An AdS/CFT analysis of gauge theory plasma

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Based on hep-th/0602010, 0607233, 06mmnnn
in collaboration w/
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AdS/QGP?

QGP experiment is underway at RHIC (Relativistic Heavy Ion Collider)
According to RHIC, QGP behaves like a liquid w/ a very low viscosity (elliptic flow)

Main theoretical challenge: QCD still strongly-coupled and pQCD does not seem to be much helpful.

AdS/CFT comes to the rescue?

One can mainly study supersym. gauge theories in AdS/CFT, however.

Q: Why AdS/CFT has anything to do w/ real QCD?

A: universality

Compute properties which are universal among gauge theories

One example: (shear viscosity)/(entropy density)

$$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$

Kovtun - Son - Starinets (2004)

The relation has been checked for many gravity duals.
There are generic proofs as well.

RHIC indeed suggests $$\frac{\eta}{s} \sim 0.1 \times \frac{\hbar}{k_B}?$$

Teaney (2003)

Chemical potential issue

However, all proofs of the universality fail w/ chemical potential

Kovtun - Son - Starinets, 0309213; 0405231
Buchel - Liu, 0311175
Buchel, 0408095

No known result for η/s

But real experiments are done in finite baryon # density
(RHIC: ion - ion collision such as ^{197}Au)

What happens to the universality?

Setup

Not easy to realize baryon # density in AdS/CFT

→ One simple alternative: charged AdS BHs instead of neutral BHs

cf. 1st law of BH thermodynamics: $dM = TdS + \underline{\Phi} dQ$

(AdS BH) × S^5 → angular momentum along S^5

→ $U(1)_R$ charge

S^5 sym → internal sym → SYM R-sym SO(6)

3 equal charge (SO(6): rank 3) → RN-AdS BH

Not a realistic finite density, but the issue is universality

Shear viscosity for charged AdS BHs was computed by 4 groups (including ours)

J. Mas, 0601144
Son and Starinets, 0601157
Saremi, 0601159
Maeda, Natsuume, and Okamura, 0602010

The result is $$\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}$$ again!

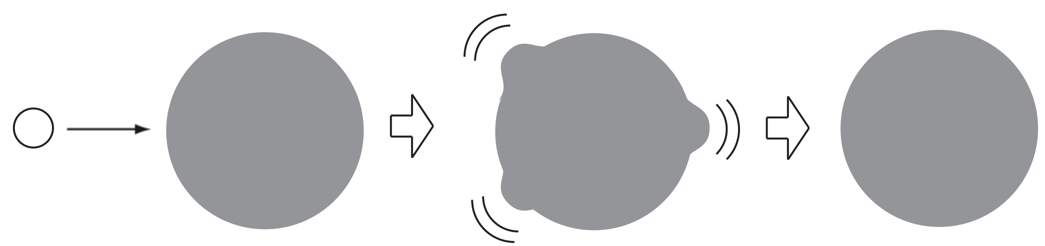
- The universality may hold even at nonzero chemical potential
  → universality even at finite baryon # density?
- η does increase but s has the same scaling (for fixed T)

Some details

BH and hydrodynamics

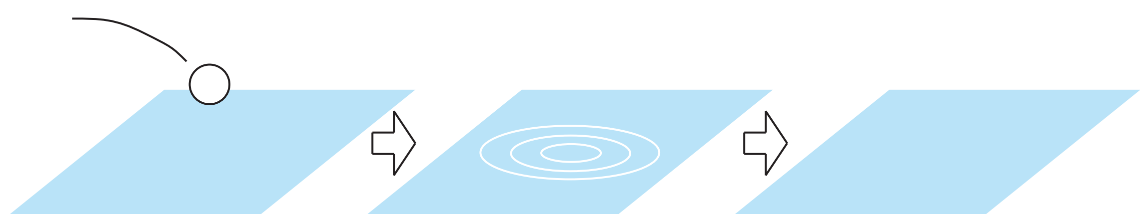
- According to RHIC experiments, QGP behaves like a liquid. AdS/CFT implies that a BH behaves like a liquid as well.
- Then, plasma viscosity must be calculable from BHs.

BH:



The diffusion: consequence of BH absorption

Water pond:

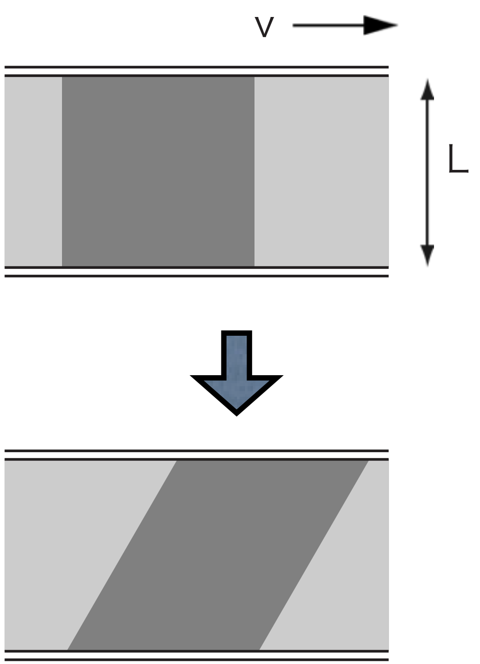


The diffusion: consequence of viscosity

Viscosity

Fluid bet. 2 plates and move the upper plate
The lower plate experiences a force

$$\frac{F}{A} = \underline{\eta} \frac{v}{L}$$



Viscosity modifies EM tensor as

$$T_{ij} = \rho v_i v_j + P\delta_{ij} - \underline{\sigma_{ij}}$$

$$\sigma_{ij} = \eta(\partial_i v_j + \partial_j v_i - \frac{2}{3}\delta_{ij}\partial_k v_k) + \varsigma \delta_{ij}\partial_k v_k$$

↑shear viscosity (traceless part) ↑bulk viscosity (trace part)

→ Navier-Stokes eq. (for imcompressible fluid)

Computation of η

Our problem is to solve perturbation eqs., but I had better explain

- How one can see the diffusion for BH

  → Diffusion is governed by quasinormal modes

- How one can extract η in general

  Which perturbations one has to study?

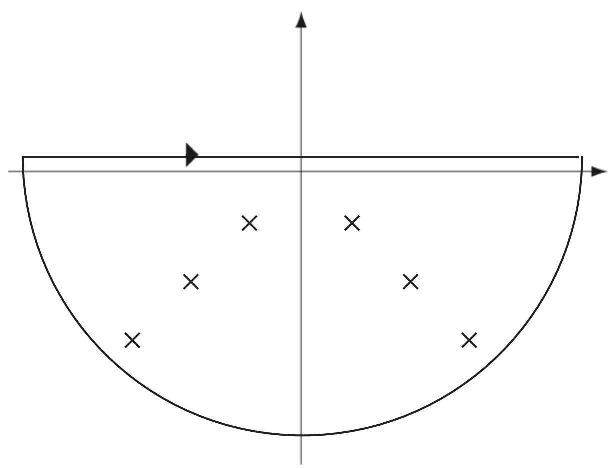
Quasinormal modes (Schwarzschild)

e.g. massless scalar perturbation:

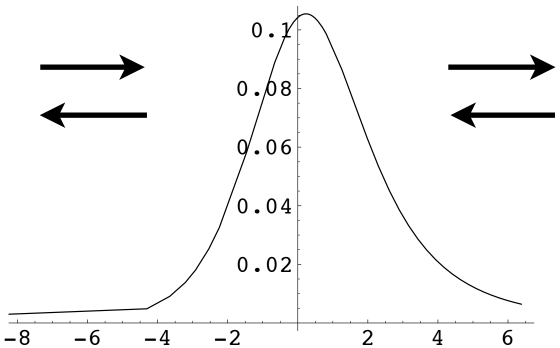
$$\nabla^2 \varphi = 0 \quad \rightarrow \quad \left\{ -\partial_t^2 + \partial_{r_*}^2 - V_l(r) \right\} \varphi_l = 0$$

2 independent solutions in each "asymptotic" region

Sew them together w/ a BC
Possible only for discrete values of ω



effective potential



← horizon

asymptotic → infinity

The perturbation decays exponentially in time $\sim e^{-\omega_I t}$ ⇨ diffusion

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Determine diffusion const.

Example: R-charge diffusion

Tensor decomposition: $J^{\mu} = (\rho, J^{i})$

Conservation eq.: $0 = \partial_{\mu} J^{\mu} = \partial_0 \rho + \partial_i J^i$

Diffusion const. is defined as ("constitutive eq") $J_i = -D \partial_i \rho$

Then,

$$\partial_0 \rho - D \partial_i^2 \rho = 0 \rightarrow \text{pole at } \omega = -iDk^2$$

Diffusion const. may be determined from the pole of an appropriate mode.

Hydrodynamic case

Similarly, tensor decomposition of $T_{\mu\nu}$ according to O(p)

scalar → sound mode e.g. T_{00}
vector → shear mode T_{0i}
tensor T_{ij}

Then, $\partial_\mu T^{\mu\nu} = 0$ + (constitutive eq.) gives

vector mode:

$$\omega = -\frac{i\eta}{\varepsilon + p} k^2$$

scalar mode:

$$\omega = v_s k - \frac{i}{2} \frac{1}{\varepsilon + p} (\varsigma + \frac{4}{3}\eta) k^2$$

speed of sound ↑ ↑ bulk viscosity

Look for h_{0i} perturbations

Outline of computations

Coupled perturbation eqs. (vector modes for RN-AdS$_5$)

⇩ Gravitational + electromagnetic

Decoupled eqs. Kodama - Ishibashi (2003)

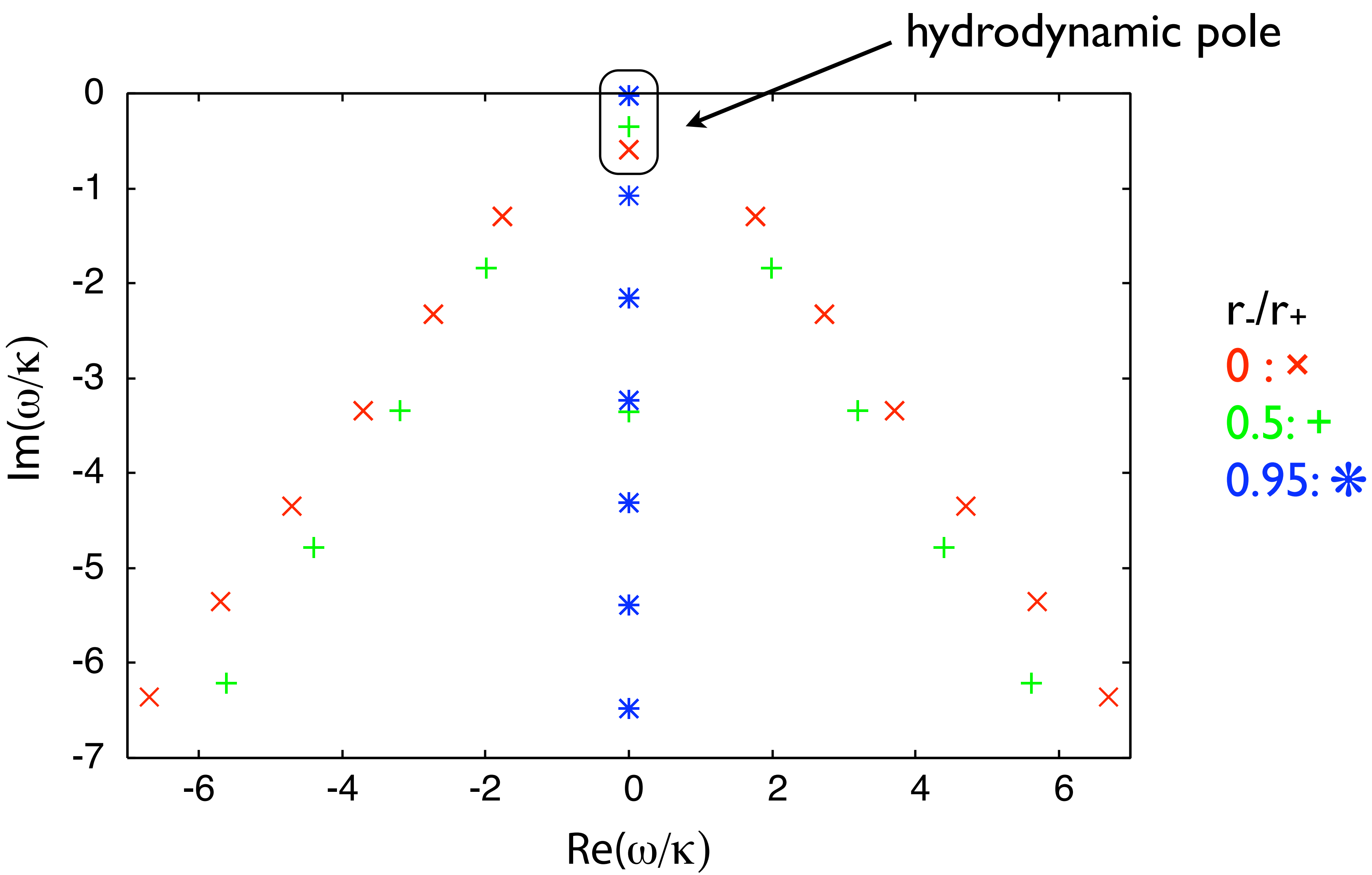
⇩ 2 gauge-inv. modes: Φ_+ and Φ_-

Numerical computations of quasinormal spectrum

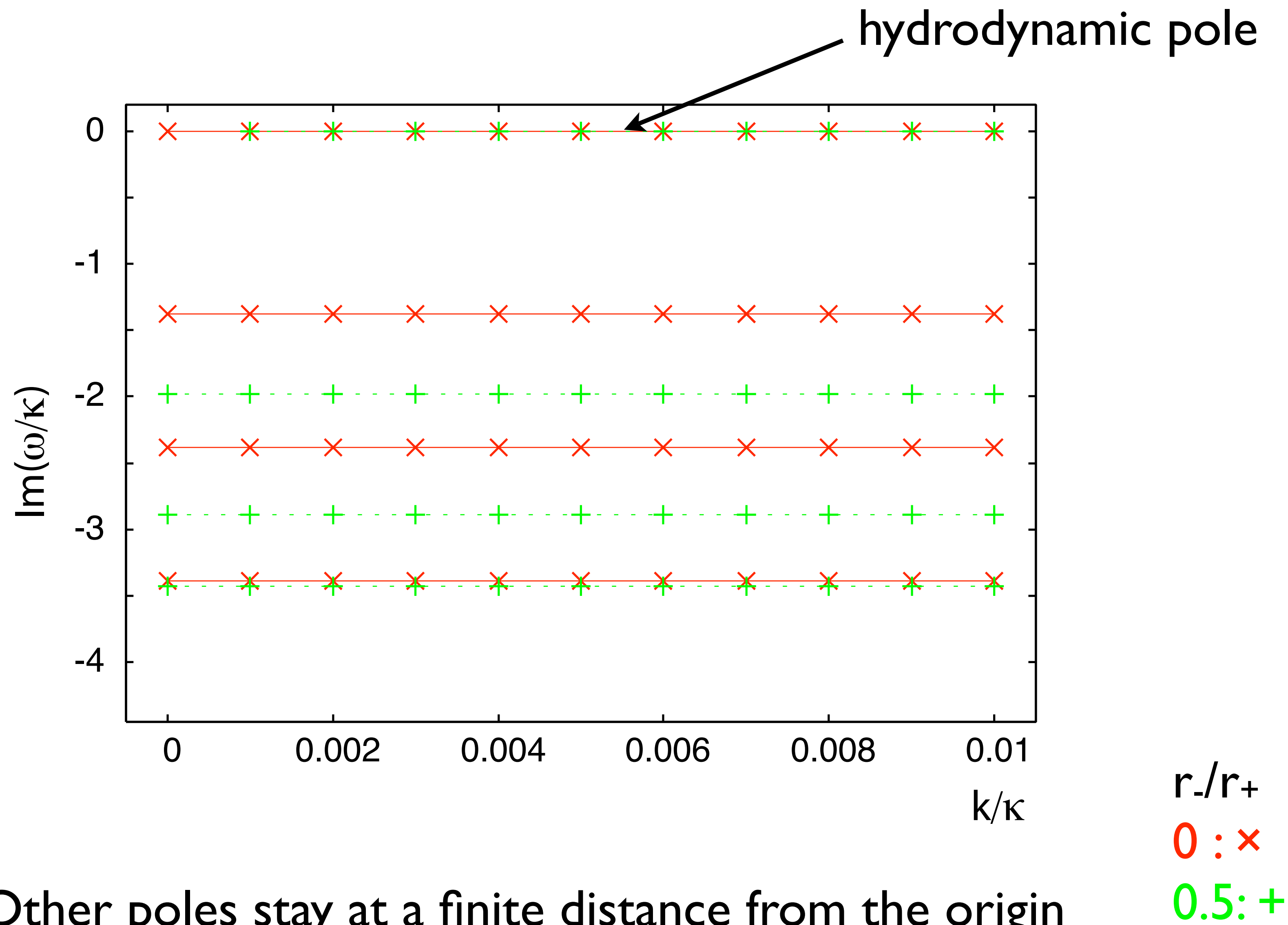
⇩ more than 3 regular singularities

Locate hydrodynamic poles (poles w/ $\omega \to 0, k \to 0$),
check dispersion relations, and find η/s

QN spectrum (Φ_-)

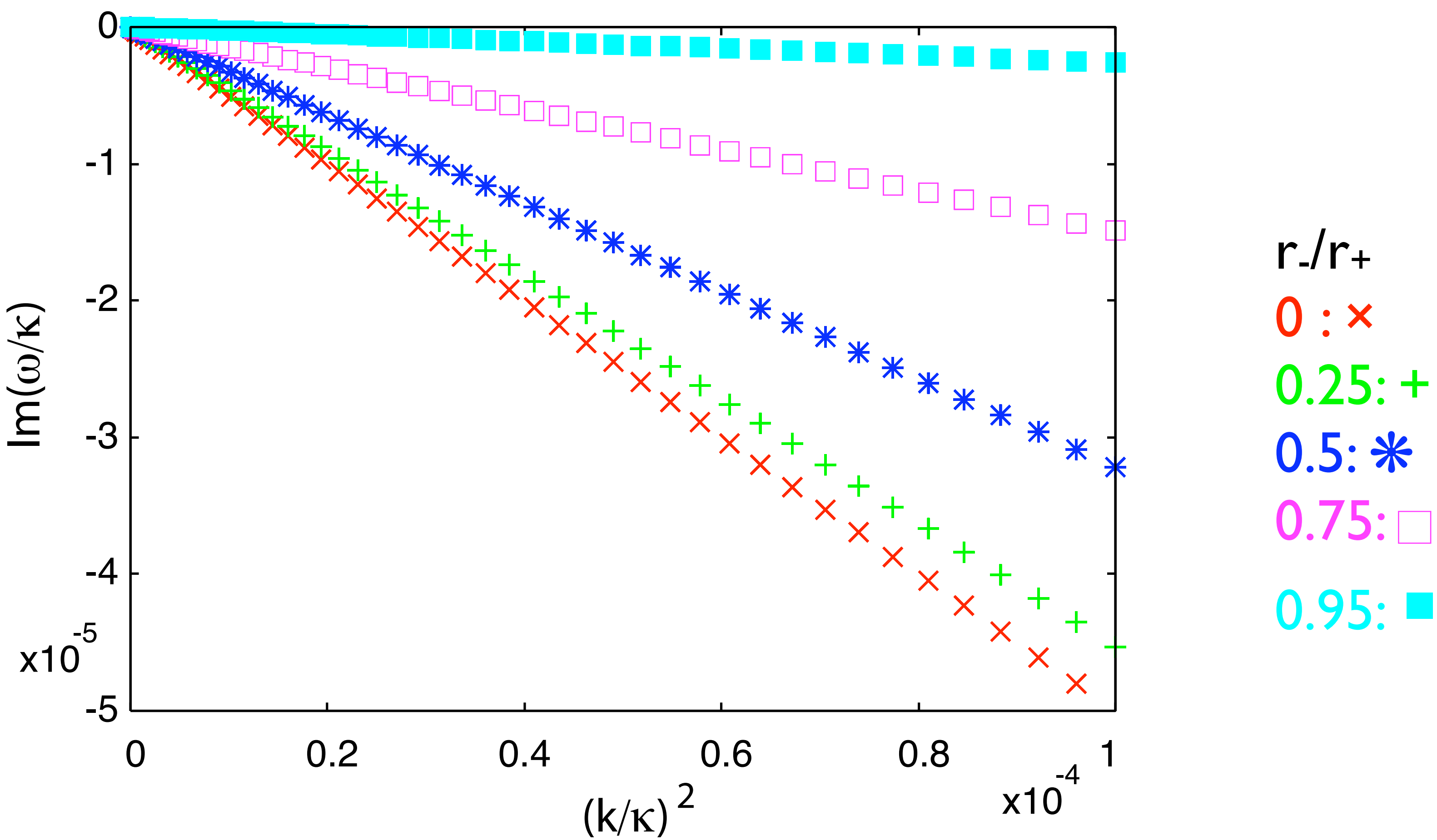


Hydrodynamic pole



Other poles stay at a finite distance from the origin

Dispersion relation



Large r- → small D

large relaxation time

⇨ $\omega_I \propto -k^2$ cf. $\omega = -\dfrac{i\eta}{\varepsilon + p}k^2$

η/s

charge to mass ratio

η/s in units of 1/(4π)

r_-/r_+	a	b	$c \times 10^{11}$	$\gamma_{\text{num.}}$
0.0	−0.500051	2.00002	−3.08339	1.000102
0.05	−0.498175	2.00002	−2.7581	1.00010035
0.1	−0.49255	2.00002	−2.94714	1.00010002
0.15	−0.483179	2.00002	−2.51994	1.00009449
0.2	−0.470089	2.00002	−2.54955	1.0000913
0.25	−0.453336	2.00002	−2.33166	1.00008669
0.3	−0.433032	2.00001	−1.86945	1.00007874
0.35	−0.409367	2.00001	−1.62585	1.0000732
0.4	−0.382616	2.00001	−1.39078	1.0000651
0.45	−0.353155	2.00001	−1.17109	1.00006151
0.5	−0.321446	2.00001	−0.971175	1.00005422
0.55	−0.288032	2.00001	−0.791317	1.00004974
0.6	−0.253502	2.00001	−0.628083	1.00004391
0.65	−0.218464	2.00001	−0.478805	1.00003943
0.7	−0.183507	2.00001	−0.343142	1.00003295
0.75	−0.149172	2.00001	−0.223444	1.00002414
0.8	−0.115927	2.0	−0.126007	1.00001675
0.85	−0.0841502	2.0	−0.0553647	1.00001171
0.9	−0.0541242	2.0	−0.0130611	1.00000517
0.95	−0.0260391	2.0	0.0549852	0.999999434

$$\omega_l = a \times k^b + c$$

b~2

c~0

η/s=1/(4π)

Our competitors

All groups essentially have done the same computations and got the same results

Mas; Son - Starinets; Saremi

- Some uses single R-charged BH, multiple R-charged BH, and RN-AdS4
- Some use different methods

h_{0i} (vector) + QN technique ⇨ diffusion: seen directly

h_{ij} (tensor) + "Kubo formula" method ⇨ indirect

1pt fn

vs

2pt fn

$$T_{ij} = p\delta_{ij} - \frac{1}{\varepsilon + p}\left[\eta(\partial_i T_{0j} + \cdots\right]$$

$$\eta \sim \int d^4x\, e^{i\omega t}\left\langle\left[T_{xy}(t,x), T_{xy}(0,0)\right]\right\rangle$$

linear response theory for BHs?

Other topics

QGP analysis extremely hard due to the strong coupling

➡ genuine signatures of QGP?

☑ Low viscosity (elliptic flow)

☐ Jet quenching

Liu - Rajagopal - Wiedemann, hep-ph/0605178
Herzog et al., 0605158
Casalderrey-Solana - Teaney, hep-ph/0605199
Gubser, 0605182

☐ J/ψ suppression

Liu - Rajagopal - Wiedemann, hep-ph/0607062
Chernicoft - Garcia - Guijosa, 0607089
Caceres - Natsuume - Okamura, 0607233; 06mmnnn

All of these are explored in AdS/CFT recently.

➡ My talk at "Hot & Dense QCD " session

Summary

- Hydrodynamic description of gauge theory plasma using AdS/CFT: very powerful due to universality
- Universality seems to hold even at finite chemical potential
- AdS/CFT may be useful to analyze experiments
  Experiments or the other theoretical tools (such as lattice) may be useful to confirm AdS/CFT
- Many loose ends