Mass Spectrum of Mesons in Second Quantized Dual String Model of QCD

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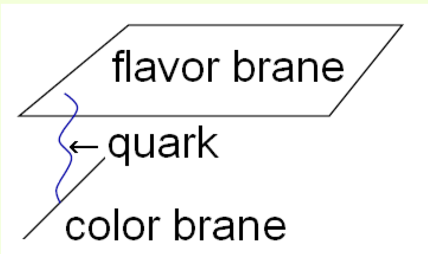
Outline

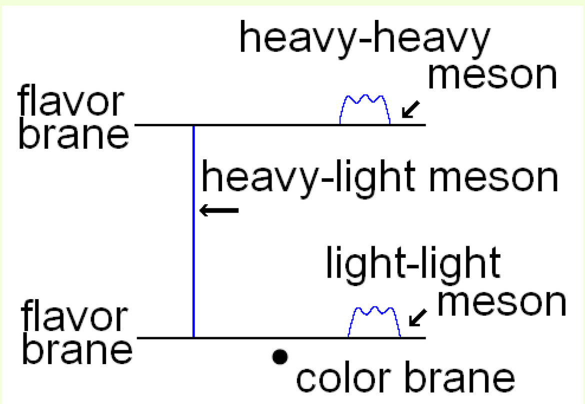
We investigate a holographic description of meson strings, especially for mesons with different flavors.
In this poster, we derive the wave equation necessary to evaluate this meson mass spectrum.
The potential between quark and anti-quark was studied by M.Bando et al. (Hep-ph/0602203)
We introduce the second quantized method has been promised by J.Erdmenger et al. (Hep-th/0605241) into our study.

Introduction

- **AdS/CFT correspondence**
  → It holographically describes the strong coupling regime of QCD using weakly coupled gravity.

Quarks are described by open strings stretching between the D4 color branes and the D6 flavor branes

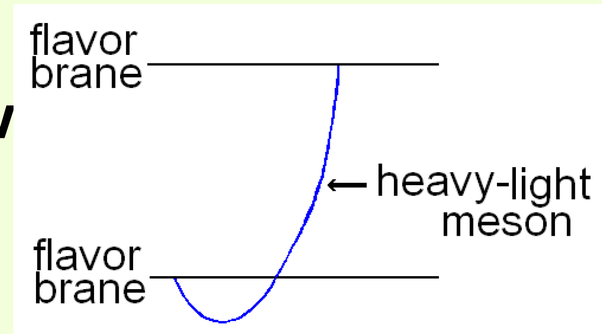




Meson is described by open strings with both ends on the D6 branes probe.

- J.Erdmenger et al. evaluated this heavy-light meson mass spectrum using the second quantized string theory, where they thought open string is rigid.

We evaluate the heavy-light meson mass spectrum using the above new method, but as string is hanging.



Calculation process

- **Deviation of constraints**

Nambu-Goto action

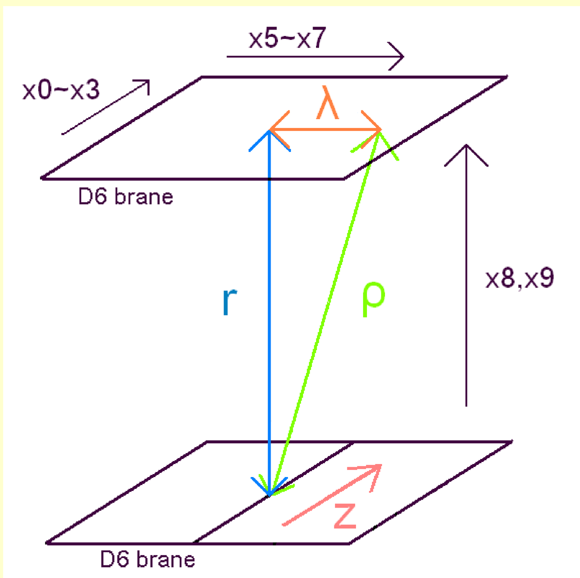
$$S_{NG} = -\frac{1}{2\pi\alpha'}\int d\tau d\sigma \sqrt{(\dot{X}\cdot X')^2 - (\dot{X}\cdot\dot{X})(X'\cdot X')}$$

Conjugate momentum

$$P_M \equiv \frac{\delta S_{NG}}{\delta \dot{X}^M}$$

Constraint

$$P^M P_M + \left(\frac{1}{2\pi\alpha'}\right)^2 (X')^2 = 0$$



quantize

$$\left[G^{MN}(x)\left(\frac{-i\delta}{\delta X^M(\tau,\sigma)}\right)\left(\frac{-i\delta}{\delta X^N(\tau,\sigma)}\right) + \left(\frac{1}{2\pi\alpha'}\right)^2 G_{MN}(x)(X'^M)(X'^N)\right] \times\Psi(X^P(\tau,\sigma)) = 0$$

- **Coordinates**

$$\begin{cases} (X^0, X^1, X^2, X^3) \longleftrightarrow x^\mu \quad (\mu = 0,1,2,3) \\ (X^5, X^6, X^7) \longleftrightarrow (\lambda, \Omega_2) : \text{horizontal directions} \\ (X^8, X^9) \longleftrightarrow (r, \phi) : \text{vertical directions} . \end{cases}$$

- **Metric**

$$ds^2 = f(u)(\eta_{\mu\nu}dx^\mu dx^\nu + h(u)d\theta^2) + K(\rho)(d\lambda^2 + \lambda^2 d\Omega_2^2 + dr^2 + r^2 d\phi^2)$$

$$f(u) = \left(\frac{u}{R'}\right)^{\frac{3}{2}} \qquad h(u) = 1 - \frac{u_{KK}^3}{u^3} \qquad K(\rho) \equiv \frac{R'^{\frac{3}{2}} u^{\frac{1}{2}}}{\rho^2}$$

$$\rho = \sqrt{\lambda^2 + r^2} \qquad u = u(\rho) = \left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{\frac{2}{3}}$$

M.Kruczanski et al. JHEP 0405 (2004) 041

- **Fluctuation**

$$X_{cl}^M(\sigma) = \begin{cases} z_{cl} = z_{cl}(\sigma) \\ \lambda_{cl} = 0 \\ r = r_{cl}(\sigma) . \end{cases} \Longrightarrow X^M(\sigma) = \begin{cases} z(\sigma) = z_{cl}(\sigma) \\ \lambda(\sigma) = \lambda_{cl} + \tilde{\lambda}(\sigma) \\ r(\sigma) = r_{cl}(\sigma) \end{cases}$$

M.Bando et al. Hep-ph/0602203

- **Wave equation to be solved**

real:

$$\Bigg[\int_0^{2\pi} d\sigma \Bigg\{ - R'^{\frac{3}{2}}\left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{-1}\Big((E_{cl}(z) - E)^2 - 2\tau P_z \partial_z E_{cl}(z) - \tau^2(\partial_z E_{cl}(z))^2 - (\mathbf{P}_\perp^2 + P_z^2) - 2(\tau\partial_z E_{cl}(z) + P_z)\partial_z B + \partial_z^2 A + (\partial_z A)^2 - (\partial_z B)^2\Big) - \rho^2 R'^{-\frac{3}{2}}\left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{-\frac{1}{3}}(\partial_\lambda^2 + \partial_r^2)A\Bigg\}$$

$$+\left(\frac{1}{2\pi\alpha'}\right)^2 \int dz \left\{\frac{1}{R'^{\frac{3}{2}}}\left(r(z)_{cl}^{\frac{3}{2}} + \frac{U_{KK}^3}{4r(z)_{cl}^{\frac{3}{2}}}\right) + R'^{\frac{3}{2}}\left(\frac{1}{r_{cl}(z)^{\frac{9}{2}}} + \frac{U_{KK}^3}{4r_{cl}(z)^{\frac{15}{2}}}\right)^{\frac{1}{3}}\left(\frac{\partial r}{\partial z}\right)_{cl}^2\right\} + \left(\frac{1}{2\pi\alpha'}\right)^2 \int d\lambda\, R'^{\frac{3}{2}}\left(\frac{1}{(r^2+\lambda^2)^{\frac{9}{4}}} + \frac{U_{KK}}{4(r^2+\lambda^2)^{\frac{15}{4}}}\right)^{\frac{1}{3}}$$

imaginary:

$$+i\int_0^{2\pi} d\sigma \left\{ - R'^{\frac{3}{2}}\left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{-1}\left(\tau\partial_z^2 E_{cl}(z) + 2(\tau\partial_z E_{cl}(z) + P_z)\partial_z A + \partial_z^2 B + 2\partial_z A \partial_z B\right) - \rho^2 R'^{-\frac{3}{2}}\left(\rho^{\frac{3}{2}} + \frac{U_{KK}^3}{4\rho^{\frac{3}{2}}}\right)^{-\frac{1}{3}}(\partial_\lambda^2 + \partial_r^2)B\right\}\Bigg]$$

$$\times e^{i\tau(E_{cl}(z)-E)} e^{ip_z z} e^{i\mathbf{P}_\perp \mathbf{X}_\perp}\tilde{\Psi}(z,\lambda,r) = 0$$

Conclusion and Discussion

- We have obtained the wave equation giving the mass and the width of the meson string which is hanging between different flavor branes.
- We need to consider the deformation of D6 brane, due to attractive force by the D4 color branes.

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