Constraints on flavor-changing $Z'$ models by $B_s$ mixing, $Z'$ production and $B_s \to \mu^+\mu^-$. 

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(with Cheng-Wei Chiang, Jing Jiang, Deshpande hep-ph/0604223)
Outline

- Motivation for flavor changing $Z'$ models.
- A flavor-changing $Z'$ model.
- Constraint from the new $B_s$ mixing data.
- Range of parameters allowed by $Z'$ production limits.
- Prediction for $B_s \rightarrow \mu^+ \mu^-$. 
Motivations for flavor changing $Z'$ models

- Heavy extra gauge bosons may exist in many extensions of the SM, including GUT, extra dimensions, little Higgs, ...
- In some string-inspired models, the 3 families are generated differently in the extra dimensions. It may happen that the coupling strength to one of the families is different.
- So when rotating to the mass eigenbasis, flavor-changing couplings are induced.
- Thus, it will affect the FCNC processes.
A Flavor Changing $Z'$ model

In some string-inspired models, it is possible to have family nonuniversal interactions, assuming flavor diagonal,

$$\mathcal{L} = -g_2 Z'_\mu \bar{\psi}_i \gamma^\mu (\epsilon_{L \ ij} P_L + \epsilon_{R \ ij} P_R) \psi_j$$

with $\epsilon_{L,R}$ being the chiral couplings of $Z'$.

- Assume

$$\epsilon_{L,R}^u = Q_{L,R}^u \mathbf{1}, \quad \epsilon_{L,R}^e = Q_{L,R}^e \mathbf{1}, \quad \epsilon_{R}^d = Q_{R}^d \mathbf{1},$$

and the only deviation from $\mathbf{1}$ is

$$\epsilon_{L}^d = Q_{L}^d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & x \end{pmatrix}$$

with $x \sim O(1) \neq 1$. 
• Rotate to the mass eigenbasis by \((V_d)_{L,R}\) (assume the up-sector is already diagonal.) So the only flavor changing effect is in the LH down-sector: \(\epsilon^d_L\):

\[
\mathcal{L} = -g_2 Z'_\mu \left( \bar{d}, \bar{s}, \bar{b} \right)_m \gamma^\mu \left( V^\dagger_{dL} \epsilon^d_L V_{dL} P_L + Q^d_R 1 P_R \right) \begin{pmatrix} d \\ s \\ b \end{pmatrix}_m
\]

where

\[
B^d_L \equiv V^\dagger_{dL} \epsilon^d_L V_{dL} = V^\dagger_{ckm} \epsilon^d_L V_{ckm}
\]

\[
= Q^d_L \begin{pmatrix}
1 & (x - 1)V_{ts}V^*_{td} & (x - 1)V_{tb}V^*_{td} \\
(x - 1)V_{td}V^*_{ts} & 1 & (x - 1)V_{tb}V^*_{ts} \\
(x - 1)V_{td}V^*_{tb} & (x - 1)V_{ts}V^*_{tb} & x
\end{pmatrix}
\]

Note that the sizes of the flavor-changing couplings are given by the CKM and satisfy the hierarchy: \(|B^{bs}_L| > |B^{bd}_L| > |B^{sd}_L|\).

Such couplings can give to observable FCNC.
\( B_s - \bar{B}_s \) Meson Mixing

- As in the \( B_d \) system, the \( B_s - \bar{B}_s \) system provides another testing ground for the SM CKM mechanism.
- The experimental ratio \( \Delta M_d / \Delta M_s \) determines \( V_{td} / V_{ts} \) if assuming no new physics.
- In the SM, the \( \Delta M_s \) is expected to be around 18 ps\(^{-1} \), and the phase \( \phi_s \) to be small.
- New physics contributions can play an important role in the \( B_s - \bar{B}_s \) mixing because of the loop nature in the SM.
New Results from DØ and CDF

- The FCNC effect in the $b$-$s$ sector of the SM was recently confirmed in the $B_s$ meson mixing observed by DØ and CDF:

  - CDF: $\Delta M_s = 17.33^{+0.42}_{-0.21} \text{ (stat.)} \pm 0.07 \text{ (syst.)} \text{ ps}^{-1}$,
  - DØ: $\Delta M_s = 19.0 \pm 1.215 \text{ ps}^{-1}$,
  - Combined: $\Delta M_s^{\exp} = 17.46^{+0.47}_{-0.30} \text{ ps}^{-1} \quad (1\sigma \text{ range})$

- Within the SM, this implies

  $$|V_{td}/V_{ts}| = 0.208^{+0.008}_{-0.007}$$

- In comparison, the latest Belle results on $b \to d\gamma$ and $b \to s\gamma$ gives $0.142 - 0.259$ (95% C.L. range) for the ratio.
Treatment of the data

- We re-evaluate the $B_s$ mass difference in the SM without referring to $\Delta M_d$ and before the recent Tevatron data:

$$\Delta m_{B_s}^{\text{SM}} = \frac{G_F^2}{6\pi^2} M_W^2 m_{B_s} f_{B_s}^2 (V_{tb} V_{ts}^*)^2 \eta_2 B S_0(x_t) \times [\alpha_s(m_b)]^{-6/23} \left[ 1 + \frac{\alpha_s(m_b)}{4\pi} J_5 \right] B_{B_s}(m_b)$$

$$= 19.52 \pm 5.28 \text{ ps}^{-1}.$$ 

- We have used

$$\eta_2 B \simeq 0.551, \quad J_5 \simeq 1.627, \quad m_{B_s} = 5.3696 \pm 0.0024 \text{ GeV}$$

$$\tau_{B_s} = 1.466 \pm 0.059 \text{ ps}^{-1}, \quad f_{B_s} \sqrt{B_{B_s}} = 0.262 \pm 0.035$$

And we use the CKMfitter results after EPS 2005:

$$\lambda = 0.22622 \pm 0.00100, \quad A = 0.825_{-0.019}^{+0.011}, \quad \bar{\rho} = 0.207_{-0.043}^{+0.036}, \quad \bar{\eta} = 0.340 \pm 0.023$$

- This is consistent with the experimental result. One can use it to constrain the parameters of the new physics models.
Z' Contributions and Constraints

- New physics contributions to $b \to s$ transition induce $|\Delta B| = |\Delta S| = 2$ operators that affects $B_s$ mixing is (LL only)

$$\mathcal{L}_{eff}^{Z'} = \frac{G_F}{\sqrt{2}} \left( \frac{g_2 M_Z}{g_1 M_{Z'}} B_{L}^{sb} \right)^2 O^{LL}(m_b) = \frac{G_F}{\sqrt{2}} \left( \rho_L^{sb} \right)^2 e^{2i\phi_L^{sb}} O^{LL}(m_b)$$

where $O^{LL} = [\bar{s}\gamma_{\mu}(1 - \gamma_5)b][\bar{s}\gamma^\mu(1 - \gamma_5)b]$.

- The effect of the LH FCNC induced by $Z'$ is

$$\frac{\Delta M_s^{\text{exp}}}{\Delta M_s^{\text{SM}}} = \left| 1 + 3.57 \times 10^5 \left( \rho_L^{sb} \right)^2 e^{2i\phi_L^{sb}} \right| = 0.894 \pm 0.243$$

- In the model that we are considering

$$\rho_L^{sb} = \left| \frac{g_2 M_Z}{g_1 M_{Z'}} B_{L}^{sb} \right| = \left| \frac{g_2 M_Z}{g_1 M_{Z'}} (x - 1) Q_L V_{tb} V_{ts}^* \right|, \quad \phi_L^{sb} = 180^\circ$$
For $\phi_L^{sb} = 0^\circ/180^\circ$, $\rho_L^{sb} < 6.20 \times 10^{-4}$.

In more general models, $\phi_L^{sb}$ can be different. For $\phi_L^{sb} = 90^\circ$, $\rho_L^{sb} < 10^{-3}$.

Note that there are regions where $\rho_L^{sb} > 10^{-3}$. E.g., at $\phi_L^{sb} = 90^\circ$, $2.15 \times 10^{-3} < \rho_L^{sb} < 2.45 \times 10^{-3}$. It corresponds to a larger $Z'$ contribution than the SM one.
Drell-Yan process:

\[ p\bar{p} \rightarrow \gamma, \ Z, \ Z' \rightarrow \mu^+\mu^- , \ e^+e^- \]

has been very useful in constraining \( Z' \) models.
95 % C.L. limits on $\sigma(Z') \cdot B(Z' \rightarrow e^+e^-)$ at the Tevatron.

<table>
<thead>
<tr>
<th>$M_{Z'}$ (GeV)</th>
<th>$\sigma \cdot B^{95}$ (pb)</th>
<th>$M_{Z'}$ (GeV)</th>
<th>$\sigma \cdot B^{95}$ (pb)</th>
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<tr>
<td>200</td>
<td>0.0505</td>
<td>600</td>
<td>0.0132</td>
</tr>
<tr>
<td>250</td>
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<td>650</td>
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<td>500</td>
<td>0.0172</td>
<td>900</td>
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<tr>
<td>550</td>
<td>0.0138</td>
<td>950</td>
<td>0.0246</td>
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</table>

Preliminary RunII 95% C.L. limits on $\sigma(Z') \cdot B(Z' \rightarrow e^+e^-)$. 
Implied constraints on various $Z'$ models

<table>
<thead>
<tr>
<th>Sequential $Z$</th>
<th>$Z_{LR}$</th>
<th>$Z_{\chi}$</th>
<th>$Z_{\psi}$</th>
<th>$Z_{\eta}$</th>
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</thead>
<tbody>
<tr>
<td>$Q^u_L$</td>
<td>0.3456</td>
<td>-0.08493</td>
<td>$\frac{-1}{2\sqrt{10}}$</td>
<td>$\frac{1}{\sqrt{24}}$</td>
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<td>$Q^d_R$</td>
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<tr>
<td>$Q^e_L$</td>
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<td>0.2548</td>
<td>$\frac{3}{2\sqrt{10}}$</td>
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<tr>
<td>$Q^e_R$</td>
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<tr>
<td>$Q^\nu_L$</td>
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<td>0.2548</td>
<td>$\frac{3}{2\sqrt{10}}$</td>
<td>$\frac{1}{\sqrt{24}}$</td>
</tr>
</tbody>
</table>

Implied constraints on various $Z'$ models:

$Z'_{SM} > 845$ GeV, $Z'_{\chi} > 720$ GeV, $Z'_{\psi} > 690$ GeV, $Z'_{\eta} > 715$ GeV,
Constraint on the present model by $Z'$ production

- The production cross section of $Z'$ followed by the leptonic decay is given by

$$\sigma(p\bar{p} \rightarrow Z' \rightarrow \ell^+ \ell^-) = \frac{g_2^4}{144} \frac{1}{s} \frac{M_{Z'}}{\Gamma_{Z'}} \left( Q_L^2 + Q_R^2 \right)$$

$$\times \sum_{q=u,d,s,c} (Q_L^2 + Q_R^2) \int_r^1 \frac{dx}{x} f_q(x) f_{\bar{q}} \left( \frac{r}{x} \right)$$

where $\sqrt{s} = 1960$ GeV, $r = M_{Z'}^2/s$ and $\Gamma_{Z'}$ is the total width.

The partial width $Z' \rightarrow f\bar{f}$ is

$$\Gamma(Z' \rightarrow f\bar{f}) = \frac{N_f g_2^2 M_{Z'}}{48\pi} \sqrt{1 - 4\mu} \left[ 2 \left( |Q_L^f|^2 + |Q_R^f|^2 \right) (1 - \mu) + 12\mu Q_L^f Q_R^f \right]$$
In the present FCNC $Z'$ model,

$$B^d_L = Q^d_L \begin{pmatrix} 1 & (x-1)V_{ts}V^*_{td} & (x-1)V_{tb}V^*_{td} \\ (x-1)V_{td}V^*_{ts} & 1 & (x-1)V_{tb}V^*_{ts} \\ (x-1)V_{td}V^*_{tb} & (x-1)V_{ts}V^*_{tb} & x \end{pmatrix}$$

one can translate the bound on $\rho^{sb}_{L} \equiv \left| \frac{g_2 M_{Z'}}{g_1 M_{Z'}} B^{sb}_L \right| < 6.2 \times 10^{-4}$ to other elements in $B^d_L$. We obtain the upper limits on $Z' - q - \bar{q}$

With the assumption

$$|Q^d_R| = |Q^d_L|, \quad |Q^u_{L,R}| = |Q^d_{L,R}|.$$

then from the bound of $\rho^{sb}_{L}$ we have the size of all $Z' - q - \bar{q}$ couplings. Then using the CDF $\sigma(Z') \cdot B(Z' \to e^+e^-)$ limits one can obtain upper limits on

$$g_2 \sqrt{(Q^e_{L}^2 + Q^e_{R}^2)/2}$$
Upper limit on $Q_L^e$, $Q_R^e$.

With $\rho_L^{sb} = 6.20 \times 10^{-4}$. 

$x = 0.1$ to $0.9$ from top to bottom.
Constraints from $B_s \to \mu^+ \mu^-$ data

- The decay rate in $Z'$ model depends on $\rho_{L,R}^{sb,\mu}$:

$$B(B_s \to \mu^+ \mu^-) = \tau(B_s) \frac{G_F^2}{4\pi} f_{B_s}^2 m_{\mu}^2 m_{B_s} \sqrt{1 - \frac{4m_{\mu}^2}{m_{B_s}^2}} |V_{tb}^* V_{ts}|^2$$

$$\times \left\{ \alpha \frac{m_{t}^2}{2\pi \sin^2 \theta_W} Y \left( \frac{m_{t}^2}{M_{W}^2} \right) + 2 \frac{\rho_{L}^{bs} \rho_{L}^{\mu\mu}}{V_{tb}^* V_{ts}} \right\}^2 + \left\{ 2 \frac{\rho_{L}^{bs} \rho_{R}^{\mu\mu}}{V_{tb}^* V_{ts}} \right\}^2$$

$$\rho_{L,R}^{\mu\mu} = \frac{g_2 M_Z}{g_1 M_{Z'}} Q_{L,R}^e$$

- In the SM, by putting all $\rho_{L,R} = 0$:

$$B(B_s \to \mu^+ \mu^-) = 4.2 \times 10^{-9}$$

- The current upper limits from CDF and DØ:

$$B(B_s \to \mu^+ \mu^-) < 1.0 \times 10^{-7} \quad \text{(CDF)}$$

$$B(B_s \to \mu^+ \mu^-) < 2.3 \times 10^{-7} \quad \text{(DØ )}.$$
In our $Z'$ model $\rho_R^{sb} = 0$, $\phi_L^{sb} = 180^\circ$, and we set $\rho_R^{\mu\mu} = \rho_L^{\mu\mu}$.

We use $B_s \to \mu^+ \mu^-$ data to constrain on the $(\rho_L^{sb} - \rho_R^{\mu\mu})$ plane.

Blue: $\rho_L^{sb} < 6.20 \times 10^{-4}$ for $\phi_L^{sb} = 180^\circ$. 
• Note that $\rho_{L}^{sb}\rho_{L,R}^{\mu\mu}$ is also constrained by the decay:

\[ b \to s \ell^+ \ell^- \]

• Using the latest average from BABAR and Belle

\[ B(B \to X_s \ell^+ \ell^-) = \left( 4.50_{-1.01}^{+1.03} \right) \times 10^{-6} \]

• We obtain the constraint:

\[ 1 \leq \left| 1 + 2 \times 10^4 \rho_{L}^{sb} \rho_{L}^{\mu\mu} \right| \leq 1.75 \]

This constraint is stronger than the CDF limit on $B(B_s \to \mu^+ \mu^-)$.

• If we fixed $\rho_{L}^{sb} = 6.20 \times 10^{-4}$, the constraint on $\rho_{L}^{\mu\mu}$ from $b \to s \ell^+ \ell^-$ is WEAKER than the direct $Z'$ production.
With the upper limit of

- \( \rho_L^{s_b} = 6.20 \times 10^{-4} \) from \( B_s - \overline{B}_s \) mixing,

- \( g_2 \sqrt{Q_L^{e2} + Q_R^{e2}} \) from \( Z' \) production

one can predict the maximal rates for \( B_s \to \mu^+ \mu^- \).
Conclusions

- In many string-inspired models, each generation is created differently. We consider a $Z'$ model that has family-nonuniversal interaction, though diagonal. It induces FCNC effects.

- The recent data on $B_s \rightarrow \overline{B}_s$ mixing data constrained

$$\rho^s_b = \left| \frac{g_2 M_{Z'}}{g_1 M_{Z'}} B_L^s \right| < 6.2 \times 10^{-4}.$$ 

- Once the upper limit of $\rho^s_b$ is fixed, the flavor-diagonal couplings $Z'-q-q$ are also fixed. We used the direct $\sigma(Z') \cdot B(Z' \rightarrow e^+e^-)$ to constrain on the $Z'-e-e$ couplings.

- We found that the same constraint on $Z'-\ell-\ell$ obtained from $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow X_s \ell^+\ell^-$ are weaker than those from $Z'$ production.

- We used the combined constraints $\rho^s_b = 6.20 \times 10^{-4}$ from $B_s \rightarrow \overline{B}_s$ mixing and $g_2 \sqrt{Q^e_L e^2 + Q^e_R e^2}$ from $Z'$ production to predict the maximally allowed $B_s \rightarrow \mu^+\mu^-$. It turns out the BR is less than $10^{-8}$. 