Twist Field as Three String Interaction Vertex in Light Cone Superstring Field Theory

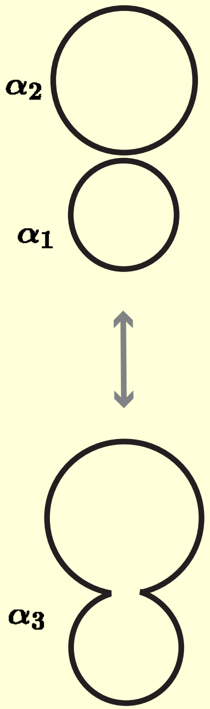
Isao Kishimoto

collaboration with **S. Moriyama, S. Teraguchi**

KMT,NPB744(2006)221; KM,hep-th/061

I. Introduction and motivation

LCSFT and MST seem to be closely related in a sense. We concentrate on their interactions.



Joining/splitting interaction

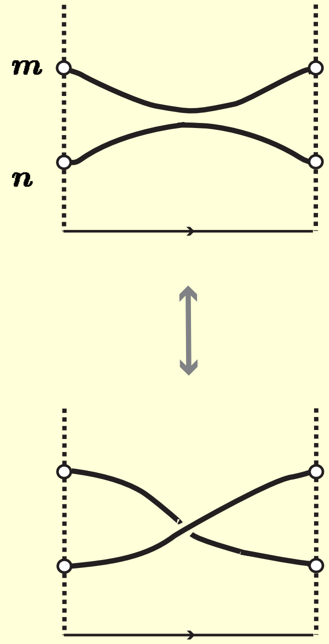
Green-Schwarz-Brink's light-cone superstring field theory (LCSFT)

3-string vertex

Given by oscillators explicitly:

$$
\begin{aligned}
|H_1\rangle &= \tilde{Z}^i Z^j v^{ij}(Y)|v_3\rangle \\
|Q_1^{\dot{a}}\rangle &= \tilde{Z}^i s^{i\dot{a}}(Y)|v_3\rangle, \\
|\tilde{Q}_1^{\dot{a}}\rangle &= Z^i \tilde{s}^{i\dot{a}}(Y)|v_3\rangle.
\end{aligned}
$$

$$
\begin{aligned}
\partial X^i(\sigma)|v_3\rangle &\sim |\sigma-\sigma_{\rm int}|^{-\frac{1}{2}} Z^i |v_3\rangle\,, \\
\bar{\partial} X^i(\sigma)|v_3\rangle &\sim |\sigma-\sigma_{\rm int}|^{-\frac{1}{2}} \tilde{Z}^i |v_3\rangle\,, \\
\lambda^a(\sigma)|v_3\rangle &\sim |\sigma-\sigma_{\rm int}|^{-\frac{1}{2}} Y^a |v_3\rangle\,.
\end{aligned}
$$



Exchange of eigen values

Dijkgraaf-Verlinde-Verlinde's matrix string theory (MST)

twist/spin field

$$(\tau^i \Sigma^i \tilde{\tau}^j \tilde{\Sigma}^j)_{mn}$$

OPEs:

$$
\begin{aligned}
\partial X^i(z)\sigma\tilde{\sigma}(0) &\sim z^{-\frac{1}{2}}\tau^i\tilde{\sigma}(0), \\
\bar{\partial} X^i(\bar{z})\sigma\tilde{\sigma}(0) &\sim \bar{z}^{-\frac{1}{2}}\sigma\tilde{\tau}^i(0), \\
\theta^a(z)\Sigma^i(0) &\sim z^{-\frac{1}{2}}\frac{1}{\sqrt{2i}}\gamma^i_{a\dot{a}}\Sigma^{\dot{a}}(0),
\end{aligned}
$$

$$
\begin{aligned}
&\Sigma^i(z)\Sigma^j(0) \sim z^{-1}\delta^{ij}, \\
&\Sigma^{\dot{a}}(z)\Sigma^{\dot{b}}(0) \sim z^{-1}\delta^{\dot{a}\dot{b}}, \\
&\Sigma^i(z)\Sigma^{\dot{a}}(0) \sim z^{-\frac{1}{2}}\frac{1}{\sqrt{2i}}\gamma^i_{c\dot{a}}\theta^c(0), \cdots.
\end{aligned}
$$

With an identification: $|v_3\rangle \leftrightarrow \sigma\tilde{\sigma}, \quad |R\rangle \leftrightarrow 1,$

in the bosonic sector, we expect the OPE correspond to

$$
\begin{aligned}
\langle R|v_3\rangle|v_3\rangle &\sim (\cdots)|R\rangle|R\rangle, \\
\langle R|\langle R|v_3\rangle|v_3\rangle &\sim (\cdots)|R\rangle.
\end{aligned}
$$

? ⟵ - - - - ⟶

$$\sigma\tilde{\sigma}(z,\bar{z})\cdot\sigma\tilde{\sigma}(0) \sim \left[\frac{1}{|z|^{1/4}(\ln|z|)^{1/2}}\right]^{d-2}$$

Investigations of the correspondence will be useful for developing both LCSFT and MST.

II. LCSFT/MST correspondence

More precise correspondence at least at the linear level with respect to $|v_3\rangle$: [Dijkgraaf-Motl]

<u>*bosonic sector*</u>

$$\begin{aligned} \tilde{Z}^i Z^j |v_3\rangle &\leftrightarrow \tau^j \tilde{\tau}^i \\ Z^i |v_3\rangle &\leftrightarrow \tau^i \tilde{\sigma} \\ \tilde{Z}^i |v_3\rangle &\leftrightarrow \sigma \tilde{\tau}^i \end{aligned}$$

<u>*ferminic sector*</u>

$$\begin{aligned} v^{ij}(Y)|v_3\rangle &\leftrightarrow \Sigma^j \tilde{\Sigma}^i \\ s^{i\dot{a}}(Y)|v_3\rangle &\leftrightarrow \Sigma^{\dot{a}} \tilde{\Sigma}^i \\ \tilde{s}^{i\dot{a}}(Y)|v_3\rangle &\leftrightarrow \Sigma^i \tilde{\Sigma}^{\dot{a}} \\ m^{\dot{a}\dot{b}}(Y)|v_3\rangle &\leftrightarrow \Sigma^{\dot{a}} \tilde{\Sigma}^{\dot{b}} \end{aligned}$$

$$\begin{aligned} |v_3\rangle \;=\; & (2\pi)^9 \delta(\alpha_1+\alpha_2+\alpha_3)\delta^8(p_1^i+p_2^i+p_3^i)\delta^8(\lambda_1^a+\lambda_2^a+\lambda_3^a) \\ & \times e^{\frac{1}{2}(a^{(r)\dagger}\tilde{N}^{rs}a^{(s)\dagger}+\tilde{a}^{(r)\dagger}\tilde{N}^{rs}\tilde{a}^{(s)\dagger})+\tilde{N}^r(a^{(r)\dagger}+\tilde{a}^{(r)\dagger})\mathrm{P}-\frac{\tau_0}{\alpha_{123}}\mathrm{P}^2} \\ & \times e^{\sum Q_{-n}^{\mathrm{II}(r)}\alpha_r^{-1}n^{\frac{1}{2}}\tilde{N}_{nm}^{rs}m^{-\frac{1}{2}}Q_{-m}^{\mathrm{I}(s)}-\sqrt{2}\Lambda\sum\alpha_r^{-1}n^{\frac{1}{2}}\tilde{N}_n^r Q_{-n}^{\mathrm{II}(r)}}|0\rangle\,. \end{aligned}$$

$$\begin{aligned} \tilde{\mathrm{Z}}^i &= \mathrm{P}^i - \alpha_{123}\sum\alpha_r^{-1}n\tilde{N}_n^r\tilde{a}_n^{(r)\dagger i}, \\ \mathrm{Z}^j &= \mathrm{P}^j - \alpha_{123}\sum\alpha_r^{-1}n\tilde{N}_n^r a_n^{(r)\dagger j}, \\ Y^a &= \Lambda^a - \frac{\alpha_{123}}{\sqrt{2}}\sum\alpha_r^{-1}n^{\frac{1}{2}}\tilde{N}_n^r Q_{-n}^{\mathrm{I}(r)a}. \end{aligned}$$

$$\alpha_{123} = \alpha_1\alpha_2\alpha_3, \quad P^i = \alpha_1 p_2^i - \alpha_2 p_1^i, \quad \Lambda^a = \alpha_1\lambda_2 - \alpha_2\lambda_1.$$

We found a concise expression of the prefactors: [KM]

$$e^{\not{Y}} = \left[e^{\not{Y}}\right]_{(i,\dot{a}),(j,\dot{b})} = \begin{pmatrix} [\cosh\not{Y}]_{ij} & [\sinh\not{Y}]_{i\dot{b}} \\ [\sinh\not{Y}]_{\dot{a}j} & [\cosh\not{Y}]_{\dot{a}\dot{b}} \end{pmatrix} = \begin{pmatrix} v^{ij}(Y) & i(-\alpha_{123})^{-\frac{1}{2}}s^{i\dot{b}}(Y) \\ (-\alpha_{123})^{-\frac{1}{2}}\tilde{s}^{j\dot{a}}(Y) & m^{\dot{b}\dot{a}}(Y) \end{pmatrix},$$

$$\not{Y} \equiv \left(\frac{2i}{-\alpha_{123}}\right)^{\frac{1}{2}} Y^a\widehat{\gamma}^a, \qquad \widehat{\gamma}^a = (\widehat{\gamma}^a)_{(i,\dot{a}),(j,\dot{b})} = \begin{pmatrix} 0 & \gamma_{a\dot{b}}^i \\ \gamma_{a\dot{a}}^j & 0 \end{pmatrix}, \qquad \widehat{\gamma}^a\widehat{\gamma}^b + \widehat{\gamma}^b\widehat{\gamma}^a = 2\delta^{ab}1_{16}\,.$$

With this prefactor, the space-time SUSY algebra is satisfied at the linear level:

$$Q_0^{\dot{a}}|Q_1^{\dot{b}}\rangle + Q_0^{\dot{b}}|Q^{\dot{a}}\rangle = \tilde{Q}_0^{\dot{a}}|\tilde{Q}_1^{\dot{b}}\rangle + \tilde{Q}_0^{\dot{b}}|\tilde{Q}_1^{\dot{a}}\rangle = 2|H_1\rangle\delta^{\dot{a}\dot{b}}, \quad Q_0^{\dot{a}}\mathcal{P}|\tilde{Q}_1^{\dot{b}}\rangle + \tilde{Q}_0^{\dot{b}}\mathcal{P}|Q_1^{\dot{a}}\rangle = 0.$$

What about the correspondence at the quadratic level with respect to $|v_3\rangle$?

III. Contractions in bosonic LCSFT [KMT]

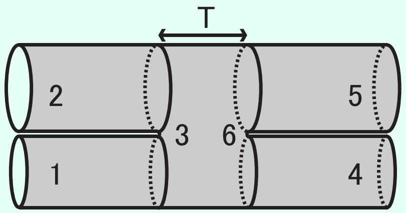
At the quadratic level, we have explicitly computed two types of contractions.

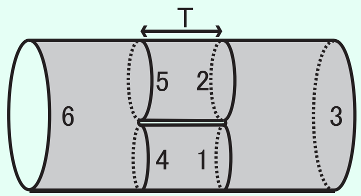
$\langle R|v_3\rangle|v_3\rangle \propto |R\rangle|R\rangle$

$\langle R|\langle R|v_3\rangle|v_3\rangle \propto |R\rangle$

can be proved by $\sum_{l,t}\tilde{N}^{rt}_{nl}\tilde{N}^{ts}_{lm}=\delta^{nm}\delta^{rs},\quad \sum_{l,t}\tilde{N}^{rt}_{nl}\tilde{N}^{t}_{l}=-\tilde{N}^{r}_{n},\quad \sum_{l,t}\tilde{N}^{t}_{l}\tilde{N}^{t}_{l}=(\alpha_{123})^{-1}2\tau_0.$

In order to evaluate the divergent coefficients, we regularize them by T :





Using the Cremmer-Gervais identity, we evaluated as

$$\left|\det{}^{-\frac{d-2}{2}}(1-\tilde{N}^{33}_{T/2}\tilde{N}^{33}_{T/2})\right|^2 \sim T^{-\frac{d-2}{4}}$$

$$e^{-\alpha_3^2\tilde{N}^3_{T/2}(1-\tilde{N}^{33}_{T/2}\tilde{N}^{33}_{T/2})^{-1}\tilde{N}^3_{T/2}(p_1+p_4)^2} \sim (\log T)^{-\frac{d-2}{2}}\delta^{d-2}(p_1+p_4)$$

We computed the 2-tachyon diagram at the one loop level using the Mandelstam map:

$$\rho(u) = |\alpha_3|(\log\vartheta_1(u-U_6|\tau)-\log\vartheta_1(u-U_3|\tau))-2\pi i\alpha_1 u,$$

$$T = \rho(u_-)-\rho(u_+),\quad \frac{d\rho}{du}(u_\pm)=0,$$

where the torus modulus is $e^{-\frac{i\pi}{\tau}} \sim \dfrac{T}{8|\alpha_3|\sin(\pi\alpha_1/|\alpha_3|)},\quad (T\to+0).$

The results:

$$\langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)}+\tilde{L}_0^{(3)})}|v_3\rangle|v_3\rangle \sim \left[T(\log T)^2\right]^{-\frac{d-2}{4}}|R\rangle|R\rangle,\quad \langle R|\langle R|e^{-\frac{T}{\alpha_1}(L_0^{(1)}+\tilde{L}_0^{(1)})}e^{-\frac{T}{\alpha_2}(L_0^{(2)}+\tilde{L}_0^{(2)})}|v_3\rangle \sim \left[T(\log T)^2\right]^{-\frac{d-2}{4}}|R\rangle.$$

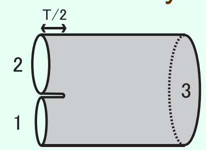
They correspond to the OPE of the twist field!

Note : We fixed α_r $(\alpha_4=-\alpha_1,\alpha_5=-\alpha_2)$. ⟷ In OPEs in MST, we fixed (m,n) (label of string bits).

Comment : In the HIKKO closed SFT (d=26), the coefficient of the idempotency relation for the boundary states is roughly a square root of the above:

$$|B\rangle_{\alpha_1} *_T |B\rangle_{\alpha_2} \sim |\alpha_{123}|\,T^{-3}\,|B\rangle_{\alpha_1+\alpha_2}$$

[I.K.-Matsuo-Watanabe, I.K.-Matsuo]



IV. Contractions in GSB LCSFT [KM]

Here we consider the fermionic sector of GSB LCSFT.

Without the prefactors, $\langle R|v_3\rangle|v_3\rangle \propto |R\rangle|R\rangle,\quad \langle R|\langle R|v_3\rangle|v_3\rangle \propto |R\rangle$ can be shown similarly *except for fermion zero mode*.

For computation of the prefactors including fermion zero mode, we have used Fourier transform and Fierz identities such as

$$[\cosh \not{Y}]_{ij}[\cosh \not{Y}]_{lk} = 2^{-4}\sum_{p=0}^{4}\frac{(-1)^p}{(2p)!}\hat{\gamma}_{ik}^{a_1\cdots a_{2p}}(\cosh \not{Y}\,\hat{\gamma}^{a_1\cdots a_{2p}}\cosh \not{Y})_{lj} = \delta_{ik}\delta_{jl}\left(\frac{4}{\alpha_{123}}\right)^4\delta^8(Y)+\mathcal{O}(Y^6),\cdots.$$

The results:

"H_1H_1":

$$\langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)}+\tilde{L}_0^{(3)})}v^{ij}|v_3\rangle v^{kl}|v_3\rangle \sim \delta^{ik}\delta^{jl}T^{-2}|R\rangle|R\rangle$$

$$\langle R|\langle R|e^{-\frac{T}{\alpha_1}(L_0^{(1)}+\tilde{L}_0^{(1)})}e^{-\frac{T}{\alpha_2}(L_0^{(2)}+\tilde{L}_0^{(2)})}v^{ij}|v_3\rangle v^{kl}|v_3\rangle \sim \delta^{ik}\delta^{jl}T^{-2}|R\rangle$$

$$\longleftrightarrow \quad \Sigma^j\tilde{\Sigma}^i(z,\bar{z})\Sigma^l\tilde{\Sigma}^k(0)\sim\frac{\delta^{ik}\delta^{jl}}{|z|^2}$$

"$Q_1^{\dot{a}}Q_1^{\dot{b}}$":

$$\langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)}+\tilde{L}_0^{(3)})}s^{i\dot{a}}|v_3\rangle s^{j\dot{b}}|v_3\rangle \sim \delta^{ij}\delta^{\dot{a}\dot{b}}T^{-2}|R\rangle|R\rangle$$

$$\langle R|\langle R|e^{-\frac{T}{\alpha_1}(L_0^{(1)}+\tilde{L}_0^{(1)})}e^{-\frac{T}{\alpha_2}(L_0^{(2)}+\tilde{L}_0^{(2)})}s^{i\dot{a}}|v_3\rangle s^{j\dot{b}}|v_3\rangle \sim \delta^{ij}\delta^{\dot{a}\dot{b}}T^{-2}|R\rangle$$

$$\longleftrightarrow \quad \Sigma^{\dot{a}}\tilde{\Sigma}^i(z,\bar{z})\Sigma^{\dot{b}}\tilde{\Sigma}^j(0)\sim\frac{\delta^{ij}\delta^{\dot{a}\dot{b}}}{|z|^2}$$

"$H_1Q_1^{\dot{a}}$":

$$\langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)}+\tilde{L}_0^{(3)})}v^{ij}|v_3\rangle s^{k\dot{a}}|v_3\rangle \sim \delta^{ik}T^{-\frac{3}{2}}\gamma^j_{c\dot{a}}(\vartheta^c_{(2)}-\vartheta^c_{(1)})(\sigma_{\rm int})|R\rangle|R\rangle \longleftrightarrow \Sigma^j\tilde{\Sigma}^i(z,\bar{z})\Sigma^{\dot{a}}\tilde{\Sigma}^k(0)\sim\frac{1}{z^{\frac{1}{2}}\bar{z}}\frac{\delta^{ik}}{\sqrt{2i}}\gamma^j_{c\dot{a}}\,\theta^c(0)$$

"$Q_1^{\dot{a}}\tilde{Q}_1^{\dot{b}}$":

$$\langle R|e^{-\frac{T}{|\alpha_3|}(L_0^{(3)}+\tilde{L}_0^{(3)})}s^{i\dot{a}}|v_3\rangle \tilde{s}^{j\dot{b}}|v_3\rangle \sim T^{-1}\gamma^j_{c\dot{a}}(\vartheta^c_{(2)}-\vartheta^c_{(1)})\gamma^i_{d\dot{b}}(\vartheta^d_{(2)}-\vartheta^d_{(1)})(\sigma_{\rm int})|R\rangle|R\rangle \longleftrightarrow \Sigma^{\dot{a}}\tilde{\Sigma}^i(z,\bar{z})\Sigma^j\tilde{\Sigma}^{\dot{b}}(0)\sim\frac{1}{2|z|}\gamma^j_{c\dot{a}}\,\theta^c\gamma^i_{d\dot{b}}\tilde{\theta}^d(0)$$

The correspondence is consistent with OPEs of the spin fields!

V. Future directions

- More detailed correspondence? $(\alpha_r,\mathcal{P}_r)\leftrightarrow(m,n,\int d\sigma,N),\cdots.$
- Higher order terms of GSB SFT and MST? $H=H_0+g_sH_1+g_s^2H_2+\cdots,\quad Q^{\dot{a}}=Q_0^{\dot{a}}+g_sQ_1^{\dot{a}}+g_s^2Q_2^{\dot{a}}+\cdots,$
  $\{Q^{\dot{a}},Q^{\dot{b}}\}=\{\tilde{Q}^{\dot{a}},\tilde{Q}^{\dot{b}}\}=2H\delta^{\dot{a}\dot{b}},\quad [Q^{\dot{a}},H]=[\tilde{Q}^{\dot{a}},H]=\{Q^{\dot{a}},\tilde{Q}^{\dot{b}}\}=0.$
- pp-wave background?
- Covariantized GSB SFT ? (using ``pure spinor"?),…