Large gauge hierarchy in gauge-Higgs unification

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contents

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I. Introduction

Theories with extra dimensions are interesting alternatives to standard 4 dim. extensions of the standard model.

An interesting scenario is the gauge-Higgs unification, [Fairlie, Manton '79]

$$A_{\hat{\mu}} \Longrightarrow \begin{cases} A_\mu \rightarrow 4\text{D EW bosons} + \text{KK modes}, \\ A_y \rightarrow 4\text{D scalars (the Higgs)} + \text{KK modes} \end{cases}$$

In 5 dim. models, the electroweak symmetry breaking occurs radiatively and is equivalent to the Wilson line phase symmetry breaking (Hosotani mechanism)

$\star$ Well-motivated for gauge hierarchy problem

(Finite Higgs mass; stable against UV effects) [C.S,Lim,$et.al.$,'99]

$\star$ Experimental tests by LHC (ILC)

(indirect evidence of extra dimensions, KK modes)

II. Gauge hierarchy in gauge-Higgs unification

Basics

The Higgs field Φ in the standard model as parts of $A_y^{(0)}$

the simplest example, $\boxed{G = SU(3) \text{ gauge theory on } M^4 \times S^1/Z_2}$

$$
\begin{aligned}
S^1 \ ; \qquad\qquad A_{\hat\mu}(x, y + 2\pi R) &= U A_{\hat\mu}(x,y) U^\dagger, \\
z_i = 0, \pi R \ \ ; \quad \begin{pmatrix} A_\mu \\ A_y \end{pmatrix}(x, z_i - y) &= P_i \begin{pmatrix} A_\mu \\ -A_y \end{pmatrix}(x, z_i + y) P_i^\dagger
\end{aligned}
$$

$$\Big(U, P_i \in G,\ z_0 = 0,\ z_1 = \pi R,\ UU^\dagger = 1,\ P_i P_i^\dagger = 1,\ \ P_{0,1}^2 = 1,\ P_{0,1}^\dagger = P_{0,1}\Big)$$

Consistency; $U = P_1 P_0$ (and $U P_i U = P_i \quad (i = 0, 1)$)

Orbifolding boundary conditions at $y = 0, \pi R$ select zero modes for $A_\mu^{(0)},\ A_y^{(0)}$

Choose $P_{0,1} = \mathrm{e}^{\pi i \sqrt{3}\lambda_8} =$ diag$(-1,-1,1)$, $(\pm,\pm) \equiv$ "parity" under $P_{0,1}$ of $A^a_{\hat{\mu}}$

$$A_\mu = \left(\begin{array}{cc|c} (+,+) & (+,+) & (-,-) \\ (+,+) & (+,+) & (-,-) \\ \hline (-,-) & (-,-) & (+,+) \end{array}\right), [\lambda^{a=1,2,3,8},\ P_{0,1}] = 0,$$

$$A_y = \left(\begin{array}{cc|c} (-,-) & (-,-) & (+,+) \\ (-,-) & (-,-) & (+,+) \\ \hline (+,+) & (+,+) & (-,-) \end{array}\right), \{\lambda^{b=4,5,6,7},\ P_{0,1}\} = 0.$$

$A_\mu^{(0)} \cdots SU(3) \to SU(2) \times U(1)$ by the orbifolding, P_0 and P_1

$A_y^{(0)} \cdots$ an $SU(2)$ doublet $\Rightarrow$ the Higgs doublet Φ

$$\Phi \equiv \sqrt{2\pi R}\ \frac{1}{\sqrt{2}}\begin{pmatrix} A_y^4 - iA_y^5 \\ A_y^6 - iA_y^7 \end{pmatrix} \Longrightarrow \langle\Phi\rangle = \sqrt{2\pi R}\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \langle A_y^6\rangle \end{pmatrix}, \text{ where } \langle A_y^6\rangle = \frac{a}{g_5 R}$$

$$\boxed{SU(3) \overset{\text{orbifolding}}{\Longrightarrow} SU(2)\times U(1) \overset{\langle\Phi\rangle}{\Longrightarrow}\ ?}$$

$$W = \mathcal{P}\text{exp}\left(ig_4 \oint_{S^1} dy \langle A_y^{(0)} \rangle \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi a) & i\sin(\pi a) \\ 0 & i\sin(\pi a) & \cos(\pi a) \end{pmatrix}$$

It follows that

$$SU(2) \times U(1) \rightarrow \begin{cases} SU(2) \times U(1) & \text{for} \quad a = 1, \\ U(1) \times U(1)' & \text{for} \quad a = 0, \\ U(1)_{em} & \text{for} \quad \text{otherwise.} \end{cases}$$

$\Phi \iff$ Wilson line phase (nonlocal quantity)

$\Longrightarrow$ a curious feature specific to the gauge-Higgs unification

$\star$ the calculability of $V_{eff}(a)$ ($\Longrightarrow$ the finite Higgs mass)

Never suffered from ultraviolet effects (the gauge invariance is crucial)

The scales, M_W and $M_c = (2\pi R)^{-1} \equiv L^{-1}$ in the gauge-Higgs unification

After $\langle \Phi \rangle$,

$$M_W = \frac{a_0}{2R} \quad \Longrightarrow \quad M_W = \frac{\pi a_0}{2\pi R} = M_c \ \times \ (\pi a_0) \qquad \therefore \quad \frac{M_W}{M_c} = \pi a_0$$

It has been known that $a_0 \simeq O(10^{-2})$ (matter content) $\Longrightarrow M_c \sim$ a few TeV

We would like to discuss the possibility that

$$\frac{M_W}{M_c} \ll 1 \Longrightarrow a_0 \ll 1$$

From an unique scale $M_c = (2\pi R)^{-1} \sim M_{Planck}, M_{GUT}$,

we have $M_W \ll M_c$ through the VEV $a_0 \ll 1 \Longrightarrow$ large gauge hierarchy

The effective potential (in one-loop approx.) $V_{eff}(a) \equiv \frac{3}{4\pi^2 L^5}\bar{V}_{eff}(a)$ goes like , where

$$
\begin{aligned}
\bar{V}_{eff}(a) &\overset{a\ll 1}{\simeq} -\frac{\zeta(3)}{2}\, C^{(2)}\, (\pi a)^2 + \frac{-C^{(3)}}{24}(\pi a)^4 \ln(\pi a) + \left[\frac{25}{288}C^{(3)} + C^{(4)}\frac{\ln 2}{24}\right](\pi a)^4 \\
C^{(2)} &= 24N_{adj}^{(+)} + 4N_{fd}^{(+)} + \frac{9d}{2}N_{adj}^{(-)s} + \frac{3}{2}N_{fd}^{(-)s} \\
&\quad - \left(18 + 6dN_{adj}^{(+)s} + 2N_{fd}^{(+)s} + 18N_{adj}^{(-)} + 3N_{fd}^{(-)}\right) \\
C^{(3)} &= 72N_{adj}^{(+)} + 4N_{fd}^{(+)} - \left(54 + 18dN_{adj}^{(+)s} + 2N_{fd}^{(+)s}\right) \\
C^{(4)} &= 48 + 16dN_{adj}^{(+)s} + 18dN_{adj}^{(-)s} + 2N_{fd}^{(-)s} - \left(64N_{adj}^{(+)} + 4N_{fd}^{(-)} + 72N_{adj}^{(-)}\right)
\end{aligned}
$$

$((+) \cdots$ periodic B.C., $(-) \cdots$antiperiodic B.C.)

⋆ Each coefficient is given by the integral values (not scale-dependent parameters)

⋆ $C^{(2)} = 0$ is not the fine tuning, but the choice of the flavor number.

Model I. Let us require that $C^{(2)} = 0$. The potential becomes

$$\bar{V}_{eff}(a) \Rightarrow \frac{-C^{(3)}}{24}(\pi a)^4 \ \mathsf{ln}(\pi a) + \left(\frac{25}{288}C^{(3)} + C^{(4)}\frac{\mathsf{ln2}}{24}\right)(\pi a)^4$$

(Coleman-Weinberg potential)

$$\pi a_0 \quad \simeq \quad \frac{M_W}{M_c} = \exp\left(-\frac{C^{(4)}}{\left|C^{(3)}\right|}\ln 2 + \frac{11}{6}\right) \quad \left(\equiv \frac{10^2\ \mathsf{GeV}}{10^p\ \mathsf{GeV}}\right)$$

$$\Longrightarrow \quad M_W = M_c \exp\left(-\left(\frac{C^{(4)}}{\left|C^{(3)}\right|}\right)\ln 2 + \frac{11}{6}\right)$$

For example,

	$p = 12$	$p = 16$	$p = 19$
$\frac{C^{(4)}}{\left\|C^{(3)}\right\|}$	35.86	49.15	59.12

$\star$ For $M_W \ll M_c$, the flavor set satisfying $C^{(2)} = 0$ and the large ratio $\frac{C^{(4)}}{|C^{(3)}|}$

(e.g.) $(k, m) = (1, 0)$ case, $\left(C^{(3)} = -6k, \quad 4N_{adj}^{(+)} - 3 - dN_{adj}^{(+)s} = -m \right)$

$$
\begin{aligned}
(N_{adj}^{(+)}, N_{adj}^{(+)s}) &= (1,1), (2,5), \cdots, \\
(N_{fd}^{(+)}, N_{fd}^{(+)s}) &= (0,3), (1,5), \cdots, \\
p = 19 \cdots (N_{adj}^{(-)}, dN_{adj}^{(-)s}) &= (0,29), (1,33), \cdots, \\
(N_{fd}^{(-)}, N_{fd}^{(-)s}) &= (42,1), (43,3), \cdots, \\
p = 16 \cdots (N_{adj}^{(-)}, dN_{adj}^{(-)s}) &= (0,24), (1,28), \cdots, \\
(N_{fd}^{(-)}, N_{fd}^{(-)s}) &= (34,0), (35,2), \cdots, \\
p = 12 \cdots (N_{adj}^{(-)}, dN_{adj}^{(-)s}) &= (0,17), (1,21), \cdots, \\
(N_{fd}^{(-)}, N_{fd}^{(-)s}) &= (24,0), (25,2), \cdots.
\end{aligned}
$$

$\star$ Many flavors are necessary to have the large gauge hierarchy

Problem for perturbative expansion, $\frac{g_4^2}{4\pi^2}\ N_{flavor}\ \ll\ 1$

$\star$ $C^{(2)} = 0$ guaranteed even against higher order loops ?

no two-loop finite corrections in 5 dim. QED on $M^4 \times S^1$ [Maru and Yamashita, '06]

- The Higgs mass in the model I,

$$m_H^2 = (g_5 R)^2 \left. \frac{\partial^2 V_{eff}}{\partial a^2} \right|_{a_0} = \frac{g_4^2}{16\pi^2}\ M_W^2 \left(\frac{1}{2} \left| C^{(3)} \right| \right) < M_W^2 \qquad \left(C^{(3)} = -6k \right)$$

The larger $\left|C^{(3)}\right|$ gives the larger Higgs mass, but it yields the smaller values for $C^{(4)}/\left|C^{(3)}\right|$, so that the large gauge hierarchy is difficult to be realized.

The heavy Higgs mass and the large gauge hierarchy are not compatible in the model I.

Model II. Add massive bulk fermions (bare mass term)

$$\text{mass term;} \quad \frac{-\zeta(3)}{2} C^{(2)} (\pi a)^2 \Longrightarrow \frac{-1}{2} \left[\zeta(3) C^{(2)} + 8 N_{bulk} B^{(2)} \right] (\pi a)^2$$

where, $(z \equiv 2\pi R M = M/M_c)$,

$$B^{(2)} = \sum_{n=1}^{\infty} \frac{1}{n^3} \left(1 + nz + \frac{n^2 z^2}{3} \right) \mathrm{e}^{-nz}, \ \text{Boltzmann-like factor} \ \ (\leftrightarrow \text{finite T})$$

Assume again that $C^{(2)} = 0$, then, the VEV is given by

$$\pi a_0 \simeq \gamma B^{(2)} \quad (\gamma \sim O(1)) \quad \Longrightarrow \quad M_W = M_c \, \mathrm{e}^{-Y}$$

$$\text{where} \quad Y \simeq z = \begin{cases} 34.54 & \text{for} \quad M_c = 10^{17}, \\ 32.24 & \text{for} \quad M_c = 10^{16}, \\ 23.03 & \text{for} \quad M_c = 10^{12}. \end{cases}$$

The massless bulk matter is not essential for the large gauge hierarchy.

The Higgs mass in the model II,

$$m_H^2 = \frac{g_4^2}{16\pi^2} M_W^2 \left(-C^{(3)} \ln\, (\pi a_0) + \frac{4}{3} C^{(3)} + C^{(4)} \ln\, 2 \right), \quad \left(\ln(\pi a_0) = -Y \right)$$

$$\begin{aligned} (N_{adj}^{(-)}, dN_{adj}^{(-)s}) &= (1,4),(2,8),\cdots, \\ (N_{fd}^{(-)}, N_{fd}^{(-)s}) &= (4,0),(5,2),\cdots, \\ (N_{fd}^{(+)}, N_{fd}^{(+)s}) &= (2,1),(3,3),\cdots, \\ (N_{adj}^{(+)}, dN_{adj}^{(+)s}) &= (1,0),(2,4),\cdots. \quad \text{satisfying} \;\; C^{(2)} = 0 \end{aligned}$$

The Higgs mass
$(g_4^2 \simeq 0.42)$

$$m_H \simeq \begin{cases} 119.9 \quad \text{(GeV)} \quad \cdots M_c = 10^{17}, \\ 115.9 \quad \text{(GeV)} \quad \cdots M_c = 10^{16}, \\ 98.2 \quad \text{(GeV)} \quad \cdots M_c = 10^{12} \end{cases}$$

The larger gauge hierarchy (larger values of $Y \leftrightarrow$ small R) gives the heavier Higgs mass.

III. Summary

⋆ Large gauge hierarchy in the gauge-Higgs unification

| Model I | the massless bulk matter, vanisihng mass term, $C^{(2)} = 0$,

The large gauge hierarchy is not compatible with the consistent Higgs mass

The too light Higgs mass, $m_H < M_W$

| Model II | The massive bulk fermions and the massless bulk matter, $C^{(2)} = 0$

Possible to obtain the large gauge hierarchy and the heavy Higgs mass

⋆ $C^{(2)} = 0$ is crucial. Is it stable against the higher loop corrections?

Finiteness of $V_{eff}(a)$ [Hosotani, '06]

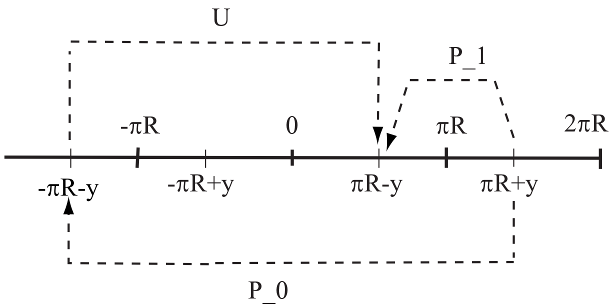
[e.g.] One-loop exact (without supersymmetry)

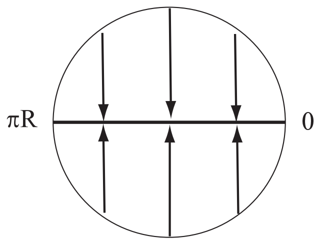
- Anomaly coefficient,
- Chern-Simons coupling ($\Leftarrow$ The invariance of the action under the large gauge transformation)

$\Longrightarrow$ The shift symmetry in the gauge-Higgs unification ?

[N.B.] An orbifold compactification, $M^4 \times S^1/Z_2$

Identifications $S^1 \cdots y \sim y + 2\pi R, \quad Z_2 \cdots y \sim -y$





$0 \leq y \leq \pi R$ with two fixed points, $y = 0, \pi R$

Specify boundary conditions of fields for the S^1 direction and at the fixed points;

$$\left(\mathcal{L}(y + 2\pi R) = \mathcal{L}(y), \quad \left\{ \begin{array}{l} \mathcal{L}(-y) = \mathcal{L}(y) \\ \mathcal{L}(\pi R - y) = \mathcal{L}(\pi R + y) \end{array} \right. \right)$$

Symmetry degrees of freedom $\Rightarrow$ twisted B.C.'s.

The kinetic term for Φ

$$\int_0^{2\pi R} dy \, -\mathrm{tr}\,(F_{\mu y}F^{\mu y}) \Rightarrow \left| \left(\partial_\mu - i g_4 A_\mu^{a(0)} \frac{\tau^a}{2} - i \frac{(\sqrt{3} g_4)}{2} A_\mu^{8(0)} \right) \Phi \right|^2 ; \;\; SU(2)\times U(1)$$

- $\mathbf{8} \in SU(3) = \mathbf{3}_0 \oplus \mathbf{2}_{1/2} \oplus \mathbf{2}^*_{-1/2} \oplus \mathbf{1}_0, \quad Q_Y \equiv \frac{1}{\sqrt{3}} \frac{\lambda^8}{2} = \mathrm{diag.}(\frac{1}{6}, \frac{1}{6}, -\frac{1}{3})$
- The Weinberg angle (problem), $g_Y = \sqrt{3} g_4$

$$\sin^2 \theta_w \equiv \frac{g_Y^2}{g_4^2 + g_Y^2} = \frac{3}{4} \neq 0.22 \quad \Rightarrow M_Z = 2 M_W \quad \text{model building}$$

¶ General discussions

Let us write

$$
\begin{aligned}
C^{(2)} &= 6\left(4N_{adj}^{(+)} - 3 - dN_{adj}^{(+)s}\right) + 2\left(2N_{fd}^{(+)} - N_{fd}^{(+)s}\right) \\
&\quad + \frac{9}{2}\left(dN_{adj}^{(-)s} - 4N_{adj}^{(-)}\right) + \frac{3}{2}\left(N_{fd}^{(-)s} - 2N_{fd}^{(-)}\right)
\end{aligned}
$$

For $C^{(2)} = 0, \quad (2N_{fd} - N_{fd}^{(+)s})$ must be an integral of 3.

Then, $C^{(3)} = 18\left(4N_{adj}^{(+)} - 3 - dN_{adj}^{(+)s}\right) + 2\left(2N_{fd}^{(+)} - N_{fd}^{(+)s}\right)$ is an integral multiple of 6, so that

$$
C^{(3)} = -6k \quad (k = \text{positive integer})
$$

We introduce an integer m by $4N_{adj}^{(+)} - 3 - dN_{adj}^{(+)s} \equiv -m$

$$
C^{(3)} = -6k \quad \Longrightarrow \quad 2N_{fd}^{(+)} - N_{fd}^{(+)s} = -3k + 9m
$$

The condition $C^{(2)} = 0$ yields

$$N_{fd}^{(-)s} - 2N_{fd}^{(-)} = -3\left(dN_{adj}^{(-)s} - 4N_{adj}^{(-)}\right) + 4k - 8m$$

The coefficient $C^{(4)}$ is written by

$$C^{(4)} = 12\left(dN_{adj}^{(-)s} - 4N_{adj}^{(-)}\right) + 8k$$

We finally obtain that

$$\frac{C^{(4)}}{\left|C^{(3)}\right|} = \frac{2}{k}\left(dN_{adj}^{(-)s} - 4N_{adj}^{(-)}\right) + \frac{4}{3}$$

Therefore, we obtain that

$$4N_{adj}^{(+)} - 3 - dN_{adj}^{(+)s} = 0 \quad \rightarrow \quad (N_{adj}^{(+)}, dN_{adj}^{(+)s}) = (1,1), (2,5), (3,9), \cdots,$$
$$2N_{fd}^{(+)} - N_{fd}^{(+)s} = -3 \quad \rightarrow \quad (N_{fd}^{(+)}, N_{fd}^{(+)s}) = (0,3), (1,5), (2,7), \cdots.$$

$(k, m, p) = (1, 0, 19)$,

$$dN_{adj}^{(-)s} - 4N_{adj}^{(-)} \simeq 29 \quad \rightarrow \quad (N_{adj}^{(-)}, dN_{adj}^{(-)s}) = (0,29), (1,33), (2,37), \cdots$$
$$N_{fd}^{(-)s} - 2N_{fd}^{(-)} = -83 \quad \rightarrow \quad (N_{fd}^{(-)}, N_{fd}^{(-)s}) = (42,1), (43,3), (44,5), \cdots.$$

$(k, m, p) = (1, 0, 16)$,

$$dN_{adj}^{(-)s} - 4N_{adj}^{(-)} \simeq 24 \quad \rightarrow \quad (N_{adj}^{(-)}, dN_{adj}^{(-)s}) = (0,24), (1,28), (2,32), \cdots$$
$$N_{fd}^{(-)s} - 2N_{fd}^{(-)} = -68 \quad \rightarrow \quad (N_{fd}^{(-)}, N_{fd}^{(-)s}) = (34,0), (35,2), (36,4), \cdots.$$