Factorization in $B \to K\pi\gamma$ and $B \to K\pi\ell\ell$

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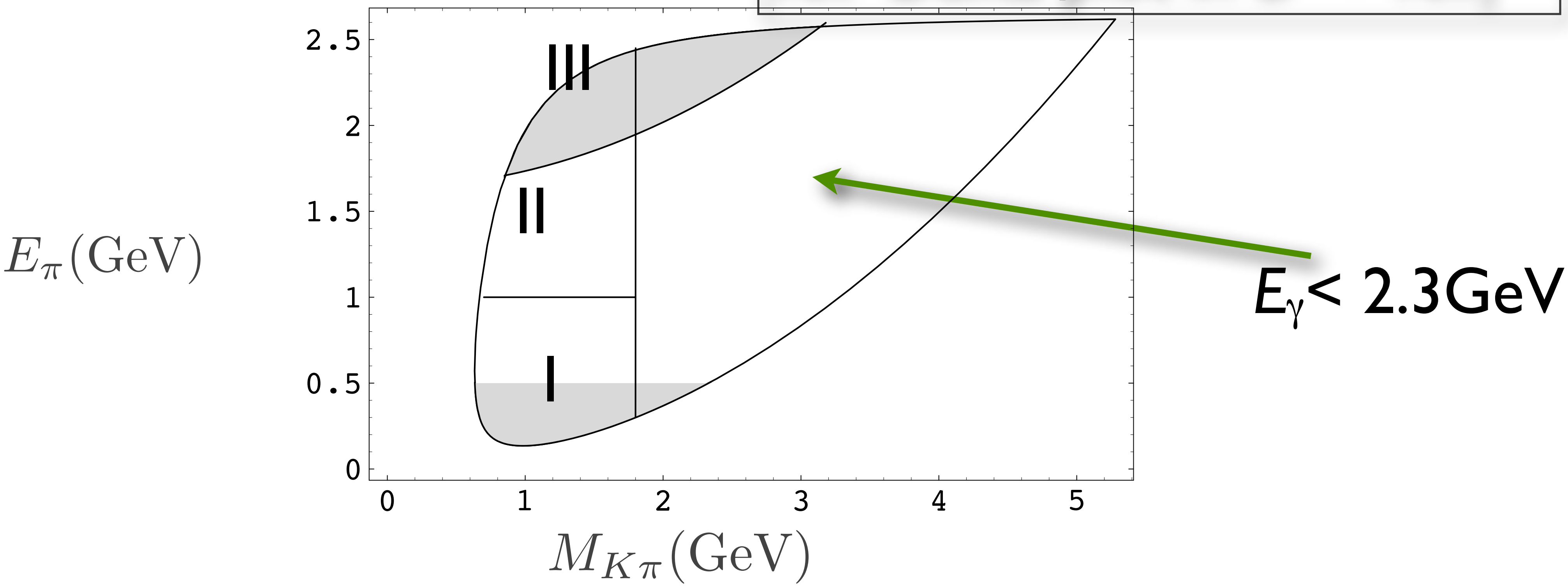
1. Phys.Lett.B615, 213 (2005)
2. Phys.Rev.D73, 094027 (2006)
3. Phys.Rev.D73, 014013 (2006)
4. Nucl. Phys. B755, 199 (2006)

Description of Problem

- Some interesting physics involve B decays into K*
- K* is seen through its decay to $K\pi$
- Normally (eg, for rates) the K* peak dominates $K\pi$ non-resonant
- What about cases where the interesting quantity involves a cancelation or interference (e.g., FBA or CPV)?
- "Background" subtraction by going to side bins is not an option

Factorization, where?

Kπ Dalitz plot in $B \to K\pi\gamma$

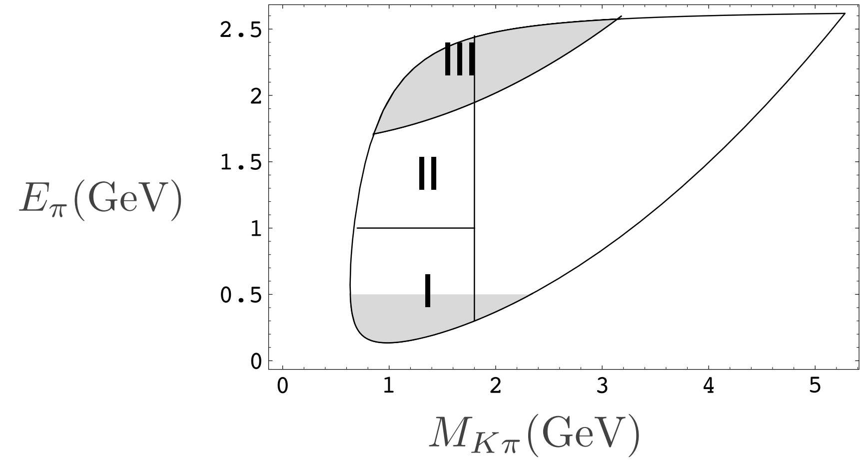


I Soft π, collinear K: $E_\pi \sim \Lambda$, $E_K \sim Q$

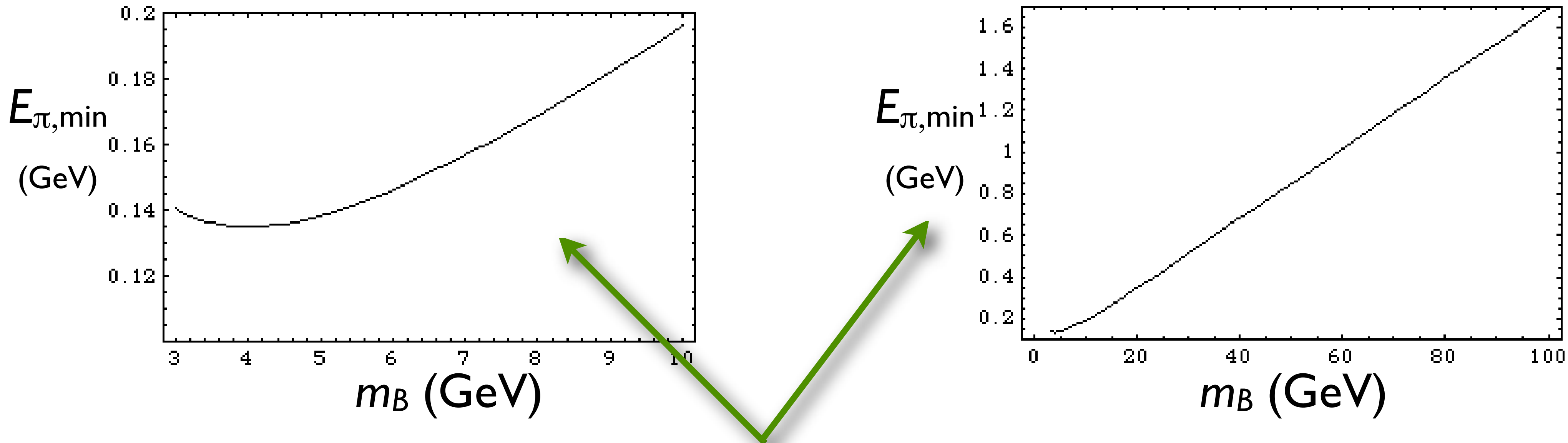
II Collinear K and : $E_\pi \sim Q$, $E_K \sim Q$

III Soft K, collinear π: $E_K \sim \Lambda$, $E_\pi \sim Q$

Shaded region in I: $E_\pi < 0.5$ GeV ($\Rightarrow E_K > 2.2$ GeV) $\Rightarrow$ ChPT OK(?)



- ChPT allows us to compute non-resonant in region I (new theory, see below)
- Do not know how to compute non-resonant elsewhere
- Problems:
    - Results depend on arbitrary cut-off (0.5GeV?)
    - $Q \sim m_b \neq \infty \Rightarrow$ regions not well separated
    - $m_b \neq \infty \Rightarrow K^* \to K\pi$ may produce soft π



Minimum energy from $B \to K^* \gamma \to K \pi \gamma$

- Cannot ignore K* in region I
- Model:
    - keep K* in regions I and II, ignore III (order $\lambda = \sqrt{\Lambda/Q}$)
    - for $K^* \to K \pi$ use Breit-Wigner (BW)
    - region I factorizable: include continuum from ChPT
    - region I non-factorizable: use K* BW
- Any better ideas??

Result 1: FBA zero

- Recall: $B \to K^* \ell\ell$ Forward-Backward Asymmetry zero at special q^2 (invariant mass of lepton pair)
  - sensitive to short distance "Wilson coefficients:"

$$q_0^2 = -2 m_B m_b \frac{\mathrm{Re}(C_7^{\mathrm{eff}}(q_0^2))}{\mathrm{Re}\ (C_9^{\mathrm{eff}}(q_0^2))}(1+\tau(\alpha_s)) + \delta_{\mathrm{fact}}$$

- For $B \to K\pi\ell\ell$ there is a zero in FBA too, but now location depends on invariant $K\pi$ mass,

$$q_0^2 = q_0^2(m_{K\pi})$$

- Slope of this function is sensitive to same short distance Wilson coefficients ⇒ fit simultaneously (better constrained experimentally?)

As we'll see, zero in FBA is from condition

γ* momentum

$$C_9 + 2m_b \frac{n \cdot q}{q^2} C_7 - a(M_{K\pi}) = 0$$

Wilson coefficients

light-cone direction of K

correction from factorizable SCET

(easy-to-include radiative corrections have been omitted here, but included in full calculation to NNLO)

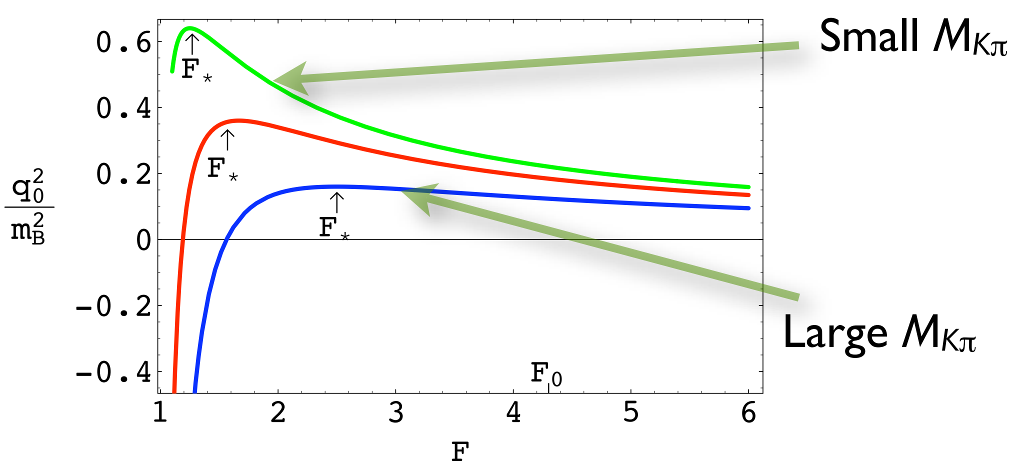
Let F be defined by

$$\frac{F}{m_B} = -\frac{1}{2m_b C_7}(C_9 - a(M_{K\pi})) = \frac{n \cdot q}{q^2}$$

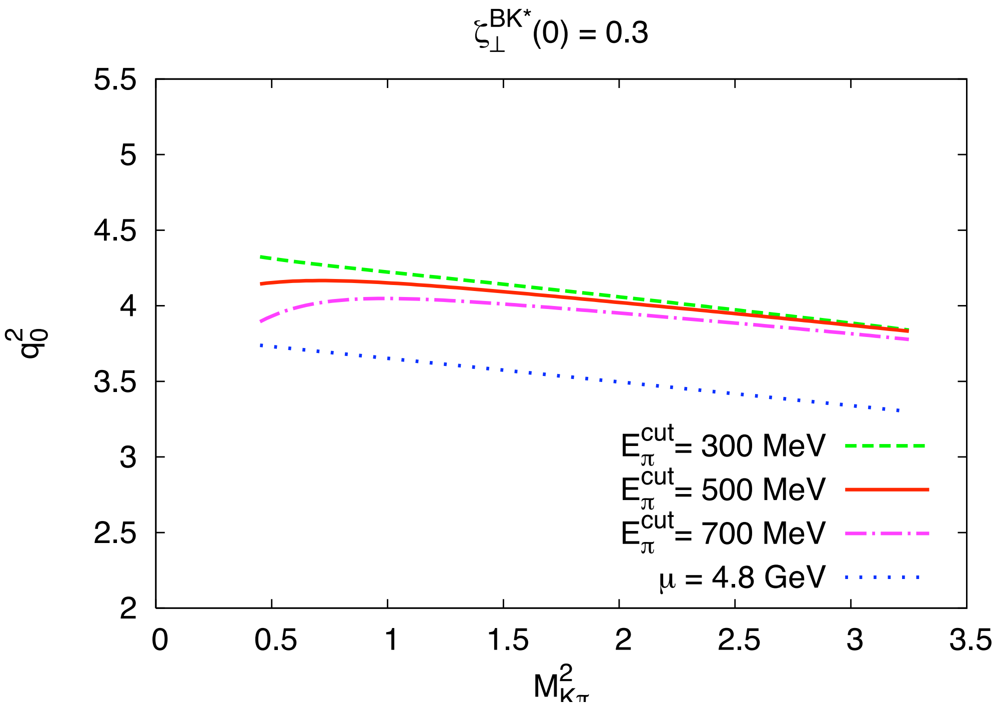
Then

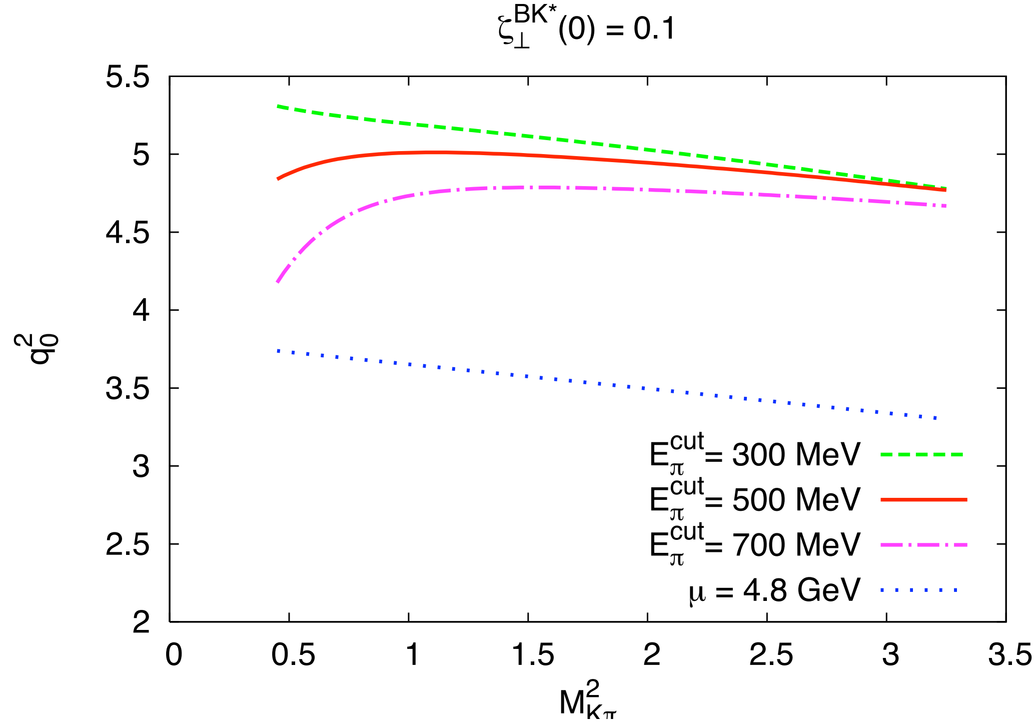
$$q_0^2(M_{K\pi}) = \frac{m_B^2}{F} - \frac{M_{K\pi}^2}{F-1} \qquad F \approx 5$$

$$\frac{dq_0^2}{dM_{K\pi}} = -\frac{1}{F-1}, \qquad F \approx \text{const}$$



More generally (see figure): if *F* increases with $M_{K\pi}$ then q_0 decreases with (increasing) $M_{K\pi}$





- These show FBA-zero as function of Kπ mass
- $\zeta_\perp^{BK^*}(0)$ normalizes leading order (right figure shows extreme case, large corrections or "hard" form factor)
- E^{cut} shows model dependence from range of validity of chiral perturbation theory
- Dotted blue: ignore factorizable corrections: still parallel!

Result 2: CPV in $B \to K\pi\gamma$

- Recall *S* defined for each γ polariztion (*i* = L,R):

$$\frac{d^2\Gamma(B^0(t) \to K_S \pi^0 \gamma_i)}{dE_\pi dM^2_{K\pi}} \propto e^{-\bar{\Gamma}t} \{1 + C_i \cos\Delta mt - S_i \sin\Delta mt\}$$

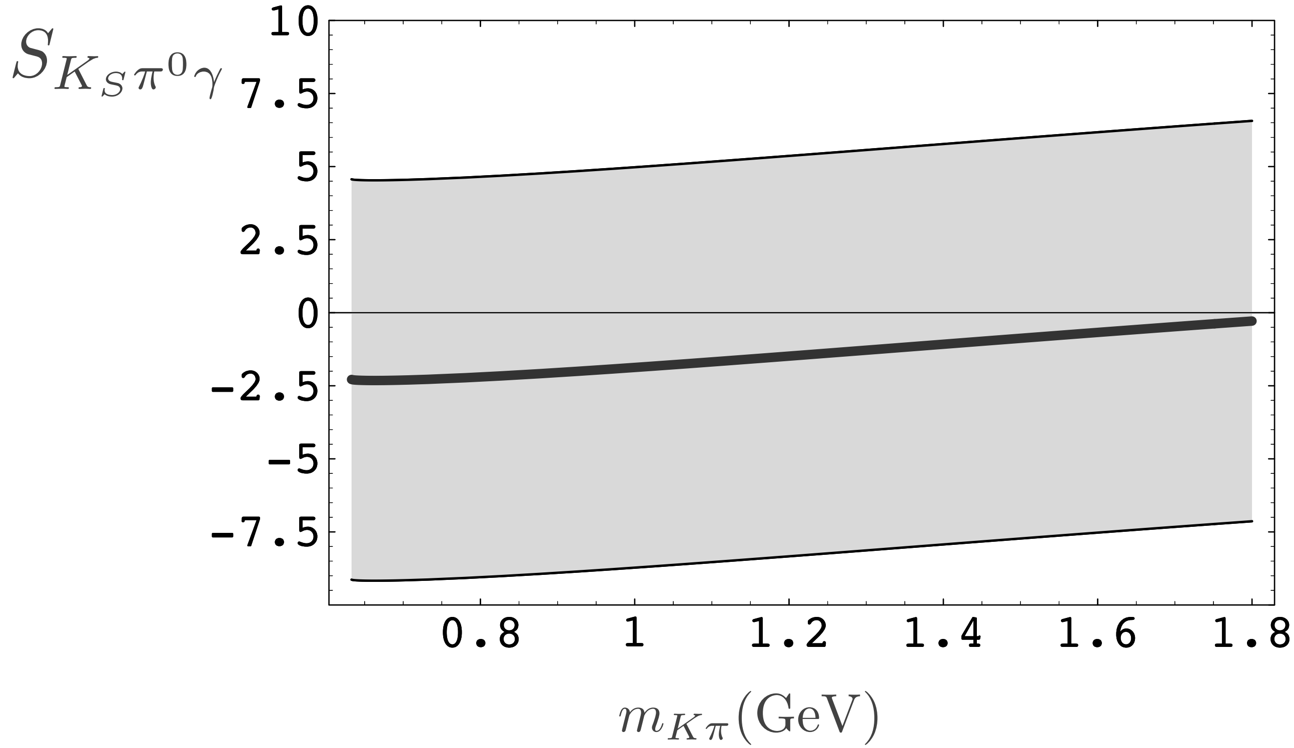
- Need interference between L and R handed photons (none for decay into K*)
Lowest order in $1/m$ ⇒ only L photons ⇒ *S*=0

- Ligeti et al (PRD 71, 011504(R) (2005)),

$$S_{K_S\pi^0\gamma} = -2\sin 2\beta \Big\{ \frac{m_s}{m_b} + h_s \cos\phi_s \Big\}$$

$h_s \sim \frac{1}{3}\frac{C_9}{C_7}\frac{\Lambda}{m_b} \sim 0.05 - 0.10$ from charm loop & power counting

Including also (small) non-resonant contribution gives some Kπ mass dependence



Solid line is $h_s = 0$

Range is $|h_s \cos(\phi_s)| < 0.05$

Plan of (rest of) Talk

- Fast review of $\mathrm{SCET_{II}}$ results/factorization ($B \to K_n X_S \gamma^{(*)}$)
- Chiral Perturbation Theory for Soft amplitudes in SCET
- Applications
  - Generic symmetry relations, and their violation
  - ITP* Zeroes in FBA
  - Will not go back to CPV in $B \to K\pi\gamma$

*if time permits

SCET - factorization and χPT (or ChPT)

Match arbitrary operator O (in H_{eff}) into SCET_{II}

$$O \to T \otimes O_S \otimes O_C + O_{\rm nf} + \cdots$$

Below: Establish Soft Pion Theorem for $B \to n\pi$ in matrix element of soft operator O_S and relations for matrix elements of $O_{\rm nf}$

Aha! Large mass in SCET

- In SCET_{II} we usually say the hadronic mass has to be small
- The reason is to make hadrons collinear
- But we can have $M_X \sim \sqrt{\Lambda Q}$

  Collinear meson with energy *E* ~ *Q* and mass ~Λ

  plus soft meson with energy *E* ~Λ and mass ~Λ

  This is "Region I" in Dalitz plot

In SCET$_\mathrm{I}$ for $b \to u l \nu$ (sim. for $b \to s\gamma$, etc), the effective current is

$$J_\mu^{\rm eff} = c_1(\omega)\,\bar{q}_{n,\omega}\gamma_\mu^\perp P_L\, b_v + \left[c_2(\omega)v_\mu + c_3(\omega)n_\mu\right]\bar{q}_{n,\omega}P_R\, b_v \qquad O(\lambda^0)$$

$$+b_{1L}(\omega_i)\,J_\mu^{(1L)}(\omega_i) + b_{1R}(\omega_i)\,J_\mu^{(1R)}(\omega_i)$$
$$+\left[b_{1v}(\omega_i)v_\mu + b_{1n}(\omega_i)n_\mu\right]\,J^{(10)}(\omega_i) \qquad O(\lambda)$$

The O(λ) operators are

$$J_\mu^{(1L,1R)}(\omega_1,\omega_2) = \bar{q}_{n,\omega_1}\,\Gamma_{\mu\alpha}^{(1L,1R)}\left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig\mathcal{B}_{\perp n}^{\alpha}\right]_{\omega_2} b_v\,,$$
$$J^{(10)}(\omega_1,\omega_2) = \bar{q}_{n,\omega_1}\left[\frac{1}{\bar{n}\cdot\mathcal{P}} ig\not{\mathcal{B}}_n^{\perp}\right]_{\omega_2} P_L\, b_v\,,$$

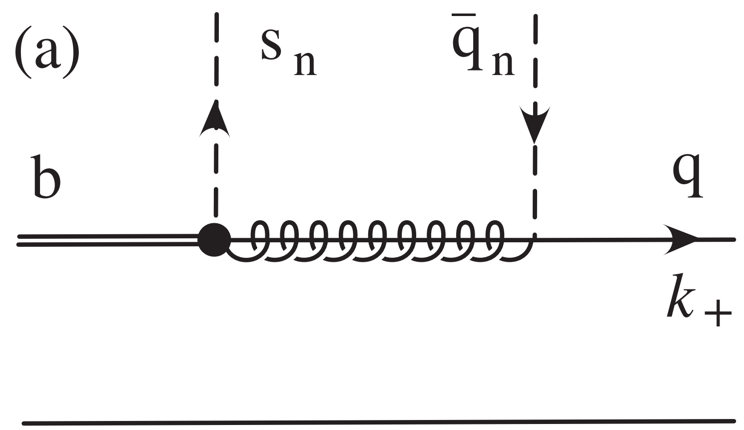
which will give (leading order) factorizable operators in SCET$_\mathrm{II}$

SCET$_{\text{II}}$ Factorizable Ops

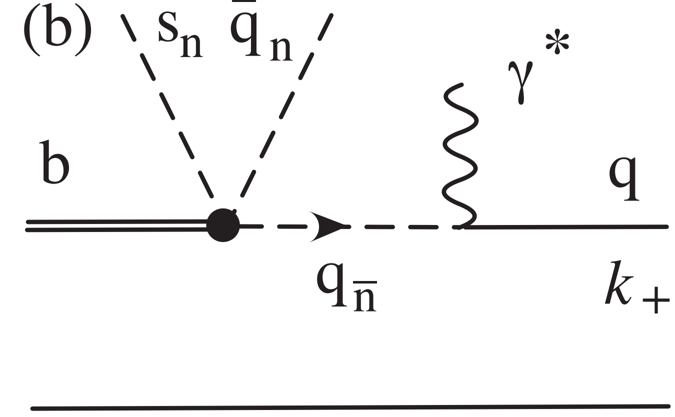
$$J_\mu^{\text{fact}} = -\frac{1}{2\omega}\int dx dz dk_+ b_{1L}(x,z) J_\perp(x,z,k_+) \big((\bar{q}Y)_{k_+} \not{n}\gamma_\mu^\perp \gamma_\perp^\lambda P_R (Y^\dagger b_v)\big)\big(\bar{q}_{n,\omega_1}\frac{\bar{\not{n}}}{2}\gamma_\perp^\lambda q_{n,\omega_2}\big)$$
$$-\frac{1}{2\omega}\int dx dz dk_+ b_{1R}(x,z) J_\parallel(x,z,k_+) \big((\bar{q}Y)_{k_+} \not{n}\gamma_\mu^\perp P_R (Y^\dagger b_v)\big)\big(\bar{q}_{n,\omega_1}\frac{\bar{\not{n}}}{2} P_L q_{n,\omega_2}\big)$$
$$-\frac{1}{\omega}\int dx dz dk_+ [b_{1v}(x,z) v_\mu + b_{1n}(x,z) n_\mu] J_\parallel(x,z,k_+) \big((\bar{q}Y)_{k_+} \not{n} P_L (Y^\dagger b_v)\big)\big(\bar{s}_{n,\omega_1}\frac{\bar{\not{n}}}{2} P_L q_{n,\omega_2}\big)$$

Focus on these: soft operators

They can mediate
B → nπ (if π's are soft)



There are also "spectator" contributions



$$J^{\mu}_{\rm sp} = \int_0^1 {\rm d}z b_{\rm sp}(z) \int dk_- J_{\rm sp}(k_- - \frac{q^2}{n\cdot q})(\bar{q}_{k_-}\gamma^\mu \bar{\not{n}}\not{n} P_L b_v)(\bar{s}_{n,z\omega}\frac{\bar{\not{n}}}{2} P_L q_{n,-\bar{z}\omega})$$

These are important corrections (for FBA-zero),
but are trivially computed:
expanding $J_{\rm sp}$ in $n.q/q^2$ then integral over k_- makes local the soft matrix element (PRL84:4545,2000)

(So ignore for now)

Light-Cone Wave Functions

Recall, for *B* to vacuum matrix element:

$$\int \frac{dz_-}{4\pi} e^{-\frac{i}{2}k_+ z_-} \langle 0|\bar{q}_i(z_-) Y_n(z_-,0) b_v^j(0)|\bar{B}(v)\rangle =$$

$$-\frac{i}{4} f_B \sqrt{m_B} \left\{ \frac{1+\not{v}}{2} [\not{\bar{n}} n\cdot v \phi_+(k_+) + \not{n} \bar{n}\cdot v \phi_-(k_+)]\gamma_5 \right\}_{ij}$$

So the (simple) questions are:

1. How is this generalized for *B* to several soft pions?
2. What new non-perturbative functions are introduced?

Answers:

Easy

None

To my knowledge this is the only known example of a computable GPD! (but my knowledge is very limited)

HMχPT: Lightning Review

M.B. Wise Phys. Rev. D45 (1992) 2188
G. Burdman and J. Donoghue Phys. Lett. B280 (1992) 287
T.-M. Yan, et al, Phys. Rev. D46 (1992) 1148

Effective theory incorporating chiral and HQ symmetries.
Use meson fields:

$$H_a^{(Q)} = \frac{1+\not{v}}{2}\left[P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right] \qquad (P_1^{(b)}, P_2^{(b)}, P_3^{(b)}) = (B^-, \bar{B}^0, \bar{B}_s),$$

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} \qquad \xi = e^{iM/f}, \quad \Sigma = \xi^2$$

Under chiral SU(3)$_L$×SU(3)$_R$

$$H_a^{(Q)} \to H_b^{(Q)}\ U_{ba}^\dagger \qquad \Sigma \to L\Sigma R^\dagger\,, \qquad \xi \to L\xi U^\dagger = U\xi R^\dagger$$

The effective Lagrangian is well known, constrained by symmetry:

$$\mathcal{L} = \frac{f^2}{8}\mathrm{Tr}\left(\partial^\mu\Sigma\partial_\mu\Sigma^\dagger\right) + \lambda_0\mathrm{Tr}\left[m_q\Sigma + m_q\Sigma^\dagger\right] - i\mathrm{Tr}[\bar{H}^{(Q)a}v_\mu\partial^\mu H_a^{(Q)}]$$
$$+\frac{i}{2}\mathrm{Tr}[\bar{H}^{(Q)a}H_b^{(Q)}]v^\mu\left[\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right]_{ba} + \frac{ig}{2}\mathrm{Tr}[\bar{H}^{(Q)a}H_b^{(Q)}\gamma_\nu\gamma_5]\left[\xi^\dagger\partial^\nu\xi - \xi\partial^\nu\xi^\dagger\right]_{ba} + \cdots$$

Symmetry also constrains representation of operators. For example, the left handed current

$$L_a^\nu = \bar{q}_a\gamma^\nu P_L Q$$

is an expansion in the HQχPT:

$$L_a^\nu = \frac{i\alpha}{2}\mathrm{Tr}[\gamma^\nu P_L H_b^{(Q)}\xi_{ba}^\dagger] + \ldots,$$

Symmetry does not give us α. Get it from B → vac: $\alpha = f_B\sqrt{m_B}$

Back to task: we want to represent

$$O^a_{L,R}(k_+) = \int \frac{dx_-}{4\pi} e^{-\frac{i}{2}k_+ x_-} \bar{q}^a(x_-) Y_n(x_-,0) P_{R,L} \Gamma b_v(0)$$

in HQχPT. These (L & R) transform as $(\overline{\mathbf{3}}_L, \mathbf{1}_R)$ and $(\mathbf{1}_L, \overline{\mathbf{3}}_R)$

so in analogy with example of current, we have (up to "...")

$$O^a_L(k_+) = \frac{i}{4}\mathrm{Tr}[\hat{\alpha}_L(k_+) P_R \Gamma H_b^{(Q)} \xi^\dagger_{ba}]$$

$$O^a_R(k_+) = \frac{i}{4}\mathrm{Tr}[\hat{\alpha}_R(k_+) P_L \Gamma H_b^{(Q)} \xi_{ba}]$$

Now for the tedious stuff: most general form: $\hat{\alpha}_{L,R}(k_+) = a_{1L,R} + a_{2L,R}\not{n} + a_{3L,R}\not{v} + \frac{1}{2}a_{4L,R}[\not{n},\not{v}]$

and use $H^{(Q)}\not{v} = -H^{(Q)}$ and take B → vac matrix element to fix remaining functions:

$$\hat{\alpha}_L(k_+) = \hat{\alpha}_R(k_+) = f_B\sqrt{m_B}[\not{\bar{n}}\phi_+(k_+) + \not{n}\phi_-(k_+)]$$

Subtleties with T-ordering ignored here, but see Nucl. Phys. B755, 199 (2006) for details

Have verified this in exact solution of ‘t Hooft model (large *N* QCD in $1{+}1$)

Nucl. Phys. B755, 199 (2006)

Incidentally, in that model

$$f(k_-) = \kappa[\psi(\bar{\Lambda} - k_-)]^2\theta(\bar{\Lambda} - k_-)$$

B-shape function = (l.c. wave function)^2

Applications

“Violations” to Symmetry Relations

- Non-factorizable operators give very specific relations between helicity amplitudes
- Some helicity amplitudes of non-factorizable operators vanish
- Corrections: from non-vanishing amplitudes from factorizable operators
- Some vanish, unless additional soft pion

So, ignore for now factorizable operators.
Then:

$$J_\mu^{\rm nf} = c_1^{(i)}(\omega)\,\bar q_{n,\omega}\gamma_\perp^\mu P_L\, b_v + [c_2^{(i)}(\omega)v^\mu + c_3^{(i)}(\omega)n^\mu]\;\bar q_{n,\omega}P_R\, b_v$$

$$(i = V, A)$$

Define $H_\lambda^{\rm nf}(M_n, X_S) = \langle M_n X_S|\epsilon_\lambda^{*\mu} J_\mu^{\rm nf}|B\rangle$

$$\varepsilon_\pm^\mu = \frac{1}{\sqrt{2}}(0,1,\mp i,0)\,,\varepsilon_0^\mu = \frac{1}{\sqrt{q^2}}(|\vec q|,0,0,q_0)\,,\varepsilon_t^\mu = \frac{1}{\sqrt{q^2}}(q_0,0,0,|\vec q|).$$

then

$$H_+^{\rm nf}(\bar B\to M_n X_S) = 0$$ (form factor relations)

$$\frac{H_t^{\rm nf}(B\to M_n X_S)}{H_0^{\rm nf}(B\to M_n X_S)} = \frac{c_2(v\cdot\varepsilon_t^*) + c_3(n\cdot\varepsilon_t^*)}{c_2(v\cdot\varepsilon_0^*) + c_3(n\cdot\varepsilon_0^*)}$$

and

$$\frac{H_-^{V-A}(B\to M_n X_S)}{H_-^{T}(B\to M_n X_S)} = \frac{c_1^{(V-A)}(E_M)}{c_1^{(T)}(E_M)}$$

$$\frac{H_0^{V-A}(B\to M_n X_S)}{H_0^{T}(B\to M_n X_S)} = \frac{c_2^{(V-A)}(v\cdot\varepsilon_0^*) + c_3^{(V-A)}(n\cdot\varepsilon_0^*)}{c_2^{(T)}(v\cdot\varepsilon_0^*) + c_3^{(T)}(n\cdot\varepsilon_0^*)}$$

Example: we already had $H_+^{\rm nf}(\bar B \to M_n X_S) = 0$

$$\epsilon_+^{*\mu} J_\mu^{\rm fact} = -\frac{1}{2\omega}\int dx dz dk_+ b_{1L}(x,z) J_\perp(x,z,k_+) ((\bar q Y)_{k_+} \not{n} \gamma_\mu^\perp \gamma_\perp^\lambda P_R (Y^\dagger b_v))(\bar q_{n,\omega_1} \frac{\not{\bar n}}{2} \gamma_\perp^\lambda q_{n,\omega_2})$$
$$- \frac{1}{2\omega}\int dx dz dk_+ b_{1R}(x,z) J_\parallel(x,z,k_+) ((\bar q Y)_{k_+} \not{n} \gamma_\mu^\perp P_R (Y^\dagger b_v))(\bar q_{n,\omega_1} \frac{\not{\bar n}}{2} P_L q_{n,\omega_2})$$
$$- \frac{1}{\omega}\int dx dz dk_+ [b_{1v}(x,z) v_\mu + b_{1n}(x,z) n_\mu] J_\parallel(x,z,k_+) ((\bar q Y)_{k_+} \not{n} P_L (Y^\dagger b_v))(\bar s_{n,\omega_1} \frac{\not{\bar n}}{2} P_L q_{n,\omega_2})$$

for 1-body, can only produce longitudinal polarized meson

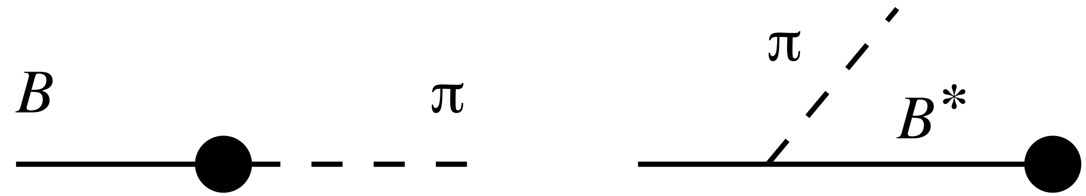
$$\Rightarrow H_+^{\rm fact}(\bar B \to K_n^* \to (K\pi)_n) = 0$$

Yet:

$$H_+^{\rm fact}(\bar B \to K_n \pi_S) \neq 0$$

Note: all of this to leading order in λ

Amplitudes can be computed using HQχPT:



$$H_+^{\mathrm{fact}}(\bar{B} \to P_n \pi_S) = C\frac{1}{2} f_B f_P m_B S_R \langle b_{1R} J_{\parallel} \phi_P \rangle$$

$$H_-^{\mathrm{fact}}(\bar{B} \to P_n \pi_S) = 0$$

$$H_+^{\mathrm{fact}}(\bar{B} \to V_n(\eta) \pi_S) = C\frac{f_B f_V m_B m_V}{2\bar{n}\cdot p_V} (\bar{n}\cdot\eta^*) S_R \langle b_{1R} J_{\parallel} \phi_V^{\parallel} \rangle$$

$$H_-^{\mathrm{fact}}(\bar{B} \to V_n(\eta) \pi_S) = -C f_B f_V^{\perp} m_B (\varepsilon_-^* \cdot \eta^*) S_L \langle b_{1L} J_{\perp} \phi_V^{\perp} \rangle$$

where

$$S_R(p_\pi) = \frac{g}{f_\pi} \frac{\varepsilon_+^* \cdot p_\pi}{v\cdot p_\pi + \Delta}$$

$$S_L(p_\pi) = \frac{1}{f_\pi} \left(1 - g\frac{e_3 \cdot p_\pi}{v\cdot p_\pi + \Delta}\right)$$

and C is an isospin factor, C=1 for charged, C=1/√2 for neutral

Use this in two examples: $B \to K\pi\ell\ell$ & $B \to K\pi\gamma$

In particular,
look at FBA in $B \to K\pi\ell\ell$
and CP violation in $B \to K\pi\gamma$

FBA: look for zero in

$$A_{FB} \propto \mathrm{Re}\,(H_-^V H_-^{A*} - H_+^V H_+^{A*})$$

where V, A = vector, axial-vector currents.
Start with nf contribution only: $H_+^V = 0, \quad H_-^V \propto c_1^{(V)}$

$$\Rightarrow \qquad \mathrm{Re}(c_1^{(V)}) = 0$$

($c_1^{(A)}$ is constant, real, and non-vanishing)

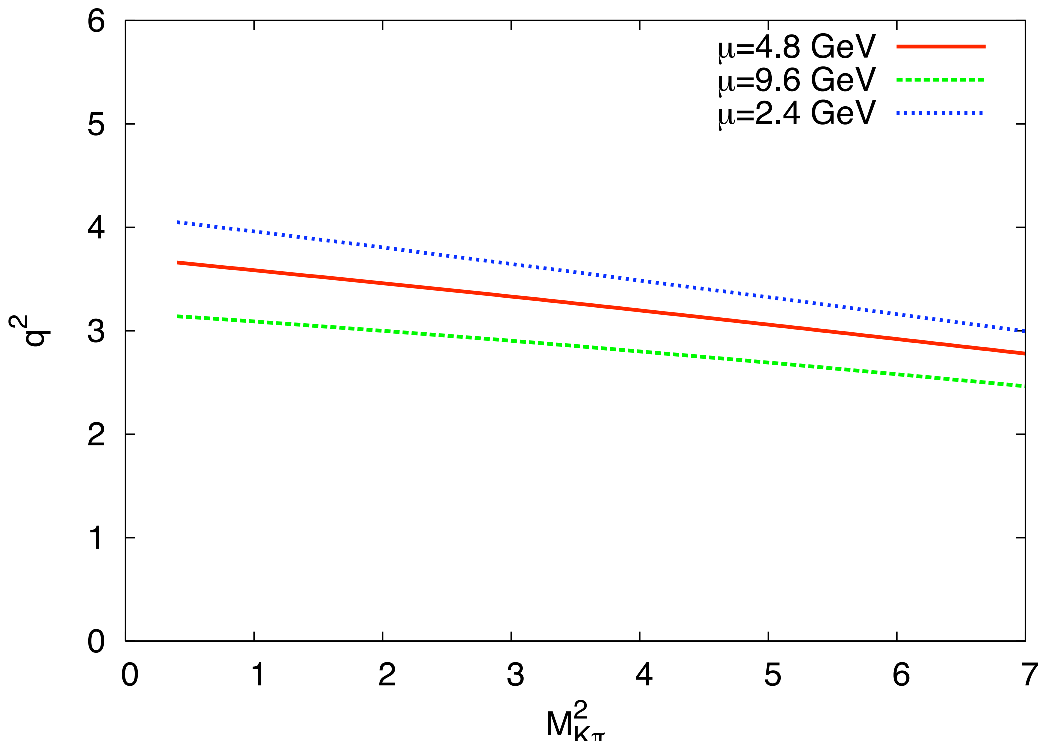
$$c_1^{(V)} = C_9^{\text{eff}} + 2m_b \frac{n \cdot q}{q^2} C_7^{\text{eff}}$$

(plus radiative corrections that are easy to include)

For $B \to K^* \ell\ell$ zero ($\text{Re}(c_1^{(V)}) = 0$) is standard result

$$q_0^2 = -2m_B m_b \frac{\text{Re}(C_7^{\text{eff}}(q_0^2))}{\text{Re}\ (C_9^{\text{eff}}(q_0^2))}$$

But for $B \to K\pi\ell\ell$ zero depends on $M_{K\pi}$



Now include factorizable terms

$$H_{\rm nf}^{(i)} = c_1^{(i)}(\bar{n}\cdot p_K,\mu)\zeta_\perp^{BK\pi}$$

$$H_{\rm f}^{(i)} = -\frac{1}{2} f_K(\varepsilon_+^*\cdot p_\pi)\int_0^1 dz dx b_{1R}^{(i)}(z)\int_{-p_\pi^+}^\infty dk_+ J_\parallel(x,z,k_+)S(k_+)\phi_K(x)$$

$$H_{\rm sp}^{(q)} = \frac{(4\pi)^2}{q^2} f_K(\bar{n}\cdot p_K)(\varepsilon_-^*\cdot p_\pi)\int_0^1 dx b_{\rm sp}^{(q)}(x)\phi_K(x)\int_{-p_{\pi-}}^\infty dk_- J_{\rm sp}(k_-)S(k_-)$$

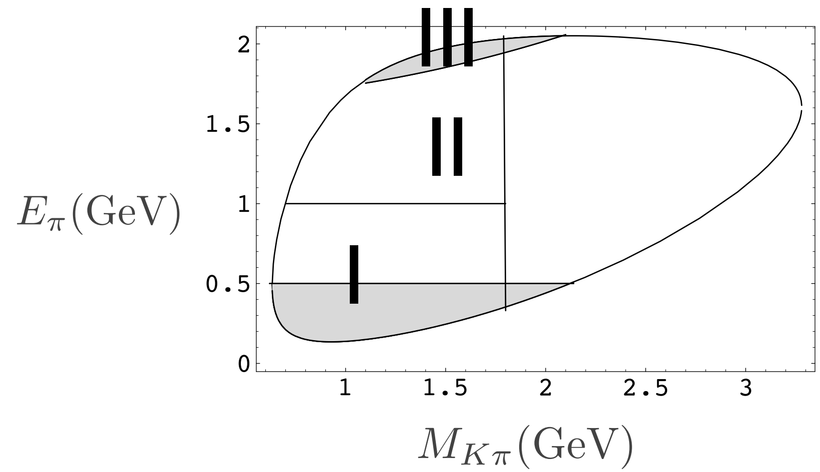
The soft functions, S, can be calculated as before, eg

$$H_{\rm f}^{(i)} = \frac{1}{2} m_B^2 S_R(p_\pi)\int_0^1 dz b_{1R}^{(i)}(z)\zeta_J^{BK}(z)$$

where $\zeta_J^{BK}(z) = \frac{f_B f_K}{m_B}\int_0^1 dx \int_0^\infty dk_+ J_\parallel(x,z,k_+)\phi_B^+(k_+)\phi_K(x)$

(Note: for sp, expand in *n.q*/*q*^{2})

Recall regions and model:



- I. Soft π, energetic K_n
- II. $(K\pi)_n$, energetic
- III. Soft K, energetic π_n: order $\lambda \sim \Lambda/Q$

In region II use resonant approximation

$$H_-^{V,A}(\bar{B}^0 \to K^-\pi^+) = H_-^{V,A}(\bar{B} \to \bar{K}^*)\frac{g_{K^*K\pi}(\varepsilon_-^* \cdot p_\pi)}{M^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}$$

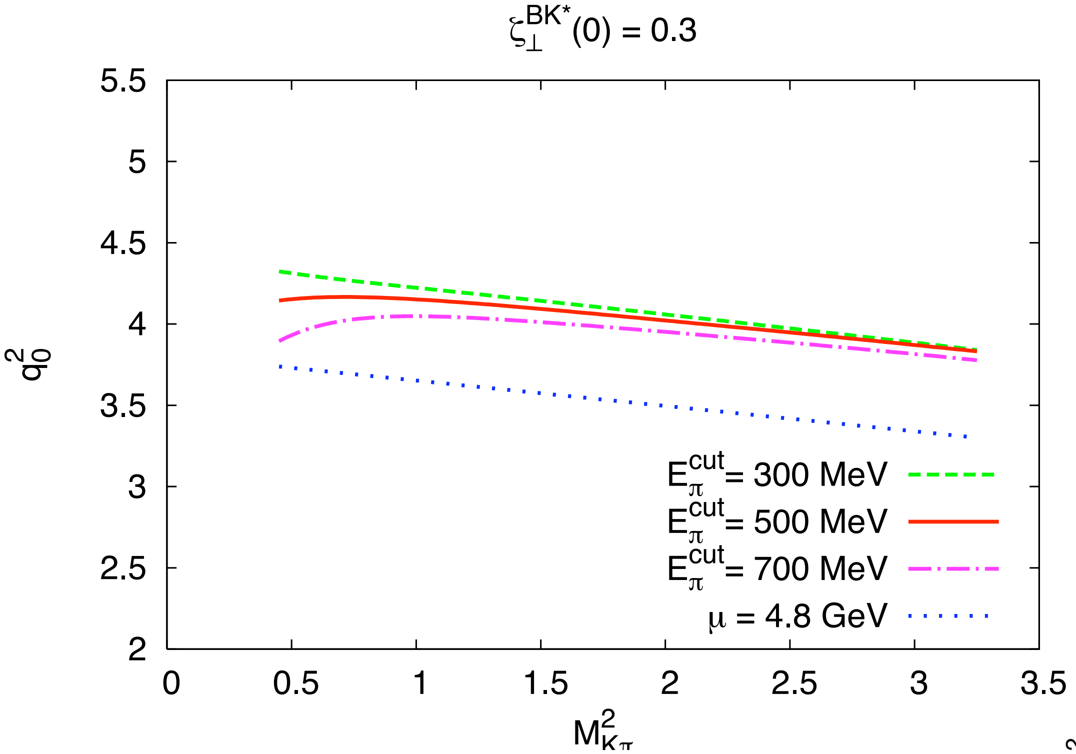
$$H_+^{V,A}(\bar{B}^0 \to K^-\pi^+) = 0$$

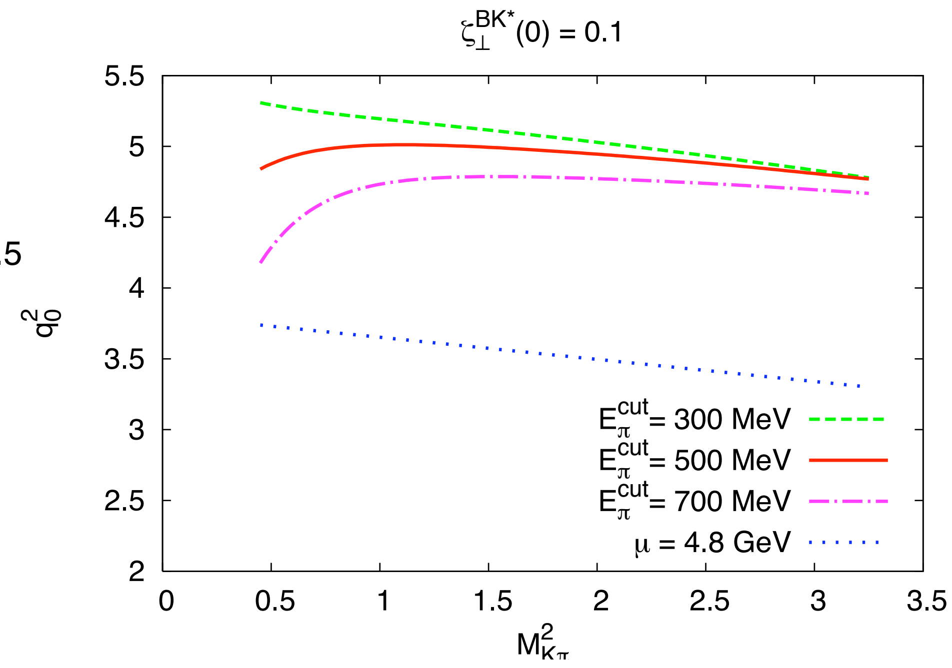
Also need in I BW model nf soft function

$$\zeta_\perp^{BK\pi}(M_{K\pi}, E_\pi) = \bar{n} \cdot p_{K^*}\zeta_\perp^{BK^*}\frac{g_{K^*K\pi}(\varepsilon_-^* \cdot p_\pi)}{M^2 - M_{K^*}^2 + iM_{K^*}\Gamma_{K^*}}$$

Solve for zeroes in FBA:

$$\mathrm{Re}\,\left[c_1^{(V)} - a_{\mathrm{sp}} - a_{\mathrm{f}}\right] = 0$$





Summary: chuck-full of results

- SCET_{II} Factorization in $B \rightarrow K\pi\gamma^{(*)}$ Studied
- Symmetry relations given, e.g., $H_+ = 0$
- SCET can do M_X large, by adding soft π's
- HQχPT formulated
- GPDs given entirely in terms of l.c. wave-functions
- Verified in 'tHooft model (and shape=(l.c.)^2 there)
- Uncertainty in CPV in $B \rightarrow K\pi\gamma$ is large, $M_{K\pi}$ dependent
- Simultaneous fit to zero and slope of zero in FBA-zero may increase accuracy of determination