THE LO HADRONIC CONTRIBUTION TO $(g-2)_\mu$

K. Maltman, DPF06/JPS06 Meeting, Honolulu

OUTLINE

- *Background (dispersive formulation, role in SM expectation)*
- *Current status of EM and/vs. EM+τ evaluations*
- *OPE constraints and the EM-τ puzzle*
- *Prospects for the near future*

BASICS

- $a_\mu \equiv \frac{(g-2)_\mu}{2}$ known to 0.5 ppm (BNL E821 $\mu^\pm$ average)

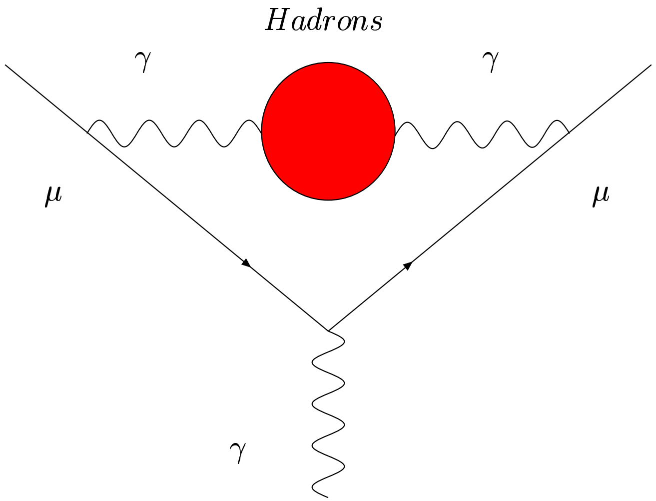
- pure QED contributions dominant: known to 4-loops (plus all 2958 enhanced among 9080 5-loop diagrams, with full 5-loop calculation in progress!) [M. Nio, Tau'06]

- next in size: LO hadronic vacuum polarization contribution $[a_\mu]^{had,LO}$

- $[a_\mu]^{had,LO}$ (at present) not computable from first principles, but related to EM hadroproduction cross-sections

$$\boxed{[a_\mu]^{had,LO} = \frac{\alpha_{EM}^2}{3\pi^2} \int_{4m_\pi^2}^{\infty} ds \, \frac{K(s)}{s} \, R(s)}$$

with $K(s)$ known, $R(s) = 3s\,\sigma[e^+e^- \to hadrons]\,/\,16\pi\,\alpha_{EM}^2$



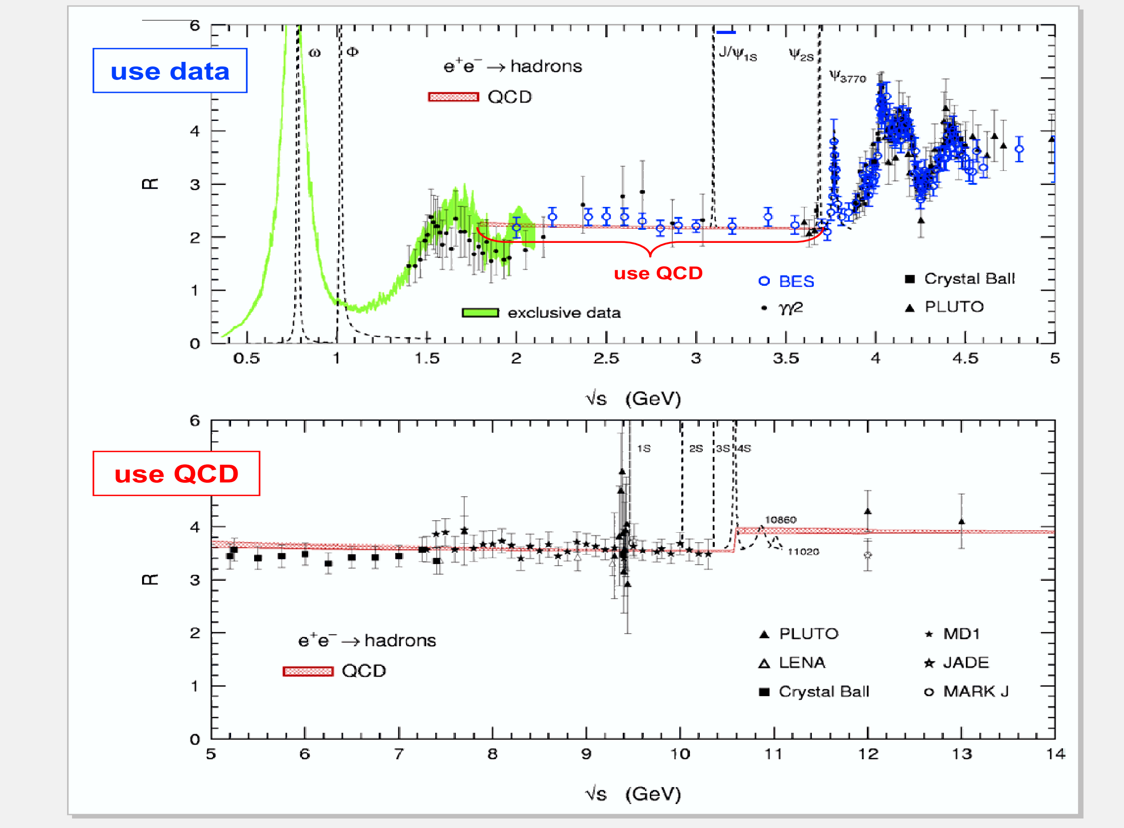
ANATOMY OF THE SM PREDICTION FOR a_μ

(see M. Passera: hep-ph/0411168)

Source	$\delta(a_\mu) \times 10^{10}$
QED	1165847.88 (3)(4)
LO had VP	$\sim$ 700 (6 $\rightarrow$ 8)??
EW	15.4 (1)(2)
HO had LBL	13.6 (2.5)
HO had VP	$-$9.79 (9)
Exp. μ^+	11659203 (8)
Exp. μ^-	11659214 (9)
Exp. $\mu^\pm$ ave	11659208 (6)

$\Rightarrow$ $[a_\mu]^{had,LO}$ has dominant impact on central value and uncertainty of SM prediction

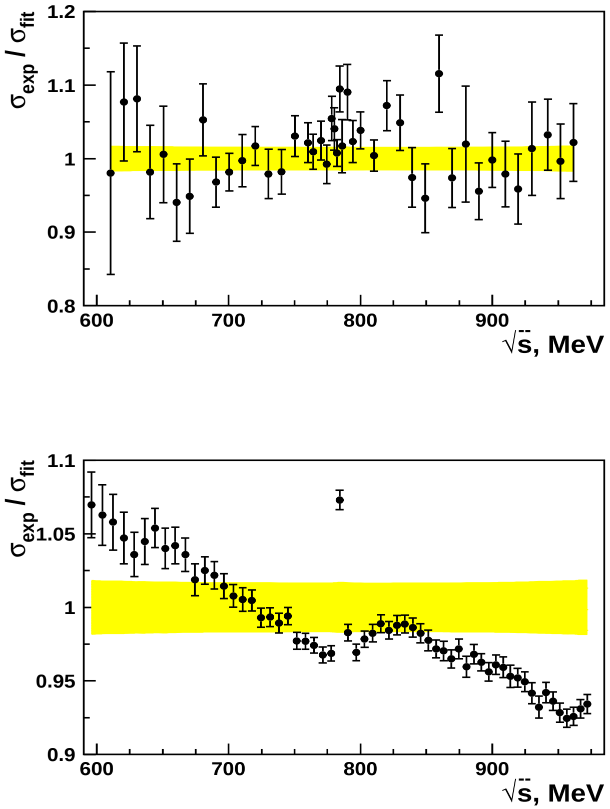
THE DISPERSIVE EVALUATION OF $[a_\mu]^{had,LO}$



- $K(s)/s = f(s)/s^2$ with $f(s)$ slowly varying ⇒ low E states ($\pi\pi$) dominant (see also Table)

- Recent EM data ($s < 1.8$ GeV) since DEHZ03
  - (corrected) SND, (corrected) CMD2 $\pi\pi$ now agree (including increased statistics hep-ex/0610021 CMD2 results) [Figure]
  - KLOE, CMD2/SND $\pi\pi$ *DISAGREE* [Figure]
  - other 2004+ small $[a_\mu]^{had,LO}$ contribution modes: CMD-2 ($\pi^0\gamma$, $\eta\gamma$, 3π, $2\pi^+2\pi^-$), SND ($\eta\gamma$), BABAR (3π, $2\pi^+2\pi^-$, 6π, $K^+K^-\pi^+\pi^-$)

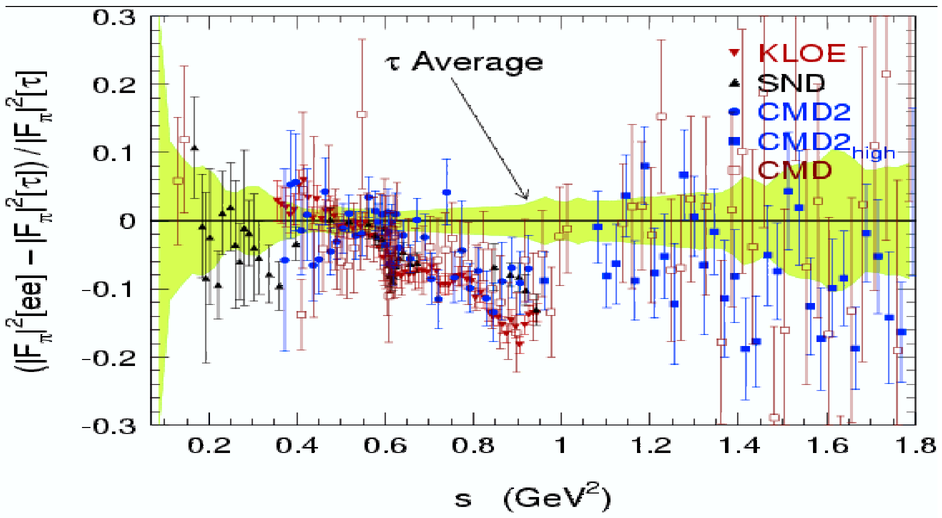
The EM $\pi\pi$ Data Situation



SND fit c.f. CMD2 (top), KLOE (bottom)

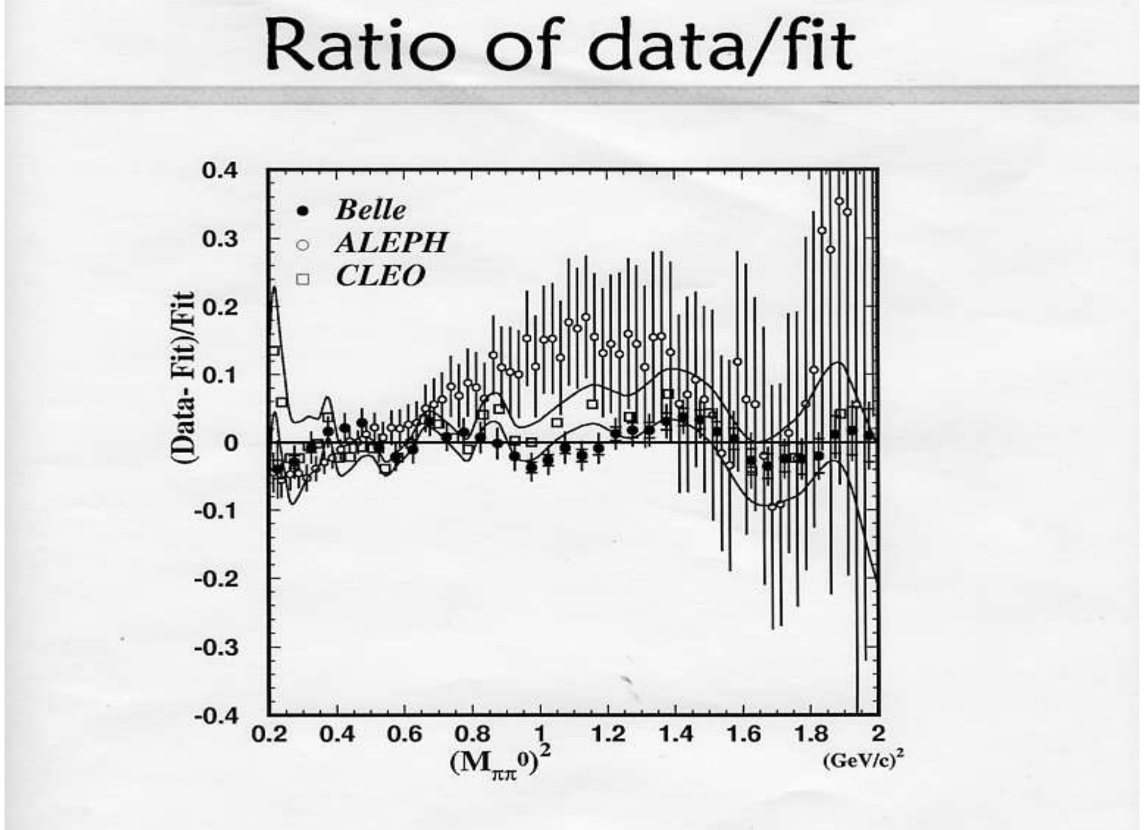
- CVC (+ IB corrections) $\Rightarrow$ alternate version of $I = 1$ contribution from non-strange hadronic τ decay data
  - IB corr'ns: $\pi\pi$: short-distance EW, $m_{\pi^\pm} - m_{\pi^0} \neq$, long-distance EM, ρ-ω mixing; 4π: only first two
  - EM, IB-corrected τ disagree (Table, Figure)
  - ALEPH, preliminary BELLE $\tau^- \to \pi^-\pi^0\nu_\tau$ differ [Figure]
  - HOWEVER, BELLE $[a_\mu]^{had,LO}_{\pi\pi}$, $B\left[\tau \to \pi^-\pi^0\nu_\tau\right] \equiv B_{\pi\pi}$ consistent with LEP, CLEO, *INCONSISTENT* with EM [Table]

EM vs τ $\pi\pi$



Preliminary BELLE τ $\pi\pi$ Data

(M. Fujikawa, Tau'06)



Ratio of data/fit

Eidelman's ICHEP'06 $[a_\mu]^{had,LO}$ Update

Contributions to a_μ^{had} [in 10^{-10}] from the different energy domains

Modes	Energy [GeV]	e^+e^-	τ
Low s expansion	$2m_\pi - 0.5$	$55.6 \pm 0.8 \pm 0.1_{rad}$	$56.0 \pm 1.6 \pm 0.3_{SU(2)}$
$\pi^+\pi^-$ **(+SND+CMD2)**	$0.5 - 1.8$	$449.0 \pm 3.0 \pm 0.9_{rad}$	$464.0 \pm 3.0 \pm 2.3_{SU(2)}$
$\pi^+\pi^- 2\pi^0$	$2m_\pi - 1.8$	$16.8 \pm 1.3 \pm 0.2_{rad}$	$21.4 \pm 1.3 \pm 0.6_{SU(2)}$
$2\pi^+ 2\pi^-$ **(+BaBar)**	$2m_\pi - 1.8$	$13.1 \pm 0.4 \pm 0.0_{rad}$	$12.3 \pm 1.0 \pm 0.4_{SU(2)}$
ω **(782)**	$0.3 - 0.81$	$38.0 \pm 1.0 \pm 0.3_{rad}$	–
ϕ **(1020)**	$1.0 - 1.055$	$35.7 \pm 0.8 \pm 0.2_{rad}$	–
Other excl. (+BaBar)	$2m_\pi - 1.8$	$24.3 \pm 1.3 \pm 0.2_{rad}$	–
J/ψ, ψ**(2S)**	$3.08 - 3.11$	$7.4 \pm 0.4 \pm 0.0_{rad}$	–
R **[QCD]**	$1.8 - 3.7$	$33.9 \pm 0.5_{theo}$	–
R **[data]**	$3.7 - 5.0$	$7.2 \pm 0.3 \pm 0.0_{rad}$	–
R **[QCD]**	$5.0 - \infty$	$9.9 \pm 0.2_{theo}$	–
Sum (w/o KLOE)	$2m_\pi - \infty$	$690.8 \pm 3.9 \pm 1.9_{rad} \pm 0.7_{QCD}$	$710.1 \pm 5.0 \pm 0.7_{rad} \pm 2.8_{SU(2)}$

MORE ON THE EM-τ DISCREPANCY

- $B^\tau_{\pi\pi}$ less sensitive to unfolding than $s_{\pi\pi}$ distribution

Source	$B^\tau_{\pi\pi}$
BELLE	$0.2515 \pm 0.0004 \pm 0.0031$
ALEPH	$0.2547 \pm 0.0010 \pm 0.0009$
CLEO	$0.2542 \pm 0.0012 \pm 0.0042$
DELPHI	$0.2529 \pm 0.0020 \pm 0.0014$
OPAL	$0.2544 \pm 0.0017 \pm 0.0029$
τ (ave)	0.2540 ± 0.0010
EM (+ IB)	0.2448 ± 0.0018

4.5σ discrepancy between τ results and EM (+ IB) expectation!!

- Similarly, isospin relations (+ IB corrections) for EM, τ 4π modes ⇒ EM expectations for $B[\tau \to 4\pi\nu_\tau]$

Mode	$[\Delta B_{4\pi}]_{\tau - e^+e^-}$
$\pi^- 3\pi^0 \nu_\tau$	-0.0008 ± 0.0011
$2\pi^-\pi^+\pi^0\nu_\tau$	0.0091 ± 0.0025
$\pi^-\pi^0\nu_\tau$	0.0092 ± 0.0021

- $\left[a_\mu^{exp} - a_\mu^{SM}\right]_{EM}^{no\ KLOE} \times 10^{10} = 27.5 \pm 8.6,$

  $\left[a_\mu^{exp} - a_\mu^{SM}\right]_{\tau}^{no\ BELLE} \times 10^{10} = 12.2 \pm 9.3$

Source	$\left[10^{10}\, a_\mu - 11659000\right]_{SM}$
DEHZ06 (e^+e^-)	$180.5 \pm 5.6^*$
DEHZ03/06 (τ)	195.6 ± 6.8
BNL E821 ($\mu^\pm$)	208.0 ± 6.3

NOTES/COMMENTS/CAUTIONS

- DEHZ06 EM averages only CMD2 and SND $\rho_{EM}(s)$, neglects KLOE (NOT a conservative approach)

- pQCD from 1.8 GeV to J/ψ (how reliable?) (BES $R(s)$ data $\Rightarrow$ effect $< O(2-3) \times 10^{-10}$)

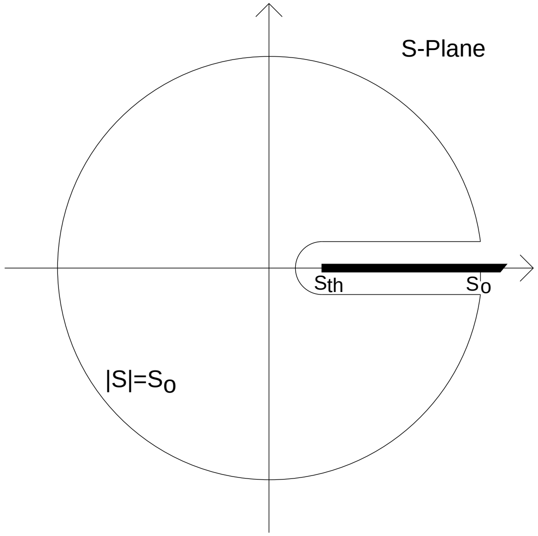
- consistency of EM $\pi^+\pi^-\pi^0\pi^0$ data not satisfactory, significant disagreement with IB-corrected τ expectations (τ yields $[a_\mu]^{had,LO}_{2\pi^0\pi^+\pi^-}$ higher by $(4.6 \pm 1.9) \times 10^{-10}$)

- τ $\pi\pi$ IB correction error underestimated (model dependence of integrated "ρ-ω interference")

OPE CONSTRAINTS AND THE EM-τ DISCREPANCY

- FESR background
  - $\Pi(s)$ (no kinematic singularities), spectral function $\rho(s)$, $w(s)$ analytic in $|s| < M$, $M > s_0 \Rightarrow$

$$\int_0^{s_0} w(s)\,\rho(s)\,ds \;=\; -\frac{1}{2\pi}\oint_{|s|=s_0} w(s)\,\Pi(s)\,ds$$



FESR OPE features

- V current correlators, $s_0 > \sim 2$ GeV^2 $\Rightarrow$ OPE *strongly* dominated by $D = 0$
- $\Rightarrow$ dominant OPE input: $\alpha_s(M_Z)$ (from independent high-scale determinations, plus 4-loop running/matching)
- good convergence of integrated $D = 0$ OPE series
- "s_0-stability tests" to check treatment of higher D contributions

WEIGHT CHOICES ETC.

FESR choices: use various pinched ($w(s = s_0) - 0$), *non-negative, monotonically decreasing* $w(y)$, $y = s/s_0$

- IB-corrected $\rho_\tau(s) > \rho_{EM}^{I=1}(s)$ in region of discrepancy

- $\Rightarrow$ if τ data correct, (i) EM spectral integrals $<$ OPE *for all* s_0 (non-negativity), (ii) slope wrt. $s_0 <$ OPE (monotonicity)

- $\Rightarrow$ if EM data correct, (i) τ spectral integrals $>$ OPE *for all* s_0, (ii) slope wrt. $s_0 >$ OPE

- *slope significantly less sensitive than norm'n to* α_s

RESULTS (also true for other $w(y)$ not shown above)

For high-scale average $\alpha_s(M_Z) = 0.1198 \pm 0.0020$ input

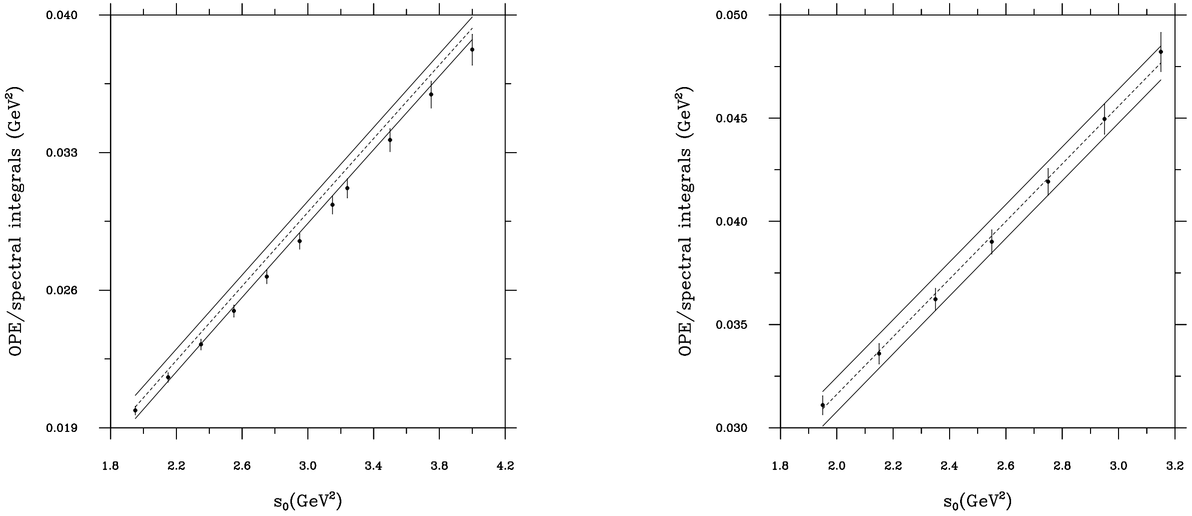
- magnitude and slope of τ spectral, OPE integrals agree for wide range of pinched, non-negative, monotonically decreasing $w(y)$, s_0

- EM spectral integrals, slopes $<$ OPE expectations for wide range of pinched, non-negative, monotonically decreasing $w(y)$, s_0

RESULTS (SELECTED WEIGHTS)

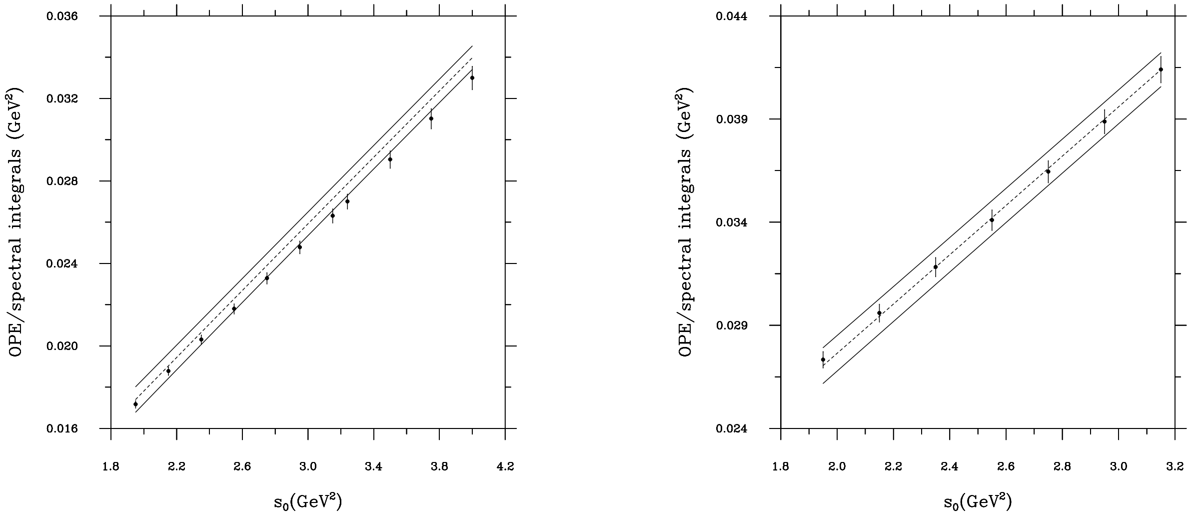
- OPE vs.spectral integrals for $w(y) = 1 - y$

LEFT: EM, RIGHT: τ



- OPE vs.spectral integrals for $w_6(y) = 1 - \frac{6y}{5} + \frac{y^6}{5}$

LEFT: EM, RIGHT: τ



(one of more general "doubly-pinched" weight family, $\{w_N(y)\}$, with $6 \to N$, $5 \to N-1$)

- more on the EM normalization problem:

$\alpha_s(M_Z)$ values required to fit EM and τ spectral integrals for $s_0 \sim m_\tau^2$

Weight	EM or τ	$\alpha_s(M_Z)$
$1-y$	EM	$0.1138^{+0.0030}_{-0.0035}$
w_3	EM	$0.1152^{+0.0019}_{-0.0021}$
w_6	EM	$0.1150^{+0.0022}_{-0.0026}$
$1-y$	τ	$0.1212^{+0.0027}_{-0.0032}$
w_3	τ	$0.1189^{+0.0018}_{-0.0021}$
w_6	τ	$0.1195^{+0.0020}_{-0.0022}$

c.f. high-scale ave (w/out lattice):

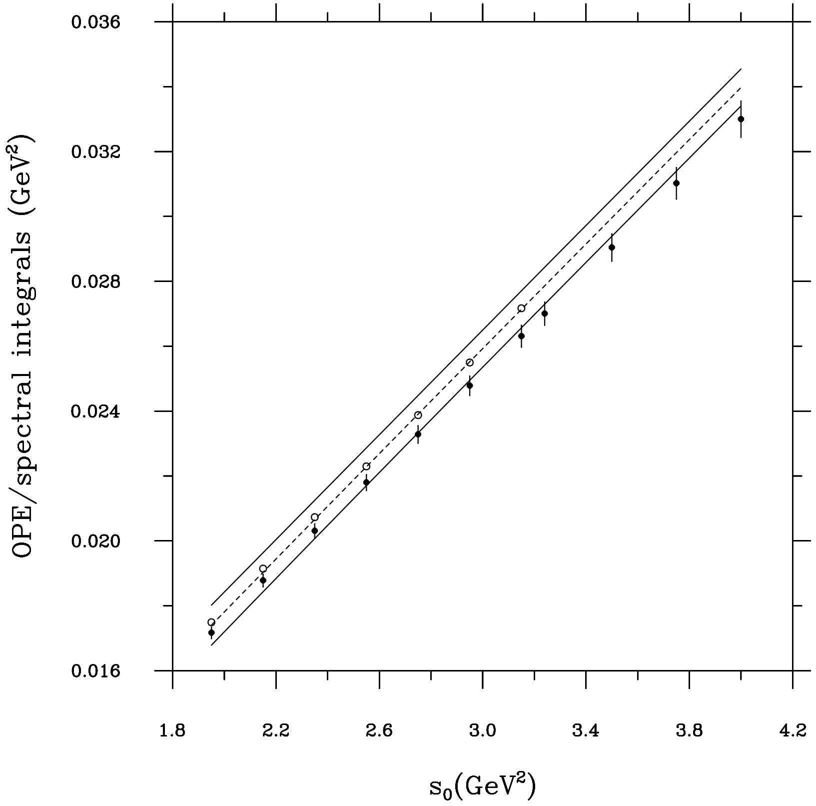
$\alpha_s(M_Z) = 0.1198 \pm 0.0020$

- more on the EM slope problem:
  - results for OPE vs. expt slope, S [*indep*: high scale $\alpha_s(M_Z)$ input (as above); *fit*: alternate $\alpha_s(M_Z)$ input from fit to EM spectral integral at $s_0 \sim 4$ GeV^2]

Weight	S_{exp}	$\alpha_s(M_Z)$	S_{OPE}
$1-y$	$.00872 \pm .00026$	indep	$.00943 \pm .00008$
		fit	$.00934 \pm .00008$
w_6	$.00762 \pm .00017$	indep	$.00811 \pm .00009$
		fit	$.00805 \pm .00009$

  - 2.6 (2.3) σ discrepancy for $w(y) = 1 - y$ with indep (fit) input, 2.5 (2.2) σ for $w_6(y)$
  - no plausible shift of $\alpha_s(M_Z)$ cures slope problem *from OPE side*

- slope, normal'n problems both "cured" if EM V $\pi\pi$, 4π $\rightarrow$ equivalent τ data ($w_6(y)$ eg. below: open circles are τ-modified EM spectral integrals)



Relative role of 2π, 4π in EM vs. τ OPE Constraints

- τ 2π, 4π contributions to effective $s_0 = 2\, GeV^2 [m_\tau^2]$ EM spectral integral shifts

Weight	$\pi\pi$	4π
$1-y$	82% [36%]	18% [64%]
$w_6(y)$	87% [45%]	13% [55%]

- impact of replacing ONLY 4π part of ρ_{EM} with τ version (slope, $\alpha_s(M_Z)$ from fitted $\alpha_s(m_\tau)$)

Weight	$\alpha_s(M_Z)$	Slope (exp)	Slope (OPE)
$1-y$	0.1186	$.00936 \pm .00026$	$.00940 \pm .00008$
$w_6(y)$	0.1176	$.00795 \pm .00017$	$.00808 \pm .00009$

COMMENTS/CONCLUSIONS/OPINIONS

- pFESR tests, high-scale OPE input favor τ over EM data for V spectral function

- with τ input, SM prediction for a_μ in agreement with current E821 result

- *NO* even remotely plausible shift in $\alpha_s(M_Z)$ cures EM slope problem from OPE side

- HOWEVER, if new EM $\pi^+\pi^-\pi^0\pi^0$ data agrees with with τ expectation, EM slope, normalization low, but compatible within errors, with OPE

- τ slope, norm'n still OK if ALEPH τ $\pi\pi \rightarrow$ BELLE $\pi\pi$ (but reduced central $\alpha_s(M_Z)$ fit value)

- a not-implausible near-term scenario:
  - BELLE τ $\pi\pi \Rightarrow$ somewhat lower $[a_\mu]_\tau^{had,LO}$
  - new EM $\pi^+\pi^-\pi^0\pi^0 \sim \tau$ 4π expectations, $R(s)$ data below J/ψ both raise $[a_\mu]_{EM}^{had,LO}$
  - BNL E969 a_μ proposal now crucial for interpretation

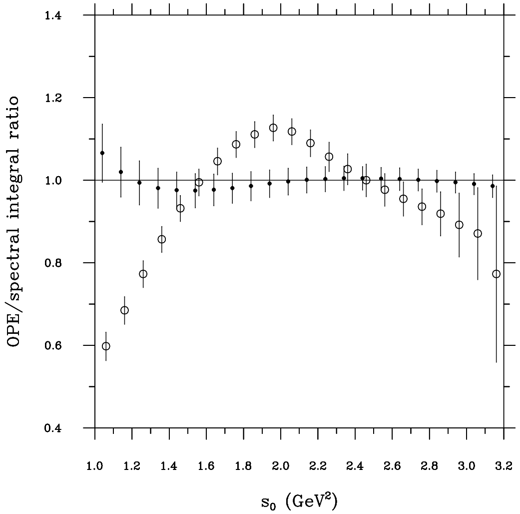
- WARNING: *minimum* plausible uncertainty in τ IB correction $\sim 4 \times 10^{-10}$ ($>$ proposed BNL E969 accuracy) [KRM, C. Wolfe, PRD73 (2006) 013004]

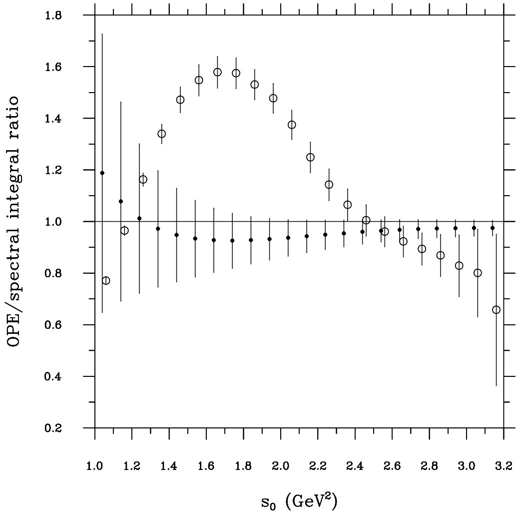
- near-future new experimental input
  - analysis of additional KLOE data ($\sim 5\times$ existing)
  - BABAR, BELLE radiative return $\sigma_{\pi\pi}$, BABAR K^+K^-, $\pi^+\pi^-\pi^0\pi^0$, $K\bar{K}\pi$, $\pi^+\pi^-3\pi^0$, $\pi^+\pi^-\pi^+\pi^-\pi^0$, $K\bar{K}\pi\pi$
  - CLEO-c $R(s)$; BABAR, BELLE hadronic τ decay with much improved statistics, K/π separation
  - Novosibirsk VEPP-2000 upgrade (luminosity, systematics, $E_{CM}^{max} \to 2$ GeV, CMD-3, SND upgrades) $\Rightarrow$ improved exclusive cross-sections (especially useful near threshold, above 1.38 GeV)
  - Beijing τ-charm upgrade

BACKUP SLIDES:

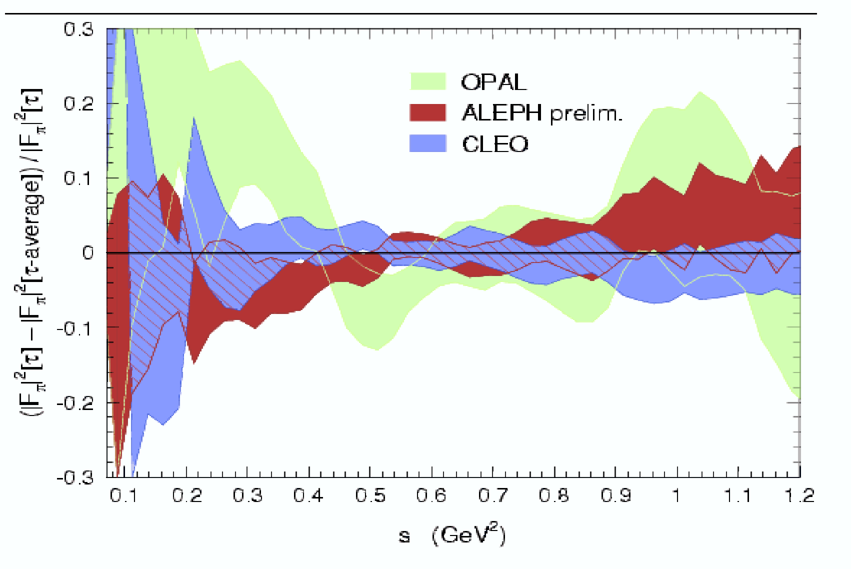
Pinched $w(y)$ OPE/spectral integral ratios

LEFT: y, $(1-y)$; RIGHT: y^3, $(1-y)^2(1+2y)$

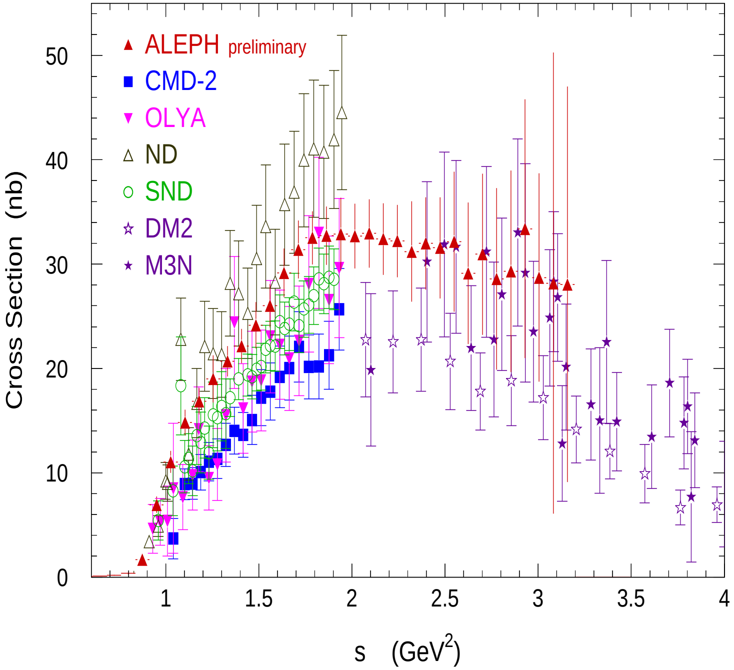




The Current τ $\pi\pi$ Situation



The $\sigma[\pi^+\pi^-\pi^0\pi^0]$ Situation



Comparison of BELLE τ $\pi\pi$ with other sources

- BELLE $[a_\mu]^{\pi\pi}_\tau \times 10^{10}$, $(0.5\mathrm{GeV})^2 < s < m_\tau^2$: $459.8 \pm 0.5 \pm 3.2 \pm 2.3_{IB}$ (Fujkikawa, Tau'06) [c.f. $464.0 \pm 3.2 \pm 2.3_{IB}$ (ALEPH+CLEO), $450.2 \pm 4.9 \pm 1.6_{rad}$ (CMD-2+KLOE)]

- τ-based determinations (no IB)

$s_{\pi\pi}$ [GeV^2]	BELLE	CLEO	ALEPH
$.25 \rightarrow .45$	119.6 ± 0.4	123.6 ± 1.7	113.8 ± 3.5
$.45 \rightarrow .75$	302.7 ± 0.3	298.5 ± 1.4	296.7 ± 2.6
$.75 \rightarrow 1.1$	32.5 ± 0.1	29.1 ± 0.3	34.4 ± 0.7
$1.1 \rightarrow 1.7$	6.1 ± 0.02	6.2 ± 0.1	6.9 ± 0.2
$1.7 \rightarrow 3.2$	0.81 ± 0.01	0.72 ± 0.03	0.78 ± 0.05