Holographic study of deformed Wilson loop operator

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based on hep-th/0609007

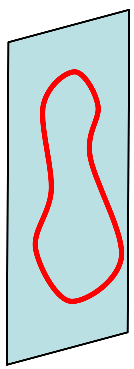
introduction and motivation

AdS/CFT correspondence: J.M.Maldacena(1997)

4-dim, $\mathcal{N}=4$, SU(N), SYM $\longleftrightarrow$ IIB superstring on $\mathrm{AdS}_5 \times \mathrm{S}^5$

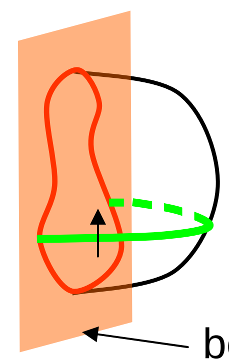
Wilson loop in AdS/CFT: S.J.Rey, J.Yee(1998), J.M. Maldacena(1998)

Wilson loop



$$\langle W[C] \rangle = e^{-A_{\mathrm{w.s.}}[C]}$$

string world sheet



present status and the aim of this talk:

status
- generic loop: difficult
- circle, straight line: well studied

this talk small deformation of circle or straight line

deformation of loop and local operator insertion:

Wilson loop: $W[C] = \mathrm{trP}\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_i \dot{y}^i\right)\right), \quad i = 1\ldots 6$

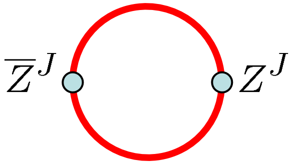
deformation of loop:

$$\frac{\delta W[C]}{\delta \dot{y}^i(s)} = \mathrm{trP}\left[\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_i \dot{y}^i\right)\right)\Phi_i(x(s))\right]$$

large R-charge local operator insertions:

N. Drukker S. Kawamoto (2006)

$$\mathrm{trP}\left[\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_4\right)\right) Z^J(x(s_1))\bar{Z}^J(x(s_2))\right]$$



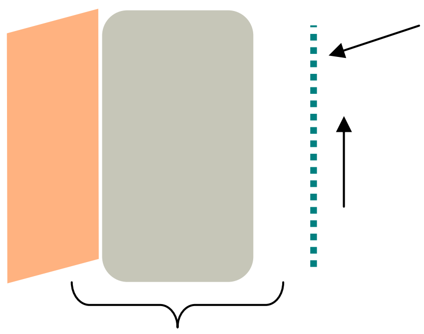
$Z = \Phi_5 + i\Phi_6, \quad J$: large, $\quad x^\mu(s)$: circle or straight line

corresponding string world sheet:

because: R-charge in SYM $\longleftrightarrow$ angular momentum in string theory

we consider: string world sheet with large angular momentum

path of large angular momentum mode:



Classical path do not reach the boundary because of potential barrier

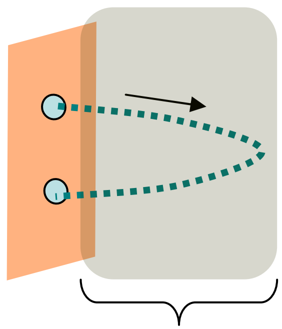
question:

SYM side: $\mathrm{trP}\left[\underline{\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_4\right)\right)}\ \underline{Z^J(x(s_1))\bar{Z}^J(x(s_2))}\right]$

string side: near boundary ←?→ separated from boundary

tunneling geodesic:

S.Dobashi, H.Shimada, T.Yoneya (2002)



potential barrier

tunneling solution

↓

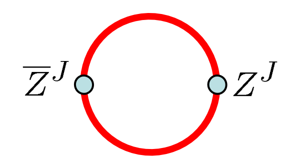
correlation function of
large R-charge local operators

ex: $\langle \mathrm{tr} Z^J(x) \mathrm{tr} \bar{Z}^J(0) \rangle$

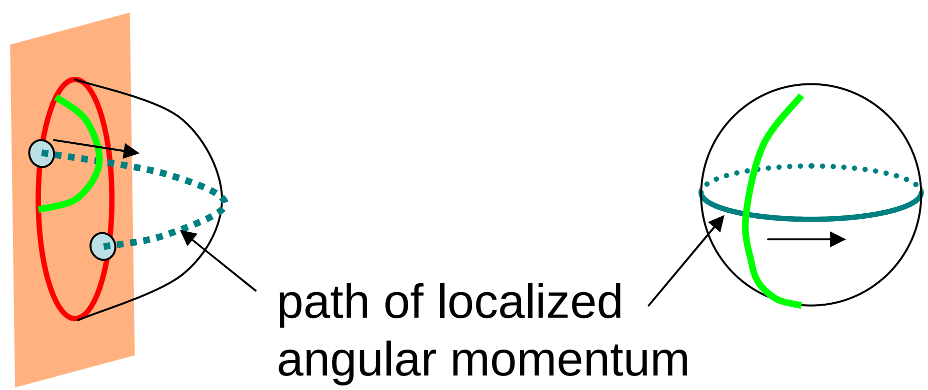
today's talk:

We want expectation value of Wilson loop operator:

$$\mathrm{trP}\left[\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_4\right)\right) Z^J \bar{Z}^J\right]$$



We will study string world sheet around tunneling geodesic



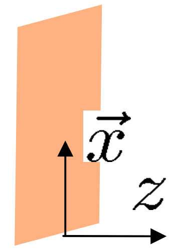
plan of the talk

- introduction and motivation
- world sheet with large angular momentum
- evaluation of “area” of world sheets
- interpretation of results
- conclusion and future work

world sheet with large angular momentum

metric:

$$ds^2 = z^{-2}(dz^2 + d\vec{x}^2) + \cos^2\theta d\psi^2 + d\theta^2 + \sin^2\theta d\tilde{\Omega}_3^2$$

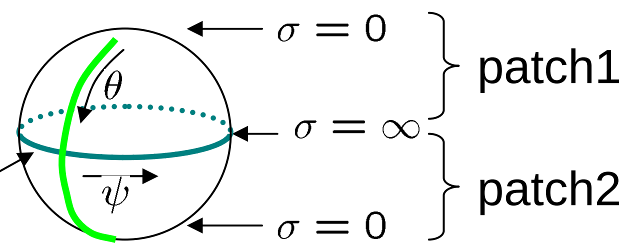


S^5 part:

solution: $\psi = \tau, \;\; \cos\theta = \tanh\sigma, \;\; \tilde{\Omega}_3 = \text{const.}$

$(-\infty < \tau < \infty, \;\; 0 \le \sigma < \infty)$

path of localized angular momentum



AdS_5 part:

Hamiltonian constraint, EOM:

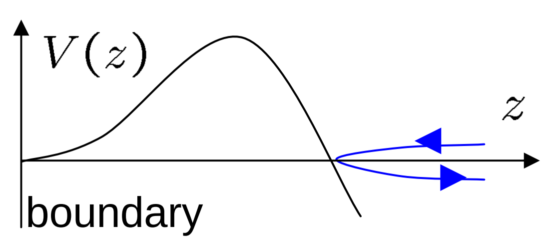
$$\begin{cases} z^{-2}(\dot{z}^2 + z'^2 + \dot{\vec{x}}^2 + \vec{x}'^2) + 1 = 0 \\ (z^{-2}\dot{\vec{x}})^{\cdot} - (z^{-2}\vec{x}')' = 0 \end{cases}$$

$\longrightarrow$ path of localized angular momentum

$$\begin{cases} \dot{z}^2 + \dot{\vec{x}}^2 + z^2 = 0 \\ z^{-4}\dot{\vec{x}}^2 = -\ell^{-2} \end{cases}$$

$$\longrightarrow \quad \dot{z}^2 + V(z) = 0$$

$$V(z) = z^2 - \ell^{-2} z^4$$

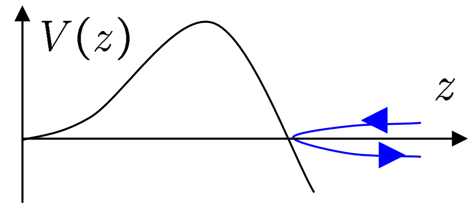


comment on non-tunneling solution:

N.Drukker, S.Kawamoto (2006)

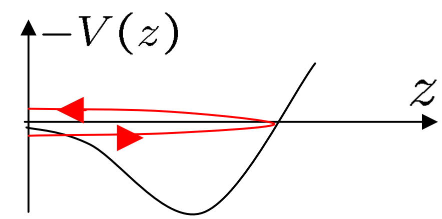
If we use non-tunneling solution, it is difficult to check following relation:

$$\langle W[C] \rangle = \mathrm{e}^{-A_{\mathrm{w.s.}}[C]}$$



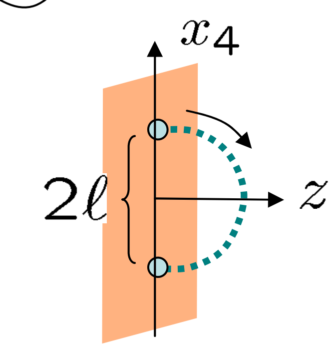
tunneling solution:

S.Dobashi, H.Shimada, T.Yoneya (2002)



$$\tau \longrightarrow -i\tau$$

$$\dot{z}^2 - V(z) = 0$$

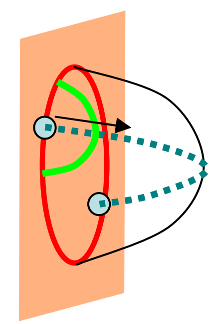


tunneling solution (path of localized R-charge)

$$z = \ell / \cosh \tau, \qquad x_4 = \ell \tanh \tau$$

What we want today:

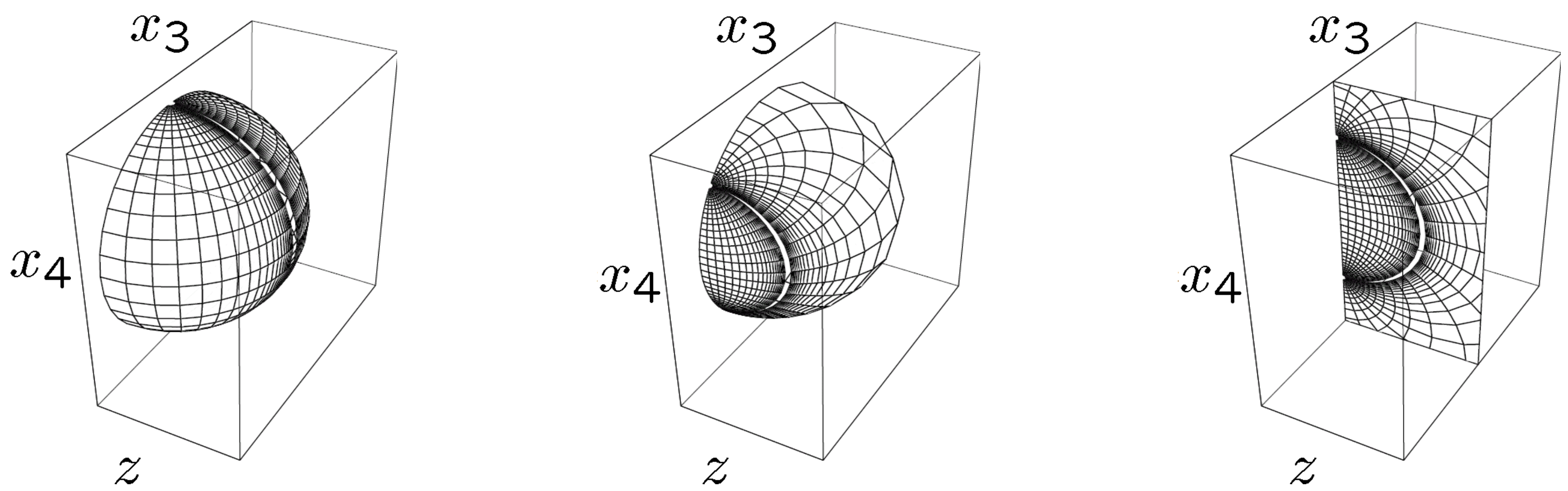
world sheet { attached to the loop on boundary
propagating along tunneling geodesic



full world-sheet solution:

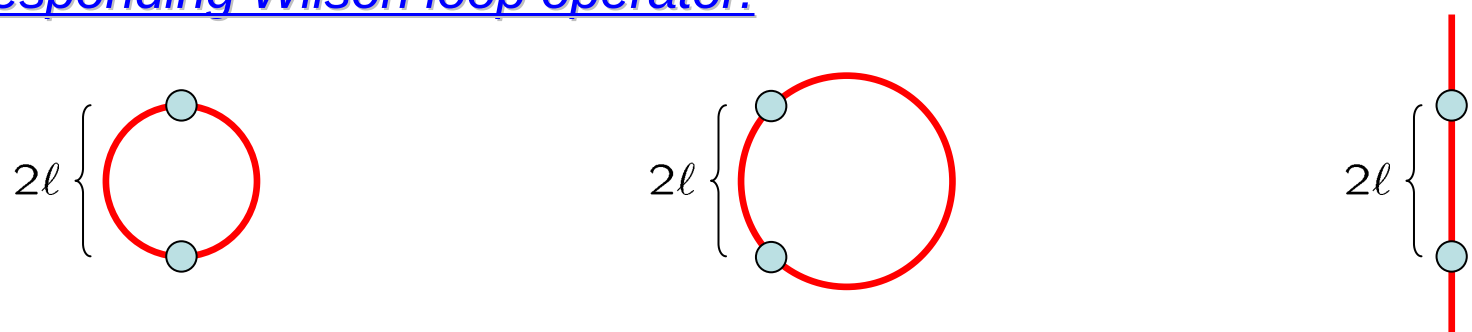
$(0 \le \alpha \le 1)$

$$(z, x_3, x_4) = \left(\frac{\ell \sinh \sigma}{\cosh \sigma \cosh \tau \pm \alpha}, \frac{\pm \ell \sqrt{1-\alpha^2}}{\cosh \sigma \cosh \tau \pm \alpha}, \frac{\ell \cosh \sigma \sinh \tau}{\cosh \sigma \cosh \tau \pm \alpha} \right)$$



$\alpha = 0$ $\quad$ $0 < \alpha < 1$ $\quad$ $\alpha = 1$

corresponding Wilson loop operator:



$\alpha = 0$ $\quad$ $0 < \alpha < 1$ $\quad$ $\alpha = 1$

evaluation of "area" of world sheets

"area" of a world sheet

N.Drukker, D.Gross, H.Ooguri (1999)

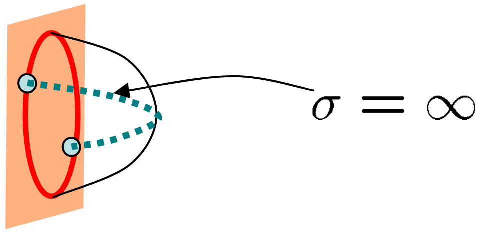
$$S_{\text{total}} = S_{\text{Polyakov}} + S_{\psi} + S_{\text{boundary}}$$

two cutoff schemes

$$\begin{cases} \text{world-sheet cutoff:} & \cosh\tau \le \ell/\epsilon, \quad \sigma > \sigma_0 \\ \text{target-space cutoff:} & z(\tau,\sigma) \ge \epsilon, \quad |x_4(\tau,\sigma)| \le L \end{cases}$$

w.s. cutoff can be rewritten as:

$$z(\tau, \sigma = \infty) \ge \epsilon$$



results:

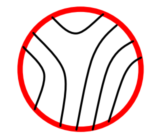
$$S_{\text{total}}\Big|_{\text{straight line}} = 2J \log\frac{2\ell}{\epsilon} - 2J + \frac{R^2}{\alpha'} + \text{const.}$$

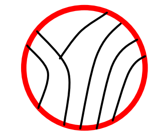
$$S_{\text{total}}\Big|_{\text{circle}} = 2J \log\frac{2\ell}{\epsilon} - 2J + 0 + \text{const.}$$

(const. depends on regularization schemes)

interpretation of results

ladder graph ***(planar graph without internal vertex):***

$$\left\langle \mathrm{P}\exp\left(\int ds\left(iA_\mu \dot{x}^\mu + \Phi_4\right)\right)\right\rangle_{\text{ladder}} = \sum$$ 

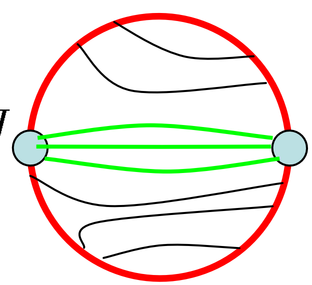
[ not ladder]

$$\begin{cases} \text{circle:} \longrightarrow \langle \mathcal{W} \rangle_{\text{ladder}} \sim e^{\sqrt{\lambda}} \\ \text{straight line:} \longrightarrow \langle \mathcal{W} \rangle_{\text{ladder}} \sim e^{0} \end{cases}$$

These results are known to be consistent with the calculation of the area of world sheet without angular momentum.

D.Berenstein, R.Corrado, W. Fischler, J.M.Maldacena (1998),
N.Drukker, D.J.Gross, H. Ooguri (1999),
J.K.Erickson, G.W.Semenoff, K.Zarembo (2000), etc.

in our case

$\bar{Z}^J$  Z^J

$$Z = \Phi_5 + i\Phi_6$$

$$\langle \mathcal{W} Z^J \bar{Z}^J \rangle \sim \langle \mathcal{W} \rangle \langle Z^J \bar{Z}^J \rangle$$
$$\sim e^{\sqrt{\lambda} \text{ or } 0} \times (2\ell)^{-2J}$$

consistent with our result

conclusion and future work

conclusion:

We have studied

- open string solution around the tunneling geodesic,
- the “area” of the world sheet in two cutoff schemes.

Main results are

- $S_{\text{total}} \sim 2J \log \frac{2\ell}{\epsilon}$ $\quad$ $S_{\text{total}}\Big|_{\text{straight line}} - S_{\text{total}}\Big|_{\text{circle}} = \sqrt{\lambda}$

  (consistent with ladder graph calculation)

- ℓ- and α- independent finite terms depend on cutoff schemes

future work:

- more complicated local operator insertions
  (more complicated deformation of string world sheet)