Precision Calculations of Radiative Corrections for ILC Physics

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Overview

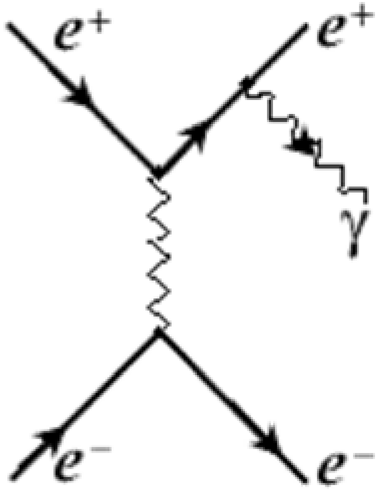
I will discuss some of the radiative corrections which will be needed for precision calculations of physics at the ILC, especially the Bhabha luminosity process, with an emphasis on $\mathcal{O}(\alpha^2)$ photonic contributions.

I will also look at the present status of photonic radiative corrections to fermion pair production, and what can be learned by comparing different versions of known results.

The Bhabha Luminosity Process

- At e^+e^- accelerators (SLC, LEP, ILC), the luminosity is calibrated using small angle Bhabha scattering

$$e^+e^- \longrightarrow e^+e^- + n\gamma$$



- This process has both experimental and theoretical advantages:
  - A large, clean signal
  - Almost pure QED
- The angle cuts were 1-3 degrees at LEP1, 3-6 degrees at LEP2.

BHLUMI Monte Carlo Program

BHLUMI was developed into an extremely precise tools for computing the Bhabha luminosity process in $e^+ e^-$ colliders.

The project was begun by S. Jadach, B.F.L. Ward, E. Richter-Was, and Z. Was and continued with contributions by S. Yost, M. Melles, M. Skrzypek, W. Placzek and others.

Historical Progress in Bhabha Scattering

Year	Expt.	Theory
1982	2%	2%
1990	0.8%	1%
1992	0.6%	0.25%
1997	0.15%	<0.11%
1999	0.05%	<0.06%

BHLUMI

for LEP 1 parameters

Theoretical Uncertainties

The leading theoretical uncertainties in BHLUMI 4.04 are shown in the table for LEP1 and LEP2 parameters.

Source of Uncertainty	LEP 1	LEP 2
Missing Photonic $\mathcal{O}(\alpha^2 L)$	0.027%	0.04%
Missing Photonic $\mathcal{O}(\alpha^3 L^3)$	0.015%	0.03%
Vacuum Polarization	0.04%	0.10%
Light Pairs	0.03%	0.05%
Z exchange	0.015%	0.0%
TOTAL	0.061%	0.122%

"big logarithm"
$L = \ln (|t| / m_e^2)$

LEP 1: E_{cms} = 92 GeV, $1^o < \theta < 3^o$ **LEP 2:** E_{cms} = 176 GeV, $3^o < \theta < 6^o$

ILC Luminosity

- The desired luminosity precision for the ILC will be 0.01%.

- The energy range is boosted to 500 – 1000 GeV and possibly beyond.

- The angle range (for LumiCal proposal) would be 28-90 mrad (1.6^o – 5.2^o)

Luminosity Uncertainty Beyond LEP

Preliminary estimates have been made (Jadach & Bardin, 2001) of the size of the terms in the luminosity error budget in the TESLA/CLIC proposal, with energies up to 3 TeV and comparable angles (25-100 mrad) – but with a target precision of 0.1%.

The size of the transfer $\sqrt{|t|} = E_{cms} \sin \theta/2$ plays an important role because t appears in the “big logarithms” $L = \ln(|t|/m_e^2)$ which determine the size of the radiative corrections.

LEP1: $\sqrt{|t|} \sim 2$ GeV

LEP2: $\sqrt{|t|} \sim 10$ GeV

ILC: $\sqrt{|t|} \sim 23\text{-}46$ GeV

Luminosity Uncertainty Beyond LEP

The TESLA/CLIC analysis points to some general features relevant also to the ILC.

Compared to LEP:

- Photonic QED corrections ~ $\alpha^2 L \ln(\theta_{max}/\theta_{min})$ increase. L is 15% larger at 1 TeV than 176 GeV.
- Vacuum polarization and its errors increase.
- Exponentiation – an integral feature of BHLUMI – will be essential for reaching ILC precision.
- Z exchange in the t channel becomes more important.

Consequences of High Transfer

The ILC transfer is intermediate between the values $\sqrt{|t|}$ ~ 10 GeV for TESLA and 75 GeV for CLIC. Jadach & Bardin investigated those cases using LabMC – a first step.

Some conclusions (2001):

- QED photonic corrections are 15-30% larger than at LEP1.
- EW uncertainty < 0.1% at 3TeV
- Hadronic vacuum polarization ~ 0.1% dominates.
- Total error < 0.1% looked feasible. No longer adequate!!

Upgrade to ILC Precision

What would need to be done to reach to the 0.01% level?

A complete analysis will require considerable effort and is beyond the scope of this talk.

I will concentrate on what improvement can be attained using known $\mathcal{O}(\alpha^2)$ photonic contributions.

Exact 2-photon bremsstrahlung corrections have been calculated to obtain the careful estimates of the missing photonic QED corrections, but these have not yet been implemented in the program.

$\mathcal{O}(\alpha^2)$ Photonic Corrections

The $\mathcal{O}(\alpha^2)$ photonic error budget at LEP2 was estimated to be 0.04% based on a calculation of exact $\mathcal{O}(\alpha^2)$ photonic corrections which were unimplemented in BHLUMI, where an expansion in the big logarithm L was used to obtain the most important contributions for LEP physics, at leading log order $\mathcal{O}(\alpha^2 L^2)$

$\mathcal{O}(\alpha^2)$ Photonic Corrections

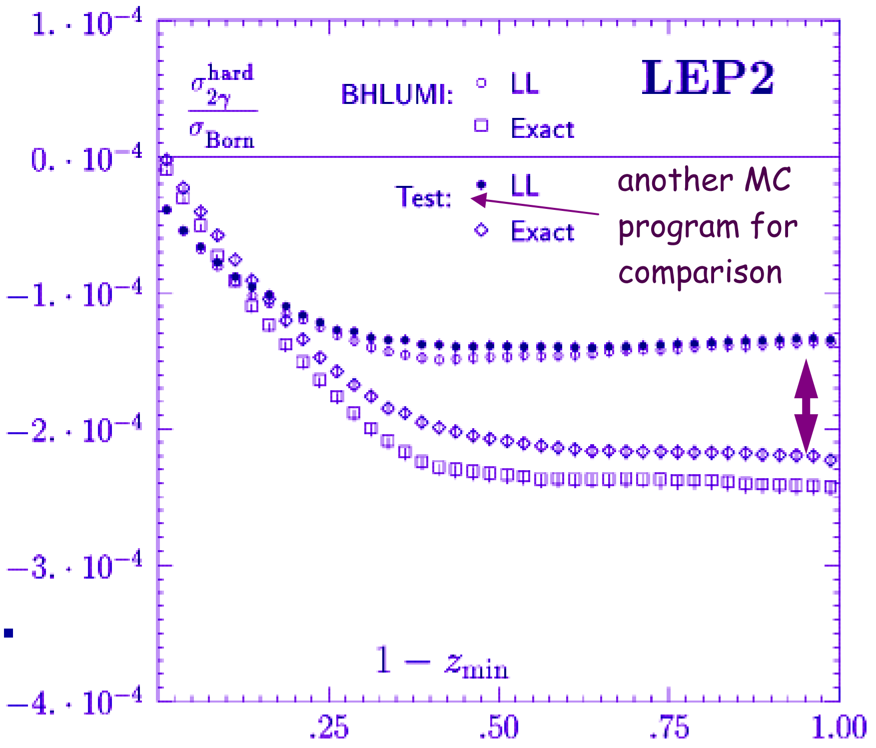
$\mathcal{O}(\alpha^2)$ exact results:

- The exact 2 real photon emission amplitudes are available (Jadach, Ward & Yost, 1993)
- The exact real + virtual e^+ and e^- line emission amplitudes are known as well (Jadach, Melles, Ward & Yost, 1996)
- The 2-loop e^+ or e^- line virtual photon correction is also known (Jadach, Melles, Ward & Yost, 1999, adapted from Berends, *et al*.)

Two Real Photons

The sub-LL contribution for two real photons is a maximum of 0.012% for LEP2 parameters

(176 GeV, 3° – 6°).



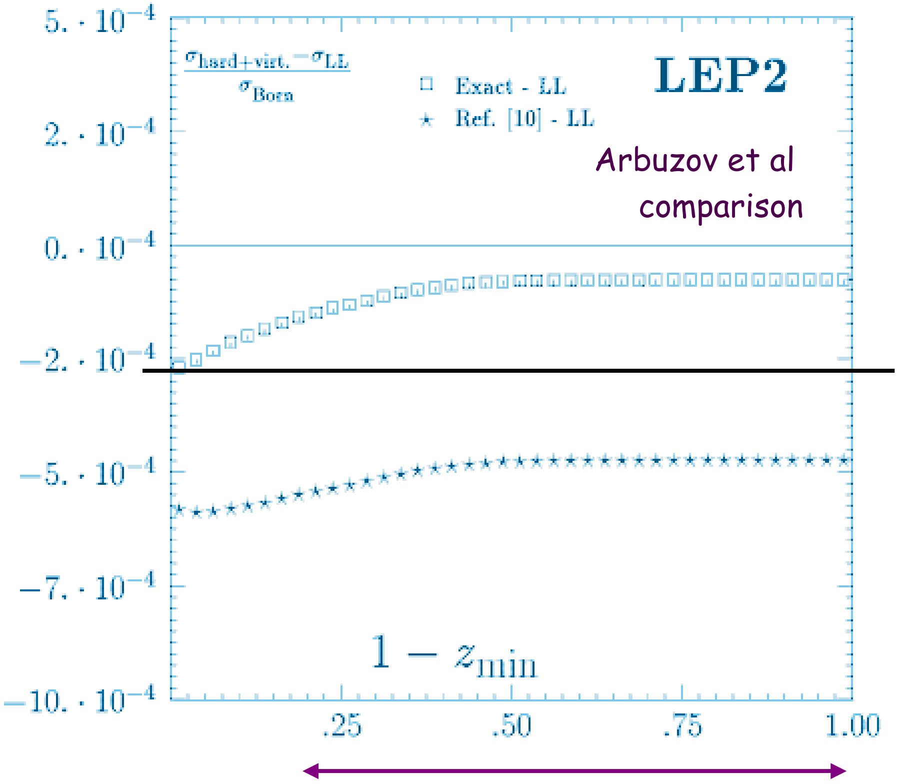
another MC program for comparison

experimentally interesting

z_{min} = minimum energy fraction in final e^+e^- pair.

Real + virtual Emission

Calculation of the NLL contribution to real+virtual e^+ or e^- line photon emission showed the effect to be bounded by 0.02% for LEP2 parameters.



Arbuzov et al comparison

experimentally interesting

Two Virtual Photon Contribution

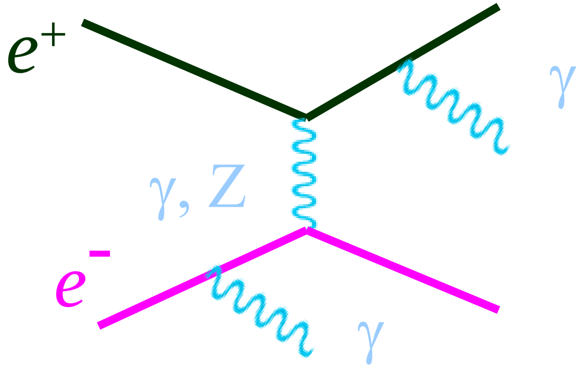
The $\mathcal{O}(\alpha^2)$ pure virtual correction to e^+ or e^- line emission, obtained by crossing from a result of Berends *et al*, gives a sub-LL contribution of 0.032%. Adding these three estimates in quadrature gives a 0.04% contribution, which was shown in the table.

Adding these available calculations to BHLUMI would eliminate this 0.04% error – except for “up-down” interference terms discussed below.

For ILC energies, these contributions would be even more essential in reaching .01% precision, since they are of order L – which is up to 15% larger than at LEP2.

What's left at $\mathcal{O}(\alpha^2)$?

An *exact* calculation of "up-down" interference effects involving simultaneous emission from both lines has not yet been included.



These contributions go to zero at small angles, and are of order $m_e^2/|t|$ *without* cuts . But they become more important at larger angles.

Typical sizes found in BHLUMI:

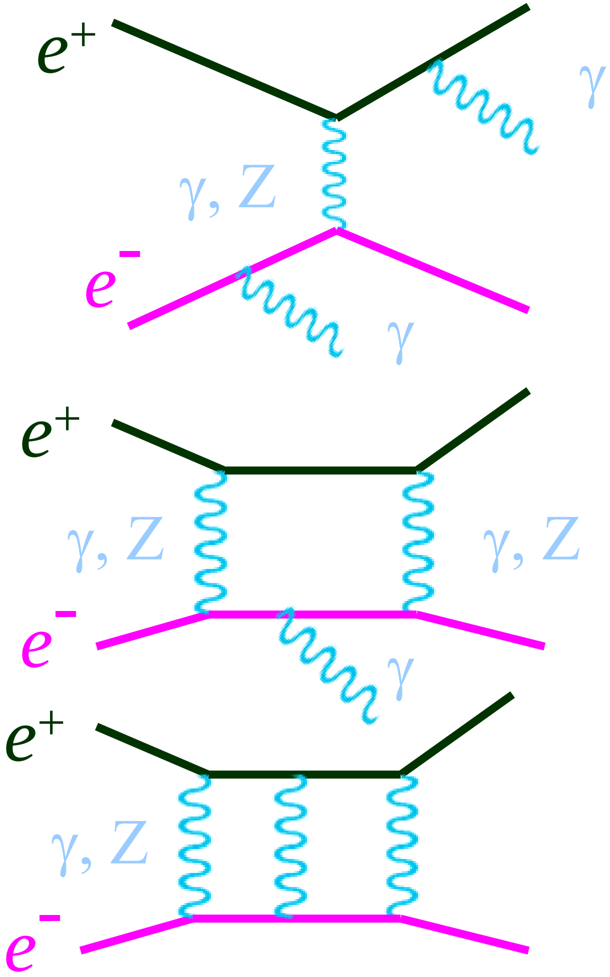
[Jadach, Richter-Was, Ward, Was, Phys. Lett. B253 (1991) 469]

Angle cut	**size**
< 1°	< 0.001%
3° – 5°	0.01%
9° – 13°	0.09%

What's left at $\mathcal{O}(\alpha^2)$?

Up-down interference could be safely neglected for LEP1 and LEP2, but might be needed for ILC physics at 0.01% accuracy.

There are several recent exact results on $\mathcal{O}(\alpha^2)$ Bhabha scattering. [*eg*, Bern, Dixon, Ghinculov, Phys. Rev. D63 (2001) 053007, A.A. Penin, Phys. Rev. Lett. 95 (2005) 056004, …]

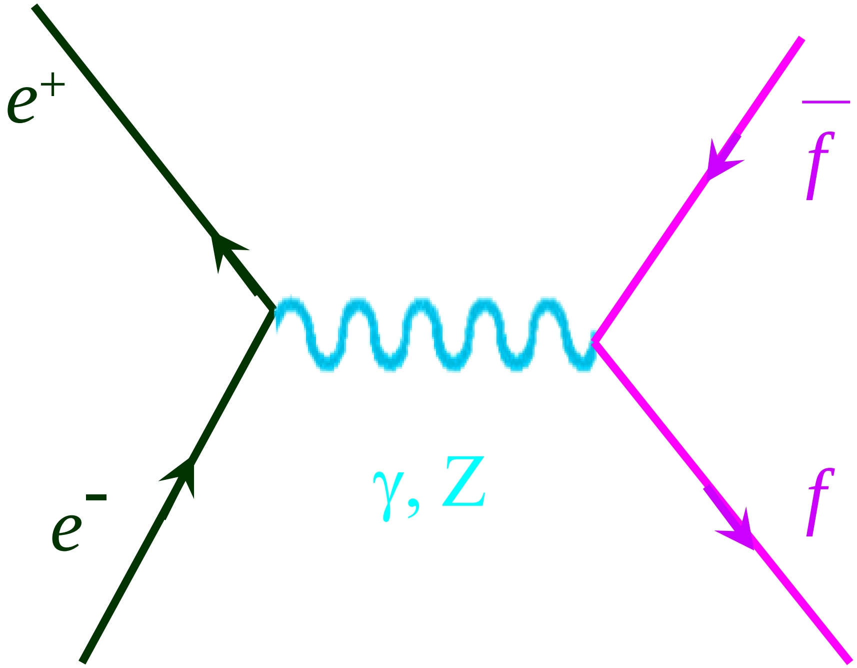


Fermion Pair Production

Fermion pair production

$$e^+ e^- \longrightarrow f \bar{f}$$

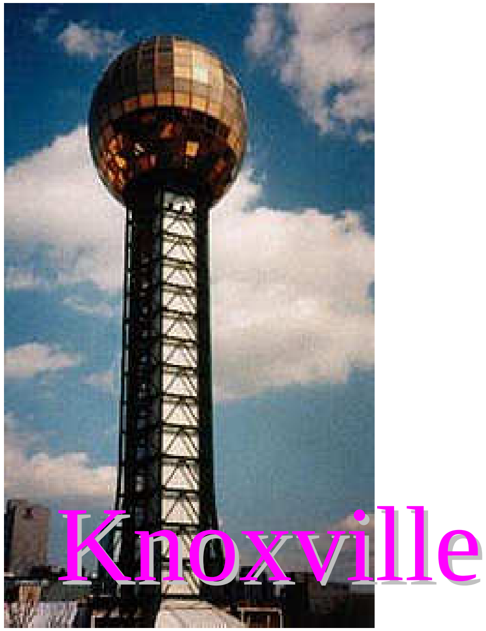
plays a critical role in extracting precision electroweak physics from e^+e^- colliders.



The KK Monte Carlo

The **KK MC** was designed for calculating fermion pair production, with radiative corrections as needed.

It also includes YFS exponentiation, and the effects of Z boson exchange.

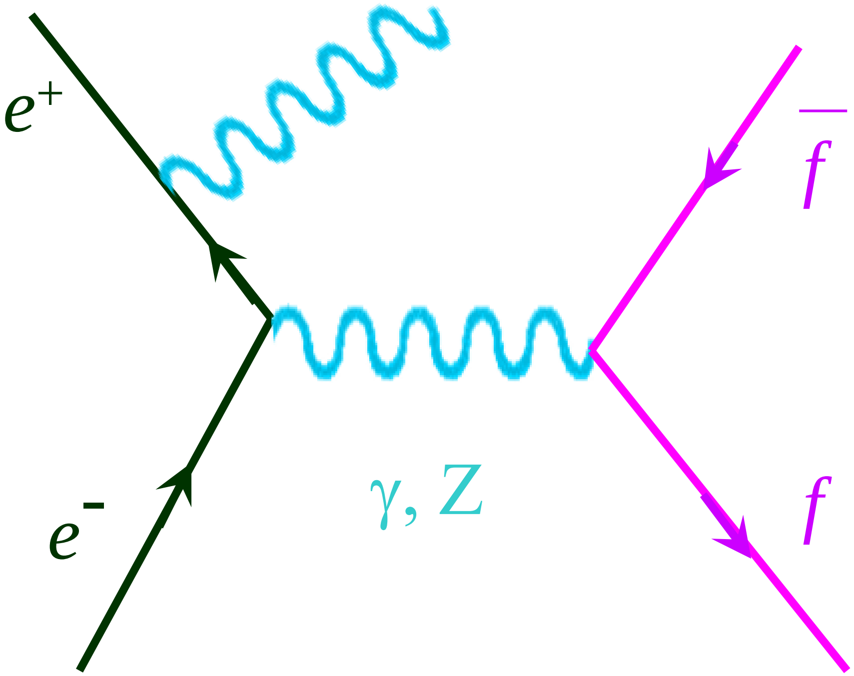


[Jadach, Ward, Was, Phys. Rev D63 (2001) 113009, Comp. Phys Comm. 130 (2000) 260]

Fermion Pairs + Bremsstrahlung

The basic process must be corrected by radiative effects, in particular Bremsstrahlung from a fermion line.

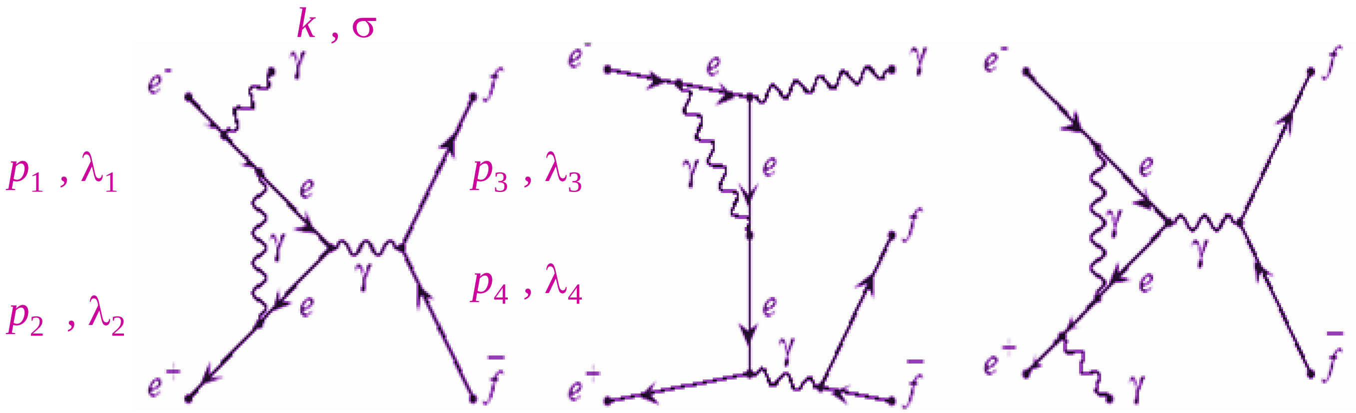
$$e^+ e^- \longrightarrow f \bar{f} \gamma$$



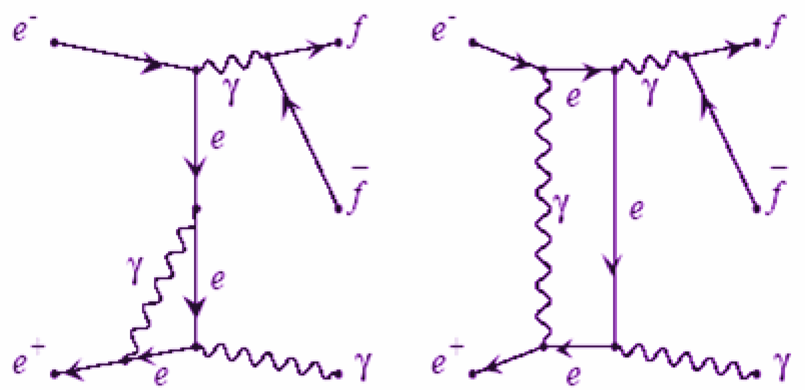
The case of initial-state radiation (ISR) is shown for a single photon.

One Loop Radiative Corrections

Virtual photons are needed for precision calculations…

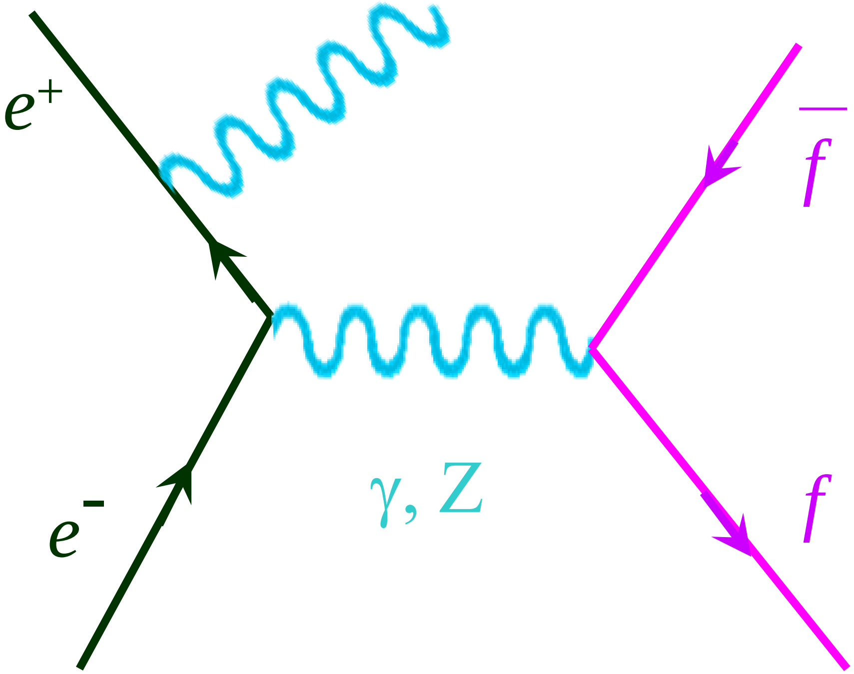


Calculations by Jadach, Melles, Ward & Yost: PRD65 (2001) 073030



Radiative Return Applications

This process is also important in radiative return experiments, where the energy carried away by the initial-state photon is used to reduce the effective energy of the collision, allowing a range of energies to be probed with a fixed-energy beam.



Radiative Correction Comparison

Another MC, **PHOKHARA** by Kuhn et al, designed for radiative return, incorporates the same hard+virtual corrections calculated by a very different means, and provides an important cross-check.

[Rodrigo, Gehrmann-De Ridder, Guilesaume, Kuhn, Eur. Phys. J. C22, 81 (2001); Kuhn, Rodrigo, Eur. Phys. J. C25, 215 (2002)].

- Both results compared claim the same degree of “exactness”.
- Both results include fermion mass corrections – but in very different ways.
- Both results have been shown to agree analytically at NLL order.

Finite e^- Mass Corrections

If the electron mass is 500 keV, and the ILC energy scale is 500 GeV, are mass effects still relevant?

$m_e^2/s = 1.0 \times 10^{-12}$ at 500 GeV cms Energy

Yes! Photons may be omitted collinearly with a fermion, leading to a large enhancement in the cross section.

- $m_e^2/(pk)^2$ is negligible away from collinear limits, and approaches $1/E_\gamma$ when k is collinear with p.
- Integrating terms of the form $m_e^2/(pk)^2$ over k gives contributions of order 1. Such contributions do not appear in LL result ($\mathcal{O}(\alpha^2L^2)$ in this case), but begin to appear at NLL.

Finite Mass Corrections

A comparison of the finite mass corrections is especially interesting, because the two calculations add them by different means.

JMWY add mass corrections following Berends, *et al* (CALCUL collaboration). The most important corrections for a photon with momentum k radiated collinearly with each incoming fermion line p_1 and p_2 are added via a simple prescription

$$\left|\mathcal{M}_{1\gamma}^{(m)}\right|^2 = -\sum_i \frac{e^2 m_e^2}{p \cdot k} \left|\mathcal{M}_{\mathrm{Born}}(p_i - k)\right|^2$$

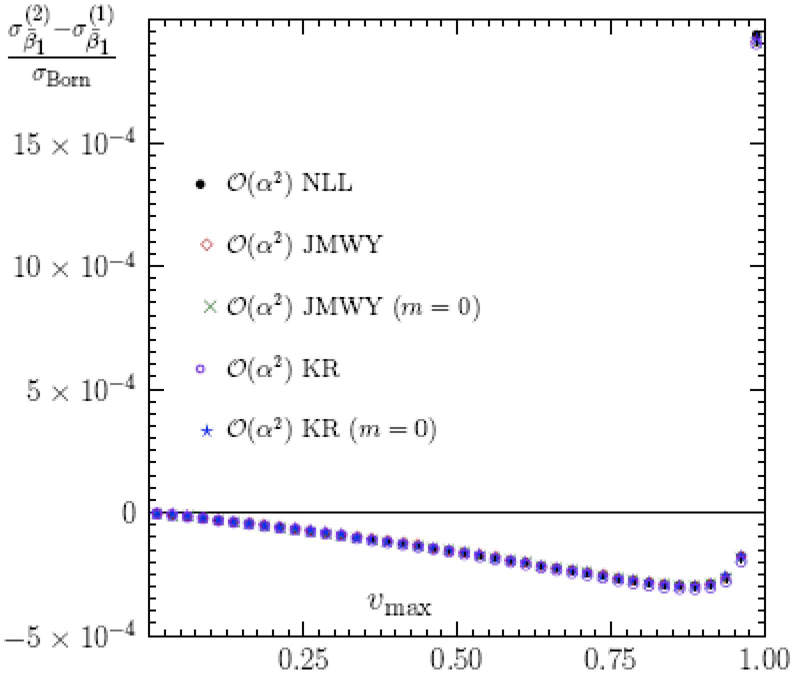
significant
when
integrated

Kuhn & Rodrigo use an expansion method in powers of $m_e^2/(p \cdot k)^2$

Monte Carlo Results

Results of KK Monte Carlo runs with 10^8 events at E_{CMS} = 500 GeV, showing only the virtual correction, with the IR contribution subtracted.

We must zoom in to see a difference – subtract the NLL part since this is known to agree analytically.



NNLL Comparison

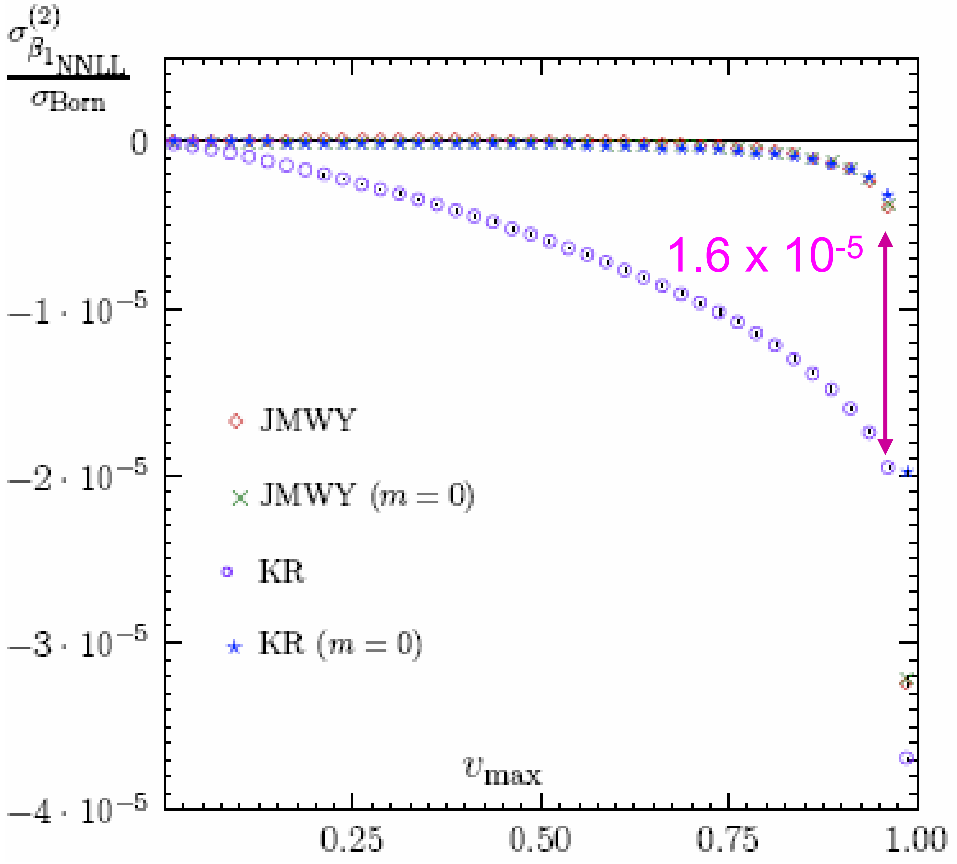
Results of a KK Monte Carlo run with 10^8 events at $E_{\mathrm{CMS}} = 500$ GeV.

Agreement of the massless results is 10^{-6} or better. The mass terms differ by as much as 1.6×10^{-5} in terms of the Born cross section.

To compare to hard photon cross section, use

$$\sigma_1^{\mathrm{ISR}} = 0.980\ \sigma_{\mathrm{Born}}$$

Massless NLL of JMWY subtracted



$v_{\max} = 1 - z_{\min}$

Summary

- Adding known exact $\mathcal{O}(\alpha^2)$ results for photonic radiative corrections will do much to bring the Bhabha luminosity process, as calculated by BHLUMI, to the level required for the ILC.
- More work must be done to say what is needed to reduce the remaining error budget to the 0.01% level.
- Comparisons of independent calculations of fermion pair production with real + virtual photon radiation suggest that these processes are understood at the level of 10^{-5} or better.
- The remaining differences are largely in the handling of mass corrections – it would be desirable to understand this better, and to what degree it may depend on the specific MC implementation vs the methods used to obtain the underlying expressions.