Recent results from Domain Wall Fermions in Full QCD

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[RBC-UKQCD Collaboration]

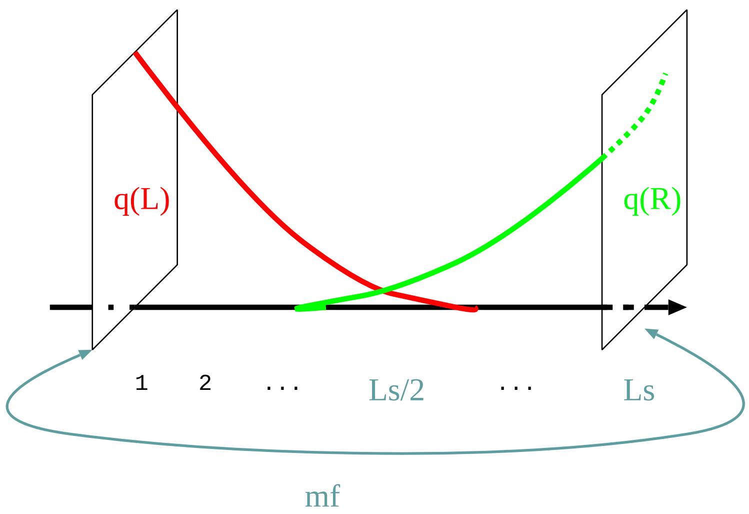
Introduction

- Plan:
  - Domain Wall Fermions
  - (RBC-UKQCD) 2+1 flavour QCD simulations
  - Two applications in kaon physics:
    - Kaon Beta decay
    - Kaon B-parameter

Rationale for Domain Wall fermions

- Traditional lattice actions break either flavour (Staggered) or chiral (Wilson) symmetry at finite lattices spacing.
- Domain Wall Fermions : Exact Flavour symmetry, greatly supressed breaking of Chiral symmetry.
- While more expensive, the hope is that the symmetry properties at finite lattice spacing outway the cost (lattice simulations $\propto 1/a^8$)
  - this is a quantity dependent question.

Domain Wall Fermions



- ▷ Action in 5th dimension asymmetric w.r.t. chirality.
- ▷ Define 4d quark fields on the wall
- ▷ Couple the two walls with a mass term

$$m_f \bar{q} q$$

- ▷ For finite L_s chiral symmetry is broken, leading to an additive shift of the mass

$$m_f \rightarrow m_f + m_{\rm res}$$

- ▷ $m_{res} \rightarrow 0$ as $L_s \rightarrow \infty$; The cost in computer time $\propto L_s$
  - ▷ Can get small m_{res} (few MeV) for reasonable L_s ($O(10)$).

Dynamical DWF on the QCDOC

- ▷ DWF work well in the quenched and two-flavour theory.
- ▷ Vital to move to full QCD

The QCDOC computers: RBRC (BNL), UKQCD (Edinburgh), US Machine (BNL).

- ▷ each ~ 10 TFlops (peak).

Joint project UKQCD, RBC using (parts of) all three machines: 2+1 flavour Dynamical DWF

- ▷ "light" dynamical masses ($\rightarrow m_s/5$)
- ▷ (at least) two lattice spacings...
- ▷ (at least) two volumes

RBC-UKQCD DWF Collaboration (probably a little out of date)

RBRC

Blum, Tom (U Conn)
Dawson, Chris
Hashimoto, Koichi
Izubuchi, Taku (Kanazawa)
Ohta, Shigemi (KEK)
Sasaki, Shoichi (KEK)
Yamazaki, Takeshi

BNL

Jung, Chulwoo
Sholtz, Enno
Soni, Amarjit

Columbia

Aubin, Christopher
Christ, Norman
Cohen, Saul
Li, Sam
Lin, Meifeng
Lin, HueyWen
Loktik, Oleg
Mawhinney, Robert

UKQCD

Allton, C.
Antonio, D. J.
Bowler, K. C.
Boyle, P. A.
Clark, M. A.
Jüttner, A.
Joo, B.
Kennedy, A. D.
Kenway, R. D.
Noaki, J.
Maynard, C. M.
Pendleton, B. J.
Sachrajda, C.
Traveni, A.
Tweedie, R. J.
Yamaguchi, A.
Zanotti, J.

Historical Document (January 2005)

m_f/m_s	L_s	$L*a$ (Fm)	$L*a*m_\pi$	Nodes	Trajs.	Time (days)	Proc. Hrs.
$16^3\times 32$, $1/a = 1.8$ GeV, $a = 0.11$ Fm:							
0.6	12	1.78	3.44	2,048	4,833	8	3.84E+05
0.5	12	1.78	3.14	2,048	5,294	12	6.13E+05
0.4	12	1.78	2.81	2,048	5,919	23	1.11E+06
$24^3\times 64$, $1/a = 1.8$ GeV, $a = 0.11$ Fm:							
0.4	12	2.67	4.22	4,096	5,919	123	1.21E+07
0.3	12	2.67	3.65	4,096	6,835	270	2.65E+07
0.2	16	2.67	2.98	6,144	8,371	758	1.12E+08
$32^3\times 64$, $1/a = 1.8$ GeV, $a = 0.11$ Fm:							
0.3	16	3.56	4.87	8,192	6,835	529	1.04E+08
$32^3\times 64$, $1/a = 2.4$ GeV, $a = 0.083$ Fm:							
0.5	12	2.67	4.71	8,192	7,059	273	5.37E+07

- Have completed $16^3\times 32$ run (at a little coarser lattice spacing, and lighter masses), and are in the process of completing $24^3\times 64$ run.
- Scale setting runs for the small lattice spacing $32^3\times 64$ lattices currently underway.

Some results

- In the remainder of this talk, I will cover two quantities in Kaon Physics
  - Kaon Beta Decay.
  - The Kaon B-parameter.

  this is an on-going calculation, so all results are preliminary
- Basic Parameters:
  - single lattice spacing : $a^{-1} \sim 1.6$ GeV
  - $m_{\mathrm{res}} \sim 5$ MeV
  - spatial extent ~ 2 fm and ~ 3 fm for $16^3 \times 32$ and $24^3 \times 32$ respectively.
  - three dynamical masses, lightest $\sim m_s/3$

Kaon Beta decay

$$K^0 \to \pi^- L^+ \nu_l ; K^+ \to \pi^0 L^+ \nu_l$$

where $l \in \{e, \mu\}$.

- Itegrating out the Weak force (and heavy quarks): $\Gamma_{K_{l3}} \propto |V_{us}|^2 |f_+(0)|^2$

  $$\langle \pi(p_f) \left| \bar{s}\gamma_\mu u \right| K(p_i) \rangle = (p_i + p_f)_\mu f_+(q) + q_\mu f_-(q) \; ; \; q = p_i - p_f$$

  where the LHS is evaluated in (3 flavour) QCD.
- Used to extract $|V_{us}|$.
  - f_K / f_π competitive (more developed calculation on the lattice)

- Why is this process so nice?
  - Insertion of vector current: favourable symmetry properties (no renormalisation in the continuum).
  - Don't need much non-perturbative input...

Calculating $f_+(0)$

- Ademollo-Gatto theorem: $f_+(0) = 1 - O\left((m_s - m_u)^2\right)$

Expand the form factors in Chiral Perturbation Theory:

$$f_+(q^2) = 1 + f_2 + f_4 + \cdots$$

with f_i of $O(M^i/f^i)$ in ChiPT

- f_2:
  - f_2 depends on no new low energy constants. Can be worked out from M_K, M_π and f_π.
    -0.023 using values from experiment.
- f_4:
  - Calculated in Chiral Perturbation Theory by Bijnens and Talavera. In *principle*, can be constrained by the experimentally measured slope of $f_0(q^2)$, but needs better experimental resolution.
  - $-0.016(8)$ from quark model [Leutwyler and Roos, 1984]

The double ratio method

Rather than work with the three-point function of interest directly, the double ratio is used. ([Becirevic et al, hep-lat/0403217] c.f. [Hashimito *et al*, 2000]).

$$\frac{\langle \pi \left| \overline{s}\gamma_0 u \right| K \rangle \langle K \left| \overline{u}\gamma_0 s \right| \pi \rangle}{\langle \pi \left| \overline{u}\gamma_0 u \right| \pi \rangle \langle K \left| \overline{s}\gamma_0 s \right| K \rangle} = \left[f_0 \left(q^2_{max} \right) \right]^2 \frac{\left(M_K + M_\pi\right)^2}{4 M_K M_\pi} \; ; \; q^2_{max} = \left(M_K - M_\pi\right)^2$$

$$f_0(q^2) = f_+(q^2) + \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

Scalar form factor:

$$f_0(0, M_\pi, M_K) = f_+(0, M_\pi, M_K)$$

This approach has several advantages:

- Small statistical error ($< 0.1\%$)
- Exactly unity, and exactly $f_+(0)$, on the lattice in $SU(3)$ limit.

The double ratio method...

This just gives $f_0(q^2, M_\pi, M_K)$. Need to

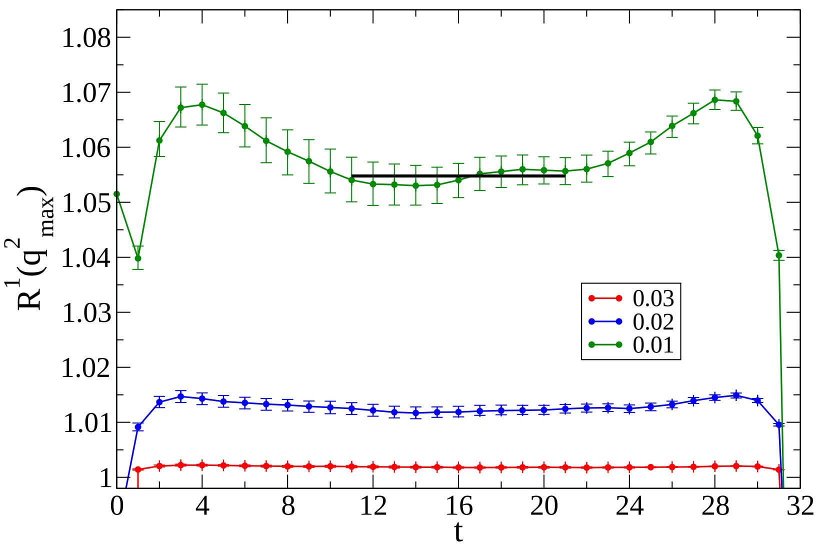
1. Extrapolate, in q^2, to $f_0(0) = f_+(0)$ at a fixed (non-physical) mass.
2. Extrapolate to physical masses.

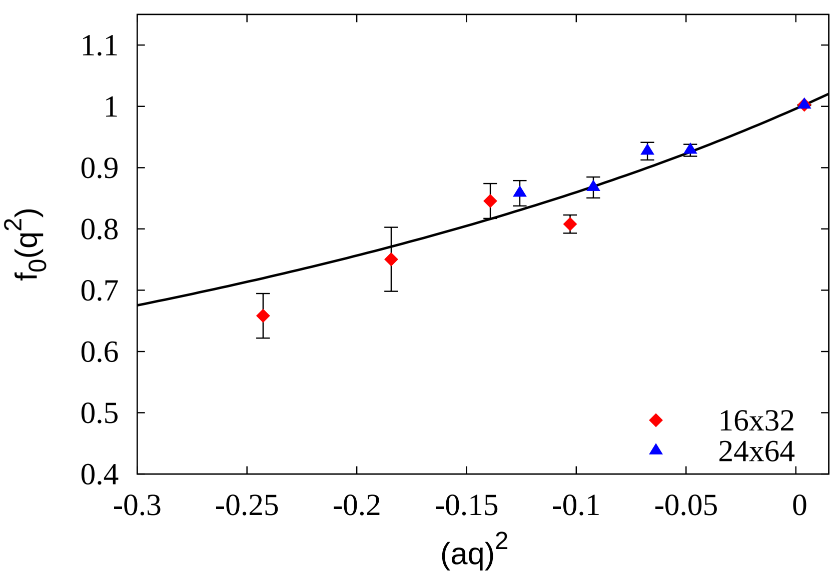
For 1) lattice data with explicit insertion of momenta is needed.

▷ [Becirevic et al, hep-lat/0403217] show how to use various ratios to allow an extraction with a small enough error-bar to be useful.

Double Ratio/Momentum Extrapolation ($N_f = 2 + 1$)

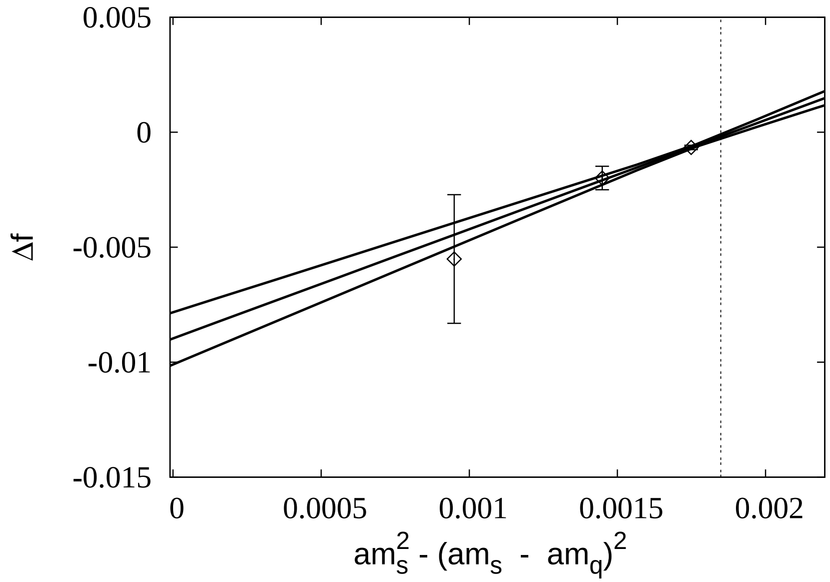
- From James Zanotti's talk at lattice 2006
- Zero momentum injection (small momentum tranfer as lattice pion and lattice kaon are close in mass).
- Several mass differences shown

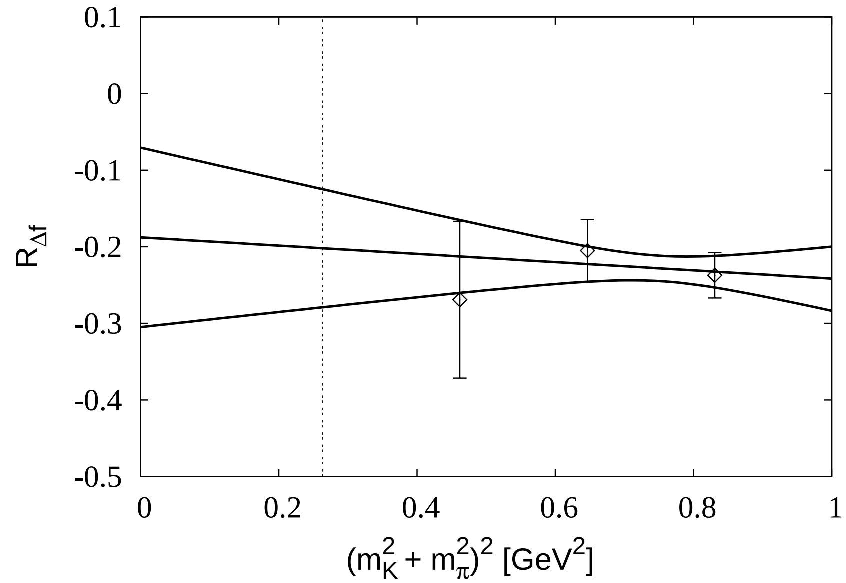




- requires some ansatz to fit to:
  - $f_0(q^2) = f_0(0)/(1 - \lambda^{(pol)} q^2)$
  - $f_0(q^2) = f_0(0)(1 + \lambda^{(1)} q^2)$
  - $f_0(q^2) = f_0(0)(1 + \lambda^{(2)} q^2 + c q^4)$
- Fit to pole form shown.
- larger lattice has smaller minimum lattice momenta

Mass extrapolation ($N_f = 2+1$)





- ▷ Unitary points shown.
- ▷ $\Delta f \propto (m_s - m_d)^2$: Ademollo-Gatto
- ▷ Higher order terms, so looks at:

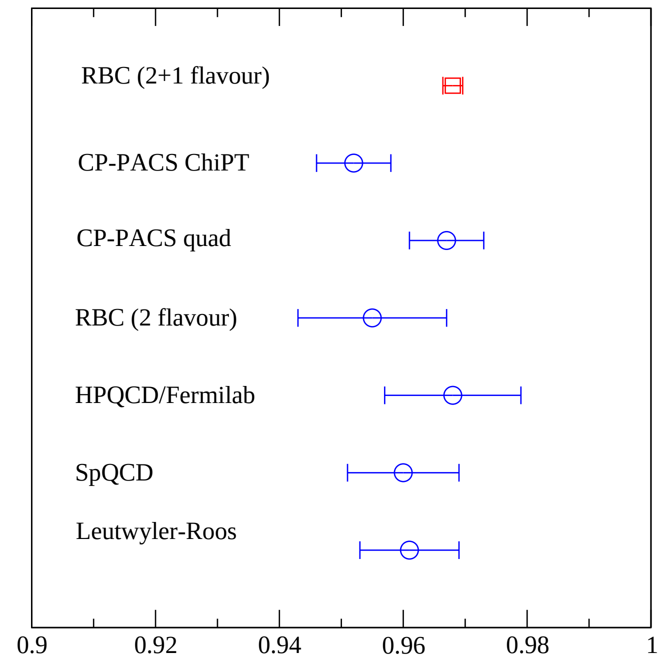
$$R = \frac{\Delta F}{m_K^2 - m_\pi^2}$$

and try

$$R = a + b\left(m_K^2 + m_\pi^2\right)$$

Lattice results

- ▷ Dramatically smaller error-bar: Larger volume certainly helping
- ▷ However:
  - ▷ Chiral extrapolation for all measurements is over a large range.
  - ▷ Ansatz used for momentum extrapolation. (systematics not yet studied)
  - ▷ Lattice spacing, Volume effects?
  - ▷ Low statistics on larger volume!



- ▷ For "fun", using $|V_{us} f_+(0)| = 0.2169(9)$:

$$|V_{us}| = 0.2241(9)_{\exp}(4)_{f_+(0)}$$

and

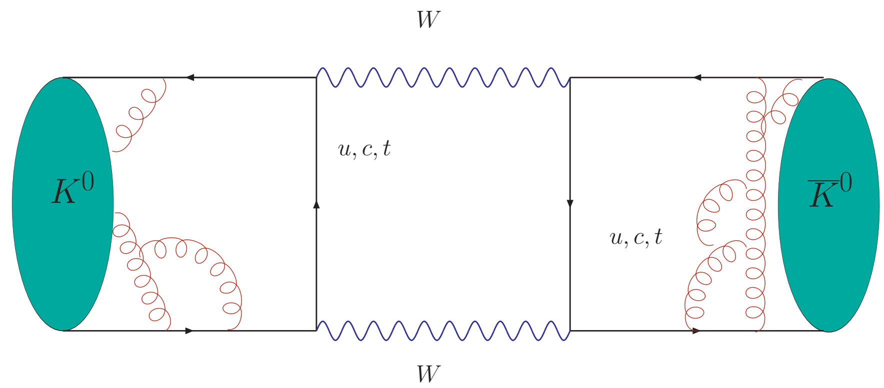
$$1 - |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.0015(7)$$

versus 0.0008(11) (PDG)

- ▷ Ecouraging preliminary result. But it is preliminary, and not including all systematics...

Kaon B-parameter

B_K is the low energy matrix element relevant to CP-violation in $K^0 - \overline{K^0}$ mixing.

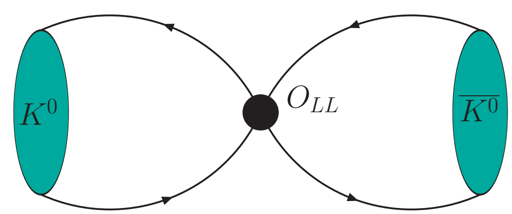


Integrate all the particles with masses $\gg \Lambda_{QCD}$:

$$|\epsilon| = C_\epsilon A^2 \lambda^6 \overline{\eta} \left[-\eta_1 S(x_c) + \eta_2 S(x_t) A^2 \lambda^4 (1 - \overline{\rho}) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

Need to calculate this on the lattice:

$$B_K = \frac{\langle \overline{K}^0 | O_{LL} | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}$$



renormalised in some scheme at some scale.

Operator Mixing and B_K

In the continuum there is one operator that contributes to B_K It if of the form:

$$O_\Gamma = \overline{s}\Gamma_i d\,\overline{s}\Gamma_i d$$

with the gamma structure:

$$VV + AA \equiv \gamma_\mu \otimes \gamma_\mu + \gamma_\mu\gamma_5 \otimes \gamma_\mu\gamma_5\ ;$$

which is simply the parity conserving part of

$$(V - A) \otimes (V - A)$$

- ▷ Staggered fermions: extra flavours (“tastes” - taste mixing problem)
  - ▷ Many other operators can mix.
  - ▷ Resolve using 1-loop perturbation theory.
- ▷ Wilson fermions: Broken Chiral Symmetry.

Operator Mixing and B_K : Broken Chiral Symmetry

- If chiral symmetry is broken, four other operators can mix (the four other possible gamma matrix structures)

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{latt}} \propto \langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle_{\text{ren}} + \sum_{i \geq 2} c_i \langle \overline{K}^0 | O_{MIX,i} | K^0 \rangle_{\text{ren}}$$

These operators, of course, have a different chiral structure.

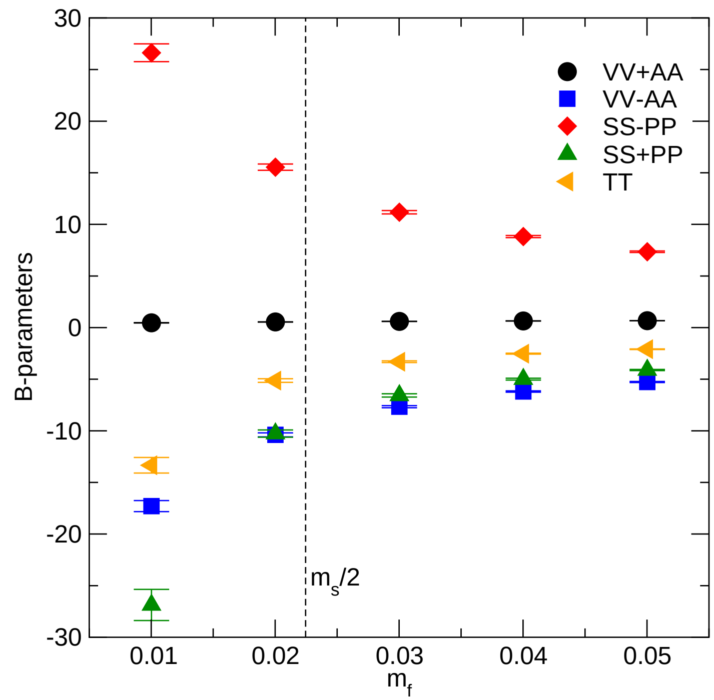
Mixing is hard to control using perturbation theory; First order chiral perturbation theory predicts that

$$\langle \overline{K}^0 | O_{VV+AA} | K^0 \rangle \propto M_k^2$$

and,

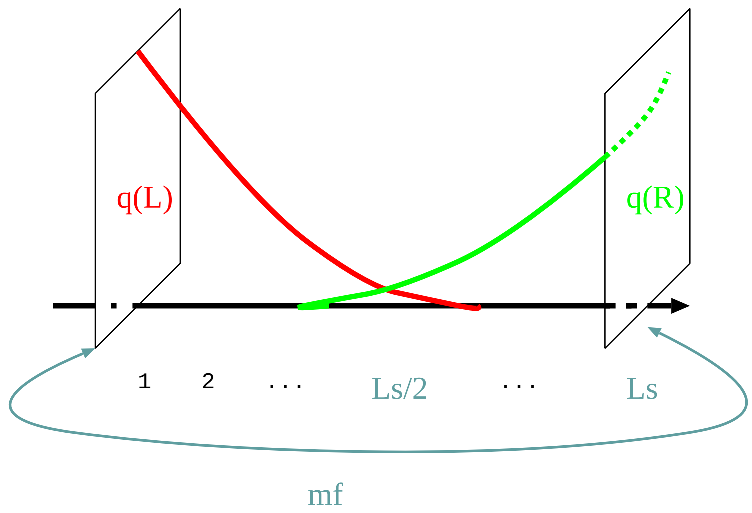
$$\langle \overline{K}^0 | O_{\text{THE REST}} | K^0 \rangle \propto \text{constant}$$

small enough mass, wrong chirality operators will dominate.



Domain Wall Fermions

- Domain Wall Fermions "almost" preserve chiral symmetry.
  - "almost" no mixing with the wrong chirality operators



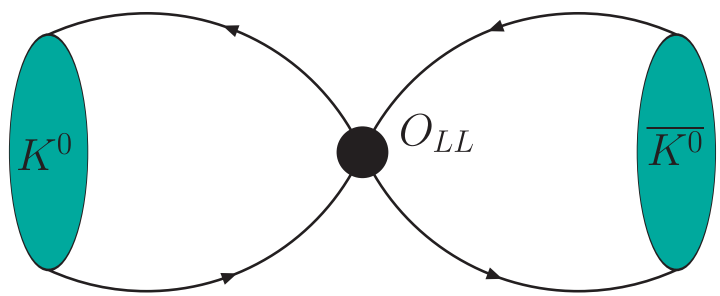
- wrong chirality matrix elements $O(10)$ times larger than signal at masses of interest.
- How much chiral symmetry is enough?
  Simple model:
  - One trip through the bulk : supression factor of $O(am_{\text{res}})$
  - Operator of interest is $(V-A)^2$: four left-handed fields.
    wrong chirality operators : two left-handed, two right-handed

$$\rightarrow O((am_{\text{res}})^2) \sim 10^{-6}$$

Approach

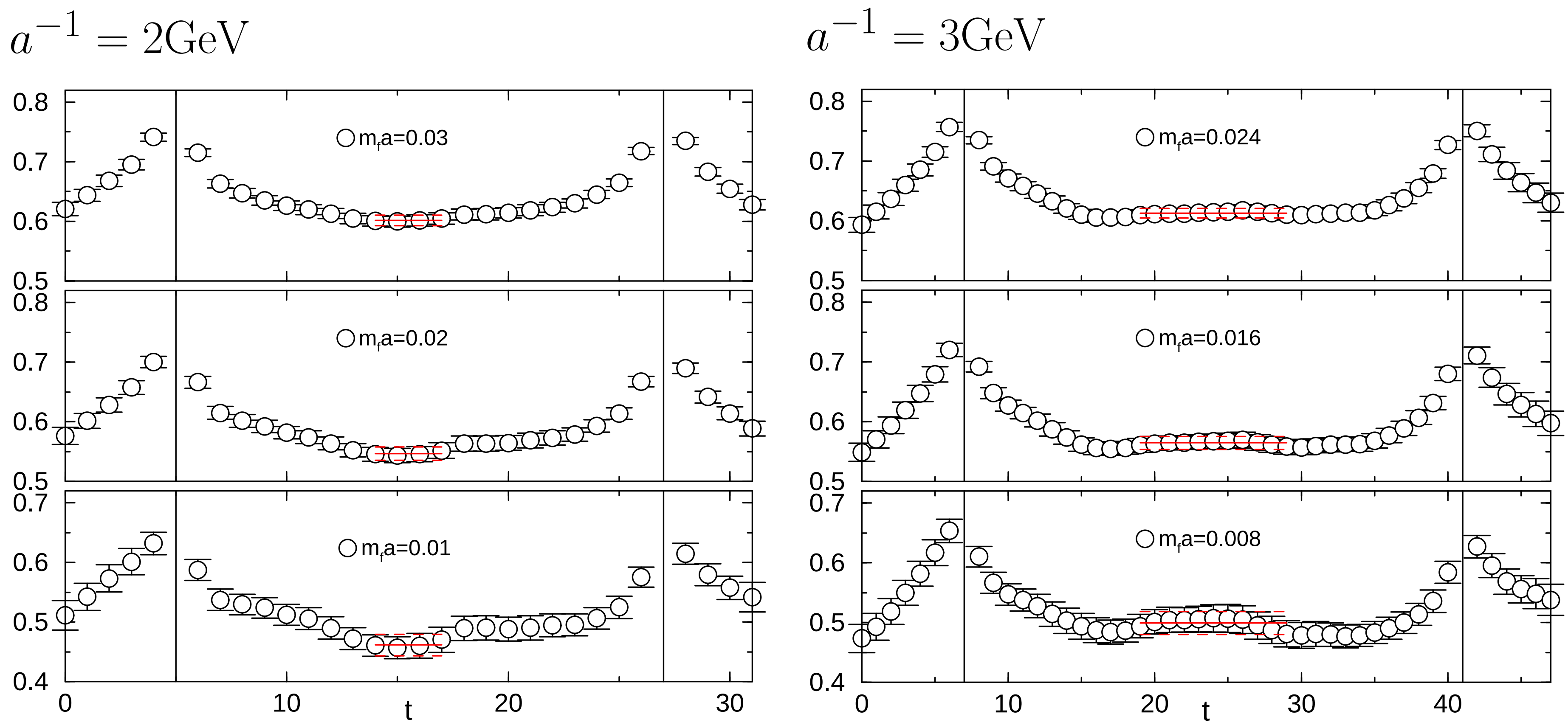
I'm going to show some RBC results in the quenched aproximation (Jun Noaki) from two different lattice spacings..

- $a^{-1} = 2$ GeV ($L_s = 16$) and 3 GeV ($L_s = 10$)
- $\sim 1.5\text{fm}^3 \times 3\text{fm}$ box
- degenerate masses



- Put operator with quantum numbers for Kaon at timeslices 4 and 28 (2 GeV)
- Move effective Weak vertex over all timeslices
  - Plateau should appear for large separation.

Bare B_K Plateus



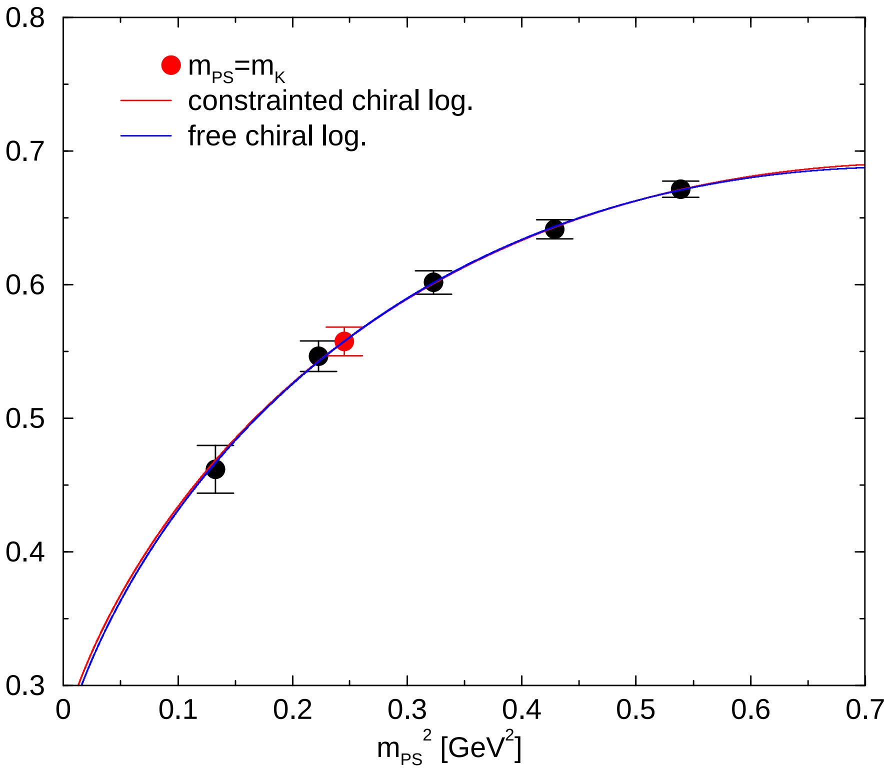
▷ Quenched Domain wall fermions results for the ratio used to extract B_K.

Chiral Fits and Extraction of B_K

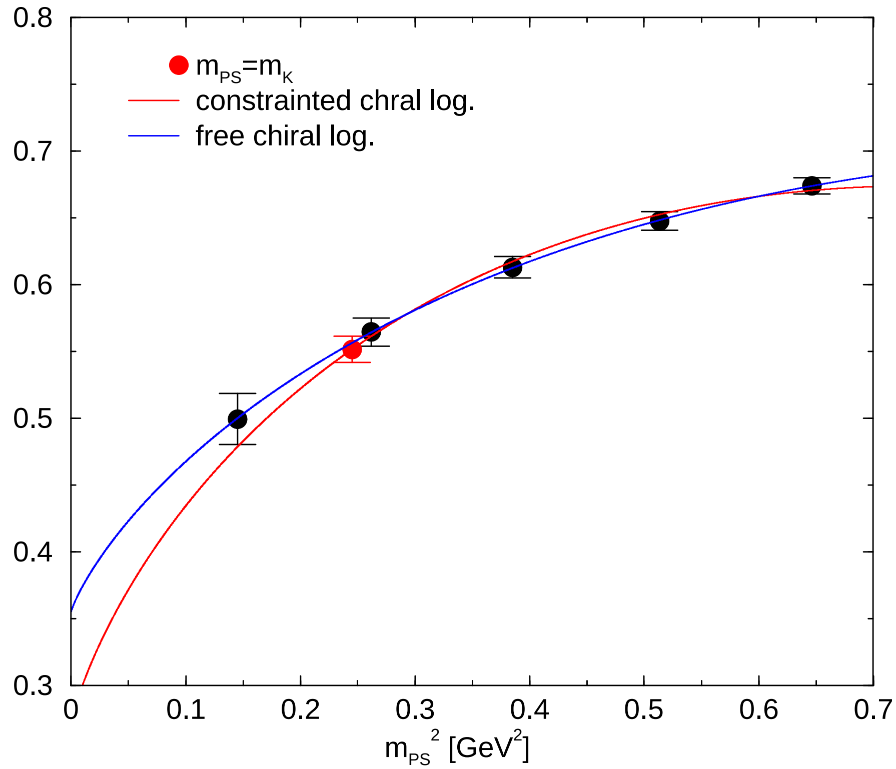
Predicted NLO ChiPT:

$$B_K = b_0 \left(1 - \frac{6}{(4\pi f)^2} M_K^2 \ln \left[\frac{M_K^2}{(4\pi f)^2} \right] \right) + b_1 M_K^2$$

DWF(RBC) $a^{-1} = 2\mathrm{GeV}$



DWF(RBC) $a^{-1} = 3\mathrm{GeV}$

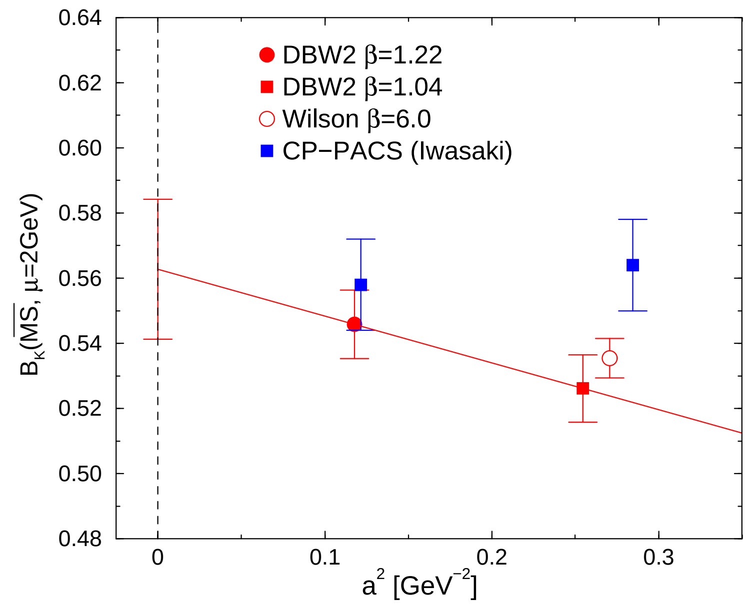


Quenched continuum limit of B_K

▷ Extrapolate to the continuum as

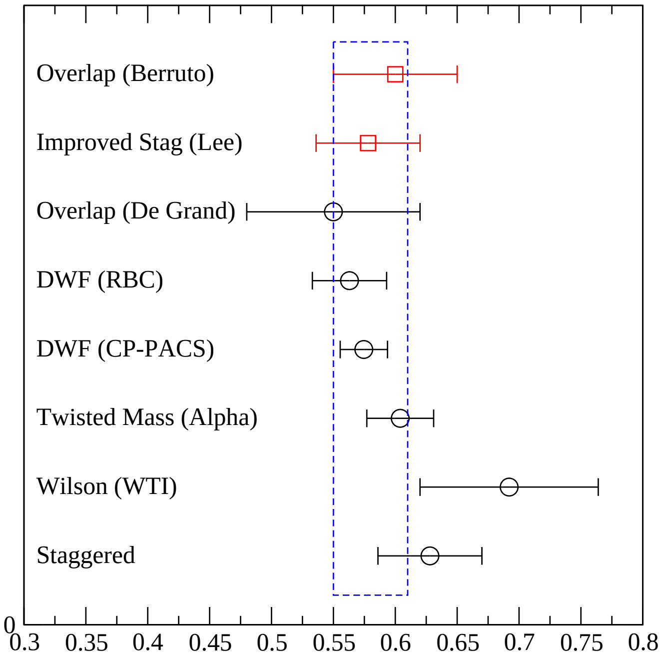
$$A + Ba^2$$

(would be linear in a without chiral symmetry).



▷ Continuum limit consistent with CP-PACs Iwasaki/DWF calculation using perturbative renormalization

Quenched World Average



Treat all the errors as statistical: all (continuum extrapolated) combined give

$$B_K^{NDR}(2\text{GeV}) = 0.587(13)$$

All a^2 extrapolated, published gives

$$B_K^{NDR}(2\text{GeV}) = 0.582(17)$$

- both with good χ^2/dof

Errors not all statistical

$$B_K^{NDR}(2\text{GeV}) = 0.58(3)$$

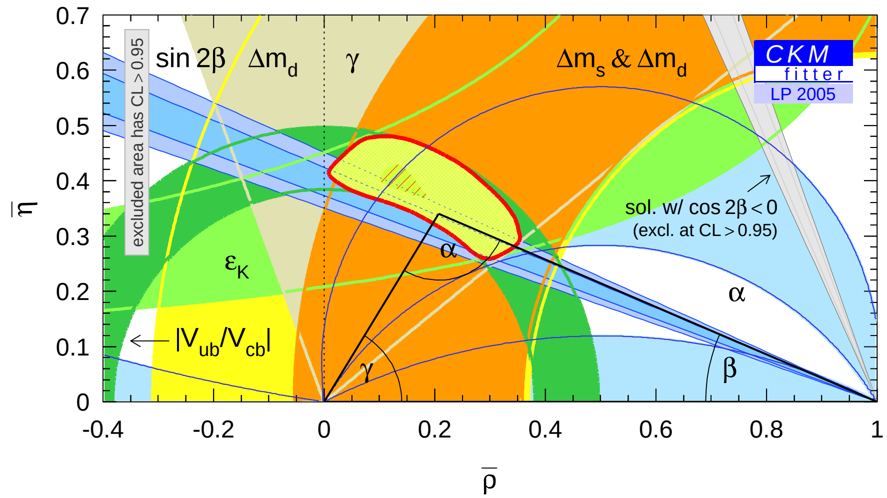
c.f $B_K^{NDR}(2\text{GeV}) = 0.58(4)$ [Shoji Hashimoto (ICHEP 2004)]

B_K and the CKM

Recall:

$$|\epsilon_K| = C_\epsilon A^2 \lambda^6 \overline{\eta} \left[-\eta_1 S(x_c) + \eta_2 S(x_t) A^2 \lambda^4 (1 - \overline{\rho}) + \eta_3 S(x_c, x_t) \right] \hat{B}_K$$

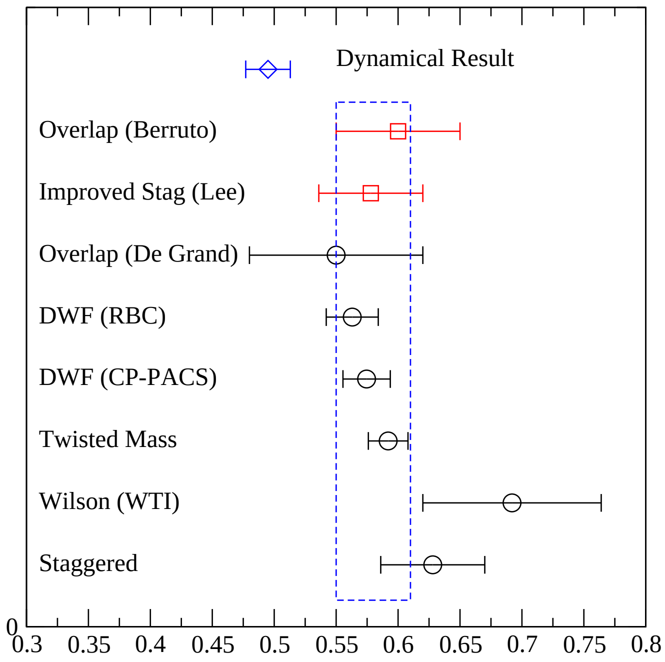
- ▷ A,λ already well known ($\sim 5\%$).
- ▷ This is the CKMfitter group's plot from EPS 2005.
- ▷ $B_K(\overline{MS}, 2\ \mathrm{GeV}) =$0.58(3)(6)



- ▷ The dominant error is from a (bad) guess of the quenching error.
- ▷ Need full QCD calculations, multiple lattice spacings, small masses...

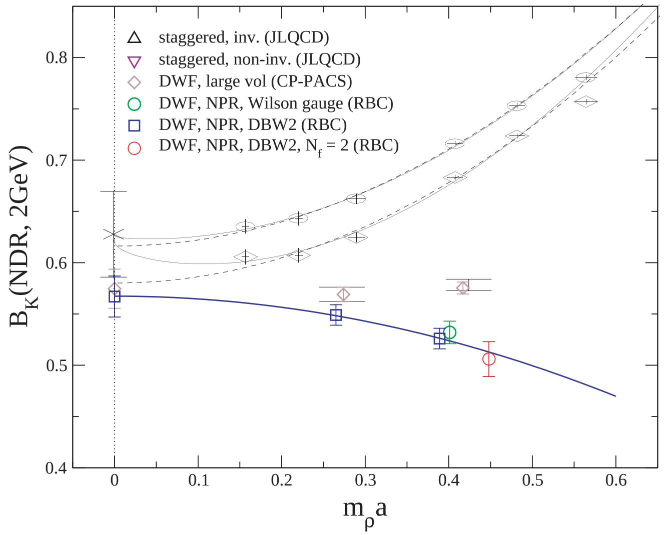
$N_f = 2$ Dynamical Domain Wall Fermions

- ▷ Repeat the same calculation for the two-flavour case
  - ▷ Now need chiral PT to perform an extrapolation! (with heavy masses)
  - ▷ small downward trend with dynamical mass resolved within statistics
  - ▷ big jump down (single lattice spacing)



- ▷ Looks very dramatic, but...

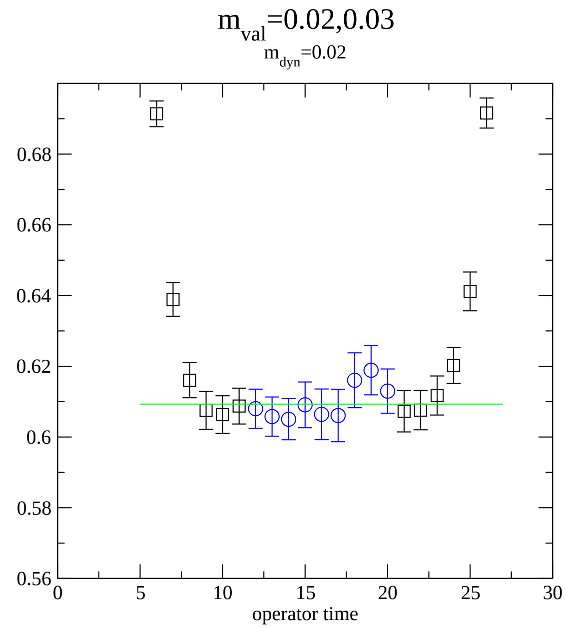
$N_f = 2$ Dynamical Domain Wall Fermions



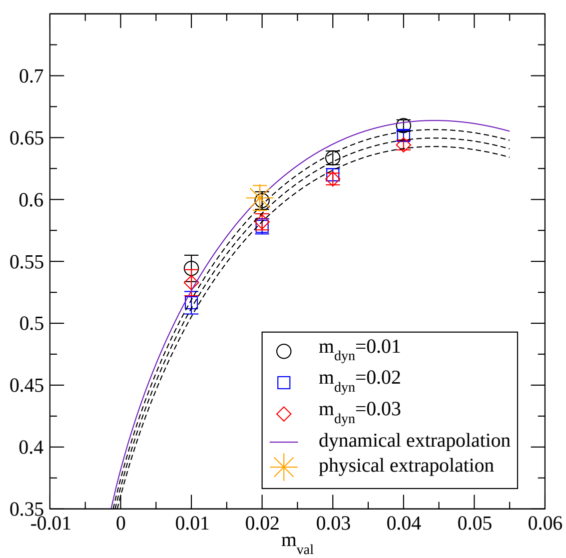
- ▷ "Suggestive" graph with the quenched and dynamical DWF results on, with the a^2 extrapolation on it.
  - ▷ (maybe) not a very sensible thing to plot
- ▷ Our dynamical result is consistent than the quenched results closest in lattice spacing.
  - ▷ This is the information used to estimate the systematic error due to quenching.

- ▷ need :
  - ▷ two lattice spacings
  - ▷ larger volumes
  - ▷ smaller masses
  - ▷ correct number of quarks

2+1 flavour Dynamical DWF; Small Volume



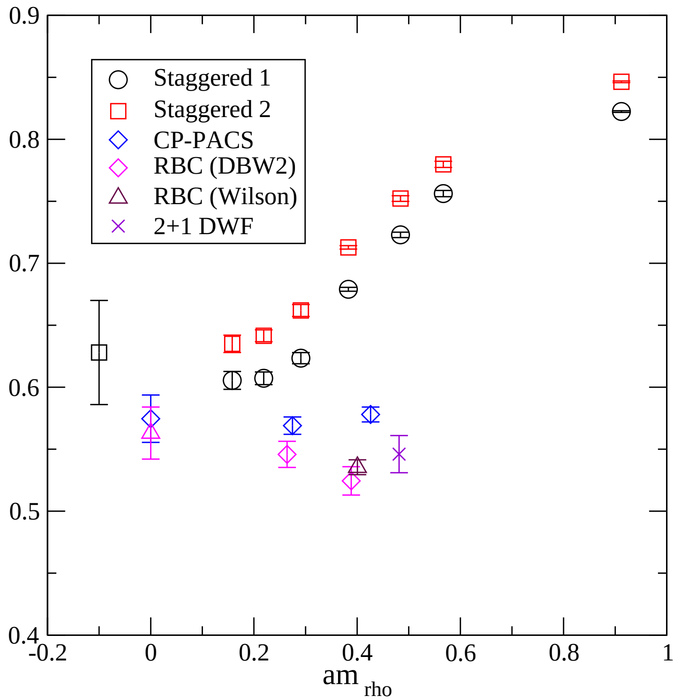
- Saul Cohens talk at lattice 2006.
- Example plateau extraction



- Chiral perturbation theory fit to the mass dependence (degenerate points)
- Doesn't fit too well
  - works well as an interpolation
  - extrapolation questionable (masses too large)

Scaling Plot

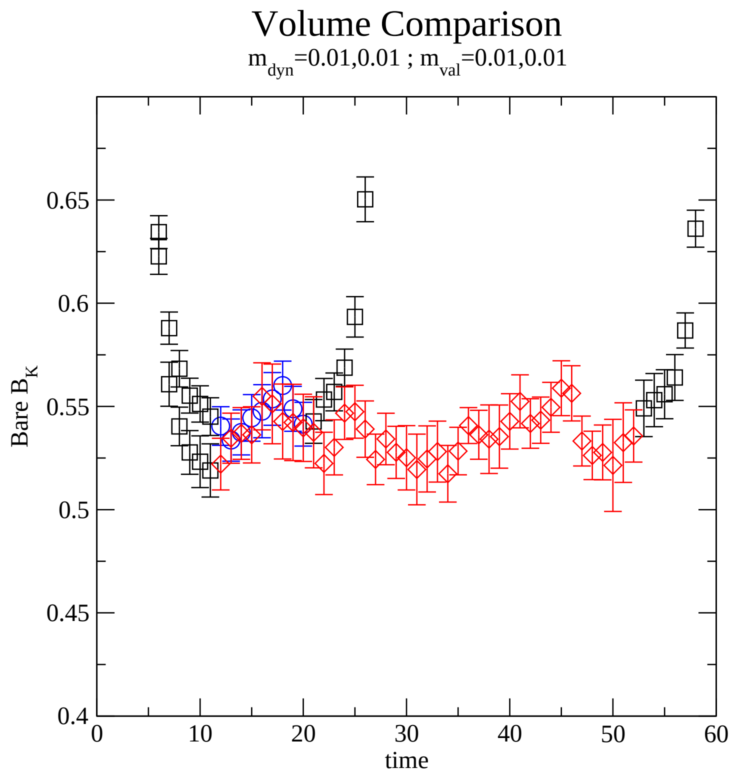
- Preliminary number : $B_K(\overline{MS}, 2GeV) = 0.546(10)(11)$
  - first error statistics, second from the renormalisation factor (conservative).
- Again, useful to look at a scaling plot:



- Moving towards the Quenched Iwasaki data (which used perturbative renormalisation)

- currently finalizing renormalisation factors, working on "error budget"

Larger Volume Work



▷ 22 configs for the large volume; 75 for small.

Conclusions/Future...

- Dynamical Domain Wall Fermion simulations are well under way
  - I've shown preliminary result for two quantities
    - Kaon Beta Decay

      relatively undeveloped quantity on the lattice

      need further study of systematics
    - Kaon B-parameter

      Good agreement in the quenched approximation between different approaches

      DWF ideally suited for this calculation because of their good symmetry properties

      Even with a single lattice spacing, useful information can be gained.
- Larger volumes, smaller lattice spacings on the way...