Light meson spectrum and strange quark mass in $N_f = 2 + 1$ domain-wall QCD

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for RBC+UKQCD Collaboeations

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Lattice QCD: conclusive study of LE physics

- **1st principle numerical simulation**
  - Correct (dynamical) #flavor
  - Good chiral property
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  - $N_f = 2 + 1$
  - Domain-wall quark + Iwasaki glue
  - $m_{\text{lightest}} \simeq 30\%$ of m_s

QCDOC @ Edinburgh Univ.

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- **Status report**
  - **C. Dawson:** Kaon matrix elements (B_K and $Kl3$ form factor)
  - **This talk:** basic study + $m_{\pi,K}$, $f_{\pi,K}$ and m_s

Numerical simulation

- **Parameters**
  - size: $16^3 \times 32$ & $24^3 \times 64$ ⇒ **finite size effect?**
  - Iwasaki action: $\beta = 2.13$
  - DWF: $L_s = 16$, $M_5 = 1.8$
  - sea quarks: $m_{u/d} = 0.01/0.02/0.03$, $m_s = 0.04$

    ⇒ **three independent ensembles**
  - statistics: $> 300 \times 10$ trajectories for each.

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- **UKQCD measurements (individual anarysis)**
  - Jacobi smearing w/ APE smeared links for source/sink
  - Average over multi-sources (4 for $m_{u/d} = 0.01, 0.02$; 2 for other)
  - Autocorrelation ≈ 50 trajs. $=$ jackknife bin
  - Mainly on $16^3 \times 32$, comparison with $24^3 \times 64$ for $m_{u/d} = 0.01$

    (Further measurements are ongoing.)

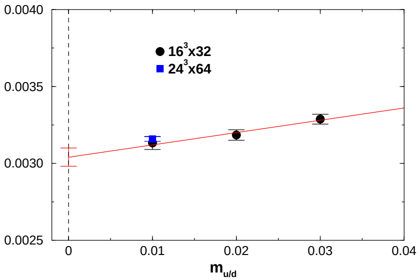
Conserved axial current

- **Residual quark mass**

Axial WTI

$$\frac{\Delta_4 \langle \mathcal{A}_4(t) P(0)\rangle}{\langle P(t) P(0)\rangle} = m_q + m_{\rm res}$$

$$m_{\rm res} \simeq 0.00304(6)/a \simeq 5\ \text{MeV}$$



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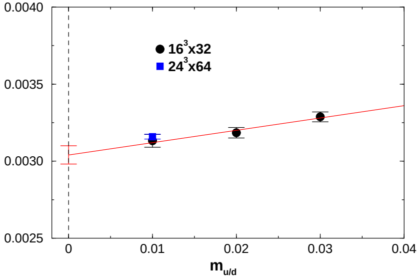
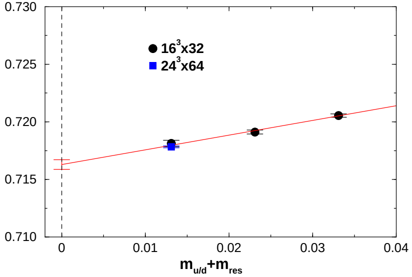


- **Renormalization factor**

$$Z_A = \frac{\langle \mathcal{A}_4(t) P(0)\rangle}{\langle A_4(t) P(0)\rangle}$$

chiral limit: $Z_A = 0.7165(6)$

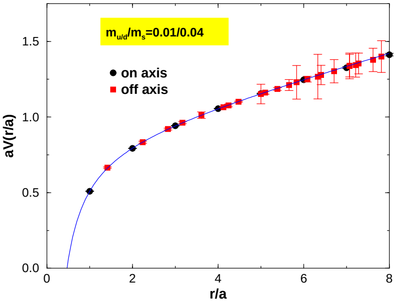


Lattice spacing

- Heavy quark potential

$$V(r) = v_0 - \frac{\alpha}{r} + \sigma r$$

$$r_0^{\mathrm{latt}} = \sqrt{\frac{1.65 - \alpha}{\sigma}} = 0.5\ \mathrm{fm}/a$$



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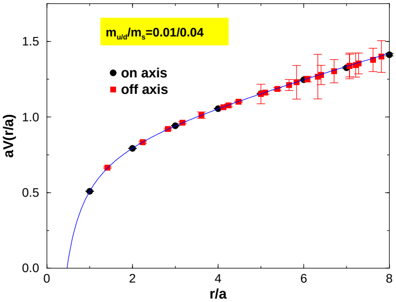
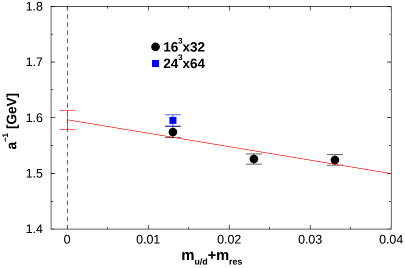


- Chiral limit

$$a^{-1} = 1.59(2)\ \text{GeV}$$

FSE : $\approx +2\%$

cf. $a^{-1} = 1.56(1)$ GeV from $m_\rho = 770$ MeV



Meson propagators

- **Simultaneous mass-fit : lgt-lgt (π) / lgt-str (K) / str-str (η_s)**

$$\begin{cases} \left\langle A_4^{(\mathrm{local})}(t) A_4^{(\mathrm{smr})}(0) \right\rangle = f_\pi^{(\mathrm{latt})}/2 \cdot \left\langle \pi \left| A_4^{(\mathrm{smr})} \right| 0 \right\rangle \cdot \cosh(m_\pi (T/2 - t)) \\ \left\langle A_4^{(\mathrm{smr})}(t) A_4^{(\mathrm{smr})}(0) \right\rangle = \frac{1}{2 m_\pi} \left| \left\langle \pi \left| A_4^{(\mathrm{smr})} \right| 0 \right\rangle \right|^2 \cdot \cosh(m_\pi (T/2 - t)) \end{cases}$$

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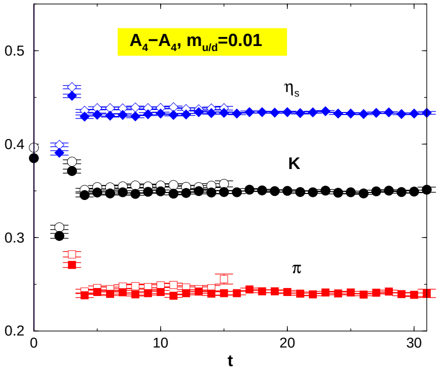
- **Effective mass plot**



FSE: ≈ −5%

PS meson mass^2

- **Chiral expansion** Gasser & Leutwyler, 1984; Aubin & Bernard, 2003

$$
\begin{aligned}
m_\pi^2 &= B_0 2m_d + \xi_1 2m_s m_d + \xi_2 2m_d(2m_d - m_s) + \alpha L_m(\pi), \\
m_K^2 &= B_0(m_s + m_d) + \xi_1(m_s + m_d)m_d + \xi_2(m_s + m_d)m_s + \alpha L_m(K), \\
m_{\eta_s}^2 &= B_0 2m_s + \xi_1 2m_s(2m_d - m_s) + \xi_2 2m_s(3m_s - 2m_d) + \alpha L_m(\eta_s)
\end{aligned}
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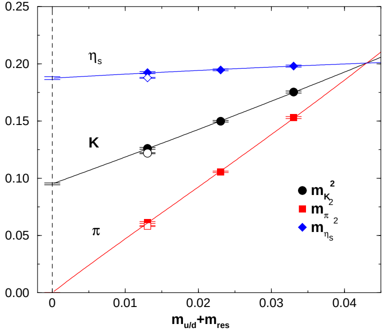
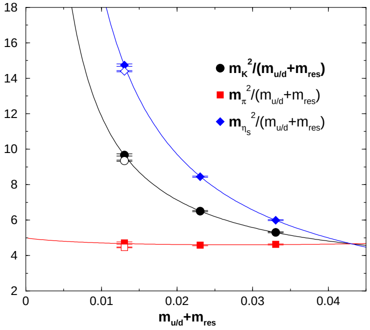
- **Simultaneous fit**





$m_K \simeq 495$ MeV $\Rightarrow m_s^{(\text{bare})} = 0.04 + m_{\text{res}}$

Decay constants

- **Chiral expansion**

$$
\begin{aligned}
f_\pi &= f + \eta_1 m_s + \eta_2 (2m_d - m_s) + \beta L_f(\pi), \\
f_K &= f + \eta_1 m_d + \eta_2 m_s + \beta L_f(K), \\
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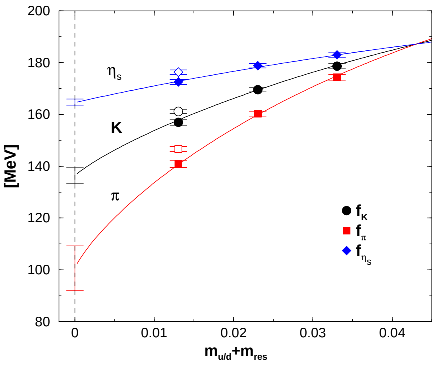
- **Simultaneous fit**



– chiral limit:

f_K = 136(3) MeV+4%

f_π = 101(9) MeV+5%

$f_K/f_\pi \simeq 1.35$

further error from a^{-1}

– inconsistency of B_0 with m^2_{PS}'s

Larger volume?

NLO effect?

Strange quark mass

- **Renormalization**

– Axial WTI:

$$\frac{\Delta_4 \left\langle \mathcal{A}_4(t) P^{(\text{bare})}(0) \right\rangle}{\left\langle P^{(\text{bare})}(t) P^{(\text{bare})}(0) \right\rangle} = m_q^{(\text{bare})} + m_{\text{res}}$$

$$\Rightarrow \ m_s^{(\text{bare})} = Z_P^{-1} \cdot (0.04 + m_{\text{res}})$$

– NPR (RI/MOM) Martinelli *et al.*, 1995

$$Z_q Z_\Gamma^{-1} \cdot \Lambda_\Gamma^{(\text{latt})} = 1$$

[$\Lambda_\Gamma(p)$: vertex func. in p-space]

– Pion-pole Giusti & Vladikus, 2000

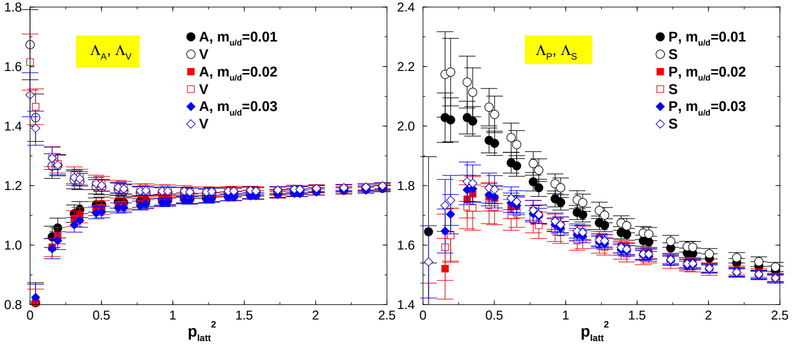
$$\Lambda_P = \frac{c}{m_f} + Z_q Z_m + \cdots \ \Rightarrow \ \tilde{\Lambda}_P = \frac{m_1 \Lambda_P(m_1) - m_2 \Lambda_P(m_2)}{m_1 - m_2}$$

$$= Z_q \cdot Z_m + \text{lattice artifacts}$$

independently, $Z_q = Z_A \Lambda_A$

Chiral symmetry

- $Z_V = Z_A?$, $Z_S = Z_P?$

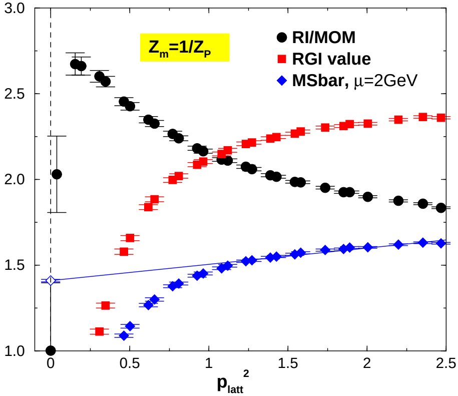


- Chiral symm. is OK.
- V/A: mass-independent; S/P: $\approx 3\%$ change
  Need more data points?/ larger volume?
- average of Z_m over $m_{u/d}$ for a preliminary study

Matching

Using 3-loop perturbation Chetyrkin & Rétey, 2000; Gracey, 2003,

$$Z_m^{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV}) = w_{\overline{\mathrm{MS}}}(\mu = 2\ \mathrm{GeV}) \cdot w^{-1}_{\mathrm{RI/MOM}}(p_{\mathrm{latt}}) \cdot Z_m^{\mathrm{RI/MOM}}(p_{\mathrm{latt}})$$



$$Z_m^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 1.411(7)(\mathrm{sys.}), \qquad m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 96.5(5)(\mathrm{sys.})\ \mathrm{MeV}$$

To do

- **Check the consistency with RBC : CPS vs. Chroma?**
  - Best fit range &, choice of source type?
  - human error?
- **More systematic estimate of FSE: $16^3 \times 32$ vs. $24^3 \times 64$**
  - Reasonable decay consts.?, Consistent B_0?
  - $V \to \infty$?
- **Lighter quark mass: $m_{u/d} = 0.005$ in $16^3 \times 32$ is ongoing.**
  - More critical check of χPT
  - Physical mass point
- **Finer lattice: $\beta = 2.23$ in $16^3 \times 32$ is ongoing**
  - Narrow down the a^{-1} ambiguity.
  - Continuum limit?