Domain Walls, the Extended Superconformal Algebra, and the Supercurrent

Tonnis ter Veldhuis

Macalester College
and
Purdue University

In collaboration with:

Thomas E. Clark
Muneto Nitta

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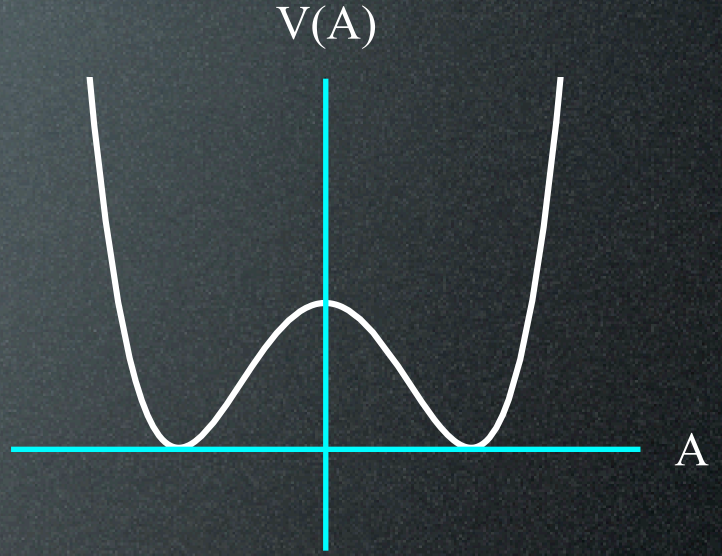
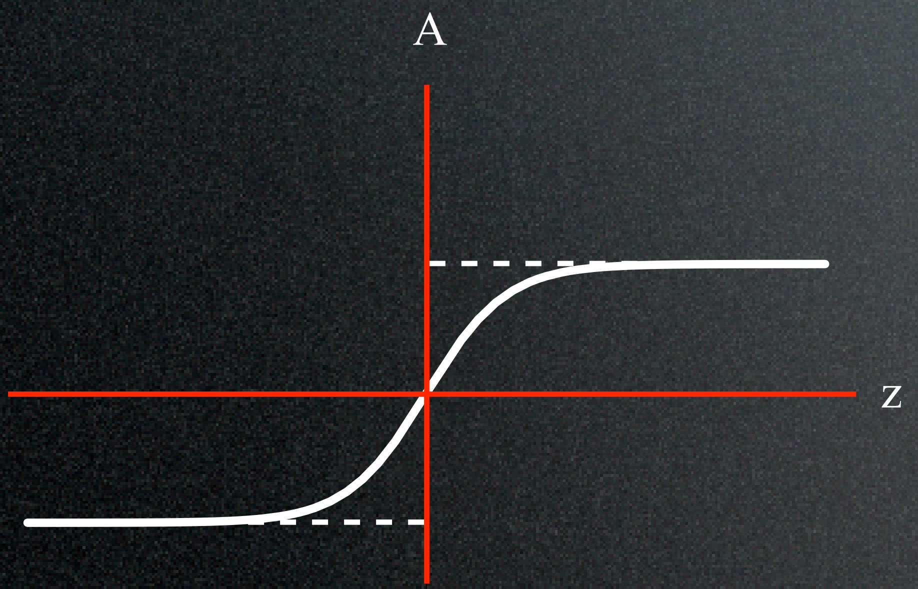
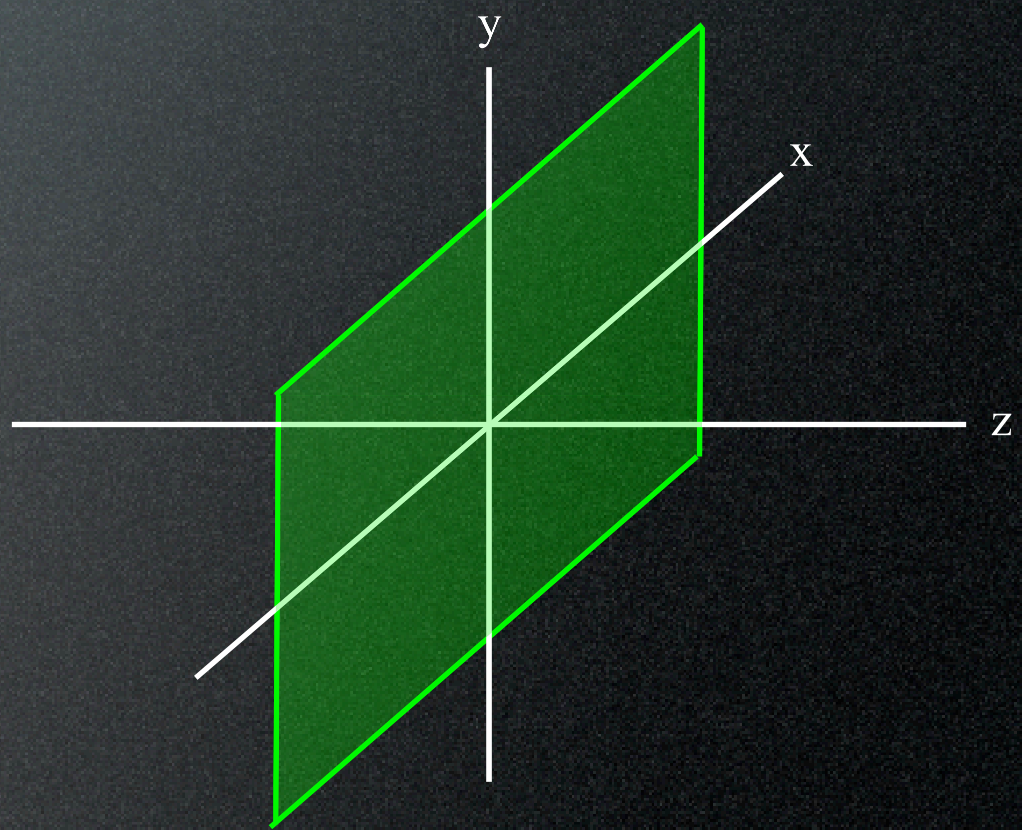
BPS domain walls

Static solution to the equations of motion that:
- saturates the BPS bound
- interpolates between two discrete supersymmetric vacua
- is invariant under half of the supersymmetry transformations.



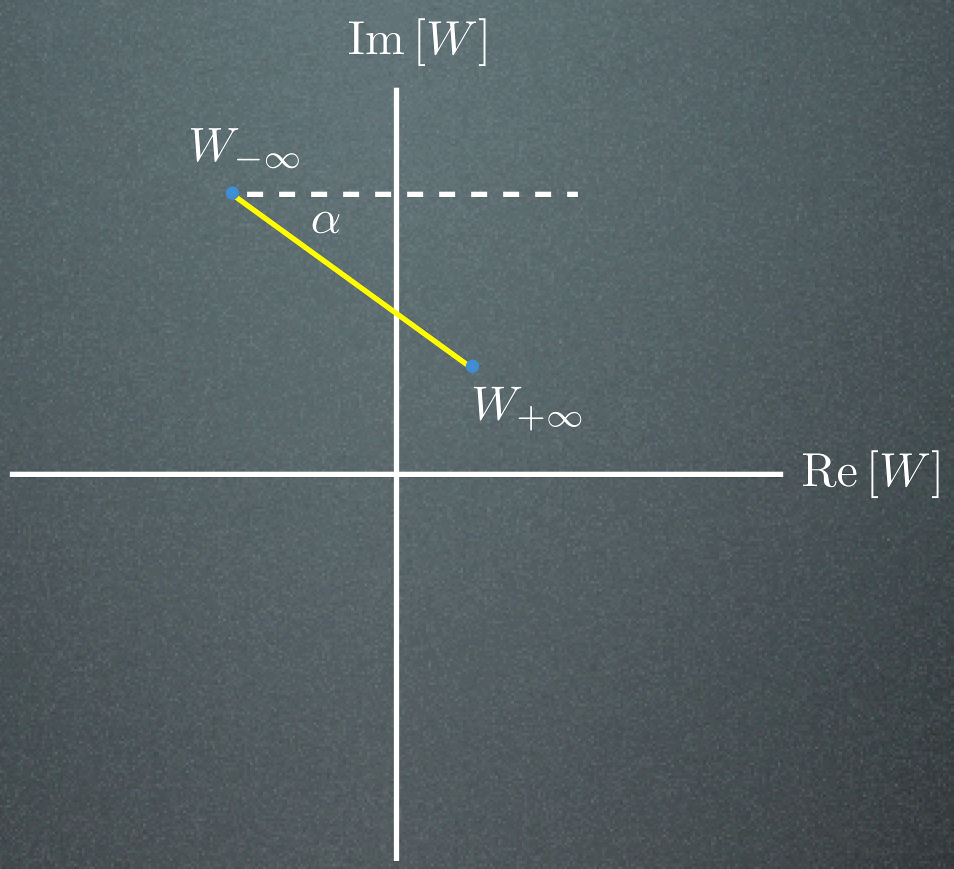


The domain wall interpolates between the two supersymmetric vacua



The orientation of the wall is perpendicular to the z-axis.

Generalized Wess Zumino model, one superfield:



BPS saturated domain walls correspond to straight trajectories in the complex W-plane.

BPS equation: $$\frac{dA}{dz} = e^{i\alpha}\bar{W}'(\bar{A})$$

anti-symmetric, trivially conserved

Topological current: $$\begin{aligned} Z^{\mu}_{N\alpha\beta} &= -32i\sigma^{\mu\nu}_{\alpha\beta}\partial_\nu \bar{W} \\ \bar{Z}^{\mu}_{N\dot{\alpha}\dot{\beta}} &= -32i\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}\partial_\nu W \end{aligned}$$

Tension: $$\sigma = 2\,|\Delta W|$$

The BPS domain wall tension is proportional to the topological charge density.

Example:

Renormalizable model, one chiral superfield:

$$
\begin{aligned}
K &= \bar{\phi}\phi \\
W &= m^2\phi - \frac{1}{3}\lambda\phi^3
\end{aligned}
$$

Lagrangian for scalar field, after eliminating auxiliary field through its equation of motion:

$$
\mathcal{L}_{\text{bos}} = \partial^\mu \bar{A}\partial_\mu A - \underbrace{\left|m^2 - \lambda A^2\right|^2}_{\text{scalar potential } V(A,\bar{A})}
$$

Two supersymmetric vacua:

$$
A = \pm\frac{m}{\sqrt{\lambda}}
$$

Solutions to the BPS equation:

inverse domain wall width

$$
A = \frac{m}{\sqrt{\lambda}}\tanh(mz)
$$

Tension: $\sigma = 2\left|\Delta W\right| = \frac{8}{3}\frac{m^3}{\sqrt{\lambda}}$

Generalized Wess-Zumino model

The action is given by:

superpotential

$$\Gamma = \int dV \; K(\phi, \bar{\phi}) + \int dS \; W(\phi) + \int d\bar{S} \; \bar{W}(\bar{\phi})$$

Kähler potential

chiral superfield

The (anti-)chiral superfields can be expanded in terms of their components:

flavor index

fermion

$$\begin{aligned} \phi^i &= e^{-i\theta \not{\partial} \bar{\theta}} \left[A^i + \theta \psi^i + \theta^2 F^i \right] \\ \bar{\phi}^{\bar{i}} &= e^{+i\theta \not{\partial} \bar{\theta}} \left[\bar{A}^{\bar{i}} + \bar{\theta} \bar{\psi}^{\bar{i}} + \bar{\theta}^2 \bar{F}^{\bar{i}} \right] \end{aligned}$$

scalar

auxiliary field

Results for generic Kähler potential can be found in hep-th/0608095. In this talk results for canonical Kähler potential are provided.

Extended D=4, N=1 Susy Algebra

Supersymmetry

Translations

$$
\begin{aligned}
\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} &= +2\sigma^{\mu}_{\alpha\dot{\alpha}} P_\mu \\
[M^{\mu\nu}, Q_\alpha] &= -\frac{1}{2}\sigma^{\mu\nu\ \beta}_{\alpha} Q_\beta \\
[M^{\mu\nu}, \bar{Q}_{\dot{\alpha}}] &= -\frac{1}{2}\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}} \bar{Q}^{\dot{\beta}} \\
[R, Q_\alpha] &= +Q_\alpha \\
[R, \bar{Q}_{\dot{\alpha}}] &= -\bar{Q}_{\dot{\alpha}}
\end{aligned}
$$

Lorentz

R-transformation

Tensorial Central Charge

$$
\begin{aligned}
\{Q_\alpha, Q_\beta\} &= Z_{\alpha\beta} \\
\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} &= \bar{Z}_{\dot{\alpha}\dot{\beta}}
\end{aligned}
$$

Construction of component currents

Noether's theorem yields the form of the conserved SUSY currents:

$$\begin{aligned} Q^{\mu}_{N\alpha} &= 16\left(\psi^i \sigma^\mu \bar{\sigma}^\lambda \partial_\lambda \bar{A}^i\right)_\alpha - 4i\bar{W}_i\left(\sigma^\mu \bar{\psi}^i\right)_\alpha \\ \bar{Q}^{\mu}_{N\dot{\alpha}} &= 16\left(\partial_\lambda A^i \bar{\sigma}^\lambda \sigma^\mu \bar{\psi}^i\right)_{\dot{\alpha}} + 4iW_i\left(\psi^i \sigma^\mu\right)_{\dot{\alpha}} \end{aligned}$$

N indicates unmodified Noether current

Calculating the SUSY variations of the SUSY Noether currents yields the expression for the currents associated with the tensorial central charges:

$$\begin{aligned} \delta^Q_\alpha Q^\mu_{N\beta} + \delta^Q_\beta Q^\mu_{N\alpha} &= Z^\mu_{N\alpha\beta} \\ \delta^{\bar{Q}}_{\dot{\alpha}} \bar{Q}^\mu_{N\dot{\beta}} + \delta^{\bar{Q}}_{\dot{\beta}} \bar{Q}^\mu_{N\dot{\alpha}} &= \bar{Z}^\mu_{N\dot{\alpha}\dot{\beta}} \end{aligned}$$

Using the variations of the currents to find the central charge currents is equivalent to doing the current algebra with canonical quantization of the fields and their conjugate momentum and Noether currents.

Up to Euler Lagrange composite field equation corrections, which are taken to vanish on shell, the result is:

$$\begin{aligned} Z^\mu_{N\alpha\beta} &= -32i\sigma^{\mu\nu}_{\alpha\beta}\partial_\nu \bar{W} \\ \bar{Z}^\mu_{N\dot{\alpha}\dot{\beta}} &= -32i\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}\partial_\nu W \end{aligned}$$

Improved currents

A Belinfante improved SUSY current can be defined:

$$\begin{aligned} Q^\mu_\alpha &= Q^\mu_{N\alpha} + \partial_\rho G^{\rho\mu}_\alpha \\ \bar{Q}^\mu_{\dot\alpha} &= \bar{Q}^\mu_{N\dot\alpha} + \partial_\rho \bar{G}^{\rho\mu}_{\dot\alpha} \end{aligned}$$

anti-symmetric, trivially conserved

where

$$\begin{aligned} G^{\rho\mu}_\alpha &= a(\psi^i \sigma^{\rho\mu})_\alpha \bar{A}^i \\ \bar{G}^{\rho\mu}_{\dot\alpha} &= \bar{a} A^i (\bar\sigma^{\rho\mu} \bar\psi^i)_{\dot\alpha} \end{aligned}$$

The constant a can be chosen so that the trace of the supersymmetry current is soft for a renormalizable model: $a = -i\frac{32}{3}$

Improved tensorial central charge currents follow from the supersymmetry variations of the improved supersymmetry charges:

scaling dimension of (anti) chiral superfield

$$\begin{aligned} \delta^Q_\alpha Q^\mu_\beta + \delta^Q_\beta Q^\mu_\alpha &= Z^\mu_{\alpha\beta} \\ \delta^{\bar Q}_{\dot\alpha} \bar{Q}^\mu_{\dot\beta} + \delta^{\bar Q}_{\dot\beta} \bar{Q}^\mu_{\dot\alpha} &= \bar{Z}^\mu_{\dot\alpha\dot\beta} \end{aligned}$$

$$\begin{aligned} Z^\mu_{\alpha\beta} &= -32 i \sigma^{\mu\nu}_{\alpha\beta} \partial_\nu \left[\bar{W} - \frac{d}{3} \bar{A}^{\bar i} \bar{W}_{,\bar i} \right] \\ \bar{Z}^\mu_{\dot\alpha\dot\beta} &= -32 i \bar\sigma^{\mu\nu}_{\dot\alpha\dot\beta} \partial_\nu \left[W - \frac{d}{3} A^i W_{,i} \right] \end{aligned}$$

improvements

The Noether form of the energy momentum tensor is:

$$T^{\mu\nu}_{\rm N} = 16[\partial^\mu A\partial^\nu\bar{A}+\partial^\nu A\partial^\mu\bar{A}]-4i[\partial^\nu\psi\sigma^\mu\bar{\psi}-\psi\sigma^\mu\partial^\nu\bar{\psi}]-g^{\mu\nu}\mathcal{L}$$

A symmetric energy-momentum tensor can be defined by means of the Belinfante improvement procedure:

$$T^{\mu\nu}_{\rm B} = T^{\mu\nu}_{\rm N}+\partial_\rho G^{\rho\mu\nu}_{\rm B} \qquad G^{\rho\mu\nu}_{\rm B} = -16\epsilon^{\mu\nu\rho\sigma}(\psi\sigma_\sigma\bar{\psi})$$

$$T^{\mu\nu}_{\rm B} = 16[\partial^\mu A\partial^\nu\bar{A}+\partial^\nu A\partial^\mu\bar{A}]+2i[\psi\sigma^\nu \overleftrightarrow{\partial^\mu}\bar{\psi}+\psi\sigma^\mu \overleftrightarrow{\partial^\nu}\bar{\psi}]-g^{\mu\nu}\mathcal{L}-2i[\psi\sigma^{\mu\nu}\frac{\delta\Gamma}{\delta\psi}+\frac{\delta\Gamma}{\delta\bar{\psi}}\bar{\sigma}^{\mu\nu}\bar{\psi}]$$

A further “new and improved” tensor with a soft trace can be obtained by adding yet another trivially conserved term:

$$T^{\mu\nu}_{\rm I} = T^{\mu\nu}_{\rm B}+\xi(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu)(32A\bar{A})$$

trace is soft for $\xi = \frac{1}{6}$

The final form of the conserved, symmetric and soft trace energy-momentum tensor is obtained by once more modifying the current by subtracting composite Euler-Lagrange equations from the improved tensor. The results is:

$$\begin{aligned}T^{\mu\nu} = {} & 16[\partial^\mu A\partial^\nu\bar{A}+\partial^\nu A\partial^\mu\bar{A}]-g^{\mu\nu}16\partial_\rho A\partial^\rho\bar{A}+\frac{1}{3}(g^{\mu\nu}\partial^2-\partial^\mu\partial^\nu)16(A\bar{A})\\ & +g^{\mu\nu}16F\bar{F}+2i[\psi\sigma^\nu \overleftrightarrow{\partial}^\mu\bar{\psi}+\psi\sigma^\mu\overleftrightarrow{\partial}^\nu\bar{\psi}]\end{aligned}$$

The Supercurrent

All improved component currents can be obtained from the supercurrent. For the generalized Wess-Zumino model with canonical Kähler potential the supercurrent takes the form:

$$V_{\alpha\dot{\alpha}} \;=\; -\frac{8}{3}D_\alpha\phi^i\bar{D}_{\dot{\alpha}}\bar{\phi}^{\bar{i}} - \frac{16}{3}i\bar{\phi}^{\bar{i}}\not{\partial}_{\alpha\dot{\alpha}}\phi^i + \frac{16}{3}i\phi^i\not{\partial}_{\alpha\dot{\alpha}}\bar{\phi}^{\bar{i}}, \qquad V^\mu = \bar{\sigma}^{\dot{\alpha}\alpha}V_{\alpha\dot{\alpha}}$$

The improved component currents are embedded in the supercurrent as

R-current

supersymmetry current

central charge current

energy momentum tensor

$$\begin{aligned}
V^\mu(x,\theta,\bar{\theta}) \;=\; & R^\mu(x) + i\theta\left[Q^\mu(x) - \frac{1}{3}Q^\rho(x)\sigma_\rho\bar{\sigma}^\mu\right] - i\bar{\theta}\left[\bar{Q}^\mu(x) - \frac{1}{3}\bar{\sigma}^\mu\sigma^\rho\bar{Q}_\rho(x)\right] \\
& + \frac{1}{24}\theta^2\sigma_\alpha^{\mu\nu\beta}Z_{\nu\beta}{}^{\alpha}(x) - \frac{1}{24}\bar{\theta}^2\bar{\sigma}^{\mu\nu\dot{\alpha}}{}_{\dot{\beta}}\bar{Z}_\nu^{\dot{\beta}}{}_{\dot{\alpha}}(x) \\
& + \theta\sigma_\nu\bar{\theta}\left\{2\left[T^{\mu\nu}(x) - \frac{1}{3}g^{\mu\nu}T^\rho{}_\rho(x)\right] + \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\partial_\rho R_\sigma(x)\right\} \\
& + \frac{1}{2}\theta^2\bar{\theta}\left[\partial^\mu\bar{\sigma}^\nu - i\epsilon^{\mu\nu\rho\sigma}\bar{\sigma}_\rho\partial_\sigma\right]\left[Q^\mu(x) - \frac{1}{3}Q^\rho(x)\sigma_\rho\bar{\sigma}^\mu\right] \\
& - \frac{1}{2}\bar{\theta}^2\theta\left[\partial^\mu\sigma^\nu - i\epsilon^{\mu\nu\rho\sigma}\sigma_\rho\partial_\sigma\right]\left[\bar{Q}^\mu(x) - \frac{1}{3}\bar{\sigma}^\mu\sigma^\rho\bar{Q}_\rho(x)\right] \\
& + \frac{1}{4}\theta^2\bar{\theta}^2\left[\partial^2 g^{\mu\nu} - 2\partial^\mu\partial^\nu\right]R_\nu(x)
\end{aligned}$$

Hence the extended super-Poincaré component currents can be obtained as derivatives of the supercurrent:

higher components of superfield

superfields

$$
\begin{aligned}
\hat{R}^{\mu} &= V^{\mu} = R^{\mu} + \cdots \\
\hat{Q}^{\mu}_{\alpha} &= -i\left[D_{\alpha}V^{\mu} - (DV^{\nu}\sigma_{\nu}\bar{\sigma}^{\mu})_{\alpha}\right] = Q^{\mu}_{\alpha} + \cdots \\
\hat{\bar{Q}}^{\mu}_{\dot{\alpha}} &= +i\left[\bar{D}_{\dot{\alpha}}V^{\mu} - (\bar{\sigma}_{\mu}\sigma^{\nu}\bar{D}V_{\nu})_{\dot{\alpha}}\right] = \bar{Q}^{\mu}_{\dot{\alpha}} + \cdots \\
\hat{Z}^{\mu}_{\alpha\beta} &= \sigma^{\mu\nu}_{\alpha\beta}DDV_{\nu} = +2i\left[\bar{\sigma}^{\mu\gamma}_{\alpha}D_{\beta} + \bar{\sigma}^{\mu\gamma}_{\beta}D_{\alpha}\right]D^{\gamma}V_{\gamma\dot{\gamma}} = Z^{\mu}_{\alpha\beta} + \cdots \\
\hat{\bar{Z}}^{\mu}_{\dot{\alpha}\dot{\beta}} &= -\bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}\bar{D}\bar{D}V_{\nu} = -2i\left[\bar{D}_{\dot{\alpha}}\bar{\sigma}^{\mu\dot{\gamma}}_{\dot{\beta}} + \bar{D}_{\dot{\beta}}\bar{\sigma}^{\mu\dot{\gamma}}_{\dot{\alpha}}\right]\bar{D}^{\dot{\gamma}}V_{\gamma\dot{\gamma}} = \bar{Z}^{\mu}_{\dot{\alpha}\dot{\beta}} + \cdots \\
\hat{T}^{\mu\nu} &= +\frac{1}{16}\left[V^{\mu\nu} + V^{\nu\mu} - 2g^{\mu\nu}V^{\rho}_{\ \rho}\right] = T^{\mu\nu} + \cdots
\end{aligned}
$$

$$V^{\mu\nu} \equiv (D\sigma^{\mu}\bar{D} - \bar{D}\bar{\sigma}^{\mu}D)V^{\nu}$$

In addition, the angular momentum tensor is constructed from the energy momentum tensor:

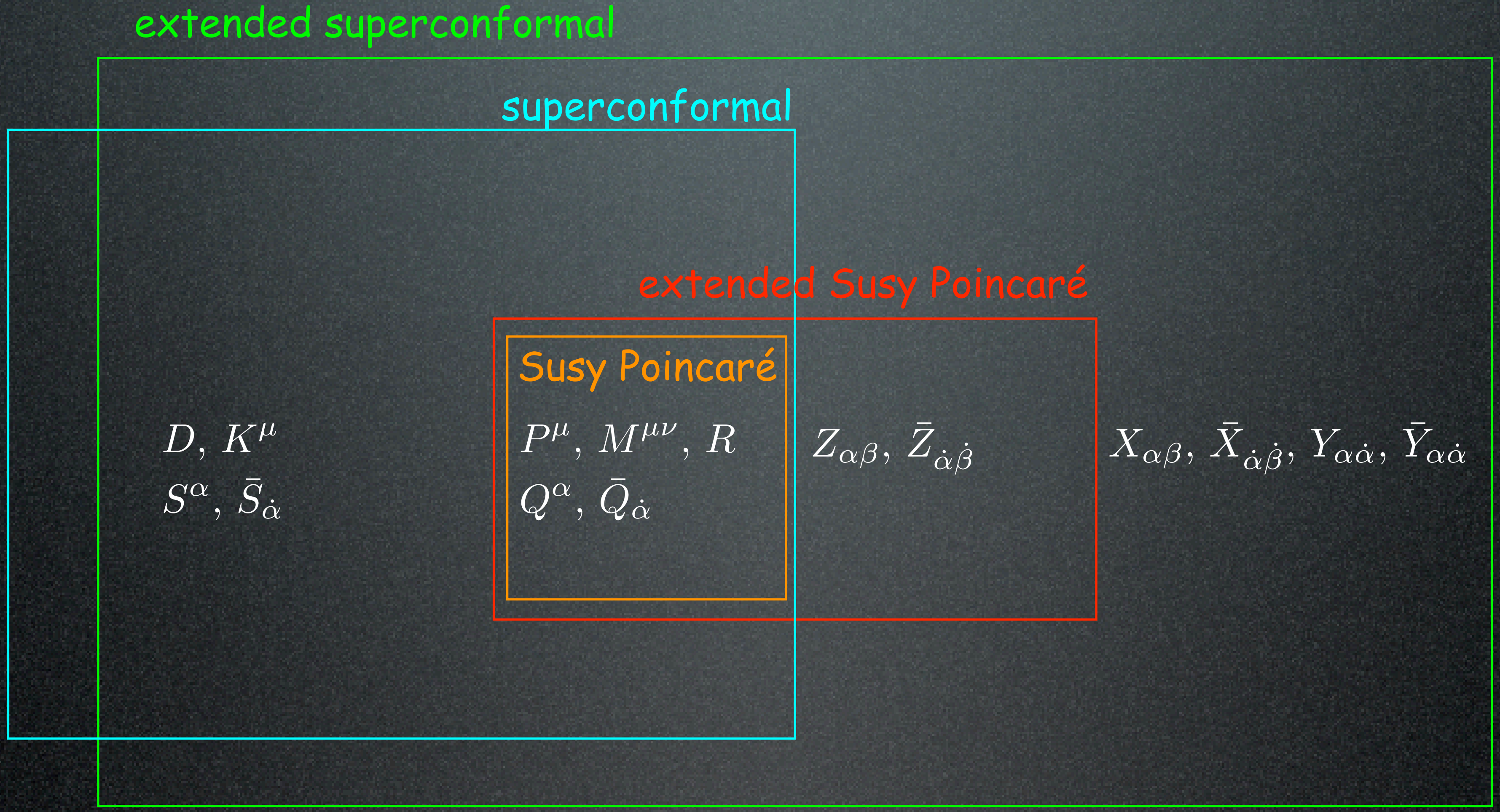
$$\hat{M}^{\mu\nu\rho} = x^{\rho}\hat{T}^{\mu\nu} - x^{\nu}\hat{T}^{\mu\rho} = M^{\mu\nu\rho} + \cdots$$

not a superfield

Extended Superconformal Algebra

In the presence of a domain wall the superconformal algebra is modified to include additional tensorial central charges:

$$\begin{aligned}\{S_\alpha, S_\beta\} &= X_{\alpha\beta} & \{Q_\alpha, \bar{S}_{\dot\alpha}\} &= Y_{\alpha\dot\alpha} = +2\sigma^\mu_{\alpha\dot\alpha} Y_\mu \\ \{\bar{S}_{\dot\alpha}, \bar{S}_{\dot\beta}\} &= \bar{X}_{\dot\alpha\dot\beta} & \{S_\alpha, \bar{Q}_{\dot\alpha}\} &= \bar{Y}_{\alpha\dot\alpha} = +2\sigma^\mu_{\alpha\dot\alpha} \bar{Y}_\mu\end{aligned}$$



$$\begin{aligned}\{Q_\alpha, S_\beta\} &= +i\left(\sigma^{\mu\nu}_{\alpha\beta} M_{\mu\nu} + 2i\epsilon_{\alpha\beta} D - 3\rho\epsilon_{\alpha\beta} R\right) \\ \{\bar{Q}_{\dot\alpha}, \bar{S}_{\dot\beta}\} &= -i\left(\bar{\sigma}^{\mu\nu}_{\dot\alpha\dot\beta} M_{\mu\nu} - 2i\epsilon_{\dot\alpha\dot\beta} D - 3\rho\epsilon_{\dot\alpha\dot\beta} R\right)\end{aligned}$$

$\rho = 1$ superconformal

$\rho = -\frac{1}{3}$ extended superconformal

Construction of component currents

The superconformal currents are found via Noether's theorem using the superconformal transformations. The result is:

$$
\begin{aligned}
S^{\mu}_{N\alpha} &= -\not{x}_{\alpha\dot\alpha}\,\bar{Q}^{\mu\dot\alpha}_{N} + 2 s^{\mu}_{\alpha} \\
\bar{S}^{\mu}_{N\dot\alpha} &= -Q^{\mu\alpha}_{N}\,\not{x}_{\alpha\dot\alpha} + 2\bar{s}^{\mu}_{\dot\alpha}
\end{aligned}
$$

with

$$
\begin{aligned}
s^{\mu}_{\alpha} &= -16 d A^{i} (\bar\sigma^{\mu}\bar\psi^{i})_{\alpha} \\
\bar{s}^{\mu}_{\dot\alpha} &= -16 d (\psi^{i}\sigma^{\mu}) \bar{A}^{i}_{\alpha}
\end{aligned}
$$

scale weight of (anti) chiral field

Just as the supersymmetry currents, the superconformal currents can also be Belinfante improved:

$$
\begin{aligned}
S^{\mu}_{\alpha} &= S^{\mu}_{N\alpha} + \partial_{\rho}[\not{x}_{\alpha}{}^{\dot\alpha}\bar{G}^{\rho\mu}_{\dot\alpha}] \\
\bar{S}^{\mu}_{\dot\alpha} &= \bar{S}^{\mu}_{N\dot\alpha} + \partial_{\rho}[\bar{\not{x}}_{\dot\alpha}{}^{\alpha} G^{\rho\mu}_{\alpha}]
\end{aligned}
$$

with

$$
\begin{aligned}
G^{\rho\mu}_{\alpha} &= a(\psi^{i}\sigma^{\rho\mu})_{\alpha}\bar{A}^{i} \\
\bar{G}^{\rho\mu}_{\dot\alpha} &= \bar{a} A^{i} (\bar\sigma^{\rho\mu}\bar\psi^{i})_{\dot\alpha}
\end{aligned}
$$

With the choice $a = -i\frac{32}{3}$, the improved superconformal currents are just the space-time moments of the supersymmetry currents:

$$
\begin{aligned}
S^{\mu}_{\alpha} &= -\not{x}_{\alpha\dot\alpha}\bar{Q}^{\mu\dot\alpha} \\
\bar{S}^{\mu}_{\dot\alpha} &= -Q^{\mu\alpha}\not{x}_{\alpha\dot\alpha}
\end{aligned}
$$

Applying the SUSY and superconformal variations to these improved currents, the improved conformal central charge currents are obtained:

$$\begin{aligned} \delta^Q_\alpha \bar{S}^\mu_{\dot\alpha} + \delta^{\bar{S}}_{\dot\alpha} Q^\mu_\alpha &= Y^\mu_{\alpha\dot\alpha} \\ \delta^{\bar{Q}}_{\dot\alpha} S^\mu_\alpha + \delta^S_\alpha \bar{Q}^\mu_{\dot\alpha} &= \bar{Y}^\mu_{\alpha\dot\alpha} \end{aligned}$$

$$\begin{aligned} \delta^S_\alpha S^\mu_\beta + \delta^S_\beta S^\mu_\alpha &= X^\mu_{\alpha\beta} \\ \delta^{\bar{S}}_{\dot\alpha} \bar{S}^\mu_{\dot\beta} + \delta^{\bar{S}}_{\dot\beta} \bar{S}^\mu_{\dot\alpha} &= \bar{X}^\mu_{\dot\alpha\dot\beta} \end{aligned}$$

The improved conformal central charge currents are just the space-time moments of the central charge currents:

$$\begin{aligned} Y^\mu_{\alpha\dot\alpha} &= (\bar{\not{x}} Z^\mu)_{\dot\alpha\alpha} \\ \bar{Y}^\mu_{\alpha\dot\alpha} &= (\not{x} \bar{Z}^\mu)_{\alpha\dot\alpha} \end{aligned}$$

$$\begin{aligned} X^\mu_{\alpha\beta} &= -(\not{x} \bar{Z}^\mu \bar{\not{x}})_{\alpha\beta} \\ \bar{X}^\mu_{\dot\alpha\dot\beta} &= -(\bar{\not{x}} Z^\mu \not{x})_{\dot\alpha\dot\beta} \end{aligned}$$

Therefore, all improved component currents are determined by the supercurrent.

Renormalization

Renormalization of the supercurrent controls radiative corrections to all superconformal charges and current conservation equations

Conjectured results:

$$
\begin{aligned}
D^{\alpha} V^{\text{ren.}}_{\alpha\dot{\alpha}} &= -2\bar{D}_{\dot{\alpha}}\bar{S}^{\text{ren.}} \\
\bar{D}^{\dot{\alpha}} V^{\text{ren.}}_{\alpha\dot{\alpha}} &= -2D_{\alpha}S^{\text{ren.}}
\end{aligned}
$$

with:

anomalous dimension

$$
\begin{aligned}
S^{\text{ren.}} &= -2[W - \frac{1}{3}\sum_i (1+\gamma_i)\phi^i W_{,i}] \\
\bar{S}^{\text{ren.}} &= -2[\bar{W} - \frac{1}{3}\sum_i (1+\gamma_i)\bar{\phi}^i \bar{W}_{,i}]
\end{aligned}
$$

The topological central charge density is not renormalized. Consider a domain wall in the simple Wess-Zumino model perpendicular to the z direction:

current $$Z_{\alpha\beta} = 16i\sigma^{\mu\nu}_{\alpha\beta}\partial_\nu \bar{S}$$

charge density

$$
\begin{aligned}
\zeta &= Z_{11} = \frac{1}{16}\int_{-\infty}^{\infty} dz Z^0_{11} \\
&= 2\left[\bar{W}\big|_{z=-\infty} - \bar{W}\big|_{z=+\infty}\right] - \frac{2}{3}(1+\gamma)\left[\bar{\phi}\bar{W}'\big|_{z=+\infty} - \bar{\phi}\bar{W}'\big|_{z=-\infty}\right] = \frac{8}{3}\frac{m^2}{\sqrt{\lambda}}
\end{aligned}
$$

vanishes in supersymmetric vacua

domain wall tension

Likewise, the first space-time moment conformal central charge density receives no radiative corrections:

$$Y^{\mu}_{\alpha\dot{\alpha}} = 16i\left[\not{x}\sigma^{\mu\nu}\partial_\nu \bar{S}^{\mathrm{ren.}}\right] \qquad \longleftarrow \text{current}$$

$$Y_{\alpha\dot{\alpha}} = \frac{1}{16}\int_{-\infty}^{+\infty} dz Y^{0}_{\alpha\dot{\alpha}} = \underbrace{-\sigma^{3}_{\alpha\dot{\alpha}}\, t\zeta}_{\text{independent of anomalous dimension}} \qquad \longleftarrow \text{charge density}$$

However, the second space-time moment conformal central charge density receives anomalous dimension radiative corrections:

$$X^{\mu}_{\alpha\beta} = -16i\left[\not{x}\bar{\sigma}^{\mu\nu}\not{x}\right]_{\alpha\beta}\partial_\nu S^{\mathrm{ren.}} \qquad \longleftarrow \text{current}$$

$$X_{\alpha\beta} = \frac{1}{16}\int_{-\infty}^{+\infty} dz X^{0}_{\alpha\beta} = -\sigma^{3}_{\alpha\beta}\left[(t^2+x^2+y^2)\zeta - \underbrace{\int_{-\infty}^{+\infty} dz z^2\partial_z S^{\mathrm{ren.}}}_{\text{depends on anomalous dimension}}\right] \qquad \longleftarrow \text{charge density}$$

Conclusions

- The presence of a domain wall requires a tensorial central charge extension of the superconformal algebra.

- The superconformal current and the superconformal tensorial central charge currents are obtained as space-time moments of the supersymmetry current and the SUSY tensorial central charge current.

- The supercurrent contains the R-symmetry current, the supersymmetry current, the tensorial central charge current, and the energy momentum tensor as its components.

- All tensorial central charge extended superconformal currents are constructed from the supercurrent.

- The SUSY tensorial central charge remains uncorrected, but one of the conformal tensorial central charges receives radiative corrections.