Selecting Gauge Theories on an Interval by 5D Gauge Transformations

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hep-th/0604121, Talk at Hawai meeting 2006.10.29-11.03,

Contents

1 Introduction

Higher Dimensional **Gauge fields** in **Brane-World**

Extra-dim. component of gauge field acts as a **Higgs** scalar

Hatanaka-Inami-Lim, Mod.Phys.Lett.**A13**, 2601 (1998); Hosotani, Phys.Lett.**B126**, 309 (1983); · · ·

Gauge symmetry breaking: $\left\{ \begin{array}{l} \textbf{Wilson Lines : (VEV of extra dim comp)} \\ \textbf{Boundary Conditions} \end{array} \right.$

Orbifold Models: [(Inner) Automorphisms of the Lie Algebra]

Rank of Gauge Group **cannot be reduced**

Kawamura, Prog.Theor.Phys.**103**, 613 (2000); **105**, 999 (2001); Hall-Nomura, Phys.Rev.**D64**, 055003 (2001); Hebecker-March-Russell, Nucl.Phys.**B625**, 128 (2002); Hall-Murayama-Nomura, Nucl.Phys.**B645**, 85 (2002); Haba-Hosotani-Kawamura, Prog.Theor.Phys.**111**, 265 (2004); Oda-Weiler, Phys.Lett.**B606**, 408 (2005); Agashe-Contino-Pomarol, Nucl.Phys.**B719**, 165 (2005); · · ·

Variational Principle $\longrightarrow$ wider class of boundary conditions

Consider Gauge theories on an **Interval**

Require **Vanishing surface terms** in varying the action

Rank of Gauge Group **can be reduced**

Csaki-Grojean-Murayama-Pilo-Terning, Phys.Rev.**D69**, 055006 (2004) $\cdots$

Are these Boundary Cond.Consistent with 5D Gauge Transformations ?

Two Characteristics of Spontaneously broken gauge theory (in 4D)

$$\begin{cases} \textbf{Ward}-\textbf{Takahashi} \quad \textbf{identity} \\ \textbf{Tree} \quad \textbf{Level} \quad \textbf{Unitarity} \end{cases}$$

Ward-Takahashi identity

Scattering amplitudes vanish for longitudinal massless gauge boson

Tree Level Unitarity

$$-i\langle n|(T-T^\dagger)|n\rangle = \sum_m \langle n|T^\dagger|m\rangle\langle m|T|n\rangle$$

$\longrightarrow$ **unitarity bound** for elastic amplitude (should not diverge)

If 2 body (in-)elastic scattering amplitudes grow with energy $\longrightarrow$

violates **unitarity bound** for elastic scattering amplitude

Llewellyn Smith, Phys.Lett.**B46**, 233 (1973); Dicus-Mathur, Phys.Rev.**D7**, 3111 (1973);

Cornwall-Levin-Tiktopoulos, Phys.Rev.Lett.**30**, 1268 (1973) ; Phys.Rev.**D10**, 1145 (1974) ;

Lee-Quigg-Thacker, Phys.Rev.Lett.**38**, 883 (1977); Chanowitz-Gaillard, Nucl.Phys.**B261**, 379 (1985);

$\cdots$

5D (Higher dim.) Gauge theories

S^1 compactification: **KK gauge bosons** cancel growing contributions

Chivukula-Dicus-He, Phys.Lett.**B525**, 175 (2002); $\cdots$

Boundary Conditions from **Variational Principle**:

Elastic scattering amplitudes of KK modes do not grow with energy

Results of Our Work:

1. **Inelastic scattering** amplitudes of KK modes are computed with gauge theory on an interval
2. **Ward-Takahashi identity** and **Tree Level Unitarity** can be violated by **Boundary contributions**
3. Variational principle allows **boundary conditions** to give boundary contributions **violating W-T identity and tree level unitarity**
4. **Consistently defining the 5D gauge transformations forbids** such boundary conditions

2 Variational Principle and Scattering Amplitudes

Boundary Conditions from Variational Principle

Warped geometry

$$ds^2 = g_{MN}dx^M dx^N = e^{-4W(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g_{55}(y)dy^2$$

$$\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+), \quad 0 \le y \le \pi R$$

$SU(N)$ pure gauge theory with a gauge fixing function G^a

$$\mathcal{S}+\mathcal{S}_{\mathrm{GF}} = \int d^4x \int_0^{\pi R} dy \sqrt{-g(y)} \left(-\frac{1}{4} F^a_{MN} F^a_{PQ} g^{MP} g^{NQ} - \frac{1}{2\xi}(G^a)^2 \right)$$

$$G^a = e^{4W} \left(\eta^{\mu\nu}\partial_\mu A^a_\nu + \xi \frac{1}{\sqrt{g_{55}}} \partial_5 \frac{e^{-4W}}{\sqrt{g_{55}}} A^a_5 \right)$$

$$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + g_5 f^{abc} A^b_M A^c_N$$

Variation of action gives equation of motion if the **surface term** vanishes

$$(\delta\mathcal{S}+\delta\mathcal{S}_{\mathrm{GF}})_{\mathrm{boundary}} = -\int d^4x \left[\frac{e^{-4W}}{\sqrt{g_{55}}} \eta^{\mu\nu} F^a_{5\mu}\, \delta A^a_\nu + \frac{e^{-8W}}{\sqrt{g_{55}}} G^a \delta A^a_5 \right]_{y=0}^{y=\pi R}$$

Possible **Boundary conditions** for **vanishing surface terms**

$$A^a_\mu\Big| = 0, \qquad A^a_5\Big| = 0, \quad \text{(inconsistent) [next sec.]}$$

$$A^a_\mu\Big| = 0, \qquad \partial_5 \frac{e^{-4W}}{\sqrt{g_{55}}} A^a_5\Big| = 0 \quad \text{(Dirichlet)}$$

$$\partial_5\, A^a_\mu\Big| = 0, \qquad A^a_5\Big| = 0 \qquad \text{(Neumann)}$$

Unbroken gauge symmetry is required to form a subgroup H

$$\partial_5\, A^a_\mu\Big| = 0, \qquad a \in H \subseteq G, \qquad y = 0, \pi R$$

Kaluza-Klein **mode decomposition** for boundary cond $D(a)$

$$A^a_\mu(x,y) = \sum_{n=0}^{\infty} A^a_{\mu n}(x) f^{D(a)}_n(y), \quad A^a_5(x,y) = \sum_{n=0}^{\infty} A^a_{5n}(x) g^{D(a)}_n(y)$$

Scattering Amplitudes

Gauge boson scattering $A^a_n A^b_m \longrightarrow A^c_l A^d_m$

Large E with fixed angle θ, longitudinal polarization ϵ except for A^c_l

Terms (of invariant amplitude) **growing in** E (assuming $\xi \ll E^2/m_k^2$)

Massive mode for the external A^c_l boson with polarization $\epsilon^*(p_3)$

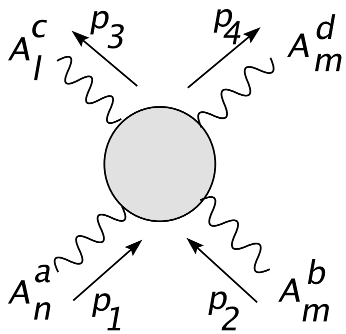

Figure 1: $A^a_n A^b_m \to A^c_l A^d_m$

Table 1: Kinematics: The energy of each mode is given as $E_n = \sqrt{p^2 + m_n^2}$. Both the longitudinal and transverse polarizations $\epsilon(p_3)$ are explored for the gauge field A^c_l.

$p_1 = (E_n, 0, 0, p)$	$\epsilon(p_1) = (p/m_n, 0, 0, E_n/m_n)$
$p_2 = (E_m, 0, 0, -p)$	$\epsilon(p_2) = (p/m_m, 0, 0, -E_m/m_m)$
$p_3 = (E_l, p' \sin\theta, 0, p' \cos\theta)$	$\epsilon(p_3)$
$p_4 = (E'_m, -p' \sin\theta, 0, -p' \cos\theta)$	$\epsilon(p_4) = (p'/m_m, -E'_m \sin\theta/m_m, 0, -E'_m \cos\theta/m_m)$

Transverse pol $\epsilon(p_3) = (0, \cos\theta, 0, -\sin\theta)$: no **growing** contribution

Transverse pol $\epsilon(p_3) = (0, 0, 1, 0)$ gives an amplitude **growing in E**

$$-ig_5^2 f^{abe} f^{cde} \frac{E \sin\theta}{4 m_n m_m^2} \mathcal{K}$$

$$\mathcal{K} = -3\sum_k \left[\frac{e^{-4W}}{\sqrt{g_{55}}} g_k^{D(e)} f_m^{D(b)} f_l^{D(c)}\right]_0^{\pi R} \left[\frac{e^{-4W}}{\sqrt{g_{55}}} g_k^{D(e)} f_m^{D(d)} f_n^{D(a)}\right]_0^{\pi R}$$

$$+2\left[\frac{e^{-4W}}{\sqrt{g_{55}}}\left(f_n^{D(a)} f_m^{D(b)} f_m^{D(d)} f_l^{D(c)}\right)'\right]_0^{\pi R}$$

which is a **boundary contribution**

Longitudinal polarization gives an amplitude **growing in E**

$$-ig_5^2 f^{abe} f^{cde} \frac{E^2(1-\cos\theta)}{8m_n m_m^2 m_l}\mathcal{K}$$

Ward-Takahashi identity: Zero mode ($l = 0$) for the gauge boson A_l^c

$\epsilon^*(p_3) \longrightarrow p'(1, \sin\theta, 0, \cos\theta)$ should give vanishing amplitude, but

$$ig_5^2 f^{abe} f^{cde}(1-\cos\theta)\frac{E^2}{8m_n m_m^2} f_l^{D(c)}$$

$$\left[\frac{e^{-4W}}{\sqrt{g_{55}}}\left(f_m^{D(b)} f_m^{D(d)} f_n^{D(a)\prime} - f_n^{D(a)} f_m^{D(b)} f_m^{D(d)\prime}\right)\right]_0^{\pi R}$$

which is also a **boundary contribution**

Coset-N/Subgroup-D Boundary Condition

Warped geometry: $ds^2 = e^{-4W(y)}\eta_{\mu\nu}dx^\mu dx^\nu + g_{55}(y)dy^2$

$m_{\rm GUT}$ scale at $y = 0$, $m_{\rm W}$ scale at $y = \pi R$ such as ($m_{\rm GUT} \gg m_{\rm W}$)

$$SU(5) \longrightarrow SU(3) \times SU(2) \times U(1) \longrightarrow SU(3) \times U(1)$$

Boundary condition at $y = 0$ ($m_{\rm GUT}$ scale)

$$\left(\begin{array}{c|c} \mathrm{N} & \mathrm{D} \\ \hline \mathrm{D} & \mathrm{N} \end{array}\right) \begin{array}{l} \} \; N_1 \\ \} \; N - N_1 \end{array} \quad \text{at} \quad y = 0,$$

$$SU(N) \longrightarrow SU(N_1) \times SU(N - N_1) \times U(1)$$

Boundary condition at $y = \pi R$ ($m_{\rm W}$ scale)

$$\left(\begin{array}{c|c} \mathrm{N} & \mathrm{N} \\ \hline \mathrm{N} & \mathrm{D} \end{array}\right) \begin{array}{l} \} \; N_1 \\ \} \; N - N_1 \end{array} \quad \text{at} \quad y = \pi R,$$

$$SU(N_1) \times SU(N - N_1) \times U(1) \longrightarrow SU(N_1) \times U(1)$$

Variational principle allows **Dirichlet condition** for $SU(N - N_1)$

$\longrightarrow$ **rank reduction**

Neumann condition for coset is allowed (already broken at $m_{\rm GUT}$)

We call this **Coset-N/Subgroup-D** boundary condition

Coset-N/Subgroup-D boundary condition gives **boundary contributions**

Group theory $[\boldsymbol{G},\boldsymbol{G}]=\boldsymbol{G},\ [\boldsymbol{W},\boldsymbol{W}]=\boldsymbol{W},\ [\boldsymbol{X},\boldsymbol{X}]=\boldsymbol{G}+\boldsymbol{W}+\boldsymbol{B}$

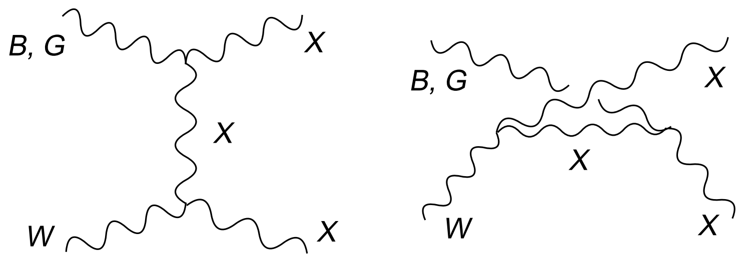

Figure 2: Typical Feynman diagrams giving the boundary contributions violating the Ward-Takahashi identity and the tree level unitarity for $\boldsymbol{W}+\boldsymbol{X}\rightarrow\boldsymbol{B}(\boldsymbol{G})+\boldsymbol{X}$.

Boundary conditions for $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}=\boldsymbol{W},\boldsymbol{X},\boldsymbol{B}(\boldsymbol{G}),\boldsymbol{X}$

$$\left[\frac{e^{-4W}}{\sqrt{g_{55}}}\left(f_n^{D(a)}f_m^{D(b)}f_m^{D(d)}f_l^{D(c)}\right)'\right]_0^{\pi R}\neq 0$$

Boundary conditions for $\boldsymbol{a},\boldsymbol{b},\boldsymbol{c},\boldsymbol{d}=\boldsymbol{X},\boldsymbol{X},\boldsymbol{X},\boldsymbol{X}$, $\boldsymbol{e}=\boldsymbol{W}$

$$\sum_k\left[\frac{e^{-4W}}{\sqrt{g_{55}}}g_k^{D(e)}f_m^{D(b)}f_l^{D(c)}\right]_0^{\pi R}\left[\frac{e^{-4W}}{\sqrt{g_{55}}}g_k^{D(e)}f_m^{D(d)}f_n^{D(a)}\right]_0^{\pi R}\neq 0$$

Violation of W-T identity: Boundary conditions for $\boldsymbol{a, b, c, d = W, X, B(G), X}$

$$\left[\frac{e^{-4W}}{\sqrt{g_{55}}}(f_m^{D(b)} f_m^{D(d)} f_n^{D(a)\prime} - f_n^{D(a)} f_m^{D(b)} f_m^{D(d)\prime})\right]_0^{\pi R} \neq 0$$

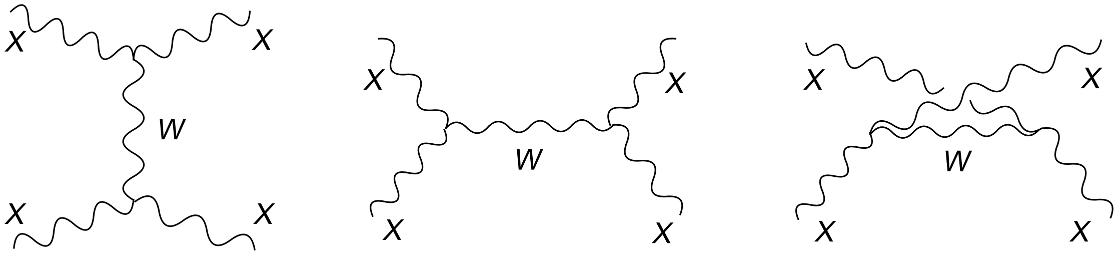

Figure 3: Another typical Feynman diagrams giving the boundary contributions violating the Ward-Takahashi identity and the tree level unitarity for $\boldsymbol{X + X \to X + X}$.

3 Consistency of 5D Gauge Transformations

5D Gauge Transformations

5D gauge transformations for 4D vector component $\boldsymbol{A^a_\mu(x, y)}$

$$\delta A^a_\mu(x, y) = \partial_\mu \epsilon^a(x, y) + g_5 f^{abc} A^b_\mu(x, y) \epsilon^c(x, y)$$

inhomogeneous term = indep of g_5,

nonlinear term = first order in g_5

5D gauge transformations for 4D scalar component $A_5^a(x,y)$

$$\delta A_5^a(x,y) = \partial_5 \epsilon^a(x,y) + g_5 f^{abc} A_5^b(x,y) \epsilon^c(x,y)$$

Opposite Boundary Cond for A_μ, A_5

inhomogeneous term $\longrightarrow$

$A_\mu^a = \epsilon^a = 0$ or $\partial_5 A_\mu^a = \partial_5 \epsilon^a = 0$ (same boundary condiiton)

$A_5^a = \partial_5 \epsilon^a = 0$ or $\partial_5 \frac{e^{-4W}}{\sqrt{g_{55}}} A_5^a = \epsilon^a = 0$ (opposite boundary condition)

$\longrightarrow$ **opposite boundary conditions** for A_μ^a and A_5^a

Coset-N/Subgroup-D Boundary Condition

Nonlinear term $g_5 f^{abc} A_\mu^b \epsilon^c \in \delta A_\mu^a$

1. If $A_\mu^b \epsilon^c \neq 0$, then $\delta A_\mu^a \neq 0$:
   Dirichlet condition is not satisfied by A_μ^a

2. If $\boldsymbol{\partial_5 A^b_\mu \epsilon^c \neq 0}$ or $\boldsymbol{A^b_\mu \partial_5 \epsilon^c \neq 0}$, then $\boldsymbol{\partial_5 \delta A^a_\mu \neq 0}$:
   Neumann condition is not satisfied by $\boldsymbol{A^a_\mu}$

Case 1. occurs if $\boldsymbol{(a, b, c) = (W_\mu, X_\mu, X_\mu)}$: $\boldsymbol{(D, N, N)}$ boundary cond.

$$\boldsymbol{X_\mu \epsilon^X \neq 0, \rightarrow \delta W_\mu \neq 0,}$$

Case 2. occurs

if $\boldsymbol{(a, b, c) = (X_\mu, X_\mu, W_\mu)}$: $\boldsymbol{(N, N, D)}$ boundary cond.

$$\boldsymbol{\partial_5 W_\mu \epsilon^X \neq 0, \rightarrow \partial_5 \delta X_\mu \neq 0,}$$

if $\boldsymbol{(a, b, c) = (X_\mu, W_\mu, X_\mu)}$: $\boldsymbol{(N, D, N)}$ boundary cond.

$$\boldsymbol{X_\mu \partial_5 \epsilon^W \neq 0, \rightarrow \partial_5 \delta X_\mu \neq 0,}$$

5D gauge transformations cannot be defined consistently
with Coset-N/Subgroup-D Boundary Condition

Similarly for $\boldsymbol{A_5}$

4 Summary and Future Problems

1. **Variational principle** allows **boundary conditions that violate the Ward-Takahashi identity and the tree level unitarity**.

2. **5D gauge transformation** parameters $\boldsymbol{\epsilon(x,y)}$ besides the gauge fields $\boldsymbol{A_M(x,y)}$ should be given a **boundary condition** which is **consistent with the 5D gauge transformations**.

3. This condition provides a stringent constraint and forbids these boundary conditions that violate the tree level unitarity and the Ward-Takahashi identity.

Future problems

1. Consistency of 5D gauge transformations $\longrightarrow$
   W-T identity and tree-level unitarity are not yet checked for all processes
2. More general boundary conditions than automorphisms in orbifolding ?
3. Nondiagonal automorphisms of the Lie algebra: interesting possibility

   Hebecker and March-Russell, Nucl.Phys.**B625**, 128 (2002); $\cdots$

4. Deconstruction: consistent 5D gauge transformations usually assured.
   Can all consistent boundary conditions be realized by deconstruction ?