Covariant Quantization of Superbranes

— An approach from the double-spinor formalism —

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1 Introduction

Super-covariant quantization of superstring (and branes)
is a long standing fundamental problem.

- ♦ **Conceptual and aesthetic desire**
- ♦ **Becoming urgent for practical reasons in various arenas**

1. **Study of D-brane physics**

D-branes generate **RR (bispinor) fields**
Difficult to describe fully in conventional formalisms

RNS: Needs spin fields, picture-changing
GS: Quantization possible only in non-covariant (and rather singular) L.C. type gauges.

2. Study of AdS/CFT

- **CFT side:** Substantial progress has been and is being made through spin-chain approach, etc.
- **String side:** Quantization in the relevant curved background, such as $AdS_5 \times S^5$, with a large RR flux is an urgent and crucial problem.
  - Description in the simplified plane-wave background has been achieved in the GS formalism in the L.C. gauge. Yet due to the lack of covariance, it is difficult to fully characterize the (SFT) interactions.

3. Study of M-theory

Beyond 11D supergravity approximation, the only formulation we have is the M(atrix) theory $\sim$ Matrix-regularized supermembrane theory
Only $SO(9)$-covariant, $\overline{D0}$ cannot be described.

It is of prime importance to construct a manageable super-covariant quantization scheme for string and membrane.

GS-type formulation appears more promising as it is classically supercovariant

But

Difficulty in quantization:

♦ **Fermionic constraints:** $d_\alpha = 0$

$$d_\alpha = p_\alpha + i\partial x_\mu(\gamma^\mu\theta)_\alpha + \frac{1}{2}(\gamma^\mu\theta)_\alpha(\theta\gamma_\mu\partial\theta)$$

Eight 1st class (κ-symmetry) and **eight** 2nd class.

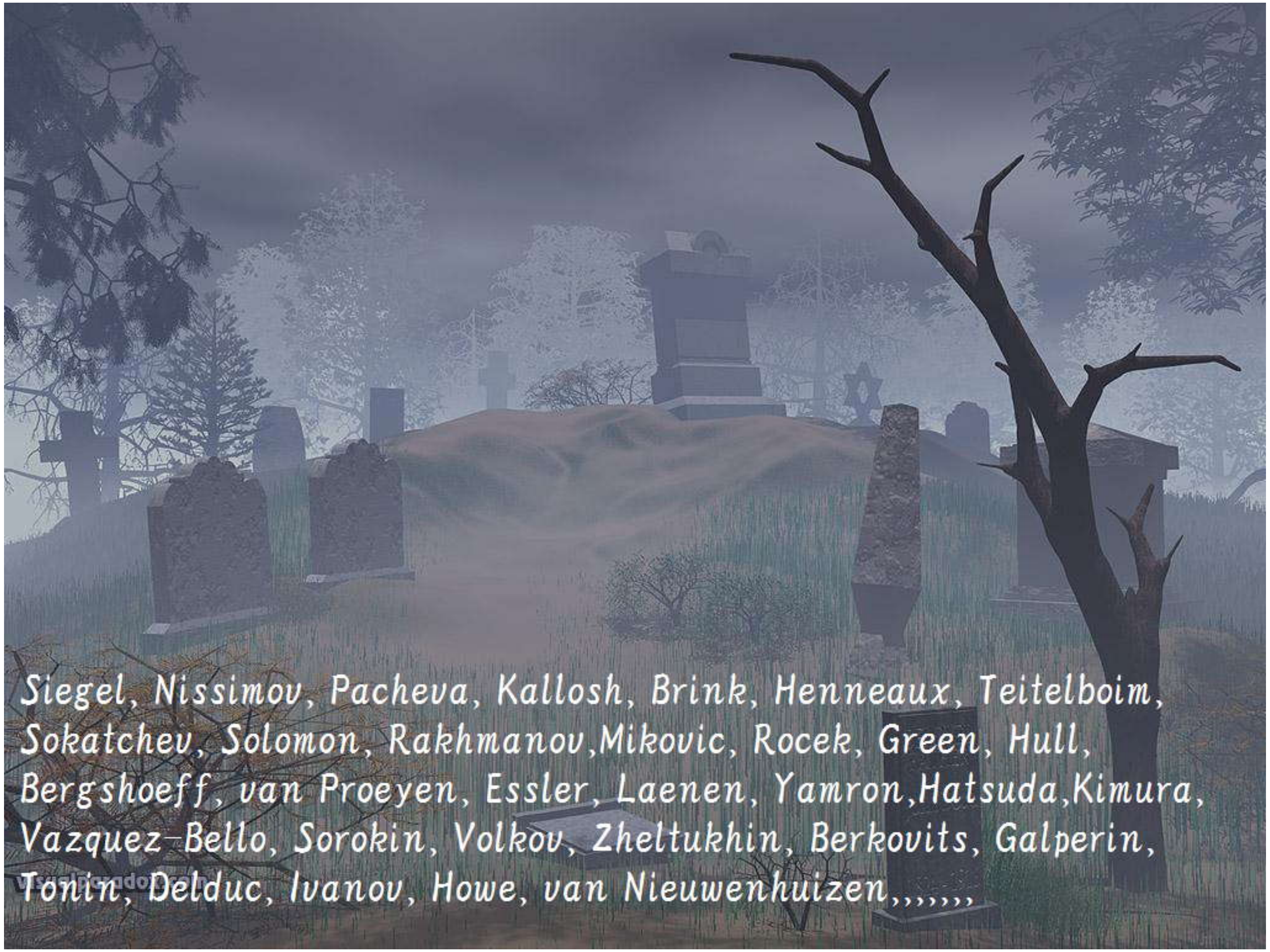
They cannot be separated in Lorentz covariant way.

(No 8-dimensional representation.)

$\Downarrow$

Quantization procedure breaks super-Poincaré covariance.

Huge number of attempts to overcome this difficulty $\sim$ 2000:



Such attempts achieved limited success but provided useful hints.

New proposal by N.Berkovits (2000):

Pure spinor formalism

Physical states of superstring = cohomology of BRST-like operator Q

$$Q = \int \frac{dz}{2\pi i} \lambda^\alpha(z) d_\alpha(z)$$

$$d_\alpha = p_\alpha + i\partial x_\mu (\gamma^\mu \theta)_\alpha + \frac{1}{2}(\gamma^\mu \theta)_\alpha (\theta \gamma_\mu \partial \theta)$$ = fermionic constraints

p_α = conjugate to θ^α ,

λ^α = bosonic “ghosts”, subject to **pure spinor** conditions $\lambda^\alpha \gamma^\mu_{\alpha\beta} \lambda^\beta = 0$

$\Rightarrow$ 11 independent components

- **All the fields are postulated to be free**

Necessity of pure spinor conditions: Nilpotency:

$$d_\alpha(z) d_\beta(w) = \frac{2i\gamma^\mu \Pi_\mu(w)}{z-w}$$

$$\Pi_\mu = \partial x_\mu - i\theta\gamma_\mu\partial\theta = \text{superinvariant momentum}$$

$$Q^2 = \int [dz] \int [dw] \lambda^\alpha(z) \lambda^\beta(w) \underbrace{d_\alpha(z) d_\beta(w)}_{2i\gamma^\mu\Pi_\mu/(z-w)}$$

$$= 2i \int [dw] \lambda^\alpha(w) \gamma^\mu_{\alpha\beta} \lambda^\beta(w) \Pi_\mu(w) = 0$$

CFT with vanishing center

Remarkable successes:

- Q-invariant vertex operators are constructed: massless, 1st massive (Berkovits, Berkovits-Chandia)
- Covariant path-integral rules for computing amplitudes to all loops are postulated (Berkovits). They yield known results and more:
  - Tree: (Berkovits, Berkovits-Vallilo, Trivedi)
  - Loop: Certain vanishing theorems to all loops, some 2-loop calculations, etc. (Berkovits)
- Equivalence with RNS and light-cone GS has been shown (Berkovits, Aisaka-Kazama)
- Action in $AdS_5 \times S^5$ background has been constructed: Classically, there exist infinite number of conserved non-local charges, as in GS formalism. (Berkovits, Vallilo)
- "Topological" formulation in an extended space (Berkovits):
  Rules for loop amplitudes $\sim$ topological string $\sim$ bosonic string

Very interesting, but unusual and mysterious

Where does the PS formalism come from ?

- Reparametrization-invariant action ? Underlying symmetry ?
  - What is Q the "BRST" charge of ?
  - Where is the Virasoro algebra ?
- Why free fields ? How to quantize, with non-linear PS constraints ?
- How to derive the covariant rules including the measure ?
- Can it be applied to supermembrane ? (Attempt by Berkovits (02))

We will answer many of these questions from the first principle by "Double Spinor Formalism"

Plan of the Talk

1. **Introduction**
2. **Pure Spinor (PS) Formalism for Superparticle in 10D**
3. **PS Formalism for Superstring**
4. **Challenge for the Supermembrane Case**
5. **Summary and Future Problems**

Based mainly on

- "Origin of Pure Spinor Superstring", JHEP 0505:046,2005 (hep-th/0502208)
- "Towards Pure Spinor Type Covariant Description of Supermembrane: — An Approach from the Double Spinor Formalism —, JHEP 0605:041,2006 (hep-th/0603004)

with **Yuri Aisaka** (U. of Tokyo, presently at Instituto de Física, UEP, Brasil)

2 Covariant PS Formalism for Superparticle in 10D

2.1 Basic Idea: “Double Spinor Formalism”

If we start from the conventional Brink-Schwarz action with x^m and $\tilde{\theta}^\alpha$, quantization becomes inevitably non-covariant.

- Introduce **an additional spinor θ^α**, together with **a compensating new local fermionic symmetry**, to keep the physical content intact.
- **Keep the local fermionic symmetry and covariance for the second spinor θ^α**.
  - BRST operator with unconstrained bosonic spinor ghosts naturally arises.
  - The non-covariant remnants produced by the quantization procedure can be decoupled.
  - This decoupling process at the same time produces the pure spinor conditions.

□ Fundamental action for type I superparticle:

Formally the same as the Brink-Schwarz, but **with crucial re-interpretation**

$$L = \frac{1}{2e}\Pi^m\Pi_m\,, \qquad \Pi^m = \dot{y}^m - i\Theta\gamma^m\dot{\Theta}$$

$$\Theta \equiv \tilde{\theta} - \theta\,, \qquad y^m \equiv x^m - i\theta\gamma^m\tilde{\theta}$$

Basic variables $= x^m, \theta^\alpha, \tilde{\theta}^\alpha\,, \qquad m = 0 \sim 9\,, \alpha = 1 \sim 16$

□ Symmetries:

- Global SUSY

$$\delta\theta = \epsilon\,, \quad \delta\tilde{\theta} = 0 \qquad \Rightarrow \delta\Theta = -\epsilon$$

$$\delta x^m = i\epsilon\gamma^m\theta \qquad \Rightarrow \delta y^m = -i\epsilon\gamma^m\Theta$$

$$\Rightarrow \qquad \delta\Pi^m = 0$$

- **Extra local fermionic symmetry** (with local fermionic parameter χ)

$$\delta\theta = \chi\,, \quad \delta\tilde{\theta} = \chi \qquad \delta\Theta = 0$$

$$\delta x^m = i\chi\gamma^m\Theta = i\chi\gamma^m(\tilde{\theta} - \theta)$$

$$\Rightarrow \qquad \delta y^m = i\chi\gamma^m\Theta - i\chi\gamma^m\tilde{\theta} + i\chi\gamma^m\theta = 0$$

Using this symmetry, one can fix $\theta = 0 \quad \Rightarrow$ Brink-Schwarz action for $\tilde{\theta}$

- κ symmetry

$$\delta\theta = 0\,, \qquad \delta\tilde{\theta} = \Pi_n \gamma^n \kappa$$
$$\delta x^m = i\tilde{\theta}\gamma^m \delta\tilde{\theta}\,, \qquad \delta e = 4ie\dot{\Theta}\kappa$$

□ **Standard Dirac Analysis**:

Momenta

$$p_m = \frac{\partial L}{\partial \dot{x}^m} = \frac{1}{e}\Pi_m$$
$$p_\alpha = \frac{\partial L}{\partial \dot{\theta}^\alpha} = \frac{1}{e}\Pi_m i(\gamma^m(\theta - 2\tilde{\theta}))_\alpha = i(\not{p}(\theta - 2\tilde{\theta}))_\alpha$$
$$\tilde{p}_\alpha = \frac{\partial L}{\partial \dot{\tilde{\theta}}^\alpha} = \frac{1}{e}\Pi_m i(\gamma^m\tilde{\theta})_\alpha = i(\not{p}\tilde{\theta})_\alpha$$
$$p_e = \frac{\partial L}{\partial \dot{e}} = 0$$

Primary constraints

$$D_\alpha = p_\alpha - i(\not{p}(\theta - 2\tilde{\theta}))_\alpha = 0$$
$$\tilde{D}_\alpha = \tilde{p}_\alpha - i(\not{p}\tilde{\theta})_\alpha = 0$$
$$p_e = 0$$

Canonical Hamiltonian

$$H = \frac{e}{2} p^m p_m$$

Secondary constraint: $\{H, p_e\}_P = 0$ gives

$$T \equiv \frac{1}{2} p^2 = 0$$

Poisson brackets for fundamental variables:

$$\{x^m, p_n\}_P = \delta_n^m$$
$$\{p_\alpha, \theta^\beta\}_P = -\delta_\alpha^\beta, \qquad \{\tilde{p}_\alpha, \tilde{\theta}^\beta\}_P = -\delta_\alpha^\beta$$

□ Constraints and their algebra:

$$D_\alpha = p_\alpha - i(\not{p}(\theta - 2\tilde{\theta}))_\alpha = 0$$

$$\tilde{D}_\alpha = \tilde{p}_\alpha - i(\not{p}\tilde{\theta})_\alpha = 0\,, \qquad T = \frac{1}{2}p^2 = 0$$

$$\{D_\alpha, D_\beta\}_P = \{\tilde{D}_\alpha, \tilde{D}_\beta\}_P = 2i\not{p}_{\alpha\beta}\,, \qquad \{D_\alpha, \tilde{D}_\beta\}_P = -2i\not{p}_{\alpha\beta}$$

rest$= 0$

Generator of local fermionic symmetry:

$$\Delta_\alpha = D_\alpha + \tilde{D}_\alpha\,, \qquad \{\Delta_\alpha, D_\beta\}_P = \{\Delta_\alpha, \tilde{D}_\beta\}_P = \{\Delta_\alpha, \Delta_\beta\}_P = 0$$

We may regard $\tilde{D}_\alpha$ and Δ_α as independent constraints.

On the constrained surface $p^2 = 0$, $\text{rank}\,\not{p} = 8$

$\Rightarrow$ **$\tilde{D}_\alpha$ consists of 8 first class and 8 second class.**

A natural way to separate them is to use $SO(8)$ decomposition:

$$p^m = (p^+, p^-, p^i)\,, \quad p^\pm = p^0 \pm p^9\,, \quad i = 1 \sim 8$$

$$\tilde{D}_\alpha = (\tilde{D}_a, \tilde{D}_{\dot{a}})\,, \qquad a, \dot{a} = 1 \sim 8$$

Further introduce **κ-generator** in place of $\tilde{D}_{\dot{a}}$

$$\tilde{K}_{\dot{a}} = \tilde{D}_{\dot{a}} - \frac{p^i}{p^+}\gamma^i_{\dot{a}b}\tilde{D}_b$$

Then

$$\{\tilde{D}_a, \tilde{D}_b\}_P = 2ip^+\delta_{ab}$$
$$\{\tilde{K}_{\dot{a}}, \tilde{D}_b\}_P = \{\tilde{K}_{\dot{a}}, T\}_P = 0$$
$$\{\tilde{K}_{\dot{a}}, \tilde{K}_{\dot{b}}\}_P = -4i\frac{T}{p^+}\delta_{\dot{a}\dot{b}}$$

So $\tilde{D}_a$ are second class and $\tilde{K}_{\dot{a}}$ and T are first class.

Note: **κ constraint $\sim \sqrt{}$ of T constraint.**

□ Semi-LC gauge, Dirac bracket and the basic constraint algebra:

Semi-LC gauge: $\tilde{\theta}^{\dot{a}} = 0$ Imposed only for $\tilde{\theta}$ to fix κ symmetry

Dirac bracket:

$$\{\tilde{\theta}_a, \tilde{\theta}_b\}_D = \frac{i}{2p^+}\delta_{ab}$$

$$S_a \equiv \sqrt{2p^+}\,\tilde{\theta}_a\,, \qquad \{S_a, S_b\}_D = i\delta_{ab}$$

We still have $\Delta_\alpha = D_\alpha$ constraint ($\Leftarrow \tilde{D}_\alpha = 0$ now)

Basic classical first class constraint algebra:

$$\{D_{\dot{a}}, D_{\dot{b}}\}_D = -4i\frac{T}{p^+}\delta_{\dot{a}\dot{b}}\,, \quad \text{rest} = 0$$

The content of the κ-symmetry algebra is transferred to $D_{\dot{\alpha}}$ algebra through the local fermionic symmetry.

□ Quantization:

Redefine $p_\alpha \to -ip_\alpha, S_a \to -iS_a, D_\alpha \to -iD_\alpha.$

Quantization is trivial:

$$[x^m, p_n] = i\delta^m_n\,, \qquad \{p_\alpha, \theta^\beta\} = \delta^\beta_\alpha\,, \qquad \{S_a, S_b\} = \delta_{ab}$$

$$D_a = d_a + i\sqrt{2p^+}\,S_a\,, \qquad D_{\dot a} = d_{\dot a} + i\sqrt{\frac{2}{p^+}}p^i\gamma^i_{\dot a b}S_b$$

$$d_\alpha \equiv p_\alpha + (\not{p}\theta)_\alpha$$

Quantum first-class constraint algebra:

$$\{D_{\dot a}, D_{\dot b}\} = -4\frac{T}{p^+}\delta_{\dot a\dot b}\,, \qquad \text{rest} = 0$$

□ BRST charge and derivation of PS formalism:

First class algebra $\Rightarrow$ **nilpotent BRST operator** (Suppress $\int$):

$$\hat{Q} = \tilde{\lambda}^{\alpha} D_{\alpha} + \tilde{\lambda}_{\dot{a}} \tilde{\lambda}_{\dot{a}} \tilde{b} + \frac{2T}{p^{+}} \tilde{c} = \tilde{\lambda}^{\alpha} D_{\alpha} + \frac{2}{p^{+}} \tilde{\lambda}_{\dot{a}} \tilde{\lambda}_{\dot{a}} b + cT$$

$$\left\{ \tilde{b}, \tilde{c} \right\} = \{b, c\} = 1 \,, \qquad \tilde{\lambda}^{\alpha} = \text{unconstrained bosonic spinor}$$

At this stage, $\hat{Q}$ contains the "energy-momentum tensor" T, as expected for a reparametrization invariant theory.
Also note the important relation $\left\{ \hat{Q}, b \right\} = T$.

We now show that $\hat{Q}$ can be transformed into $Q = \lambda^{\alpha} d_{\alpha}$ of the PS formalism without changing its cohomology by a quantum similarity transformation:

<u>**Step 1**</u>:

- Disappearance of T, b, c $\quad\Rightarrow$ explains why T is absent in Q.
- and the appearance of a **quadratic constraint** $\tilde{\lambda}_{\dot{a}} \rightarrow \lambda_{\dot{a}}, \lambda_{\dot{a}} \lambda_{\dot{a}} = 0$

Introduce an auxiliary field $l_{\dot{a}}$ with the properties $\tilde{\lambda}_{\dot{a}} l_{\dot{a}} = 1$ and $l_{\dot{a}} l_{\dot{a}} = 0$. Then, one can construct **another (composite) b-ghost b_B** in addition to the original b:

$$b_B \equiv -\frac{p^+}{4} l_{\dot{a}} D_{\dot{a}} \quad \Rightarrow \quad \{b_B, b_B\} = 0\,, \qquad \left\{\hat{Q}, b_B\right\} = T$$

Perform the following similarity transformation: T disappears and we get

$$e^{b_B c} \hat{Q} e^{-b_B c} = \tilde{\lambda}_a D_a + \underbrace{\left(\tilde{\lambda}_{\dot{a}} - \frac{1}{2} l_{\dot{a}} \tilde{\lambda}_{\dot{b}} \tilde{\lambda}_{\dot{b}}\right)}_{\lambda_{\dot{a}}} D_{\dot{a}} + \underbrace{(2/p^+) \tilde{\lambda}_{\dot{a}} \tilde{\lambda}_{\dot{a}} b}_{\delta_b:\ decouples}$$

$$= \tilde{Q} + \delta_b \nearrow$$

where

$$\tilde{Q} = \tilde{\lambda}_a D_a + \lambda_{\dot{a}} D_{\dot{a}}\,,$$

$\lambda_{\dot{a}} \lambda_{\dot{a}} = 0:$ one of the PS constraints

Step 2: Decoupling of S_a and a part of $\tilde{\lambda}$: $\tilde{\lambda}_a \longrightarrow \lambda_a$

Split S_a and $\tilde{\lambda}_a$ into two parts by introducing projection operators

$$P^1_{ab} + P^2_{ab} = \delta_{ab}\,, \qquad P^1_{ab} \equiv \frac{1}{2}(\gamma^i\lambda)_a(\gamma^i l)_b \;= P^2_{ba}$$

$$S_a = S^1_a + S^2_a \equiv (P^1S)_a + (P^2S)_a$$
$$\tilde{\lambda}_a = \lambda^1_a + \lambda^2_a \equiv (P^1\tilde{\lambda})_a + (P^2\tilde{\lambda})_a$$
$$\lambda^1_a S^1_a = \lambda^2_a S^2_a = 0\,, \quad etc.$$

- λ^1_a satisfies the remaining 4 PS conditions $\lambda^1_a\gamma^i_{a\dot{b}}\lambda_{\dot{b}} = 0$.

$\Rightarrow$ $\boxed{\lambda^\alpha \equiv (\lambda^1_a, \lambda_{\dot{a}}) \text{ satisfies } \lambda\gamma^m\lambda = 0}$

- (S^1, S^2) forms a "conjugate" pair:

$$\{S^1_a, S^1_b\} = \{S^2_a, S^2_b\} = 0\,, \quad \{S^1_a, S^2_b\} = P^1_{ab}\,, \quad \{S^2_a, S^1_b\} = P^2_{ab}$$

In terms of these variables, $\tilde{Q}$ takes the form

$$\tilde{Q} = \underbrace{\lambda^\alpha d_\alpha}_{Q} + \underbrace{i\sqrt{2p^+}\,\lambda^2_a S^1_a}_{\delta} + \lambda^2_a d_a + i\left(\sqrt{2p^+}\,\lambda^1_b + \sqrt{\frac{2}{p^+}}\,p_i\lambda_{\dot{a}}\gamma^i_{\dot{a}b}\right) S^2_b$$

Now perform another similarity transformation:

$$e^{Y}\tilde{Q}e^{-Y} = Q + \cancel{\delta}$$

$$\text{where} \qquad Y = \frac{i d_a S_a^2}{\sqrt{2p^+}}\,, \qquad \{\delta, \delta\} = \{\delta, \lambda^\alpha d_\alpha\} = 0$$

Thus we finally obtain

$$\boxed{Q = \lambda^\alpha d_\alpha\,, \qquad \lambda\gamma^m\lambda = 0}$$

Note: Once we decouple non-covariant S_a, we are bound to get PS condition $\lambda\gamma^m\lambda = 0$, since it is the only way that Q remains nilpotent.

Thus we have derived the PS formalism for superparticle from the first principle.

3 PS Formalism for Superstring

The basic idea is exactly the same as for the superparticle case.
However we will encounter **several new non-trivial complications**:

- Action is **more non-linear with the WZ term**.
- More complicated structures with the **derivatives** ∂_σ.
- For type II string, **left-right separation will be non-trivial** due to extra spinors.
- Fundamental fields are apparently **not free under the Dirac bracket**.
- **Quantum singularities** will produce corrections.

□ **Reparametrization Invariant Fundamantal Action (for type IIB)**:

Formally the same as the GS action:

$$S = \int d^2\xi (\mathcal{L}_K + \mathcal{L}_{WZ}) \,,$$

$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}\, g^{ij}\Pi_i^m \Pi_{mj} \,,$$

$$\mathcal{L}_{WZ} = \epsilon^{ij}\Pi_i^m (W_{mj}^1 - W_{mj}^2) - \epsilon^{ij} W_i^{1m} W_{mj}^2 \sim \mathcal{O}(\Theta^4) \,,$$

where

$$\Pi_i^m \equiv \partial_i y^m - \sum_A W_i^{Am} \,, \qquad W_i^{Am} \equiv i\Theta^A \gamma^m \partial_i \Theta^A \qquad (A = 1, 2)$$

$$\Theta^A \equiv \tilde{\theta}^A - \theta^A \,, \quad y^m \equiv x^m - \sum_A i\theta^A \gamma^m \tilde{\theta}^A$$

Symmetries: Reparametrization, Global SUSY,
extra local fermionic sym. and κ symmetry.

Procedure:

- Perform Dirac analysis to get constraints
- Identify κ generator, and separate the constraints into 1st class and the 2nd class
- Adopt the semi-LC gauge for $\tilde{\theta}^\alpha$ and compute the algebra of constraints under the appropriate Dirac bracket

$$\Downarrow$$

Constraint algebra governing the entire classical dynamics: (for the left-moving sector)

$$\{D_{\dot{a}}(\sigma), D_{\dot{\beta}}(\sigma')\}_D = -8i\mathcal{T}(\sigma)\delta_{\dot{a}\dot{b}}\delta(\sigma-\sigma')\,, \qquad \text{rest} = 0$$

$$\mathcal{T} \equiv \frac{T}{\Pi^+}\,, \qquad T = \text{Virasoro generator}$$

□ **Free field basis:**

♠ **Problem:** Except for the self-conjugate field $S_a = \sqrt{2\Pi^+}\,\tilde{\theta}_a$, the other basic fields no longer satisfy canonical relations under the Dirac bracket:

Examples:

$$\{x^m(\sigma), k^n(\sigma')\}_D = \eta^{mn}\delta(\sigma-\sigma') + (i/2\Pi^+)(\gamma^m\tilde{\theta})_a(\gamma^n)_a(\sigma)\delta'(\sigma-\sigma')$$
$$\{k^m(\sigma), k^n(\sigma')\}_D = -(i/2)\partial_\sigma[(1/\Pi^+)(\gamma^m\Theta)_a(\gamma^n\Theta)_a\delta'(\sigma-\sigma')]\,, \quad etc.$$

♡ **Solution:** We have found a systematic redefinition of the original momenta $(k^m, k_a^A, k_{\dot{a}}^A) \longrightarrow (p^m, p_a^A, p_{\dot{a}}^A)$ so that the new fields satisfy canonical bracket relations:

$$p^m \equiv k^m - i\partial_\sigma(\tilde{\theta}\gamma^m\theta) + i\partial_\sigma(\hat{\tilde{\theta}}\gamma^m\hat{\theta})\,,$$
$$p_a^A \equiv k_a^A - i\eta_A(\partial_\sigma x^+ - i\theta^A\gamma^+\partial_\sigma\theta^A)\tilde{\theta}_a^A$$
$$\qquad + \eta_A\big[2(\gamma^i\partial_\sigma\theta^A)_a\tilde{\theta}^A\gamma^i\theta^A + (\gamma^i\theta^A)_a\partial_\sigma(\tilde{\theta}^A\gamma^i\theta^A)\big]\,,$$
$$p_{\dot{a}}^A \equiv k_{\dot{a}}^A + i\eta_A(\gamma^m\theta^A)_{\dot{a}}\big[-2i\tilde{\theta}^A\gamma_m\partial_\sigma\theta^A + i\tilde{\theta}^A\gamma_m\partial_\sigma\tilde{\theta}^A - i\partial_\sigma(\tilde{\theta}^A\gamma_m\theta^A)\big]$$
$$\qquad - i\eta_A(\gamma^i\tilde{\theta}^A)_{\dot{a}}\big[\partial_\sigma x^i - 3i\theta^A\gamma^i\partial_\sigma\theta^A + 2i\theta^A\gamma^i\partial_\sigma\tilde{\theta}^A + i\partial_\sigma(\tilde{\theta}^{A'}\gamma^i\theta^{A'})\big]$$

In terms of these “free” fields $(x^m, p^m, \theta^\alpha, p_\alpha, S_a)$
(= the canonical basis for the supersymplectic structure on the constraint surface)
the constraints simplify considerably and become very similar to those for the superparticle case:

$$D_a = d_a + i\sqrt{2\Pi^+}\, S_a \,,$$

$$D_{\dot{a}} = d_{\dot{a}} + i\sqrt{\frac{2}{\Pi^+}}\Pi^i(\gamma^i S)_{\dot{a}} + \frac{2}{\Pi^+}(\gamma^i S)_{\dot{a}}(S\gamma^i\partial_\sigma\theta) \,,$$

$$\mathcal{T} = \frac{1}{4}\frac{\Pi^m\Pi_m}{\Pi^+} \,.$$

where $\quad d_\alpha \equiv p_\alpha - i(\gamma^m\theta)_\alpha(p_m + \partial x_m) - (\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)$

□ Quantization:

Radial quantization is straightforward, except that we need to add a few quantum corrections due to multiple contractions and normal-ordering.

$$D_a = d_a + i\sqrt{2\pi^+}S_a \,,$$

$$D_{\dot{a}} = d_{\dot{a}} + i\sqrt{\frac{2}{\pi^+}}\pi^i(\gamma^i S)_{\dot{a}} - \frac{1}{\pi^+}(\gamma^i S)_{\dot{a}}(S\gamma^i\partial\theta)$$

$$+\frac{4\partial^2\theta_{\dot{a}}}{\pi^+} - \frac{2\partial\pi^+\partial\theta_{\dot{a}}}{(\pi^+)^2}$$

$$\mathcal{T} = \frac{1}{2}\frac{\pi^m\pi_m}{\pi^+} - \frac{1}{2\pi^+}S_c\partial S_c + i\sqrt{\frac{2}{\pi^+}}S_c\partial\theta_c + i\frac{\sqrt{2}}{(\pi^+)^{3/2}}\pi^i(S\gamma^i\partial\theta)$$

$$-\frac{1}{(\pi^+)^2}(S\gamma^i\partial\theta)^2 + \frac{4\partial^2\theta_{\dot{c}}\partial\theta_{\dot{c}}}{(\pi^+)^2} - \frac{1}{2}\frac{\partial^2\ln\pi^+}{\pi^+}$$

where $\pi^m \equiv i\partial x^m + \theta\gamma^m\partial\theta$

$$d_\alpha = p_\alpha + i\partial x^m(\gamma_m\theta)_\alpha + \frac{1}{2}(\gamma^m\theta)_\alpha(\theta\gamma_m\partial\theta)$$

This coincides with the system constructed by hand by Berkovits and Marchioro, hep-th/0412198 ! **We have derived it from the first principle.**

Under OPE, they close as

$$D_{\dot{a}}(z)D_{\dot{b}}(w) = \frac{-4\delta_{\dot{a}\dot{b}}\mathcal{T}(w)}{z-w}, \qquad \text{other OPE's} = \text{regular}$$

□ **BRST operator**:

$$\hat{Q} = \int \frac{dz}{2\pi i}\left(\tilde{\lambda}^\alpha D_\alpha + cT - (\tilde{\lambda}\gamma^+\tilde{\lambda})\frac{b}{\Pi^+}\right)$$

$\tilde{\lambda}^\alpha$ = unconstrained bosonic spinor ghosts

□ Derivation of Q for PS formalism:

Essentially the same as for the superparticle case (slightly more involved).

1. Remove b, c and $\mathcal{T}$: (omit $\int dz/2\pi i$)

$$e^{X}\hat{Q}e^{-X} = \not{\delta_b} + \tilde{Q}$$

$$\delta_b = (2/\Pi^+)\tilde{\lambda}_{\dot{a}}\tilde{\lambda}_{\dot{a}}b\,, \quad \tilde{Q} = \tilde{\lambda}_a D_a + \lambda_{\dot{a}} D_{\dot{a}}\,, \qquad \lambda_{\dot{a}}\lambda_{\dot{a}} = 0$$

$$X = b_B c\,, \qquad b_B = -\frac{\Pi^+}{4}(l_{\dot{a}} D_{\dot{a}})\,, \quad \tilde{\lambda}_{\dot{a}} l_{\dot{a}} = 1\,, \quad l_{\dot{a}} l_{\dot{a}} = 0$$

2. Make further similarity transformations

$$e^{Z}e^{Y}\hat{Q}e^{-Y}e^{-Z} = Q + \not{\delta}$$

$$Y = -\frac{1}{2}S^1_a S^2_a \ln \pi^+\,, \qquad Z = i\frac{d_a S^2_a}{\sqrt{2}} + \frac{4(\partial\theta_{\dot{a}}\lambda_{\dot{a}})(\partial\theta_{\dot{b}} r_{\dot{b}})}{\pi^+}$$

$$\delta = \sqrt{2}\, i\lambda^2_a S^1_a$$

$$\boxed{Q = \lambda^\alpha d_\alpha\,, \qquad \lambda\gamma^m\lambda = 0}$$

4 Challenge for the Supermembrane Case

It is extremely challanging and intersting to see if our idea can be applied to the supermembrane in 11D.

4.1 Constraint algebra at the classical level

Fundamental Action

$$S = \int d^3\xi \mathcal{L} \, , \qquad \mathcal{L} = \mathcal{L}_K + \mathcal{L}_{WZ} \quad (\xi^I = (t, \sigma_i) \, , \quad I = 0 \sim 2 \, , \; i = 1, 2)$$

$$\mathcal{L}_K = -\frac{1}{2}\sqrt{-g}\,(g^{IJ}\Pi_I^M \Pi_J^N - 1) \, , \qquad (M = 0 \sim 10)$$

$$\mathcal{L}_{WZ} = -\frac{1}{2}\epsilon^{IJK} W_{IMN} \left(\Pi_J^M \Pi_K^N + \Pi_J^M W_K^N + \frac{1}{3} W_I^M W_J^N \right) \sim \mathcal{O}(\Theta^6)$$

where

$$\Pi_I^M \equiv \partial_I y^M - W_I^M \, , \quad W_I^M \equiv i\bar{\Theta}\Gamma^M \partial_I \Theta \, , \quad W_I^{MN} \equiv i\bar{\Theta}\Gamma^{MN}\partial_I\Theta$$

$$\boxed{\Theta \equiv \tilde{\theta} - \theta \, , \qquad y^M \equiv x^M - i\bar{\theta}\Gamma^M\tilde{\theta}}$$

First class algebra of the constraints in semi-LC gauge $\Gamma^+\tilde{\theta} = 0$:

$$\{D_{\dot\alpha}(\sigma), D_{\dot\beta}(\sigma')\}_D = (\mathcal{T}\delta_{\dot\alpha\dot\beta} + \mathcal{T}_m\gamma^m_{\dot\alpha\dot\beta})\delta(\sigma - \sigma')\,, \qquad \text{all the rest } = 0$$

where $\mathcal{T}$ and $\mathcal{T}_{m=1\sim 9}$ are linear in the bosonic reparametrization constraints

$$T = \mathcal{K}^M\mathcal{K}_M + \det\,(\Pi_i^M\Pi_{jM}) = 0\,, \qquad T_i = \mathcal{K}_M\Pi_i^M = 0$$

where $\quad \mathcal{K}_M \equiv k_M - \epsilon^{ij}W_{iMN}\left(\Pi_j^N + \frac{1}{2}W_j^N\right)$

- $(\mathcal{T}, \mathcal{T}_m)$ define the same constraint surface as (T, T_i), but the system is **first-order reducible**: $\ ^\exists Z^m_{\bar p}\mathcal{T}_m = 0$, $(\bar p = 1 \sim 7)$
  $\Rightarrow$ One needs care in constructing the BRST operator in such a case.
- Unfortunately it is **hard to construct the "free field basis"**.

5 Summary and Future Problems

Summary:

♦ **Basic idea**

8 second class
8 κ-sym
8 semi-LC conditions

$x^m, (\theta^\alpha, p_\alpha)$

$\downarrow$

$(\tilde{\theta}^\alpha, \tilde{p}_\alpha) \longrightarrow S_a$

32 $\quad$ 8

$(\tilde{\lambda}^\alpha, \tilde{\omega}_\alpha) = (\lambda^\alpha, \omega_\alpha) \oplus$ 5 bosonic pairs

16 pairs $\quad$ 11 pairs

$\Updownarrow \qquad (b, c)$

$\lambda\gamma^m\lambda = 0$

form quartet and decouple

♦ κ **sym.** $\sim \sqrt{}$ **of** T **sym.** $\xrightarrow{\text{fermionic local sym}}$ Q_{BRST} **sym.**

∴ κ invariance is more generic

and explains

– why T need not appear explicitly in PS formalism

– why background field eq. is obtained either from κ invariance or from conformal invariance.

♦ The basic idea seems to work also for the supermembrane.

Quantization requires more work.

More conditions on λ_A than just $\lambda C \Gamma^M \lambda = 0$.

Future Problems:

- Path-integral derivation of the covariant rules, including the multiloop measure.
- Understand the origin and the structure of the new non-minimal "topological" PS formalism (Berkovits, hep-th/0509120; Berkovits and Nekrasov, hep-th/0609012) .
- Further analysis of supermembrane. Dimensional reduction to string case ?
- Extract physics: Application to curved backgrounds, in particular $\boldsymbol{AdS_5 \times S^5}$.
  - Spectrum: superparticle, superstring.
  - Integrability in $\boldsymbol{AdS_5 \times S^5}$ background.

Wish to report on further progress in the near future