An AdS/CFT analysis of gauge theory plasma

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Based on hep-th/0602010, 0607233, 06mnnnn
in collaboration w/
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AdS/QGP?

QGP experiment is underway at RHIC (Relativistic Heavy Ion Collider). According to RHIC, QGP behaves like a liquid with a very low viscosity (elliptic flow).

Main theoretical challenge: QCD still strongly-coupled and pQCD does not seem to be much helpful.

AdS/CFT comes to the rescue?

One can mainly study supersym. gauge theories in AdS/CFT, however.
Q: Why AdS/CFT has anything to do with real QCD?

A: universality

Compute properties which are universal among gauge theories

One example: (shear viscosity)/(entropy density)

\[
\frac{\eta}{s} = \frac{\hbar}{4\pi k_B}
\]

Kovtun - Son - Starinets (2004)

The relation has been checked for many gravity duals. There are generic proofs as well.

RHIC indeed suggests

\[
\frac{\eta}{s} \sim 0.1 \times \frac{\hbar}{k_B}
\]

Teaney (2003)

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However, all proofs of the universality fail w/ chemical potential

Kovtun - Son - Starinets, 0309213; 0405231
Buchel - Liu, 0311175
Buchel, 0408095

No known result for $\eta/s$

But real experiments are done in finite baryon # density
(RHIC: ion - ion collision such as $^{197}$Au)

What happens to the universality?
Not easy to realize baryon # density in AdS/CFT

→ One simple alternative: charged AdS BHs instead of neutral BHs

cf. 1st law of BH thermodynamics:  \( dM = TdS + \Phi dQ \)

\((\text{AdS BH}) \times S^5 \rightarrow \text{angular momentum along } S^5 \)

\( \rightarrow U(1)_R \text{ charge} \)

\( S^5 \text{ sym} \rightarrow \text{internal sym} \rightarrow \text{SYM R-sym SO(6)} \)

3 equal charge (SO(6): rank 3) \( \rightarrow \text{RN-AdS BH} \)

Not a realistic finite density, but the issue is universality
Shear viscosity for charged AdS BHs was computed by 4 groups (including ours)

\[ \frac{\eta}{s} = \frac{\hbar}{4\pi k_B} \]

The result is again!

- The universality may hold even at nonzero chemical potential
  \[ \rightarrow \text{ universality even at finite baryon \# density?} \]

- \( \eta \) does increase but \( s \) has the same scaling (for fixed \( T \))

J. Mas, 0601144
Son and Starinets, 0601157
Saremi, 0601159
Maeda, Natsuume, and Okamura, 0602010
Some details
BH and hydrodynamics

- According to RHIC experiments, QGP behaves like a liquid. AdS/CFT implies that a BH behaves like a liquid as well.
- Then, plasma viscosity must be calculable from BHs.

**BH:**

The diffusion: consequence of BH absorption

**Water pond:**

The diffusion: consequence of viscosity
Fluid bet. 2 plates and move the upper plate
The lower plate experiences a force

\[
\frac{F}{A} = \eta \frac{v}{L}
\]

Viscosity modifies EM tensor as

\[
T_{ij} = \rho v_i v_j + P\delta_{ij} - \sigma_{ij}
\]

\[
\sigma_{ij} = \eta (\partial_i v_j + \partial_j v_i - \frac{2}{3} \delta_{ij} \partial_k v_k) + \zeta \delta_{ij} \partial_k v_k
\]

\text{↑ shear viscosity (traceless part) \quad ↑ bulk viscosity (trace part)}

→ Navier-Stokes eq. (for incompressible fluid)
Computation of $\eta$

Our problem is to solve perturbation eqs., but I had better explain

How one can see the diffusion for BH

$\rightarrow$ Diffusion is governed by quasinormal modes

How one can extract $\eta$ in general

Which perturbations one has to study?
Quasinormal modes (Schwarzschild)

e.g. massless scalar perturbation:

\[ \nabla^2 \varphi = 0 \rightarrow \left\{ -\partial_t^2 + \partial_{r^*}^2 - V_l(r) \right\} \varphi_l = 0 \]

2 independent solutions in each “asymptotic” region

Sew them together w/ a BC
Possible only for discrete values of \( \omega \)

The perturbation decays exponentially in time

\[ \sim e^{-\omega t} \]

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Computation of $\eta$

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How one can see the diffusion for BH

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Which perturbations one has to study?
Example: R-charge diffusion

Tensor decomposition: \( J^\mu = (\rho, J^i) \)

Conservation eq.: \( 0 = \partial_\mu J^\mu = \partial_0 \rho + \partial_i J^i \)

Diffusion const. is defined as ("constitutive eq") \( J_i = -D \partial_i \rho \)

Then,

\[
\partial_0 \rho - D \partial_i^2 \rho = 0 \rightarrow \text{pole at } \omega = -iDK^2
\]

Diffusion const. may be determined from the pole of an appropriate mode.
Hydrodynamic case

Similarly, tensor decomposition of $T_{\mu\nu}$ according to $O(p)$

- scalar $\rightarrow$ sound mode e.g. $T_{00}$
- vector $\rightarrow$ shear mode $T_{0i}$
- tensor $T_{ij}$

Then, $\partial_\mu T^{\mu\nu} = 0$ + (constitutive eq.) gives

vector mode:

$$\omega = -\frac{i\eta}{\varepsilon + p} k^2$$

scalar mode:

$$\omega = v_s k - \frac{i}{2} \frac{1}{\varepsilon + p} (\zeta + \frac{4}{3} \eta) k^2$$

speed of sound $\uparrow$ $\uparrow$ bulk viscosity

Look for $h_{0i}$ perturbations
Outline of computations

Coupled perturbation eqs. (vector modes for RN-AdS$_5$)

Gravitational + electromagnetic

Decoupled eqs.

Kodama - Ishibashi (2003)

2 gauge-inv. modes: $\Phi_+$ and $\Phi_-$

Numerical computations of quasinormal spectrum

more than 3 regular singularities

Locate hydrodynamic poles (poles w/ $\omega \to 0, k \to 0$), check dispersion relations, and find $\eta/s$
QN spectrum (Φ−)

hydrodynamic pole

0 : x
0.5: +
0.95: ★
Hydrodynamic pole

Other poles stay at a finite distance from the origin

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Dispersion relation

\[ \text{Large } r_+ \rightarrow \text{small } D \]

large relaxation time

\[ \omega_I \propto -k^2 \quad \text{cf.} \quad \omega = -\frac{i\eta}{\varepsilon + \rho} k^2 \]
\[ \eta/s \]

**charge to mass ratio**

**\( \eta/s \) in units of \( 1/(4\pi) \)**

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<th>( b )</th>
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\[ \omega_I = a \times k^b + c \]

\[ b \sim 2 \quad c \sim 0 \]

\[ \eta/s = 1/(4\pi) \]

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Our competitors

All groups essentially have done the same computations and got the same results

- Some uses single R-charged BH, multiple R-charged BH, and RN-AdS₄
- Some use different methods
  - $h_{0i}$ (vector) + QN technique $\Rightarrow$ diffusion: seen directly
  - $h_{ij}$ (tensor) + “Kubo formula” method $\Rightarrow$ indirect

\[
T_{ij} = p\delta_{ij} - \frac{1}{\epsilon + p} \left[ \eta(\partial_i T_{0j} + \cdots) \right]
\]

linear response theory for BHs?
Other topics

QGP analysis extremely hard due to the strong coupling

genuine signatures of QGP?

☑ Low viscosity (elliptic flow)
☐ Jet quenching
☐ J/ψ suppression

Liu - Rajagopal - Wiedemann, hep-ph/0605178
Herzog et al., 0605158
Casalderrey-Solana - Teaney, hep-ph/0605199
Gubser, 0605182
Liu - Rajagopal - Wiedemann, hep-ph/0607062
Chernicoff - Garcia - Guijosa, 0607089
Caceres - Natsuume - Okamura, 0607233; 06mnnnn

All of these are explored in AdS/CFT recently.

My talk at “Hot & Dense QCD“ session

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Summary

Hydrodynamic description of gauge theory plasma using AdS/CFT: very powerful due to universality

Universality seems to hold even at finite chemical potential

AdS/CFT may be useful to analyze experiments
Experiments or the other theoretical tools (such as lattice) may be useful to confirm AdS/CFT

Many loose ends