Proton Decay Matrix Elements

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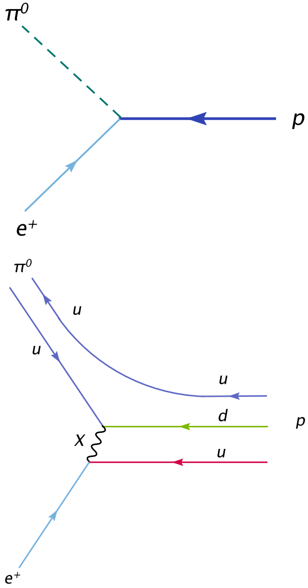
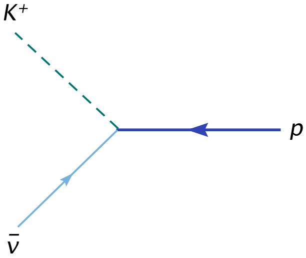
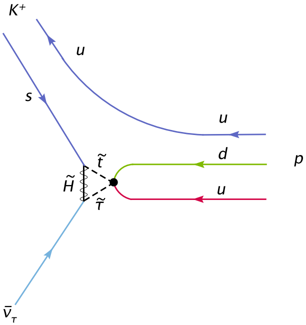
Proton decay



- Smoking gun evidence of Beyond SM
  (B is conserved in SM.)
- Natural in GUT
- Dimension 6 operator suppressed by $1/M_X^2$
- Dominant mode in non-SUSY GUT
  $p \rightarrow \pi^0 + e^+$

Proton decay





- Smoking gun evidence of Beyond SM
- Occurs in SUSY GUT
- Dimention 5 operator suppressed by $1/M_c$
- Dominant mode ?
  $p \to K^+ + \overline{\nu}$

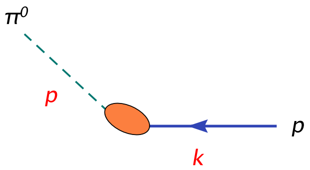
Low energy effective interaction

- $\mathcal{L}_{\not B} = C[\mu] \cdot (qq)(ql)[\mu]$
- operator type determined by the conditions
  - Lorentz scalar
  - $SU(3)$ gauge singlet
- $(qq)_{L/R}(lq)_{L/R} = \epsilon^{ijk}(\overline{q^c}^i P_{L/R} q^j)(\overline{l^c} P_{L/R} q^k)$
- ~~$\epsilon^{ijk}(\overline{q^c}^i \gamma_\mu q^j)(\overline{l^c} \gamma_\mu q^k)$~~ can be written in terms of the above operators.
- $A = C[\mu] \cdot \langle e^+, \pi^0 | (ud)(eu)[\mu] | p \rangle$

$$\langle \pi^0 | (ud)u[\mu] | p \rangle$$

- $\mu \simeq 1$ GeV
- 2 tasks
  - $(ud)u^{ren}[\mu] = Z[\mu] \cdot (ud)u^{bare}$
  - $\langle \pi^0 | (ud)u^{bare} | p \rangle$

The Relevant Form Factor



- $q = k - p$: $\rightarrow$ momentum of e^+

$$\langle \pi^0; \vec{p} | (ud) u_L | p; \vec{k}, s \rangle = P_L [W_0(q^2) - i \not{q} W_q(q^2)] u_p(\vec{k}, s)$$

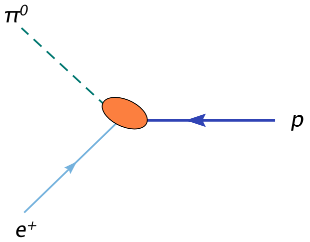
- W_0: relevant form factor, W_q: irrelevant form factor [JLQCD].

$$\langle e^+; \vec{q} | \langle \pi^0; \vec{p} | (ud)(eu)_L | p; \vec{k}, s \rangle = W_0 \overline{v_e^c} P_L u_p + W_q \overline{v_e^c} (-i \not{q}) P_R u_p$$

- $\overline{v_e^c}(-i\not{q}) = \overline{v_e^c} m_e$: negligible

Partial Width

written in terms of Wilson Coefficient and the relevant form factor W_0



$$\Gamma(p \to \pi^0 + e^+) = \frac{m_p}{32\pi^2}\left[1 - \left(\frac{m_\pi}{m_p}\right)^2\right]^2 \left|\sum_i C^i W_0{}^i(p \to \pi^0)\right|^2,$$

$$\mathcal{L}_{\not B} = \sum_i C^i[\mu] \cdot \mathcal{O}^i[\mu]$$

Hadronic Matrix Elements

- Parity invariance of (lattice) QCD yields:

$$\begin{aligned}\langle PS;\vec{p}|(qq)_R q_R|N;\vec{k},s\rangle &= \gamma_4\langle PS;-\vec{p}|(qq)_L q_L|N;-\vec{k},s\rangle,\\ \langle PS;\vec{p}|(qq)_L q_R|N;\vec{k},s\rangle &= \gamma_4\langle PS;-\vec{p}|(qq)_R q_L|N;-\vec{k},s\rangle.\end{aligned}$$

- By $m_N \geq m_{PS}$ & $\Delta S \leq 0$, $(p,n) \to (\pi^{0,\pm}, K^{+,0}, \eta)$. All possible $N \to PS$ matrix elements. Isospin symmetry ($u \leftrightarrow d$) reduces the number of MEs.

$$\begin{aligned}\langle\pi^0|(ud)_{R/L}u_L|p\rangle &= \langle\pi^0|(du)_{R/L}d_L|n\rangle,\\ \langle\pi^+|(ud)_{R/L}d_L|p\rangle &= -\langle\pi^-|(du)_{R/L}u_L|n\rangle,\\ \langle K^0|(us)_{R/L}u_L|p\rangle &= -\langle K^+|(ds)_{R/L}d_L|n\rangle,\\ \langle K^+|(us)_{R/L}d_L|p\rangle &= -\langle K^0|(ds)_{R/L}u_L|n\rangle,\\ \langle K^+|(ud)_{R/L}s_L|p\rangle &= -\langle K^0|(du)_{R/L}s_L|n\rangle,\\ \langle K^+|(ds)_{R/L}u_L|p\rangle &= -\langle K^0|(us)_{R/L}d_L|n\rangle,\\ \langle\eta|(ud)_{R/L}u_L|p\rangle &= -\langle\eta|(du)_{R/L}d_L|n\rangle.\end{aligned}$$

- isospin limit

$$\langle\pi^+|(ud)d|p\rangle = \sqrt{2}\langle\pi^0|(ud)u|p\rangle$$

How to calculate W_0

- direct method: direct measurement of three point function
- indirect method: Chiral Perturbation Theory + Low Energy Constants evaluated on the lattice
  - $\mathcal{L}_B^\chi(f, D, F, , ,) + \mathcal{L}_{\not B}^\chi(\alpha, \beta)$, $(D + F = g_A)$
  - On the lattice, measure α and β

$$\langle 0|(ud)_R u_L|p\rangle = \alpha P_L u_p,$$

$$\langle 0|(ud)_L u_L|p\rangle = \beta P_L u_p$$

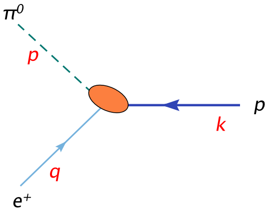
  - Reduction formula for $p \to \pi^0$:

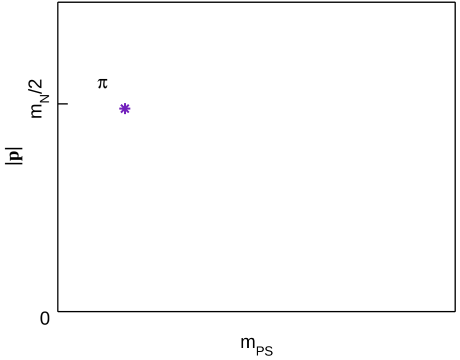
$$W_0(\langle \pi^0|(ud)_R u_L|p\rangle) = \alpha(1 + D + F)/\sqrt{2}f,$$

$$W_0(\langle \pi^0|(ud)_L u_L|p\rangle) = \beta(1 + D + F)/\sqrt{2}f$$

  - Reduction formula available for all the matrix elements [Claudson-Wise-Hall, JLQCD].

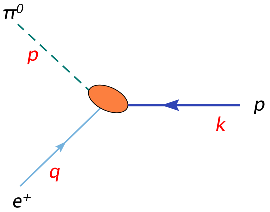
Direct $\leftrightarrow$ Indirect method

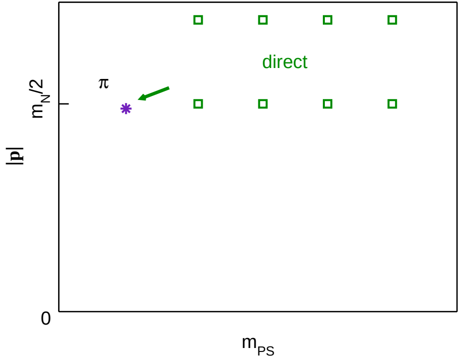




Similar for $p \to K$. Direct method is preferable

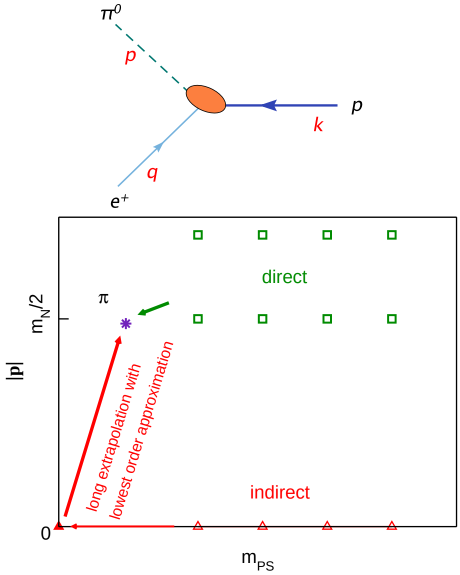
Direct $\leftrightarrow$ Indirect method





Similar for $p \to K$. Direct method is preferable

Direct ↔ Indirect method



Similar for $p \to K$. Direct method is preferable

History: indirect method

		$\lvert\alpha\rvert$ [GeV^3]	$\lvert\beta\rvert$ [GeV^3]	
QCD model calculation	Donoghue and Goldwich	0.003		Bag model
	Thomas and McKellar	0.02		Bag model
	Meljanac et al.	0.004		Bag model
	Ioffe	0.009		Sum rule
	Krasnikov et al.	0.003		Sum rule
	Ioffe and Smilga	0.006		Sum rule
	Tomozawa	0.006		Quark model
	Brodsky et al.	0.03		
Lattice QCD $N_f = 0$	Hara et al.	0.03		WF, $a = 0.11$ fm
	Bowler et al.	0.013	0.010	WF, $a = 0.22$ fm
	Gavela et al.	0.0056(8)	$\simeq \lvert\alpha\rvert$	WF, $a = 0.09$ fm
	JLQCD	0.015(1)	0.014(1)	WF, $a = 0.09$ fm
	CP-PACS & JLQCD	$0.0090(09)(^{+5}_{-19})$	$0.0096(09)(^{+6}_{-20})$	WF, continuum limit
	This work	0.0100(19)	0.0108(21)	DWF, $a = 0.15$ fm
Lattice QCD $N_f = 2$	This work	0.0118(21)	0.0118(21)	DWF, $a = 0.12$ fm

History: Direct method

- Gavela et al (89), Direct/Indirect$\simeq 2 - 3$
  What they calculated is $W_0 + E_\pi W_q$: not useful!
- JLQCD (00), correct direct method, but
  - Wilson fermion ($\beta = 6$): large scaling violation
  - lattice perturbation: systematic error involved
  - quench

We do

- Domain-wall fermion: small scaling violation, no operator mixing.
- Non-perturbative renormalizatoin: very small systematic error.
- Try $N_f = 2$ dynamical calculation for α and β.

Non-Perturbative Renormalization (NPR)

- MOM scheme on the lattice with scale set by off-shell p^2,
- match to $\overline{\mathrm{MS}}$ with NLO perturbation.
- Biggest problem of the lattice perturbation theory is the bad convergence due to tadpole, which is solved by NPR completely.
- This is the 1st time to apply NPR to nucleon decay operator.

Non-Perturbative Renormalization (NPR)

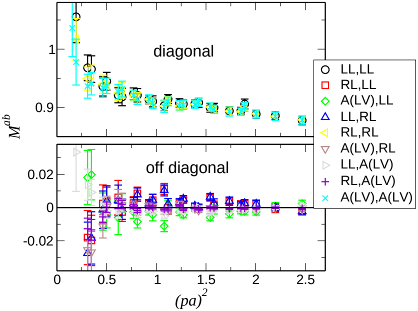
Quench, $a = 0.15$ fm

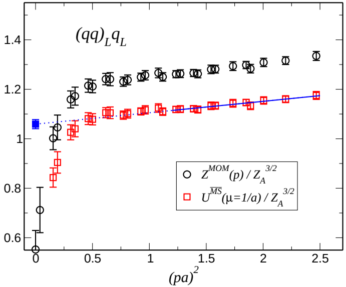
$$\mathcal{O}^{MOM,i} = (Z^{MOM}_{latt})^{ij}\mathcal{O}^{latt,j}$$

$$M = Z^{-1}$$

$$\mathcal{O}^{\overline{MS}}(\mu) = U^{\overline{MS}\leftarrow latt}(\mu)\mathcal{O}^{latt},$$

$$U^{\overline{MS}\leftarrow latt}(\mu) = U^{\overline{MS}}(\mu;p)\frac{Z^{\overline{MS}}(p)}{Z^{MOM}_{cont}(p)}Z^{MOM}_{latt}(p).$$





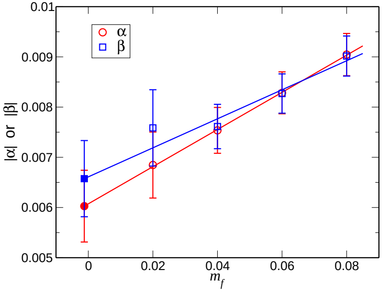
$$U^{\overline{MS}\leftarrow latt}(2\text{GeV}) = \begin{cases} 0.751(13)(45) & \text{for } \mathcal{O}^{LL}, \\ 0.755(15)(45) & \text{for } \mathcal{O}^{RL}, \end{cases}$$

$$\text{error} = (stat)(sys).$$

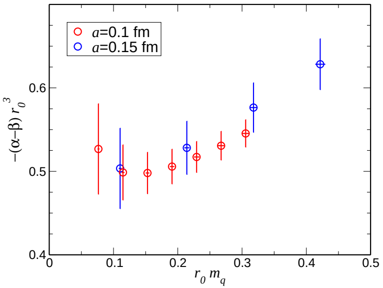
Low energy parameters α and β

Quench, $a = 0.15$ fm

- chiral extrapolation



- scaling violation negligible for $-(\alpha - \beta)$



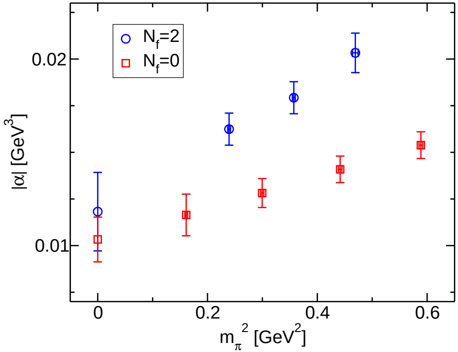
- $\mu = 2$ GeV, $\overline{\text{MS}}$

$$-\alpha = 0.0100(12)(14)(6)\ \text{GeV}^3,$$
$$\beta = 0.0108(13)(15)(7)\ \text{GeV}^3,$$

$$\text{error} = (stat)(sys)(Z).$$

How large is the quench systematic error ?

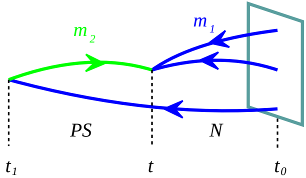
- comparison with $N_f = 2$ domain-wall fermion ($a = 0.12$ fm)

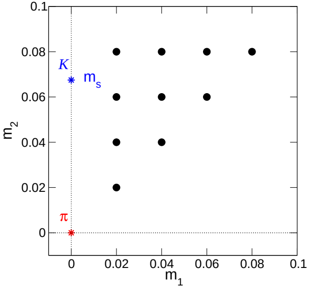


- So far, no dynamical quark effect is observed.
- Error needs to be reduced.

Direct Method

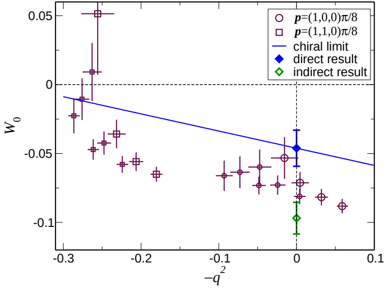
Quench, $a = 0.15$ fm





Momentum injection:
$(1,0,0)\pi/8$, $(1,1,0)\pi/8$

- $W_0(\langle\pi^0|(ud)_R u_L|p\rangle)$



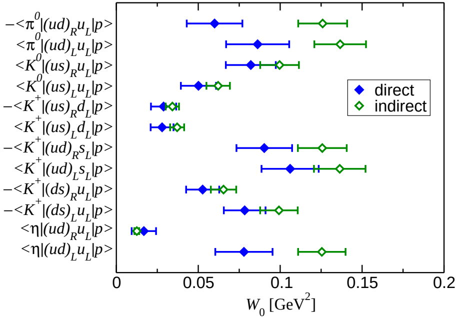
$$W_0\ [\mathrm{GeV}^2] = \begin{cases} -0.060(18)(17) & \text{direct,} \\ -0.123(19)(12) & \text{indirect,} \end{cases}$$

$$\text{error} = (\mathit{total})(\mathit{stat}).$$

- Large difference observed.
- Statistical error dominates for direct.

Summary for all the matrix elements

Quench, $a = 0.15$ fm



Errors are statistical only. For a complete table, see hep-lat/0607002.

- Indirect method tends to overestimate W_0, results in underestimating proton lifetime.
- Largest difference observed for $p \to \pi$ decay.
  - $\tau(\text{direct})/\tau(\text{indirect}) = 2 - 4$.
- (If you do not care a factor 10 for the proton lifetime, you can safely use indirect results.)

Summary and Outlook

- Summary
  - Nucleon decay matrix elements calculated with DWF.
  - NPR successfully applied for the nucleon decay operators.
  - Problem of the indirect method emphasized.
  - Direct method applied in the quenched approximation, and all the possible $N \to PS$ matrix elements were calculated. Results tend to prolong the proton life time than indirect method.
  - Importance of reducing the statistical error on the direct method was emphasized.
  - Scaling violation and quenching errors were investigated for low energy parameters α & β, which are negligible compared with the statistical error.
- Outlook
  - Extend the calculation to $N_f = 3$ dynamical domain-wall fermions, to include dynamics of all u, d, s quarks. Perform direct method calculation.
  - Statistical error could be reduced by appropriate tuning of the hadron-interpolating operators.