infinitesimal gauge transformation $g^{-1}\delta g = \epsilon$, its variation reads for either defintion as

$$\Delta F_U(g) = -2\langle \partial A^g | \epsilon \rangle + \langle \epsilon | - \partial D(U^g) | \epsilon \rangle + \cdots,$$

- Stationality(Landau gauge), Local minimum(Gribov Region), Global minimum(Fundamental modular(FM) region)

Kugo-Ojima parameter c of quenched SU(3)

$U-$linear(left) and $\log U$(right). $\beta = 6.0$ and 6.4.

β	L	c_1	e_1/d	h_1	c_2	e_2/d	h_2
6.0	16	0.576(79)	0.860(1)	-0.28	0.628(94)	0.943(1)	-0.32
6.0	24	0.695(63)	0.861(1)	-0.17	0.774(76)	0.944(1)	-0.17
6.0	32	0.706(39)	0.862(1)	-0.15	0.777(46)	0.944(1)	-0.16
6.4	32	0.650(39)	0.883(1)	-0.23	0.700(42)	0.953(1)	-0.25
6.4	48	0.739(65)	0.884(1)	-0.15(7)	0.793(61)	0.954(1)	-0.16
6.4	56	0.758(52)	0.884(1)	-0.13(5)	0.827(27)	0.954(1)	-0.12
6.45	56				0.814(89)	0.954(1)	-0.14

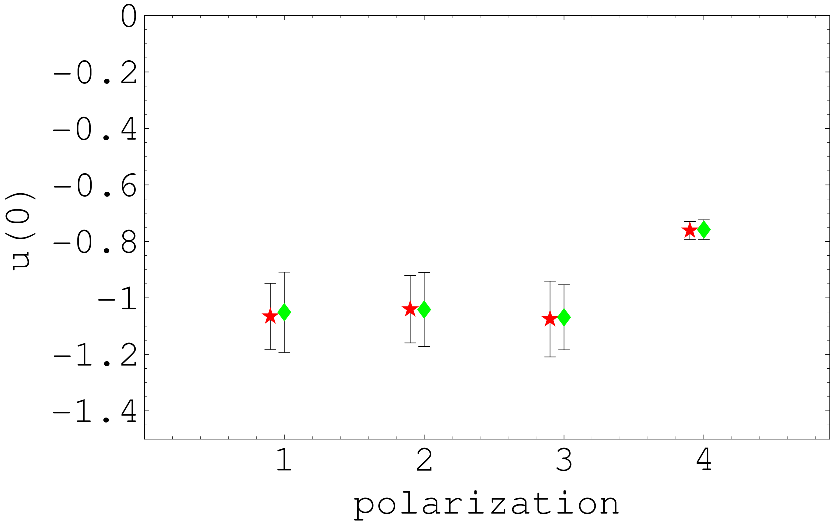
MILC configurations used in our simulation

	β_{imp}	am_{ud}^{VWI}/am_s^{VWI}	N_f	$1/a$(GeV)	L_s	L_t	aL_s(fm)
MILC$_c$	6.83	0.040/0.050	2+1	1.64	20	64	2.41
	6.76	0.007/0.050	2+1	1.64	20	64	2.41
MILC$_f$	7.11	0.0124/0.031	2+1	2.19	28	96	2.52
	7.09	0.0062/0.031	2+1	2.19	28	96	2.52
MILC$_{ft}$	5.65	0.008	2	1.716	24	12	2.76
	5.725	0.008	2	1.914	24	12	2.47
	5.85	0.008	2	2.244	24	12	2.11

The Kugo-Ojima parameter of unquenched SU(3)

Table 1:The Kugo-Ojima parameter for the polarization along the spacial directions c_x and that along the time direction c_t and the average c, trace divided by the dimension e/d, horizon function deviation h of the unquenched KS fermion (MILC$_c$,MILC$_f$,MILC$_{ft}$) with use of$\log U$ definition.

	β_{imp}	c_x	c_t	c	e/d	h
MILC$_c$	6.76	1.04(11)	0.74(3)	0.97(16)	0.9325(1)	0.03(16)
	6.83	0.99(14)	0.75(3)	0.93(16)	0.9339(1)	-0.00(16)
MILC$_f$	7.09	1.06(13)	0.76(3)	0.99(17)	0.9409(1)	0.04(17)
	7.11	1.05(13)	0.76(3)	0.98(17)	0.9412(1)	0.04(17)
MILC$_{ft}$	5.65	0.72(13)	1.04(23)	0.80(21)	0.9400(7)	-0.14
	5.725	0.68(15)	0.77(16)	0.70(15)	0.9430(2)	-0.24
	5.85	0.63(19)	0.60(12)	0.62(17)	0.9465(2)	-0.33



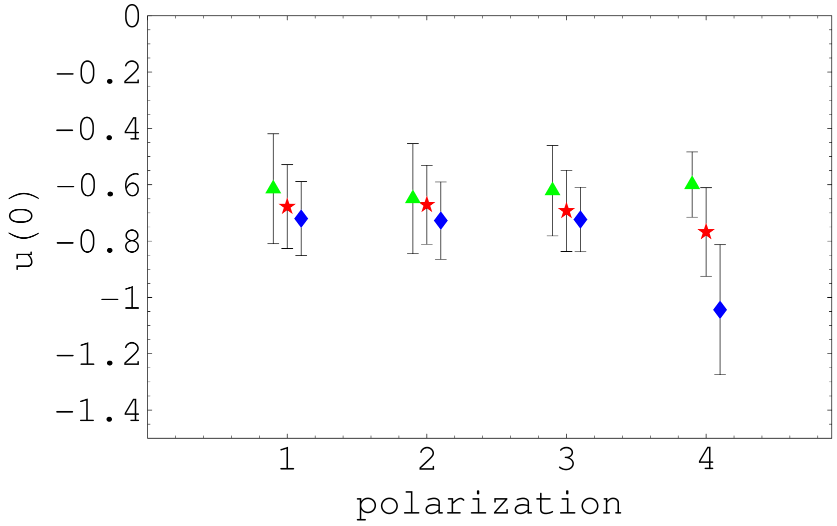


Fig. 1: Kugo-Ojima parameter $u(0)$ of MILC$_f$ $N_f = 2+1$ KS fermion unquenched configurations of $\beta_{imp} = 7.11$(green diamonds), $\beta_{imp} = 7.09$(red stars) .

Fig. 2: Kugo-Ojima parameter $u(0)$ of MILC finite temperature configurations of $\beta = 5.65$(blue diamonds), $\beta = 5.725$(red stars) and $\beta = 5.85$(green triangles).

IV. The Ghost propagator

$$\begin{aligned} FT[D_G^{ab}(x,y)] &= FT\langle tr(\Lambda^a\{(\mathcal{M}[U])^{-1}\}_{xy}\Lambda^b\rangle, \\ &= \delta^{ab}D_G(q^2), \end{aligned}$$

$$\mathcal{M} = -\partial_\mu D_\mu.$$

- Ghost dressing function $G(q^2) = q^2 D_G(q^2)$.

- Solve the equation with plane wave sources.

$$-\partial_\mu D_\mu f_s^b(\mathbf{x}) = \frac{1}{\sqrt{V}}\Lambda^b \sin \mathbf{q}\cdot\mathbf{x} \tag{1}$$

$$-\partial_\mu D_\mu f_c^b(\mathbf{x}) = \frac{1}{\sqrt{V}}\Lambda^b \cos \mathbf{q}\cdot\mathbf{x}. \tag{2}$$

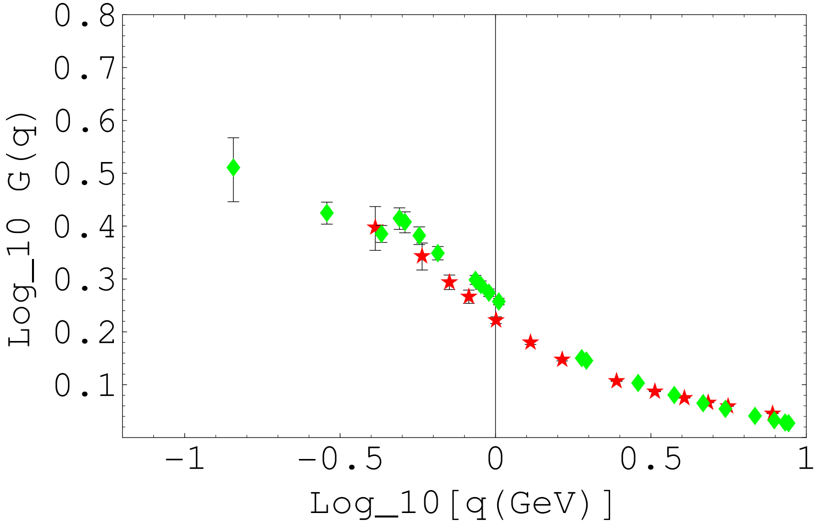
- Color diagonal ghost propagator

$$D_G(q) = \frac{1}{N_c^2 - 1}\frac{1}{V} \times\delta^{ab}(\langle\Lambda^a \cos\mathbf{q}\cdot\mathbf{x}|f_c^b(\mathbf{x})\rangle + \langle\Lambda^a \sin\mathbf{q}\cdot\mathbf{x}|f_s^b(\mathbf{x})\rangle) \tag{3}$$

- Color anti-symmetric ghost propagator

$$\phi^c(q) = \frac{1}{\mathcal{N}}\frac{1}{V} \times f^{abc}(\langle\Lambda^a \cos\mathbf{q}\cdot\mathbf{x}|f_s^b(\mathbf{x})\rangle - \langle\Lambda^a \sin\mathbf{q}\cdot\mathbf{x}|f_c^b(\mathbf{x})\rangle) \tag{4}$$

where $\mathcal{N} = 2$ for SU(2) and 6 for SU(3).



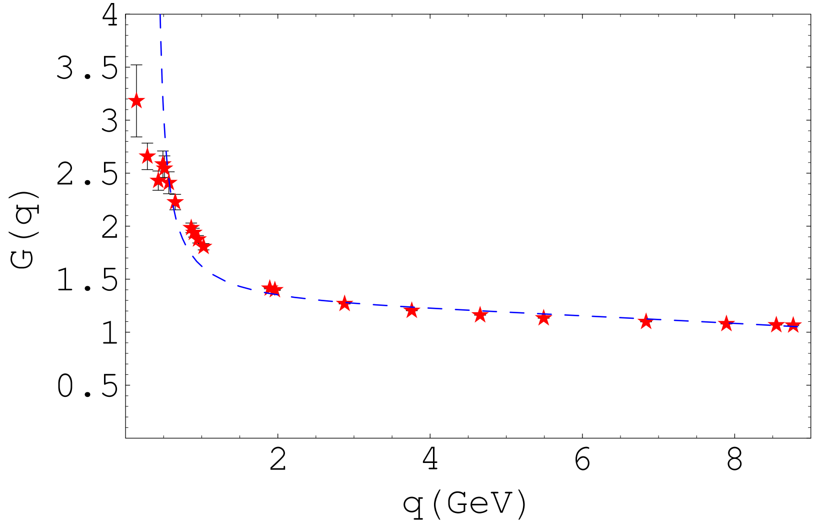


Fig. 3: Log of the ghost dressing function $\log_{10} G(q)$ as a function of $\log_{10} q$(GeV) of MILC$_f$ $\beta_{imp} = 7.09$ (diamonds) and that of quenched $\beta = 6.45$ 56^4 (stars).

Fig. 4: The ghost dressing function of MILC$_f$ $\beta_{imp} = 7.09$(stars) Dashed line is the 4-loop $N_f = 2$ pQCD result ($\lambda_G = 3.01, y = 0.0246100$)

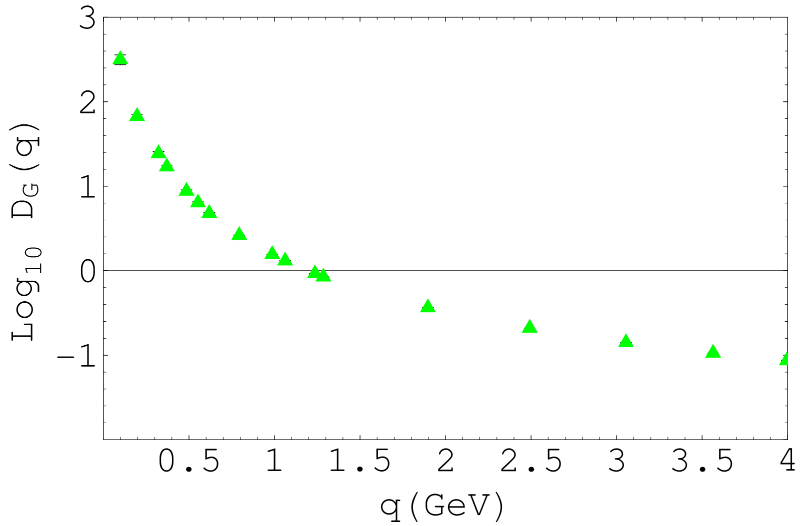


Fig. 5: Log of the color diagonal ghost propagator of $\log_{10}[D_G(q)]$ as a function of q(GeV)MILC$_c$.

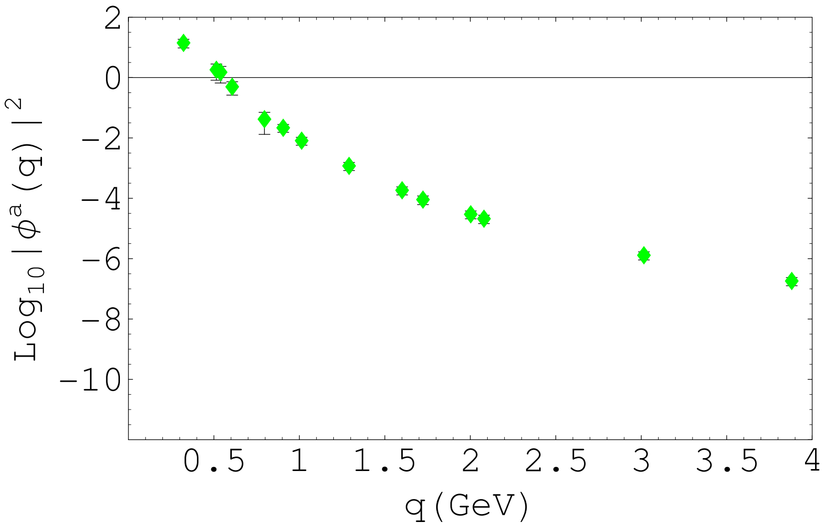


Fig. 6: Log of the color anti-symmetric ghost propagator squared $\log_{10}[\phi(q)^2]$ as a function of q(GeV). $20^3 \times 64$ MILC$_c$.

The ghost condensate

- The ghost condensate $\langle f^{abc}\bar{c}^b c^c\rangle$ in the color anti-symmetric ghost propagator is parametrized by v, r and z.

- Expression of the finite size effect in the $L_x^3 L_t$ lattice for q_μ under cylinder cut is difficult. Our choice is,

$$\frac{1}{N_c^2-1}\sum_a \frac{\sqrt{L_x^3 L_t}}{\cos(\pi\tilde{q}/8\sqrt{L_x L_t})}\langle|\phi^a(q)|\rangle = \frac{r}{q^z}, \tag{5}$$

$$\tilde{q}^2 = \sum_{i=1}^{3}(2\sin\frac{\pi\bar{q}_i}{L_x})^2 + (2\sin\frac{\pi\bar{q}_4}{L_t})^2 \tag{6}$$

$$\frac{1}{N_c^2-1}\sum_a\langle|\phi^a(q)|\rangle = \frac{r/\sqrt{L_x^3 L_t}+v}{q^4+v^2} \tag{7}$$

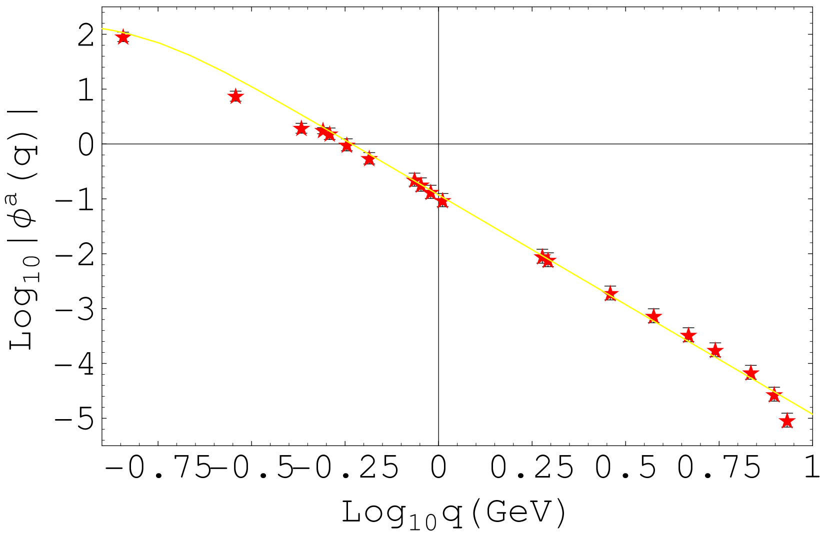


Fig. 7: The logarithm of the color antisymmetric ghost propagator of MILC$_f$ as the function of $\log_{10}(q(\mathrm{GeV}))$ and the fit using $r = 134$ and $v = 0.026\mathrm{GeV}^2$.

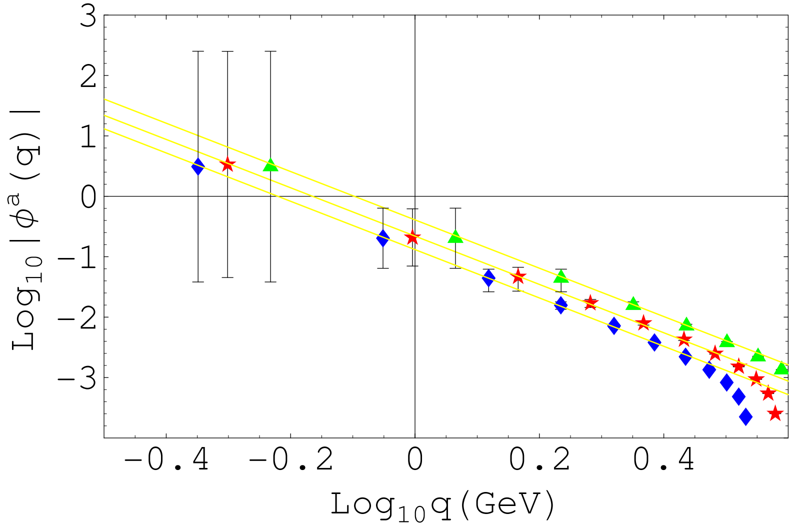


Fig. 8: The logarithm of the color antisymmetric ghost propagator of MILC$_{ft}$ of T = 143MeV(blue diamonds), T = 159.5MeV (red stars) and T = 187MeV (green triangles) and their fits.

The Binder cumulant

- Binder cumulant of the color antisymmetric ghost propagator

$$U(q) = 1 - \frac{\langle \vec{\phi}(q)^4 \rangle}{3\langle \vec{\phi}(q)^2 \rangle^2}.$$

- Suppression away from the d-dimensional gaussian distribution $\frac{\langle \vec{\phi}^4 \rangle}{(\langle \vec{\phi}^2 \rangle)^2} = \frac{d+2}{d}$ may imply presence of ghost condensate.

- Quenched SU(2) is compatible with gaussian. MILC$_c$ shows a larger randomness than the Gaussian.

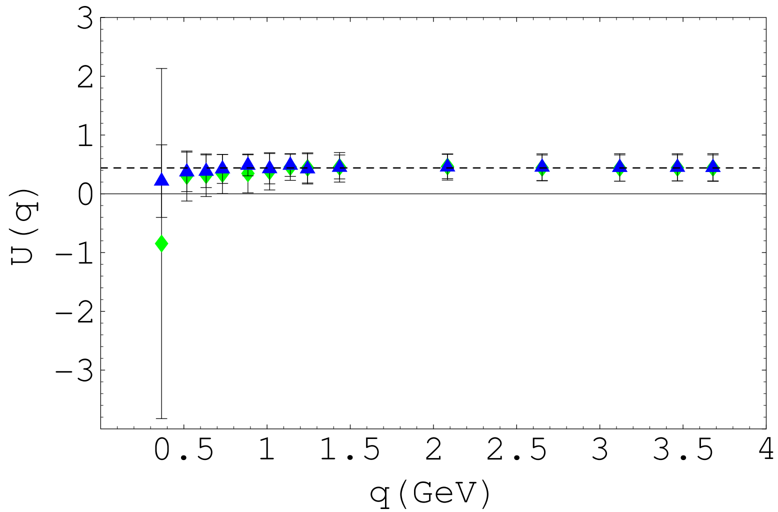


Fig. 9: The momentum dependence of Binder cumulant $U(q)$ of SU(2), $\beta = 2.2$, $a = 1.07\mathrm{GeV}^{-1}$ of PT samples(blue triangles) and first copy samples(green diamonds).

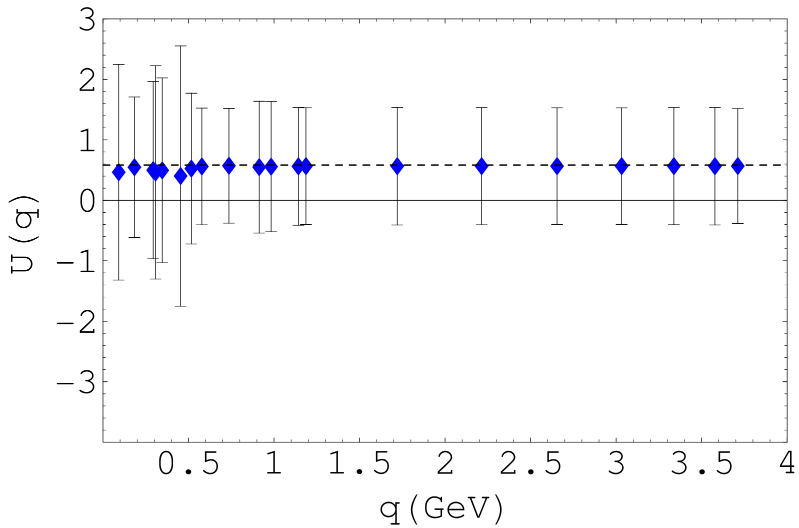


Fig.10: The momentum dependence of Binder cumulant $U(q)$ of unquenched SU(3), $a = 1.64\mathrm{GeV}^{-1}$ MILC$_c$.

Exceptional samples

- In $\beta = 6.4, 56^4$ quenched SU(3) configurations, we found a copy whose $\alpha_G = 0.272$ v.s. α_G(average)=0.223, and whose gluon propagator has an axis along which the reflection positivity is manifestly violated.

- In the $\beta_{imp} = 6.76, 20^3 \times 64$ MILC$_c$ configurations (21 samples), we find a similar exceptional sample.

- The exceptional sample makes the sample average of the Binder cumulant of color anti-symmetric ghost propagator in the infrared small, and the standard deviation large.

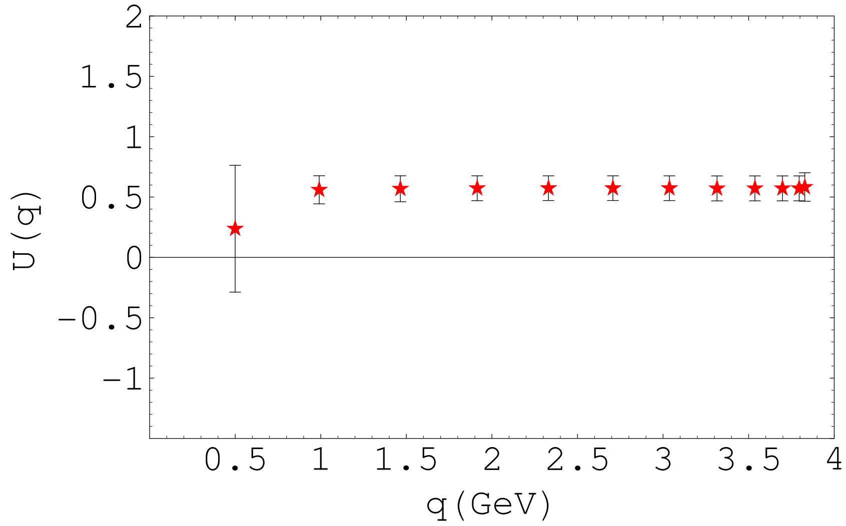


Fig.11: The Binder cumulant of the color antisymmetric ghost propagator of MILC$_{ft}$ $N_f = 2$ configurations of $\beta = 5.725$(red stars)

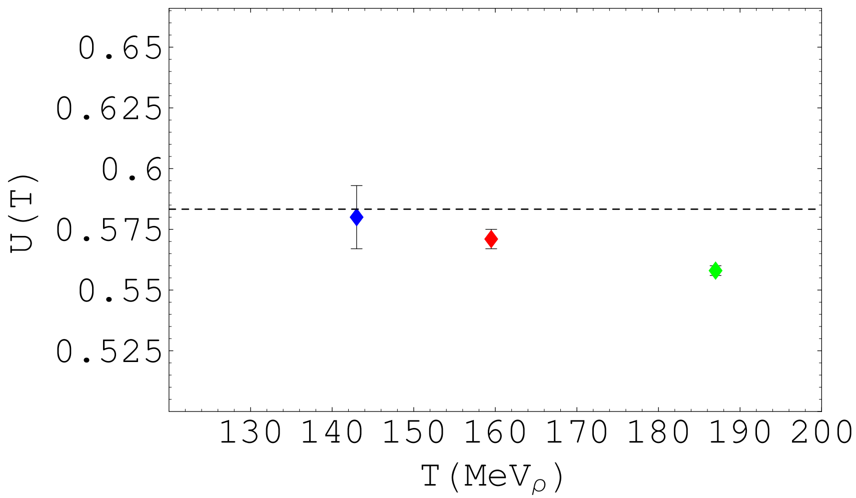


Fig.12:Averages over momenta excluding the lowest momentum point of the Binder cumulants of MILC finite temperature configurations. $\beta = 5.65$(blue diamonds), $\beta = 5.725$(red stars) and $\beta = 5.85$(green triangles).

- The fitted parameters r, z, v of $|\phi(q)|$, $\bar{r}, \bar{z}, \bar{v}$ of $\phi(q)^2$ and U of MILC$_c$ and MILC$_f$ samples. (U of MILC$_f$ corresponds to the average below $q = 1$GeV and the average above 1GeV, respectively)

β_{imp}	m_0(MeV)	r	z	v	$\bar{r}$	$\bar{z}$	$\bar{v}$	U
6.76	11.5/82.2	37.5	3.90	0.012	33.5	7.6	0.045	0.53(5)
6.83	65.7/82.2	38.7	3.85	0.007	33.5	7.6	0.048	0.57(4)
7.09	13.6/68.0	134	3.83	0.026	251	7.35	0.044	0.57(4)/0.56(1)
7.11	27.2/68.0	112	3.81	0.028	164	7.34	0.002	0.58(2)/0.52(1)

- The data of MILC$_f$ $\beta_{imp} = 7.11$ are rather noisy. $U(q)$ below 1GeV and above 1GeV are different. The anomaly is correlated with that of the dynamical quark mass function.

The infrared exponents of ghost and gluon

	β/β_{imp}	α_G	α_D	$\alpha_D + 2\alpha_G$
quench	6.40	0.22	-0.32	0.12
MILC$_c$	6.76	0.25	-0.60	-0.10
	6.83	0.23	-0.57	-0.11
MILC$_f$	7.09	0.24	-0.67	-0.19
	7.11	0.23	-0.65	-0.19

VII. The quark propagator

- The statistical average over Landau-gauge-fixed samples

$$S_{\alpha\beta}(p) = \left\langle \langle \chi_{p,\alpha} | \frac{1}{i \not{D}(U) + m} | \chi_{p,\beta} \rangle \right\rangle$$

The inversion, $\frac{1}{i \not{D}(U)+m}$, is performed via conjugate gradient method after preconditioning.

$$S_{\alpha\beta}(q) = Z_2(q) \frac{-i\gamma q + M(q)}{q^2 + M(q)^2}$$

- The mass function in large q.

$$
\begin{aligned}
M(q) &= -\frac{4\pi^2 d_M \langle\bar{\psi}\psi\rangle_\mu [\log(q^2/\Lambda_{QCD}^2)]^{d_M-1}}{3q^2[\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}} \\
&+ \frac{m(\mu^2)[\log(\mu^2/\Lambda_{QCD}^2)]^{d_M}}{[\log(q^2/\Lambda_{QCD}^2)]^{d_M}},
\end{aligned}
$$

$$d_M = 12/(33-2N_f)$$

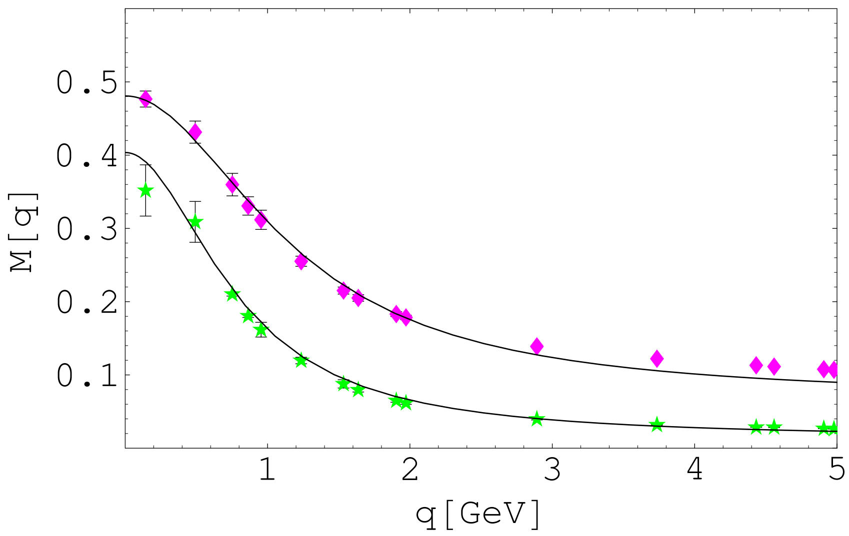
- The mass function in the infrared region.

$$M(q) = \frac{\tilde{c}\Lambda^3}{q^2+\Lambda^2} + m_0$$

The mass function

The parameters $\tilde{c}$ and Λ of the Staple+Naik action(left) and the Asqtad action.

β_{imp}	m_0(MeV)	$\tilde{c}$	Λ(GeV)	$\tilde{c}\Lambda$(GeV)	$\tilde{c}$	Λ(GeV)	$\tilde{c}\Lambda$(GeV)
6.76	11.5	0.44(1)	0.87(2)	0.38	0.45(1)	0.91(2)	0.41
	82.2	0.30(1)	1.45(2)	0.43	0.33(1)	1.36(1)	0.46
6.83	65.7	0.33(1)	1.28(2)	0.42	0.35(1)	1.25(1)	0.44
	82.2	0.30(1)	1.45(2)	0.43	0.33(1)	1.34(1)	0.45
7.09	13.6	0.45(1)	0.82(2)	0.37	0.50(2)	0.79(2)	0.39
	68.0	0.30(1)	1.27(4)	0.38	0.35(1)	1.19(1)	0.41
7.11	27.2	0.43(1)	0.89(2)	0.38	0.20(2)	1.04(3)	0.21
	68.0	0.32(1)	1.23(2)	0.40	0.36(1)	1.15(1)	0.42



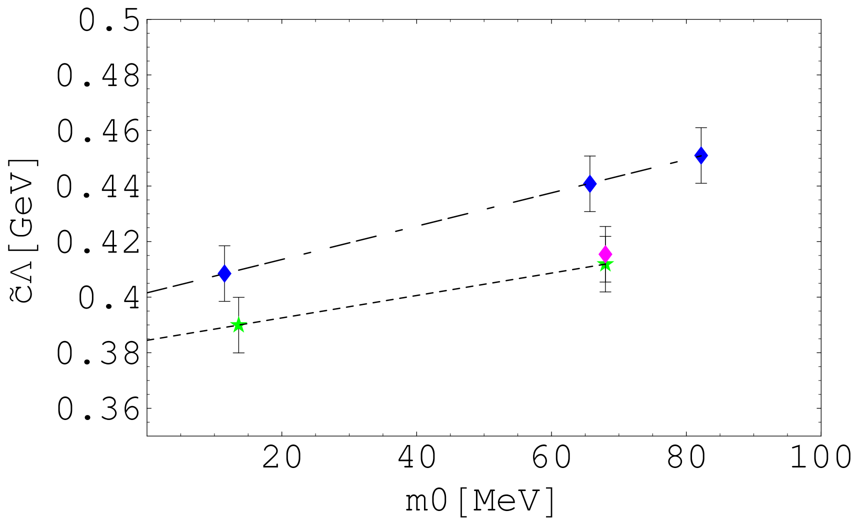


Fig.13: The mass function $M(q)$ of the Asqtad action of MILC$_f$ with the bare quark mass $m_0 = 13.6$MeV (green stars) and with the bare quark mass $m_0 = 68$MeV (red diamonds).

Fig.14: The chiral symmetry breaking mass $\tilde{c}\Lambda$ as a function of bare mass and its chiral limit. Dotted line is an extrapolation of MILC$_f$ and the dash-dotted line is that of MILC$_c$, Asqtad action.

VIII. Summary and Discussion

- The Kugo-Ojima parameter c saturated at about 0.8 in the quenched 56^4 lattice, but it is consistent with 1 in the zero temperature MILC configurations.
- The quark has the effect of quenching randomness.
- The condensate parameter v in the color anti-symmetric ghost propagator in quenched SU(2) is consistent with 0 and also small in MILC$_c$ and MILC$_f$.
- The Binder cumulants of zero temperature MILC are close to those of Gaussian distributions except the lowest momentum point.
- The Binder cumulants of MILC finite temperature($T > T_c$) show larger randomness than those of Gaussian distributions.

- The gluon propagator is infrared finite?
- Kugo-Ojima confinement criterion
  - $\dfrac{Z_1}{Z_3} = \dfrac{o((q2)^{-\alpha_D + s})}{O((q^2)^{-\alpha_D})}$ $[s > 0]$?
  - $\dfrac{\tilde{Z}_1}{\tilde{Z}_3} = \dfrac{1}{\infty}$
  - $\dfrac{Z_1^{\psi}}{Z_2} = \dfrac{o((q^2)^s)}{O(1)}$ $[s > 0]$?
- How does the true vacuum manifest itself in the $V \to$ limit of the simulation?

References

[1] T. Kugo and I. Ojima, Prog. Theor. Phys. Suppl. **66**, 1 (1979).

[2] V.N. Gribov, Nucl. Phys. **B 139**1(1978).

[3] D. Zwanziger, Nucl. Phys. **B 364** ,127 (1991), idem B **412**, 657 (1994).

[4] P.O. Bowman et al., Phys. Rev. D**71**,054507(2005).

[5] A. Cucchieri, Nucl. Phys. **B521**,365(1998).

[6] A. Cucchieri, T. Mendes and A. Mihara, Phys. Rev. D**72**,094505(2004).

[7] Ph. Boucaud et al., hep-lat/0504017.

[8] S. Furui and H.Nakajima, Phys. Rev. D**69**,074505(2004), hep-lat/0305010.

[9] S. Furui and H.Nakajima, Phys. Rev. D**70**,094504(2004), hep-lat/0403021.

[10] S. Furui and H. Nakajima, Few Body Systems**42**(2006), hep-lat/0503029.

[11] H. Nakajima and S. Furui, PoS **LAT2005** 302 (2005),hep-lat/0509033.

[12] S. Furui and H. Nakajima, PoS **LAT2005** 291 (2005),hep-lat/0509035.

[13] S. Furui and H. Nakajima, Phys. Rev. D**73**,074503(2006),hep-lat/0511045.

[14] S. Furui and H. Nakajima, Phys. Rev. D**73**,094506(2006),hep-lat/0602027.