# Dimension-six top-Higgs interaction and its effect in collider phenomenology

**Koji TSUMURA (Osaka Univ.)**

**with S. Kanemura (Univ. of Toyama) and D. Nomura (KEK)**

**APS-DPF2006+JPS2006… , Sheraton Waikiki Hotel, Honolulu, Hawaii**

**Oct. 29-Nov. 3, 2006**

**This talk is based on Phys. Rev. D 74, 076007 (2006) .**

# Motivation

- **Why the top ?**
  - **Large $Y_t$ is essentially important at future colliders.**

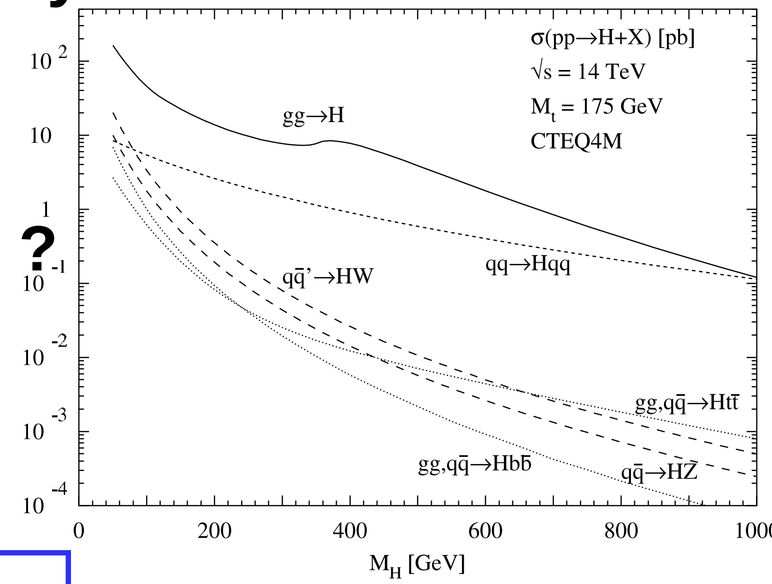
    The main H production at LHC is gluon fusion via the top- loop.

  - **Why the top exceptionally heavy ?**

    $$m_i \ll m_t$$

  - **Are the top and EWSB related ?**

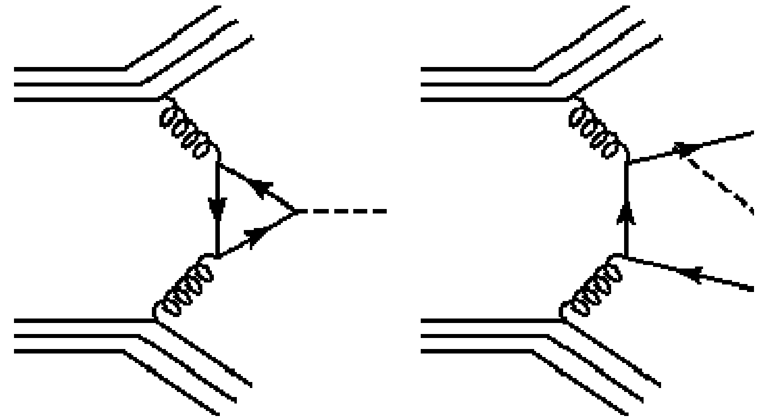
    $$\frac{v}{\sqrt{2}} \sim m_t$$




**Top might have a special role !?**

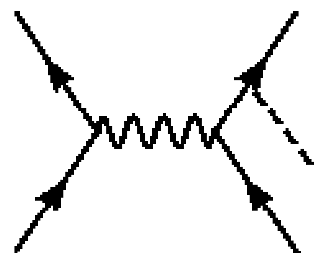
# Top Yukawa coupling @ future colliders

- **Gluon fusion**
  - Light Higgs
  - Large QCD Background



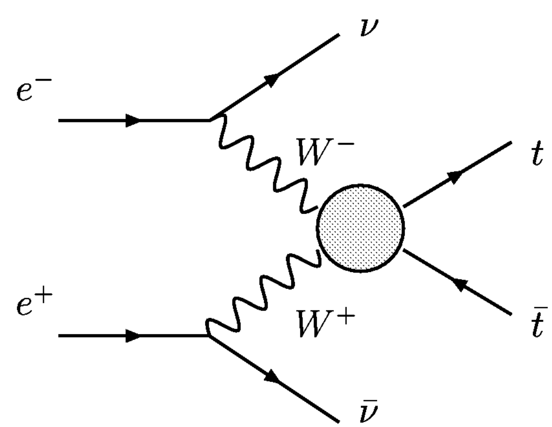
- **Associate production**
  - Light Higgs

  $m_H \lesssim 150\text{GeV}$



T. Han, T. Huang, Z. H. Lin, J. X. Wang, and X. Zhang, Phys. Rev. D 61, 015006 (2000)

- **W boson fusion**
  - Heavy Higgs



# Effective theory approach

– **Effective Lagrangian below the new physics scale $\Lambda$**

$$\mathcal{L}^{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{dim.6}} + \mathcal{L}_{\text{dim.8}} + \cdots$$

$$\mathcal{L}_{dim.6} = \frac{1}{\Lambda^2}\sum_i C_i \mathcal{O}_i$$

The non-SM int. is characterized by dim.6 operators at leading order.

– **Dimension-six top-Higgs interaction**

W. Buchmuller and D. Wyler, Nucl. Phys. B268, 621 (1986)

$$\mathcal{O}_{t1} = \left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)\left(\bar{q}_L t_R \tilde{\Phi} + \text{h.c.}\right)$$

- Modefy the relation between $m_t$ and $Y_t$.
- $m_t$ renormalization is taken into account.

$$\mathcal{O}_{Dt} = (\bar{q}_L D_\mu t_R)\left(D^\mu \tilde{\Phi}\right) + \text{h.c.}$$

- Covariant derivatives introduce gauge int. and energy dependence.

# Constraint on dimension-six operators

– **Perturbative unitarity**

G. J. Gounaris, D. T. Papadamou, and F. M. Renard, Z. Phys. C 76, 333 (1997)

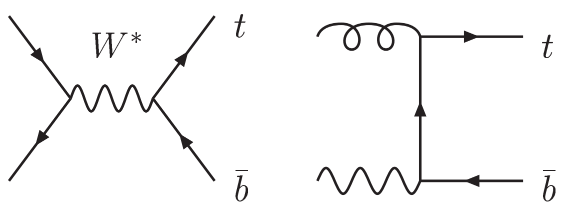
$$|C_{t1}| \leq \frac{16\pi}{3\sqrt{2}}\left(\frac{\Lambda}{v}\right)$$

$$-6.2 \leq C_{Dt} \leq 10.2$$

– **Direct search** K. i. Hikasa, K. Whisnant, J. M. Yang, and B. L. Young, Phys. Rev. D 58, 114003 (1998)

- No experimental bound for $C_{t1}$.
- $C_{Dt}$ can be constrained by Tevatron.

$|C_{Dt}| \leq 9.8$ for $\int \mathcal{L}dt = 100\mathrm{fb}^{-1}$.




– **Indirect search** G. J. Gounaris, F. M. Renard, and C. Verzegnassi, Phys. Rev. D 52, 451 (1995)

$\mathcal{O}_{Dt}$ can give contributions to rho-parameter. Heavier Higgs boson can be allowed in the SM-like situation with $C_{Dt}$.

$$\Delta\rho_{Dt} \sim -\frac{N_c}{16\pi^2}\left(\frac{m_t^2}{\Lambda^2}\right)\left\{-\frac{\sqrt{2}m_t}{v}C_{Dt}\ln\frac{\Lambda^2}{m_t^2} + C_{Dt}^2\right]$$

Ex. $m_H = 500\mathrm{GeV}$ corresponds $C_{Dt} \sim 1.5$ with $\Lambda = 1\mathrm{TeV}$.

# Effects of dimension-six coupling

## – Effective top-Yukawa coupling

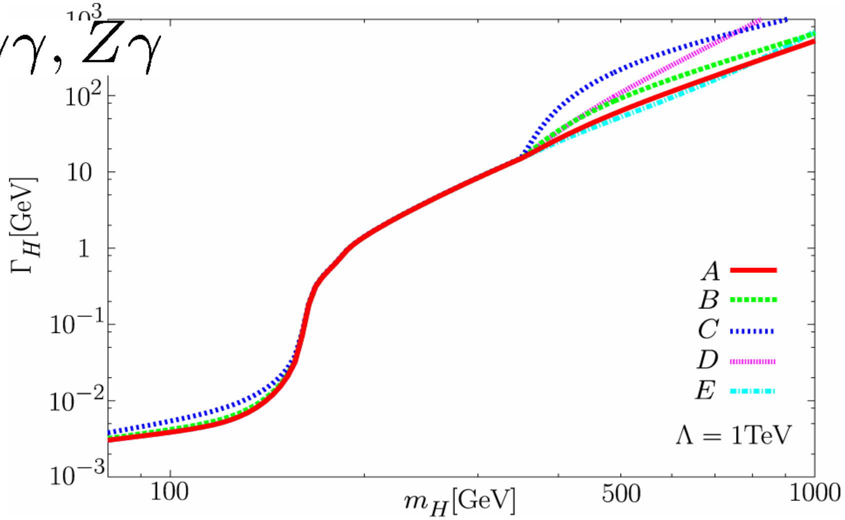
$$y_t^{\text{eff}}(-q^2,\Lambda) = y_t^{\text{SM}} - v^2\frac{C_{t1}}{\Lambda^2} - q^2\frac{C_{Dt}}{2\Lambda^2}$$

- $y_t^{\text{SM}} \sim 1$ which is restricted by $m_t$ .
- Dim.6 couplings are only constrained by unitarity, its allowed values can reach $|y_{t1,Dt}| \sim 3$ under the unitarity bounds.

## – Decay width for Higgs boson

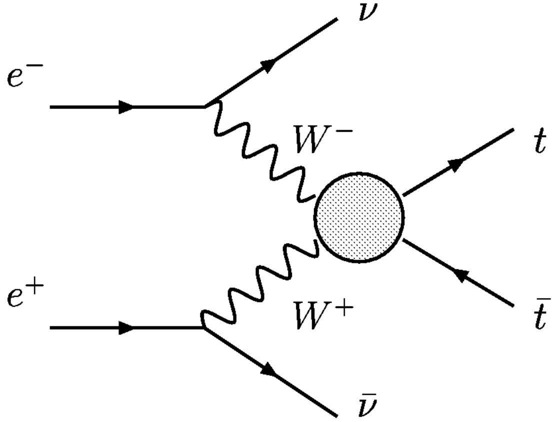
- At the tree level, top-pair production $H \to t\bar{t}$ can be modified.
- Loop induced decays $H \to gg, \gamma\gamma, Z\gamma$ can be enhanced.

| $C_i$ | Set A | Set B | Set C | Set D | Set E |
|---|---|---|---|---|---|
| $C_{t1}$ | 0 | $-\frac{16\pi}{3\sqrt{2}}\frac{\Lambda}{v}$ | $+\frac{16\pi}{3\sqrt{2}}\frac{\Lambda}{v}$ | 0 | 0 |
| $C_{Dt}$ | 0 | 0 | 0 | $+10.2$ | $-6.2$ |



# W boson fusion

– At the high energy LC, W-fusion is an important probe.
– A few thousands of top-pair events produced via vector boson fusion.
– BGs have been studied.




$e^-e^+ \to t\bar{t}$ J. Alcaraz and E. Ruiz Morales, Phys. Rev. Lett. 86, 3726 (2001)

$M_{t\bar{t}} \neq \sqrt{s}$

$e^-e^+ \to t\bar{t}\gamma$ F. Larios, T. Tait, and C. P. Yuan, Phys. Rev. D 57, 3106 (1998)

$p_T \geq$ 20GeV

$e^-e^+ \to t\bar{t}\nu\bar{\nu}$ J. Alcaraz and E. Ruiz Morales, Phys. Rev. Lett. 86, 3726 (2001)

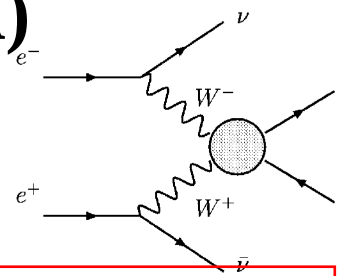
$E_T \geq$ 50GeV

$e^-e^+ \to t\bar{t}\nu\bar{\nu}$

$M_{\nu\bar{\nu}} \neq m_Z$

S. Dawson, Nucl. Phys. B249, 42 (1985)

# Effective W approximation (EWA)




– W-bosons are treated as a parton which are emitted by initial electron and positron.

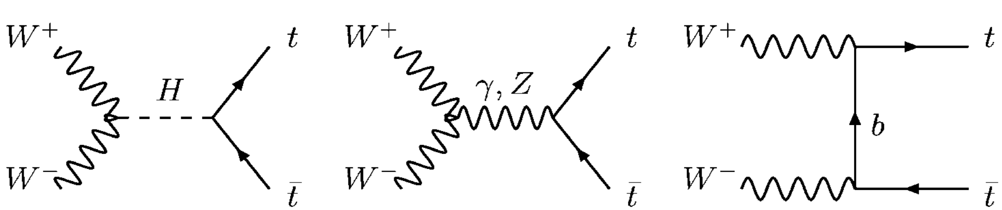
$$\sigma(e^-e^+ \to W^-W^+\nu\bar{\nu} \to t\bar{t}\nu\bar{\nu}; \sqrt{s})$$
$$= \int_{m_W/E}^{1} dx_1 \int_{m_W/E}^{1} dx_2 f_{e/W_\lambda}(x_1) f_{e/W_{\lambda'}}(x_2) \sigma(W^-W^+ \to t\bar{t}; \sqrt{\hat{s}})$$

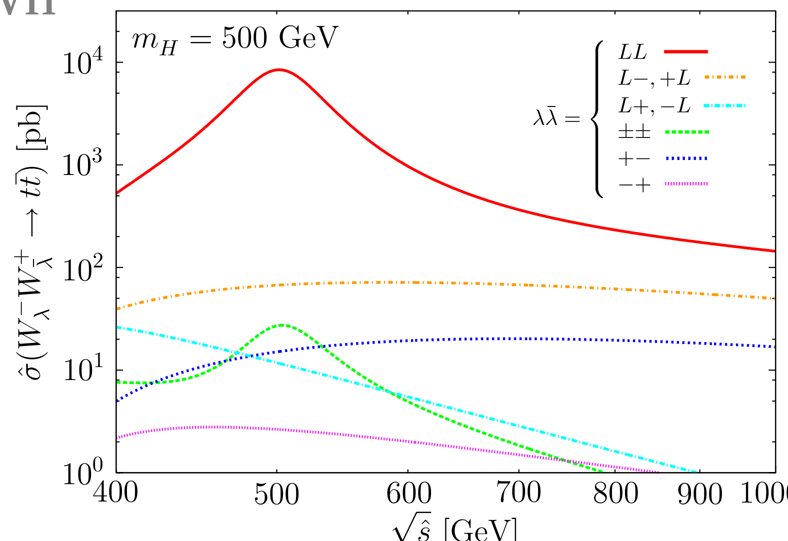
– At the leading order, parton distributions for $W_L$ consists $e^-e^+ \to H\nu\bar{\nu}$ with approximation $p_T \ll \sqrt{s}$ .

- The validity of EWA will be shown by using the package CalcHEP.

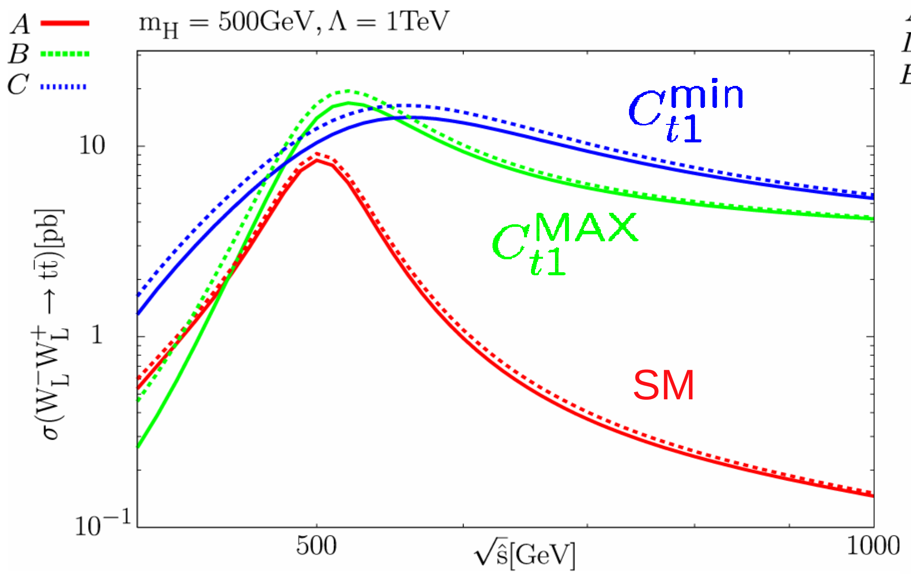
– **At first, we discuss sub-process**

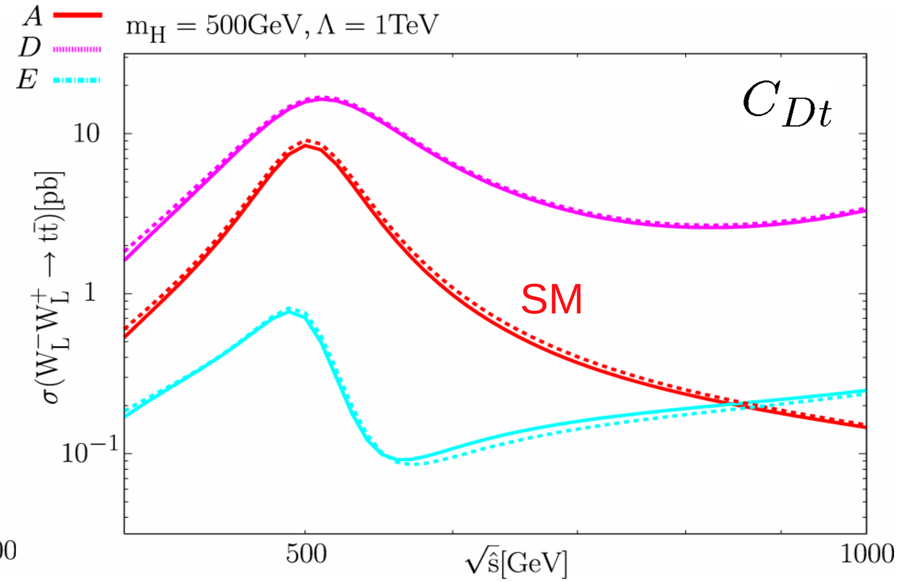
$$W^-W^+ \to t\bar{t}$$



# $W^-W^+ \to t\bar{t}$ with dimension-six operators

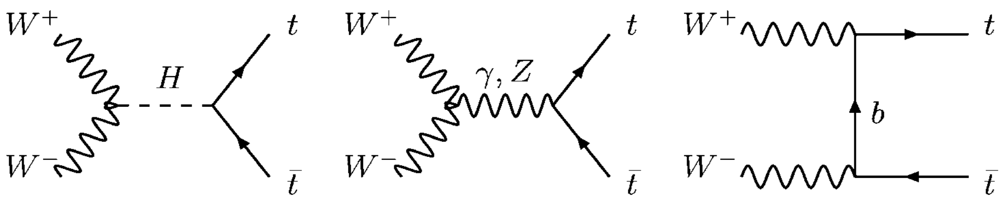


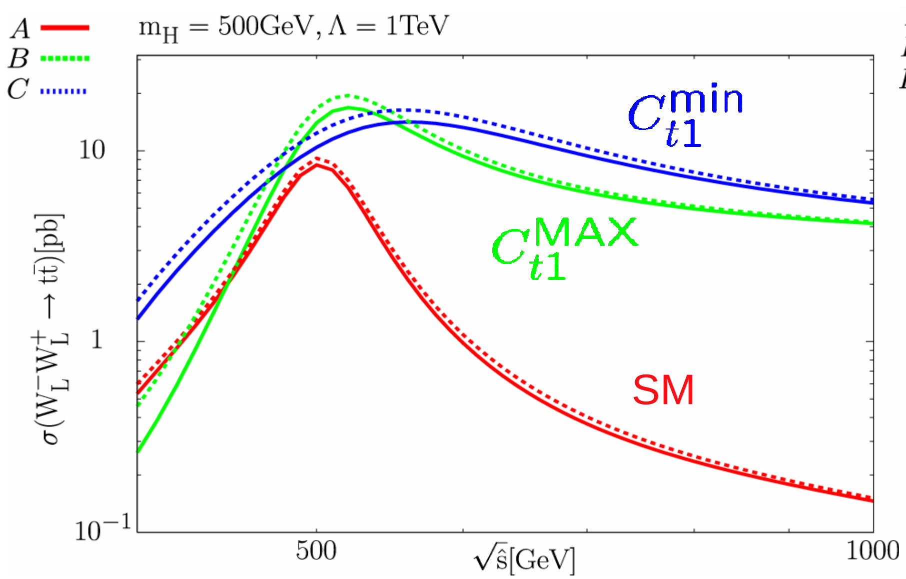

– Cutoff scale $\Lambda$ is set to be 1TeV.

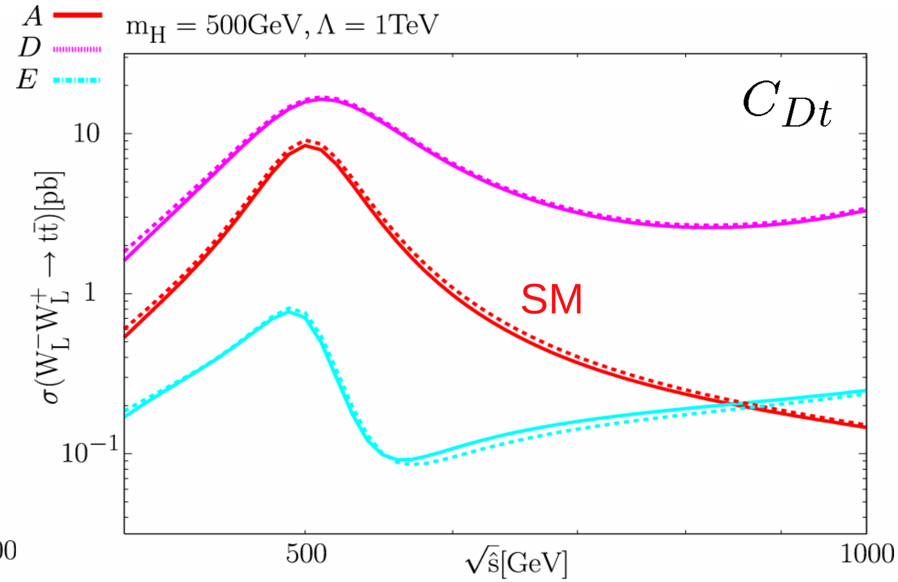
– Dim.6 couplings can change the cross sections by a factor.

– Since Higgs decay width grow wider, cross sections can be enhanced not only the sub-process energy equal to $m_H$ pole.



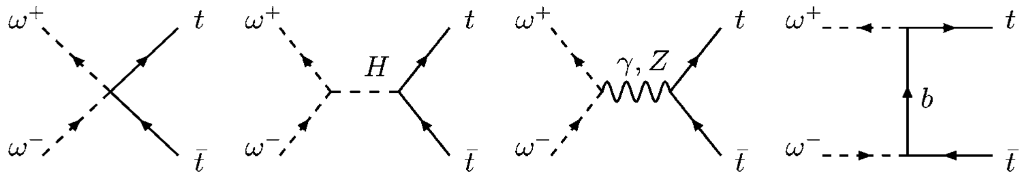
# Equivalence theorem (ET)



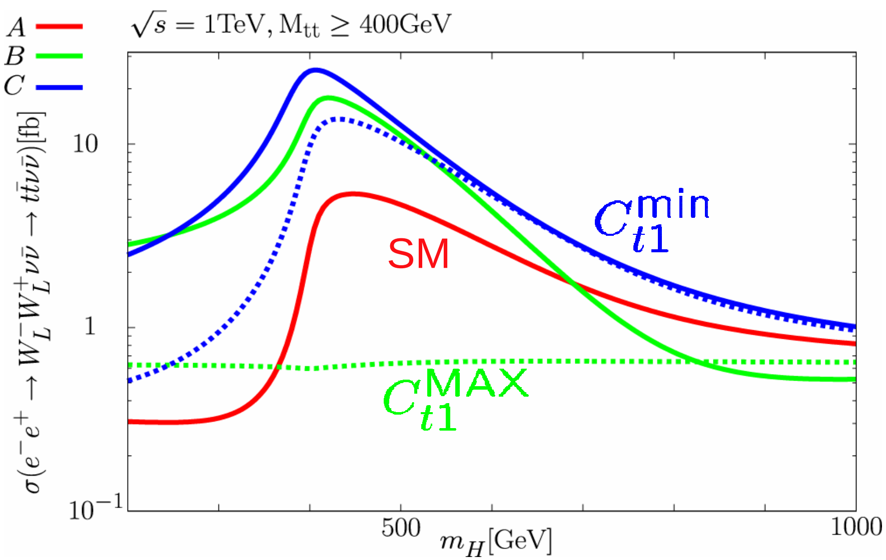



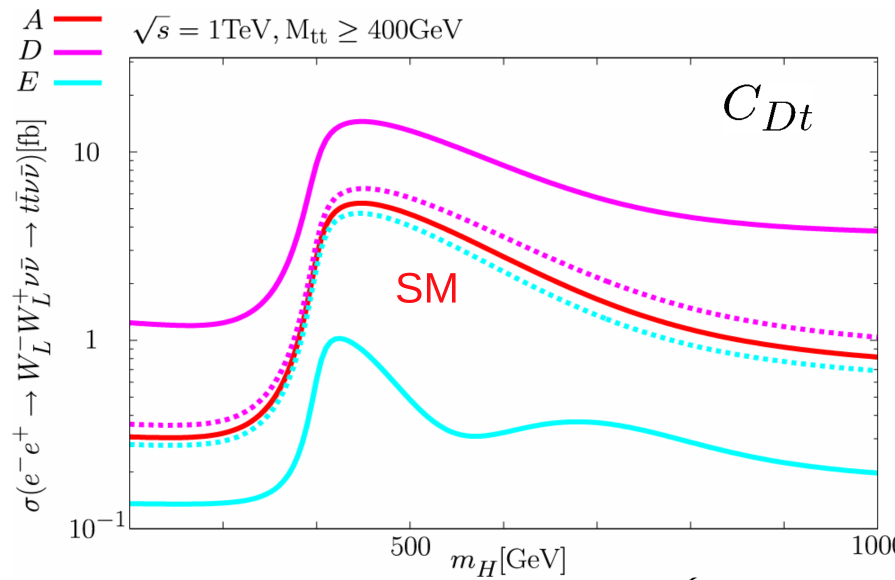

- **Our calculations can be checked by ET.**
  - S-matrix for $W_L$ scattering is equivalent to those for NG-boson up to m/E. Dotted curves indicate the process $\omega\omega \to t\bar{t}$.

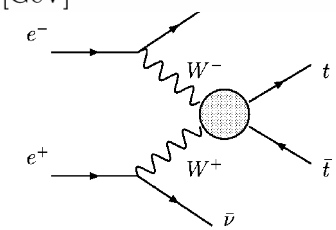
$$\mathcal{M}(W_L W_L \to t\bar{t}) \simeq \mathcal{M}(\omega\omega \to t\bar{t}) + \mathcal{O}(m_W/\sqrt{s})$$



# Total cross sections $\sigma(e^-e^+ \to W^-W^+\nu\bar{\nu} \to t\bar{t}\nu\bar{\nu})$

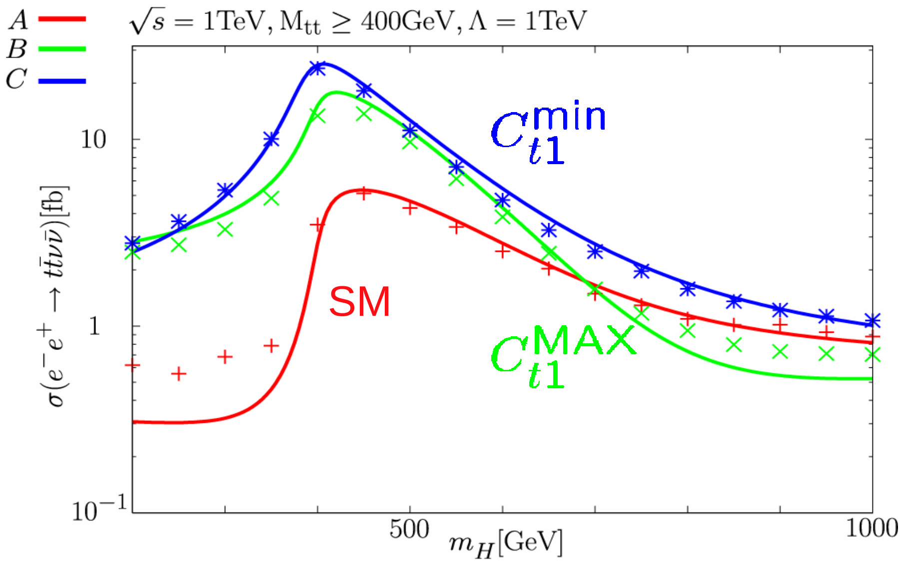


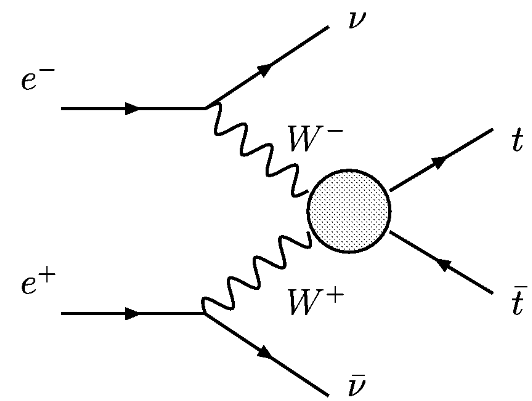




- Solid $\Lambda = 1\text{TeV}$, dotted $\Lambda = 3\text{TeV}$
- We only impose cut $M_{t\bar{t}} \geq 400\text{GeV}$.
- The total cross section can be enhanced by factor of 2 in the range $400\text{GeV} \leq m_H \leq 500\text{GeV}$.
- The effects of $\mathcal{O}_{Dt}$ become large for heavier Higgs compare to those of $\mathcal{O}_{t1}$.

# The validity of EWA




– Dotted curves are calculated by using the package CalcHEP.

- The EWA results agree with those of CalcHEP in about 20-30 % error for heavier Higgs boson.

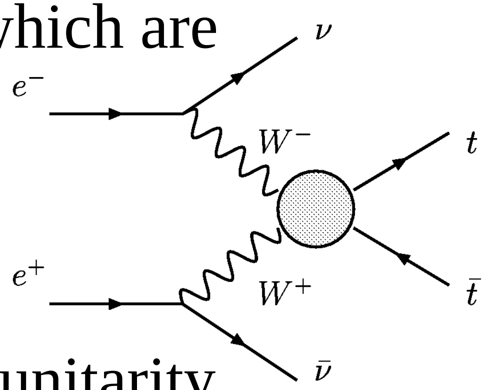
A. Pukhov, hep-ph/0412191

**At the ILC with $\sqrt{s} = 500\text{GeV}$, and $\int \mathcal{L}dt = 500\text{fb}^{-1}$, several hundred events are expected. Statistical error is less than 10 %. Therefore the effect of dim.6 couplings can be observed.**

# Summary

– We discuss the W-fusion with the non-SM $Y_t$ which are characterized by dim.6 operators.

$$y_t^{\rm eff}(-q^2,\Lambda) = y_t^{\rm SM} - v^2\frac{C_{t1}}{\Lambda^2} - q^2\frac{C_{Dt}}{2\Lambda^2}$$




– They are constrained by experimental data and unitarity.

- Our calculations have been checked by ET.
- The W-fusion has been analyzed by EWA and CalcHEP.

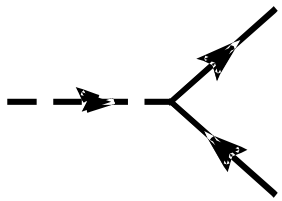
**Conclusion**

- Dim.6 couplings can enhance $\sigma(e^-e^+ \to W^-W^+\nu\bar{\nu} \to t\bar{t}\nu\bar{\nu})$

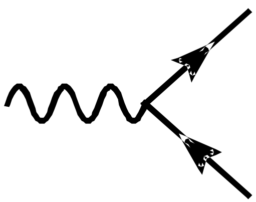
**At the ILC with $\sqrt{s} = 500\mathrm{GeV},\ \text{and}\ \int\mathcal{L}dt = 500\mathrm{fb}^{-1}$, several hundred events are expected. Statistical error is less than 10 %. Therefore the effect of dim.6 couplings can be observed.**

# Feynman rules

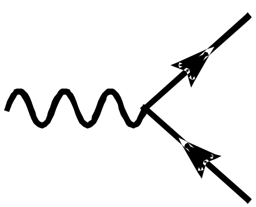
- **Feynman rules for dimension-six operators.**



$$-i\left(\frac{m_t}{v}-\frac{C_{t1}}{\sqrt{2}}\frac{v^2}{\Lambda^2}+\frac{C_{Dt}}{2\sqrt{2}}\frac{s}{\Lambda^2}\right)$$

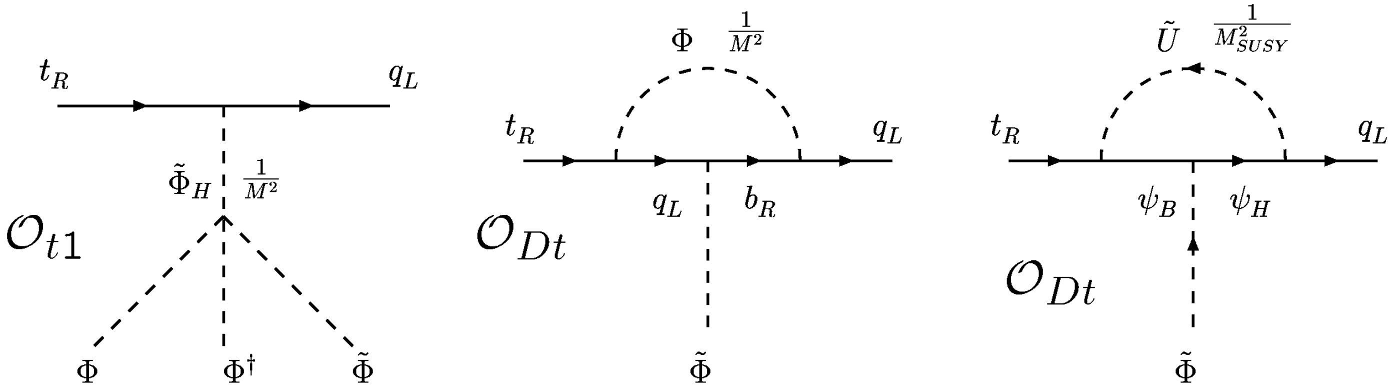


$$-\frac{g_Z}{2}\left\{\gamma^\mu(v_t+a_t\gamma_5)-i\frac{C_{Dt}}{\Lambda^2}\frac{v}{\sqrt{2}}K^\mu\right\}$$



$$-\frac{g}{2}\left(\gamma^\mu-ik^\mu\frac{C_{Dt}}{\sqrt{2}}\frac{v}{\Lambda^2}\right)$$

# Origin of dim.6 operators




- **MSSM (with the light lightest Higgs boson)**

$$\mathcal{L}_{dim.6} = \frac{1}{M_A^2} \sum_i C_i \mathcal{O}_i$$

$$C_{t1} = \frac{g^2 + g'^2}{2} \mathrm{Re}(h_U^{33}) s_\beta c_\beta^2 (c_\beta^2 - s_\beta^2), \;\; C_{Dt} = \cdots$$

PRD69,115007 Feng, Li, Maalampi

- **Little Higgs, Extra dim., Top color, etc.**
  - These models can also include the structure of 2HDM, dim.6 couplings can occur.

JHEP0208, 021 Arkani, Cohen, Katz, Nelson, Gregorie, Wacker
PRD64, 035002, Appelquist, Cheng, Dbrescu
PRD65, 055066, He, Hill, Tait

# Unitarity bounds

- **Amplitudes**

$$i\mathcal{M}^{t\bar{t}\to t\bar{t}} \sim \frac{i}{s-m_h^2}\left(\frac{m_t}{v} - \frac{v^2}{\sqrt{2}}\frac{C_{t1}}{\Lambda^2} + \frac{s}{\sqrt{2}}\frac{C_{Dt}}{\Lambda^2}\right)^2 \bar{v}u\bar{u}v + \cdots$$

$$i\mathcal{M}^{t\bar{t}\to hh} \sim \frac{1}{s-m_h^2}\left(\frac{m_t}{v} - \frac{v^2}{\sqrt{2}}\frac{C_{t1}}{\Lambda^2} + \frac{s}{\sqrt{2}}\frac{C_{Dt}}{\Lambda^2}\right)(-i\lambda)\bar{u}v3! + i\frac{C_{t1}}{\sqrt{2}}\frac{v}{\Lambda^2}\bar{u}v2!$$

  - Imposing unitarity @ $\sqrt{s} = \Lambda$

$$|C_{t1}| \leq 8\pi\left(\frac{\Lambda}{v}\right), \;\; |C_{Dt} + \frac{\sqrt{2}m_t}{v}| \leq \sqrt{8\pi}$$

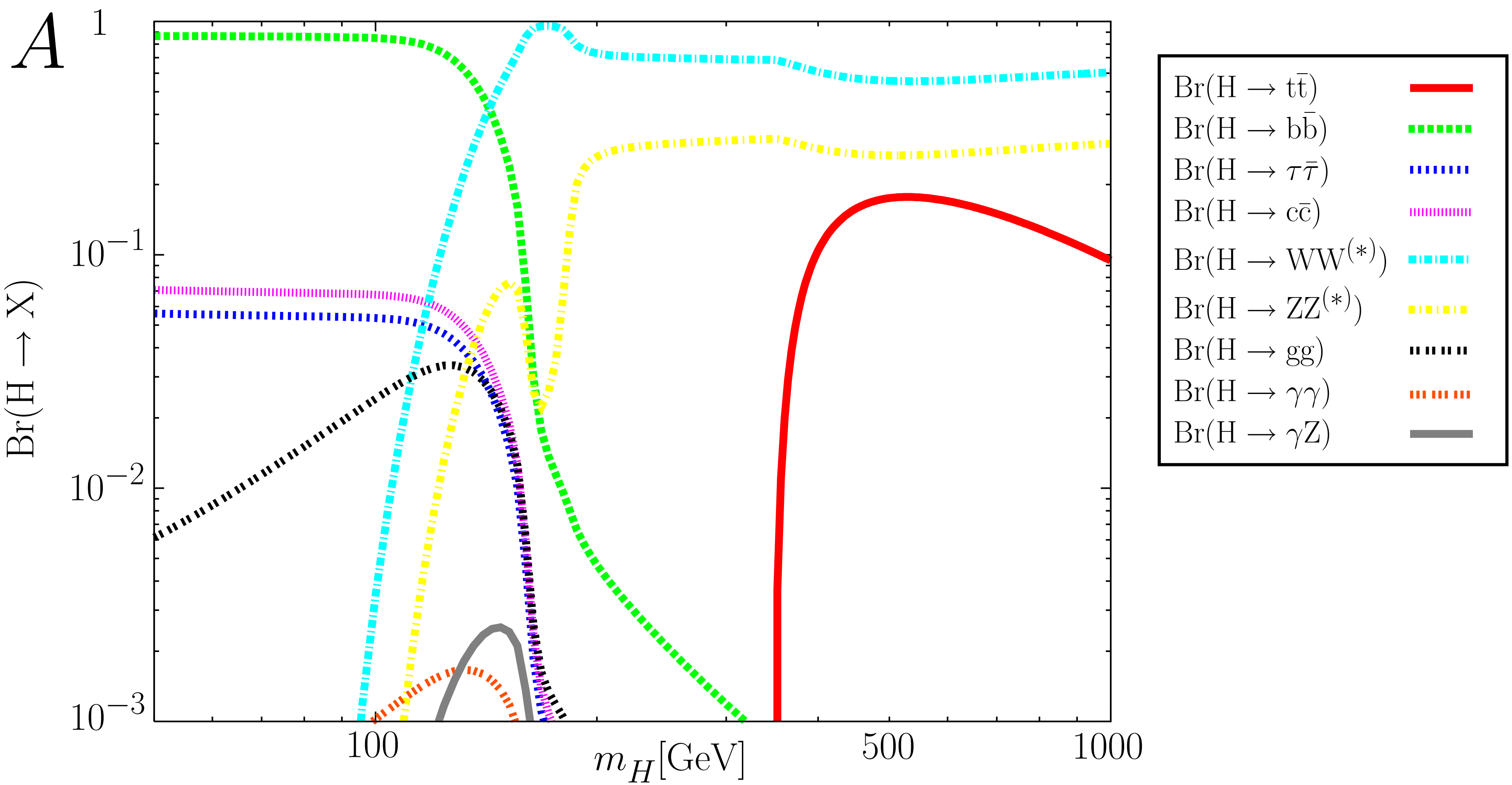
  - Considering 2-body scattering channels (hh, $W_L W_L$, $Z_L Z_L$, and t anti-t), chirality and color factors, then we obtained

$$|C_{t1}| \leq \frac{16\pi}{3\sqrt{2}}\left(\frac{\Lambda}{v}\right)$$

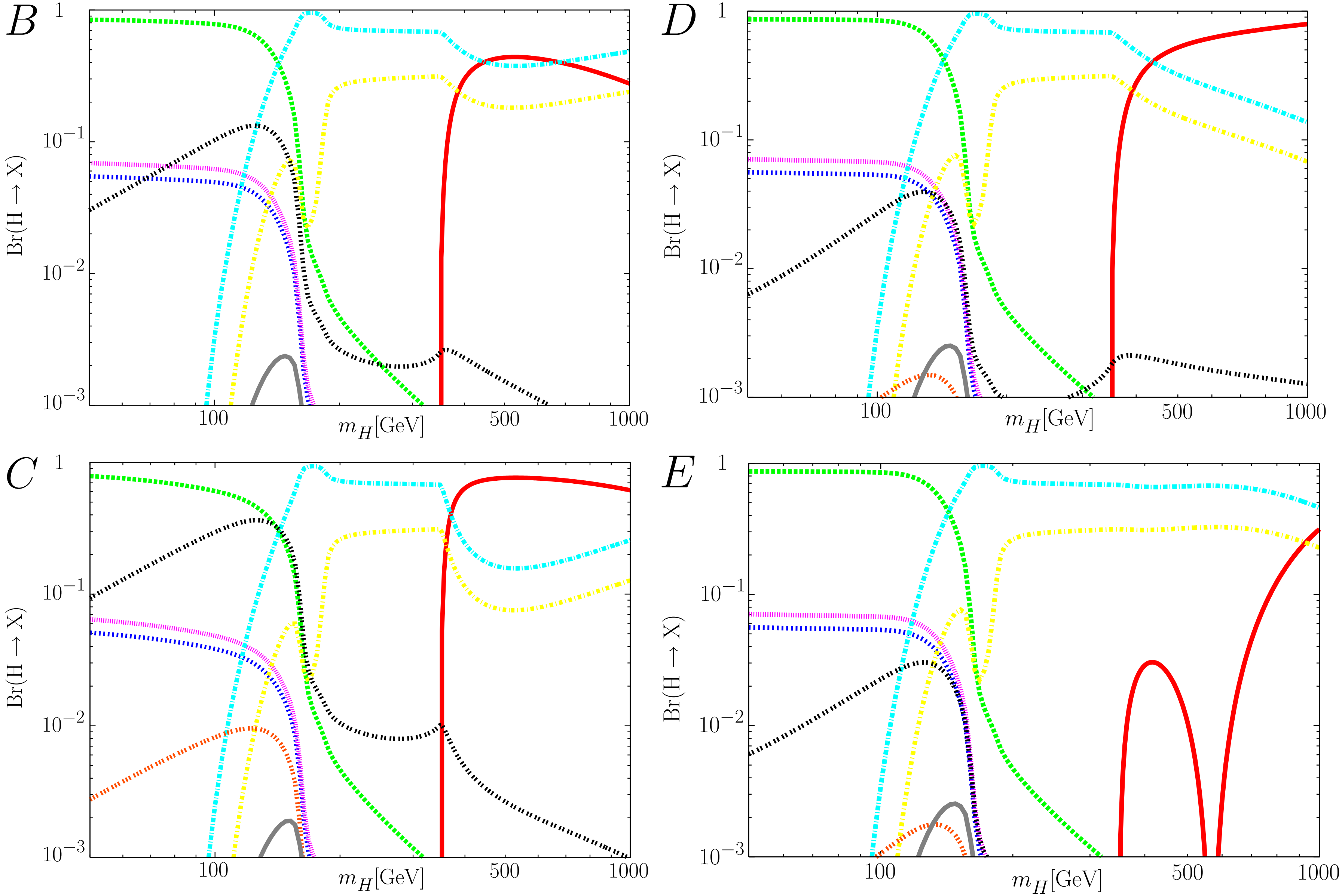
$$-6.2 \leq C_{Dt} \leq 10.2$$

# Higgs decay branching ratios

- Higgs decay branching ratio in the SM



# Higgs decay branching ratios with dim.6 coupling



# Amplitudes with dim.6 couplings for WW scattering

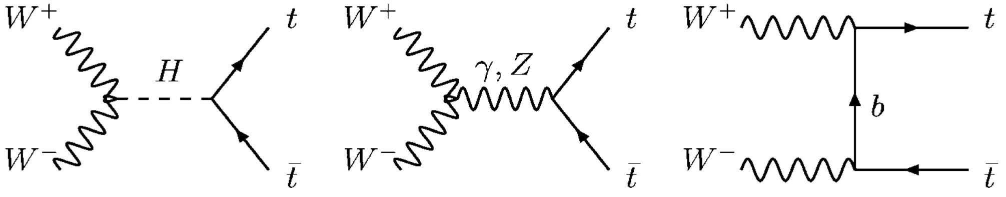
$$\mathcal{M}_h = \frac{2m_W^2/v}{s - m_h^2 + im_h\Gamma_h}\left(\frac{m_t}{v} - \frac{C_{t1}}{\sqrt{2}}\frac{v^2}{\Lambda^2} + \frac{C_{Dt}}{2\sqrt{2}}\frac{s}{\Lambda^2}\right)(e_\lambda \cdot \bar{e}_{\bar{\lambda}})\bar{u}v.$$

$$\mathcal{M}_\gamma = -\frac{iQ_t e^2}{s} A^\mu_{\lambda\bar{\lambda}} \bar{u}\gamma_\mu v.$$

$$\mathcal{M}_Z = -\frac{2im_W^2/v^2}{s - m_Z^2} A^\mu_{\lambda\bar{\lambda}} \bar{u}\left[\gamma_\mu(v_t + a_t\gamma_5) - iK_\mu \frac{C_{Dt}}{\Lambda^2}\frac{v}{2\sqrt{2}}\right]v.$$

$$\mathcal{M}_b = -\frac{2im_W^2/v^2}{u - m_b^2} e^\mu_\lambda \bar{e}^\nu_{\bar{\lambda}} \bar{u}\left[\left(\gamma_\nu - i\frac{C_{Dt}}{\sqrt{2}}\frac{v}{\Lambda^2}k_\nu\right) P_L \gamma_\rho (p - k')^\rho P_R \left(\gamma_\mu + i\frac{C_{Dt}}{\sqrt{2}}\frac{v}{\Lambda^2}p_\mu\right)\right]v.$$

$$A^\mu_{\lambda\bar{\lambda}} = (e_\lambda \cdot \bar{e}_{\bar{\lambda}})P^\mu + 2(e_\lambda \cdot q)\bar{e}^\mu_{\bar{\lambda}} - 2(\bar{e}_{\bar{\lambda}} \cdot q)e^\mu_\lambda$$



# Amplitudes with dim.6 couplings (NG-bosons)

$$\mathcal{M}_\times = -\frac{C_{t1}}{\Lambda^2}\frac{v}{\sqrt{2}}\bar{u}v.$$

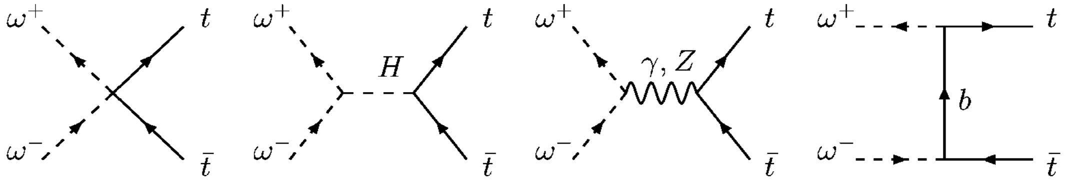
$$\mathcal{M}_h = \frac{2\lambda v}{s - m_h^2 + i m_h \Gamma_h}\left(\frac{m_t}{v} - \frac{C_{t1}}{\sqrt{2}}\frac{v^2}{\Lambda^2} + \frac{C_{Dt}}{2\sqrt{2}}\frac{s}{\Lambda^2}\right)\bar{u}v.$$

$$\mathcal{M}_\gamma = -\frac{iQ_t e^2}{s}P^\mu \bar{u}\gamma_\mu v.$$

$$\mathcal{M}_Z = -\frac{2im_Z^2/v^2}{s - m_Z^2}A^\mu_{\lambda\bar{\lambda}}\bar{u}\left[\gamma_\mu(v_t + a_t\gamma_5) - iK_\mu \frac{C_{Dt}}{\Lambda^2}\frac{v}{2\sqrt{2}}\right]v.$$

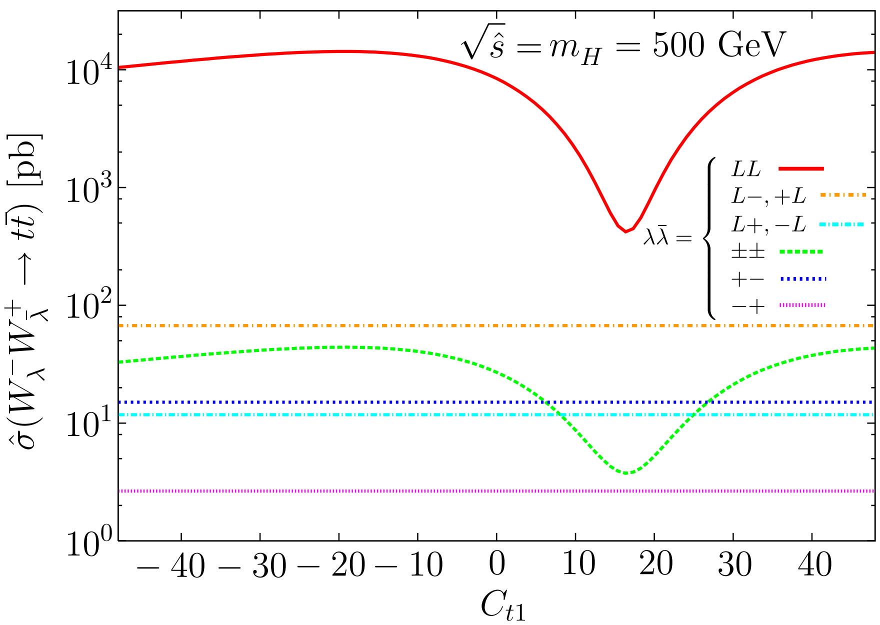
$$\mathcal{M}_b = -\frac{2i}{u - m_b^2}e^\mu_\lambda \bar{e}^\nu_{\bar{\lambda}}\bar{u}\left[\left(\frac{m_t}{v} + \frac{C_{Dt}}{\sqrt{2}\Lambda^2}p'\cdot k\right)P_L\gamma_\rho(p-k')^\rho P_R\left(\gamma_\mu + \frac{C_{Dt}}{\sqrt{2}\Lambda^2}p\cdot k'\right)\right]v.$$

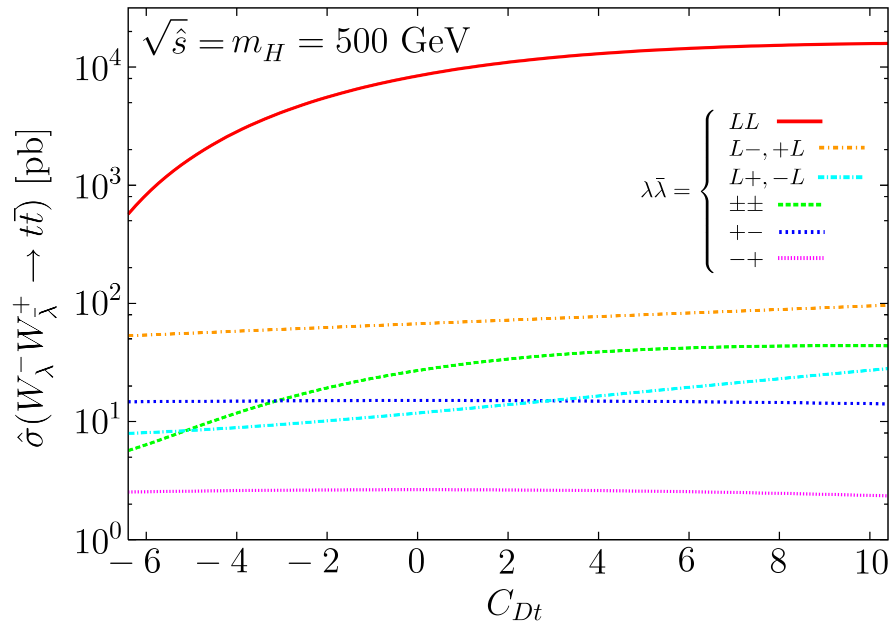
$$A^\mu_{\lambda\bar{\lambda}} = (e_\lambda\cdot\bar{e}_{\bar{\lambda}})P^\mu + 2(e_\lambda\cdot q)\bar{e}^\mu_{\bar{\lambda}} - 2(\bar{e}_{\bar{\lambda}}\cdot q)e^\mu_\lambda$$



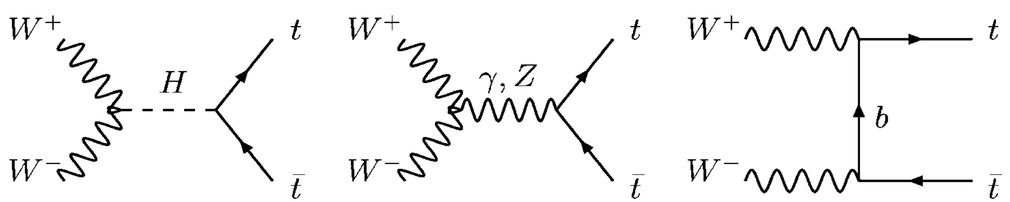
# Sub-process cross section vs dimension-six couplings

- Cross sections for the sub-process $W^-W^+ \to t\bar{t}$




  – Dim.6 coupling can give negative contributions.



# Equivalence theorem

- **Our calculations can be checked by equivalence theorem.**
  - S-matrix for longitudinally polarized W boson scattering is equivalent to those for NG-boson up to mass/energy. Dotted curves indicate the process $\omega\omega \to t\bar{t}$ .

$$\mathcal{M}(W_L W_L \to t\bar{t}) \simeq \mathcal{M}(\omega\omega \to t\bar{t}) + \mathcal{O}(m_W/\sqrt{s})$$

- BRS identity

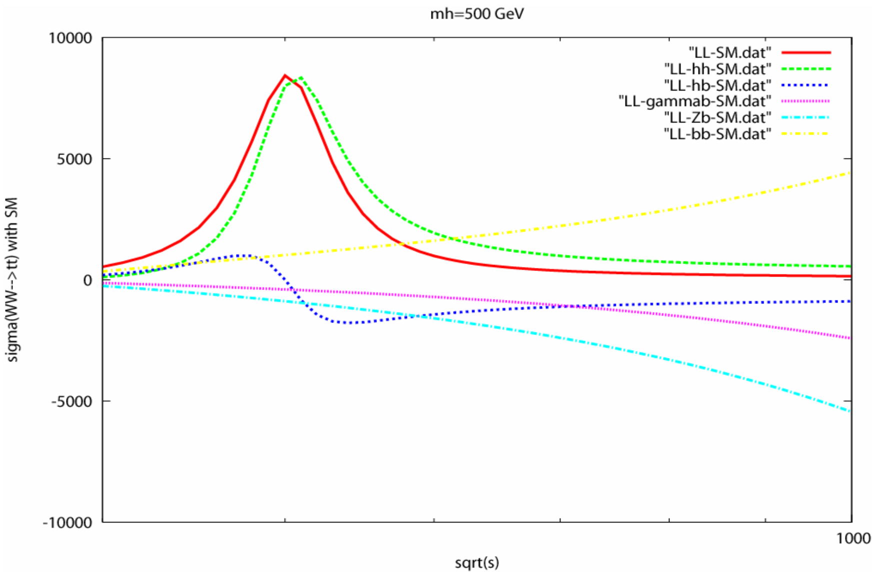
$$\partial^\mu W_\mu^\pm - i m_W \omega^\pm = 0$$

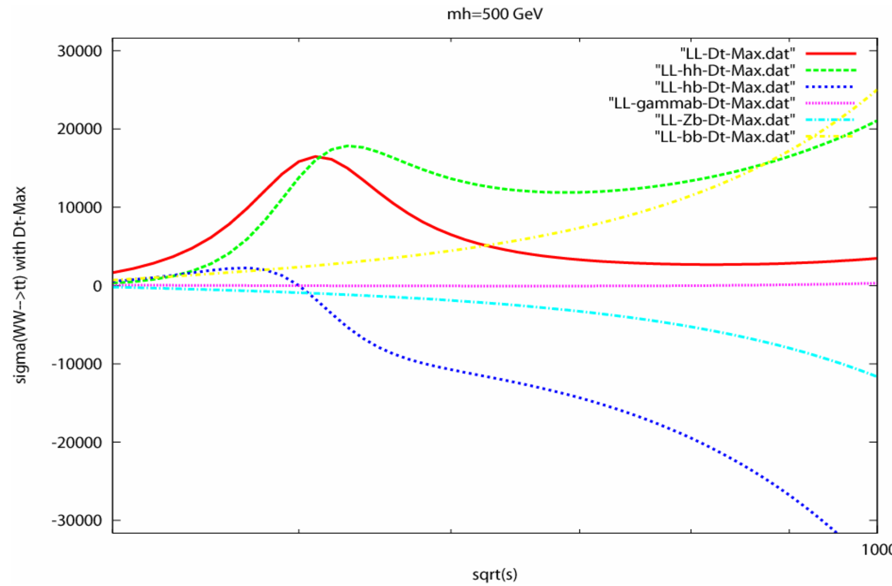
- Equivalence between longitudinal pol. and scalar pol. at high energies.

$$e_S^\mu = \frac{1}{m_W} p^\mu = \gamma_W (1, 0, 0, \beta_W), \;\; e_L^\mu = \gamma_W (\beta_W, 0, 0, 1)$$

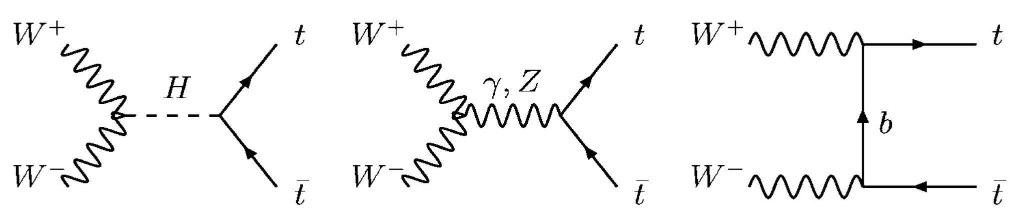
$$\gamma_W = \frac{\sqrt{s}}{2 m_W}, \beta_W = \sqrt{1 - \frac{4 m_W^2}{s}}$$

# Unitarity cancellation




- In the SM, the amplitude for WW scattering does not divergent at high energies due to unitarity cancellation.
- The effects of dimension-six operator $\mathcal{O}_{Dt}$ rapidly grow higher energy.
- However, the SM-like unitarity cancellation can take place, the total cross section is stabilized up to the cutoff scale.



# Total cross section vs dimension-six couplings

- Total cross section for the process $e^-e^+ \to W^-W^+\nu\bar{\nu} \to t\bar{t}\nu\bar{\nu}$