Minimal Lepton Flavor Violation

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Rationale for MFV (Minimal Flavor Violation)

- Buchmuller & Wyler: order $1/\Lambda^2$ terms in H_{eff}
  Nucl.Phys.**B268**,621(1986)
- If Λ is related to EW-breaking, or higgs hierarchy problem, expect $\Lambda \sim 10^{3\text{-}4}$ GeV
- All fine, except processes with FCNC
- To avoid FCNC need small coefficients $C \sim 10^{-(2\text{-}3)}$, or large scale $\Lambda \sim 10^{6\text{-}7}$ GeV
- If $\Lambda \sim 10^{6\text{-}7}$ GeV then no relation to hierarchy
- Can we make C naturally small?

R.S.Chivukula, H.Georgi, Phys. Lett. **B188**(1987)99
A.J.Buras, *et al.*, Phys. Lett. **B500**(2001)16
G.D'Ambrosio, *et al.*, Nucl. Phys. **B645**(2002)155

Minimal Flavor Violation (MFV)

- Premise: Unique source of flavor braking
- Quark sector in SM, in absence of masses has large flavor (global) symmetry: $G_F = SU(3)^3 \times U(1)^2$
- Symmetry is only broken by Yukawa interactions, parametrized by couplings λ_U and λ_D
- MFV: all breaking of G_F must transform as these
- When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic

How does this work?

Consider $K_L \to \pi\nu\bar{\nu}$

Recall, G_F breaking from: $\mathcal{L}_{\text{Yuk}} = H\bar{q}_L\lambda_U u_R + \tilde{H}\bar{q}_L\lambda_D d_R$

G_F, using spurion method:

$$q_L \to V_L q_L \qquad \lambda_U \to V_L \lambda_U V_u^\dagger$$
$$u_R \to V_u u_R \qquad \lambda_D \to V_L \lambda_D V_d^\dagger$$
$$d_R \to V_d d_R$$

Effective lagrangian $\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2}\sum C_i O_i$

where the operator is, for example

$$O = \bar{q}_L(\lambda_U\lambda_U^\dagger)\gamma_\mu q_L\,\bar{\nu}_L\gamma^\mu\nu_L$$

In mass basis $\Rightarrow \left(\sum_{q=u,c,t} V_{qs}^* V_{qd}\frac{m_q^2}{v^2}\right)\bar{s}_L\gamma_\mu d_L\,\bar{\nu}_L\gamma^\mu\nu_L$

Since $|V_{ts}^* V_{td}|m_t^2/v^2 \approx A^2\lambda^5 \approx 5\times10^{-4}$ so effectively C ~10^{-3}

Minimal Lepton Flavor Violation (MLFV)

- Rationale: elevate MFV to a principle
- If leptons acquire Dirac masses (mimic quark sector) copy from above. But uninteresting: flavor violation proportional to tiny neutrino masses
- More attractive/interesting: Small neutrino masses from see-saw mechanism.
- What are the restrictions from MLFV in see-saw models?
- Two cases. Field content (below LNV scale):
  - Minimal: three L_i and e_{Ri}
  - Extended: three L_i, e_{Ri}, and ν_{Ri}

A Note on: LN vs LF

- Distinguish Lepton Number (LN) violating interactions from Lepton Flavor (LF) violating interactions
- LN is a $U(1)$ symmetry, assigning unit charge to all leptons.
  - Neutrino Majorana mass breaks LN
- LF is an $SU(3)$ symmetry, mixing different flavors
  - It commutes with $U(1)_{\text{LN}}$, *ie*, preserves the LN charge

MLFV: Minimal Field Content

Ex: SUSY Triplet Model, A. Rossi, PRD**66**(2002)075003

Assumptions:

1. The breaking of the $U(1)_{\text{LN}}$ is independent from the breaking of lepton flavor G_{LF}, with large Λ_{LN} (associated with see-saw)

2. There are only two irreducible sources of G_{LF} breaking, λ_e and g_ν, defined by

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij}\, \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}}\, g_\nu^{ij} (\bar{L}_L^{ci} \tau_2 H)(H^T \tau_2 L_L^j) + \text{h.c.}$$

3. The scale of LNV is large compared to the scale of LFV, $\Lambda_{\text{LF}} \gg \Lambda_{\text{LN}}$

$$\mathcal{L}_{\text{LFV}} = \frac{1}{\Lambda_{\text{LFV}}^2} \sum C_i O_i$$

Implementation of MLFV in Minimal Field Content Case

- Add all possible terms to the lagrangian consistent with assumptions (also: Lorentz invariance, gauge symmetry, locality, ...)
- Characterize allowed operators by spurion method

$$L_L \to V_L\, L_L \qquad e_R \to V_R\, e_R$$
$$\lambda_e \to V_R\, \lambda_e V_L^\dagger \qquad g_\nu \to V_L^*\; g_\nu V_L^\dagger$$

(recall: $\mathcal{L}_{\rm Sym.Br.} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}}\,g_\nu^{ij}(\bar{L}_L^{ci}\tau_2 H)(H^T\tau_2 L_L^j) + \text{h.c.}$)

- All ops for $\mu \to e\gamma, \quad \mu + N \to e + N'$, have at least one factor of $\Delta \equiv g_\nu^\dagger g_\nu$ (neglect Δ^2); $\quad m_\nu \sim v^2 g_\nu/\Lambda_{LN}$

- Amplitudes are given in terms of
  - Λ_{LN} and Λ_{LFV} (actually only ratio $\Lambda_{LN}/\Lambda_{LFV}$)
  - Coefficients, C, of order 1
  - Low energy measured (or measurable) masses and mixing angles
- In particular, the following two combinations appear in the operators (δ appears in 4L ops, eg, $\mu \to ee\bar{e}$):

$$\Delta = \frac{\Lambda_{LN}^2}{v^4} U m_\nu^2 U^\dagger \qquad \delta = \delta^T = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger$$

U= PMNS mixing matrix, m_ν diagonal

MLFV: Extended Field Content

Ex: SUSY with RH degenerate N's, J. Hisano et al, Phys. Rev. D **53**, 2442–2459 (1996)

Recall, now we include RH neutrinos, flavor group has additional $SU(3)_{\nu R}$ factor

Assumptions:

1. The right handed neutrino mass is flavor neutral, *ie*, it breaks $SU(3)_{\nu R}$ to $O(3)_{\nu R}$. Denote $M_\nu^{ij} = M_\nu \delta^{ij}$
2. The right handed neutrino mass is the only source of LN breaking and $M_\nu \gg \Lambda_{LFV}$
3. Remaining LF-symmetry broken only by λ_e and λ_ν defined by

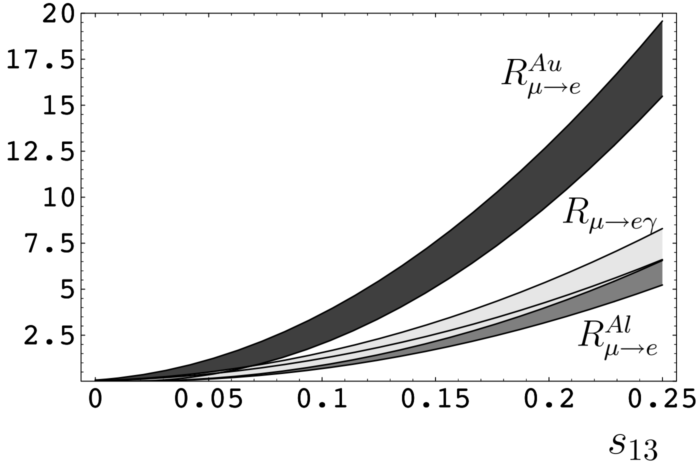
$$\mathcal{L}_{\mathrm{Sym.Br.}} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) + i\lambda_\nu^{ij}\,\bar{\nu}_R^i(H^T\tau_2 L_L^j) + \mathrm{h.c.}$$

MLFV: Phenomenology

- Future experiments will (continue to) look for flavor changing neutral interactions in the charged lepton sector:
  - MECO ... was cancelled
  - PRIME at the PRISM muon facility at JPARC will measure μ-to-e conversion at 10^{-18} sensitivity
  - MEG at PSI looks for $\mu^+ \to e^+ \gamma$ at 10^{-13} single event sensitivity

COBRA(Constant Bending Radius Spectrometer)

Sample results, minimal field content



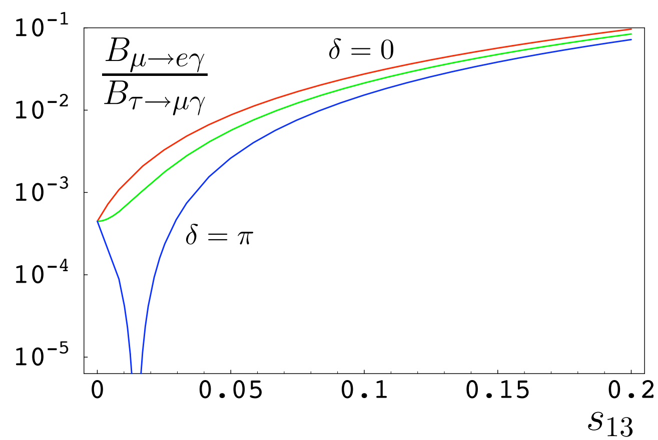
$$B_{\ell_i\to\ell_j(\gamma)} = 10^{-50}\left(\frac{\Lambda_{\rm LN}}{\Lambda_{\rm LFV}}\right)^4 R_{\ell_i\to\ell_j(\gamma)}(s_{13},\delta;c^{(i)})$$

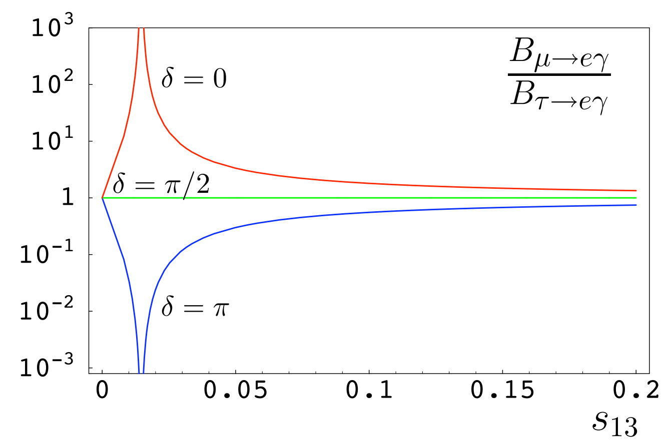
- s_{13} and δ unknown PMNS parameters (scan on δ)
- choose $c^{(i)}$ of order one for the estimate
- ratio of scales can be large

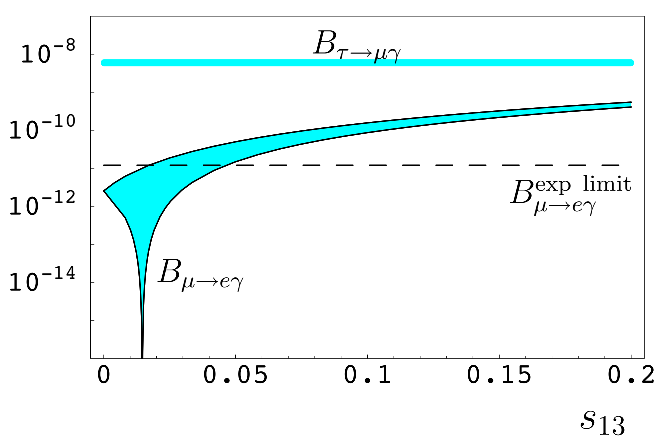
  perturbative $g_\nu \Rightarrow$ $\Lambda_{\rm LN} \lesssim 3\times 10^{13}(1\ {\rm eV}/m_\nu)$ GeV

  so $\Lambda_{\rm LFV}\sim 1$ TeV $\Rightarrow \Lambda_{\rm LN}/\Lambda_{\rm LFV}\lesssim 10^{10}$

Predictive:
patterns are independent
of unknown input parameters







If s_{13} is small, look at tau modes.
Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c^{(1)}_{RL} - c^{(2)}_{RL} = 1$

quick example (probably out of time by now):

$$\tau \to \mu\gamma, \quad \tau \to e\gamma \quad \& \quad \mu \to e\gamma$$

$$\Delta\mathcal{L}_{\text{eff}} = \frac{v}{\Lambda^2}\bar{e}_R\left[\, c_1 \lambda_e \lambda_1^\dagger \lambda_1 + c_2 \lambda_u \lambda_u^\dagger \lambda_e + c_3 \lambda_u \lambda_u^\dagger \lambda_d^T \right] \sigma^{\mu\nu} e_L F_{\mu\nu}$$

just like MLFV above

Generalizes Barbieri & Hall

New mixing structures

$$C = V_{e_R}^T V_{d_L}$$

$$G = V_{e_L}^T V_{d_R}$$

Independent of M_ν

Hierarchical

$$\left(\frac{m_t^2}{v^2}\right) \times \begin{cases} \lambda^2 (m_\tau / v), & (\tau \to \mu) \\ \lambda^3 (m_\tau / v), & (\tau \to e) \\ \lambda^5 (m_\mu / v), & (\mu \to e) \end{cases}$$

$(\lambda = 0.22)$

Large: for Λ=10TeV

$$\mathrm{Br}(\mu \to e\gamma) \sim 10^{-12}$$

MFV/MLFV and GUTs

- Quarks/leptons in same reps of G_{GUT}, eg., $\psi \sim \bar{5}, \chi \sim 10$
- Flavor symmetry smaller: $SU(3)^2$
- Broken by Yukawas, including trans-PL term for light quark/lepton mass relations $(H \sim 5, \Sigma \sim 24)$

$$\mathcal{L}_{\rm sym.br.} = \psi^T \lambda_5 \chi H^* + \chi^T \lambda_{10} \chi H + \frac{1}{M} \psi^T \lambda_5' \Sigma \chi H^*$$

- Right handed neutrino for see-saw (not mandatory); extends G_F to $SU(3)^3$, and

$$\Delta\mathcal{L}_{\rm sym.br.} = N^T \lambda_1 \psi H + N^T M_R N$$

- MFV/MLFV: GF broken only by

$$\lambda_5, \lambda_5', \lambda_{10}, \lambda_1, M_R$$

Implementation similar,

$$Q_L \rightarrow V_{10}\ Q_L$$
$$u_R \rightarrow V_{10}^*\ u_R$$
$$d_R \rightarrow V_{\bar{5}}^*\ d_R$$
$$L_L \rightarrow V_{\bar{5}}\ \ L_L$$
$$e_R \rightarrow V_{10}^*\ e_R$$

$$\lambda_{10} \rightarrow V_{10}^*\ \ \lambda_{10}\ V_{10}^\dagger$$
$$\lambda_5 \rightarrow V_{\bar{5}}^*\ \ \lambda_5\ V_{10}^\dagger$$
$$\lambda_5' \rightarrow V_{\bar{5}}^*\ \ \lambda_5'\ \ V_{10}^\dagger$$
$$\lambda_1 \rightarrow V_1^*\ \ \lambda_1\ V_{\bar{5}}^\dagger$$
$$M_R \rightarrow\ V_1^*\ \ M_R\ V_1^\dagger$$

but trade parameters for low energy ones, eg.,

$$\lambda_u = a_u \left[\lambda_{10} + \epsilon_{1u}\lambda_{10}\lambda_{10}^\dagger\lambda_{10} + \epsilon_{2u}^{('')}\lambda_{10}(\lambda_5^{(')}\ )^\dagger\lambda_5^{(')}\ \ + \ldots\right]$$
$$\lambda_d = a_d \left[\left(\lambda_5 + \lambda_5'\ \right) + \epsilon_{d1}'\lambda_5'\ + \epsilon_{d2}^{(')}\lambda_5^{(')}\ \lambda_{10}^\dagger\lambda_{10} + \epsilon_{d3}^{(')}\lambda_1^T\lambda_1^*\lambda_5^{(')}\ \ + \ldots\right]$$
$$\lambda_e^T = a_e \left[\left(\lambda_5 - \frac{3}{2}\lambda_5'\ \right) + \epsilon_{e1}'\lambda_5'\ + \epsilon_{e2}^{(')}\lambda_5^{(')}\ \lambda_{10}^\dagger\lambda_{10} + \epsilon_{e3}^{(')}\lambda_1^T\lambda_1^*\lambda_5^{(')}\ \ + \ldots\right]$$

FCNCs: Bilinears as building blocks of operators

Old

quarks: $\bar{Q}_L \lambda_u^\dagger \lambda_u Q_L\,, \qquad \bar{d}_R \lambda_d \lambda_u^\dagger \lambda_u Q_L$

leptons: $\bar{L}_L \lambda_1^\dagger \lambda_1 L_L\,, \qquad \bar{e}_R \lambda_e \lambda_1^\dagger \lambda_1 L_L$

New (plus replace anywhere $\lambda_e \leftrightarrow \lambda_d^T$)

quarks: $\bar{Q}_L (\lambda_e \lambda_e^\dagger)^T Q_L$

$\bar{d}_R \lambda_e^T (\lambda_e \lambda_e^\dagger)^T Q_L\,, \qquad \bar{d}_R (\lambda_e \lambda_1^\dagger \lambda_1)^T Q_L$

$\bar{d}_R (\lambda_e^\dagger \lambda_e)^T d_R\,, \qquad \bar{d}_R (\lambda_1^\dagger \lambda_1)^T d_R$

leptons: $\bar{L}_L (\lambda_d \lambda_d^\dagger)^T L_L$

$\bar{e}_R (\lambda_d \lambda_d^\dagger \lambda_d)^T L_L, \qquad \bar{e}_R \lambda_u \lambda_u^\dagger \lambda_d^T L_L$

$\bar{e}_R \lambda_u \lambda_u^\dagger e_R\,, \qquad \bar{e}_R (\lambda_d^\dagger \lambda_d)^T e_R$

Summary and Conclusions

- Notion of MLFV for see-saw models formulated
- Two approaches: minimal and extended field content
- MFV in GUTs formulated
- Predictive (in extended f.c. case need CP limit and M-diagonal assumptions)
- All or nothing: effects observable in near future rare process experiments iff LFV scale low (LHC) and LNV scale as large as possible (perturbative couplings)
- A lot to study: mixing, non-CP limit and leptogenesis, LHC physics, other versions...

The End

backup material

Write all operators of dimension 5, 6, ... consistent with assumptions.

For $\mu \to e\gamma, \quad \mu + N \to e + N'$, need two lepton field ops:

Ops with LL

$$O_{LL}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \; H^\dagger i D_\mu H$$
$$O_{LL}^{(2)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \; H^\dagger \tau^a i D_\mu H$$
$$O_{LL}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{Q}_L \gamma_\mu Q_L$$
$$O_{LL}^{(4d)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{d}_R \gamma_\mu d_R$$
$$O_{LL}^{(4u)} = \bar{L}_L \gamma^\mu \Delta L_L \; \bar{u}_R \gamma_\mu u_R$$
$$O_{LL}^{(5)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \; \bar{Q}_L \gamma_\mu \tau^a Q_L$$

Ops with RL

$$O_{RL}^{(1)} = g' H^\dagger \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, B_{\mu\nu}$$
$$O_{RL}^{(2)} = g H^\dagger \bar{e}_R \sigma^{\mu\nu} \tau^a \lambda_e \Delta L_L \, W^a_{\mu\nu}$$
$$O_{RL}^{(3)} = (D_\mu H)^\dagger \bar{e}_R \lambda_e \Delta D_\mu L_L$$
$$O_{RL}^{(4)} = \bar{e}_R \lambda_e \Delta L_L \, \bar{Q}_L \lambda_D d_R$$
$$O_{RL}^{(5)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, \bar{Q}_L \sigma_{\mu\nu} \lambda_D d_R$$
$$O_{RL}^{(6)} = \bar{e}_R \lambda_e \Delta L_L \, \bar{u}_R \lambda_U^\dagger i \tau^2 Q_L$$
$$O_{RL}^{(7)} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \, \bar{u}_R \sigma_{\mu\nu} \lambda_U^\dagger i \tau^2 Q_L$$

Have used $\Delta \equiv g_\nu^\dagger g_\nu$ with transformation $\Delta \to V_L \Delta V_L^\dagger$

Also neglected Δ^2

We have neglected $\sim (\lambda_e)^2$, hence no RR operators

For $\mu \to ee\bar{e}$ need four lepton operators

$$O_{4L}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \, \bar{L}_L \gamma_\mu L_L$$

$$O_{4L}^{(2)} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \, \bar{L}_L \gamma_\mu \tau^a L_L$$

$$O_{4L}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \, \bar{e}_R \gamma_\mu e_R$$

$$O_{4L}^{(4)} = \delta_{nj} \delta_{mi}^* \bar{L}_L^i \gamma^\mu L_L^j \, \bar{L}_L^m \gamma^\mu L_L^n$$

$$O_{4L}^{(5)} = \delta_{nj} \delta_{mi}^* \bar{L}_L^i \gamma^\mu \tau^a L_L^j \, \bar{L}_L^m \gamma^\mu \tau^a L_L^n$$

where we used $\delta = g_\nu$ (so we can use same expressions for extended field content case)

Up to dimension 6 operators, the new interactions are

$$\mathcal{L}_{\rm eff} = \frac{1}{\Lambda^2_{\rm LFV}} \sum_{i=1}^{5} \left(c^{(i)}_{LL} O^{(i)}_{LL} + c^{(i)}_{4L} O^{(i)}_{4L} \right) + \frac{1}{\Lambda^2_{\rm LFV}} \left(\sum_{j=1}^{2} c^{(j)}_{RL} O^{(j)}_{RL} + \text{h.c.} \right)$$

with coefficients naively $c \sim 1$

We can now study the phenomenology of MLFV with minimal field content.

Useful to look at parameters first

Use G_{LF} symmetry to rotate to the mass eigenstate basis (v = Higgs vev)

$$\lambda_e = \frac{m_\ell}{v} = \frac{1}{v}\,\mathrm{diag}(m_e, m_\mu, m_\tau)$$

$$g_\nu = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger = \frac{\Lambda_{LN}}{v^2} U^* \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$$

U is the PMNS matrix. It is determined from neutrino mixing:

$$U \approx \begin{pmatrix} c e^{i\alpha_1/2} & s e^{i\alpha_2/2} & s_{13} e^{-i\delta} \\ -s e^{i\alpha_1/2}/\sqrt{2} & c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\ s e^{i\alpha_1/2}/\sqrt{2} & -c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

Here $c \equiv \cos\theta_{\rm sol} \quad s \equiv \sin\theta_{\rm sol} \quad \theta_{\rm sol} \simeq 32.5^\circ$

s_{13} is poorly known, $s_{13} < 0.3$

Implementation of MLFV in Extended Field Content Case

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda_e^{ij}\,\bar{e}_R^i(H^\dagger L_L^j) + i\lambda_\nu^{ij}\bar{\nu}_R^i(H^T\tau_2 L_L^j) + \text{h.c.}$$

Same as before, but now transformations are:

$$L_L \to V_L\,L_L \qquad e_R \to V_R\,e_R \qquad \nu_R \to O_\nu\,\nu_R$$

$$\lambda_e \to V_R\,\lambda_e V_L^\dagger \qquad \lambda_\nu \to O_\nu\,\lambda_\nu V_L^\dagger$$

As before $\Delta = \lambda_\nu^\dagger \lambda_\nu \qquad \Delta \to V_L \Delta V_L^\dagger$

but now not directly related to mass matrix $m_\nu = \frac{v^2}{M_\nu}\lambda_\nu^T\lambda_\nu$

However $\delta = \lambda_\nu^T\lambda_\nu \qquad \delta \to V_L^*\,\delta\,V_L^\dagger$

In CP limit $\lambda_\nu^* = \lambda_\nu$ and $\Delta = \lambda_\nu^T\lambda_\nu$

- Same operator basis as before (chose Δ and δ by transformation properties)

- Same effective lagrangian, but with $\Lambda_{NL} \rightarrow M_\nu$

- Summary: In mass eigenstate basis

$$\Delta = \begin{cases} \frac{\Lambda_{\rm LN}^2}{v^4}\; U m_\nu^2 U^\dagger & \text{minimal field content} \\ \frac{M_\nu}{v^2}\; U m_\nu U^\dagger & \text{extended field content, CP limit} \end{cases}$$

$$\delta = \delta^T = \begin{cases} \frac{\Lambda_{\rm LN}}{v^2}\, U^* m_\nu U^\dagger & \text{minimal field content} \\ \frac{M_\nu}{v^2}\, U^* m_\nu U^\dagger & \text{extended field content} \end{cases}$$

MFV by example: consider $K_L \to \pi\nu\nu$

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$\mathcal{C}^\ell = \frac{\alpha X(\frac{m_t}{M_W})}{2\pi \sin^2\theta_W} V_{ts}^* V_{td}$$

CKM factor

1 loop factor, $X \sim 1$

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$V_{\rm CKM} \approx \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\bar\rho - i\bar\eta) \\ -\lambda(1+iA^2\lambda^4\bar\eta) & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\bar\rho-i\bar\eta) & -A\lambda^2(1+i\lambda^2\bar\eta) & 1 \end{pmatrix} + \mathcal{O}(\lambda^6).$$

$$\lambda \simeq 0.22 \qquad |V_{ts}V_{td}| \sim A^2\lambda^5$$

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

New physics

$$\mathcal{H}_{\rm eff} = \frac{1}{\Lambda_F^2} \sum_{\ell=e,\mu,\tau} \mathcal{C}^\ell_{\rm new} \bar{s}_L \gamma_\mu d_L \, \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

with $\mathcal{C}^\ell_{\rm new} \sim 1$

Assume sensitivity to fractional deviation *r* from SM rate:

$$1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2$$

For example, *r* = 4% gives sensitivity to $\Lambda_F \sim 10^6$ GeV

(r=4% is KOPIO's would be sensitivity)

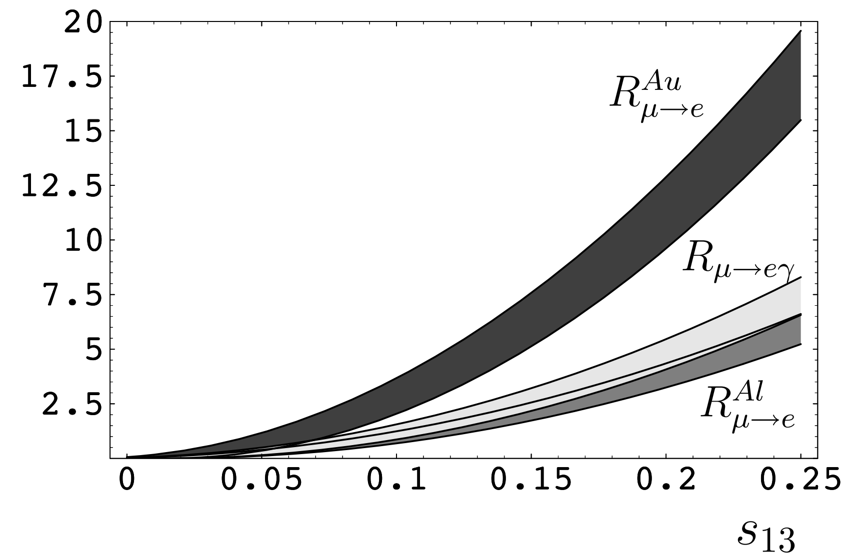
- The small factor comes from CKM
- Generalized GIM mechanism
  - old GIM: smallness of $V_{cs}V_{cd}^*(m_c^2 - m_u^2)/M_W^2$
  - new GIM: smallness of 1-to-3 generation jump
- If new physics respects this then the same small CKM factor appears. New estimate

$$1 + r \sim \left|1 + \frac{(M_W/\Lambda)^2}{1/(16\pi^2)}\right|^2$$

And now *r* = 4% gives sensitivity to $\Lambda_F \sim 10^{3\text{-}4}$ GeV

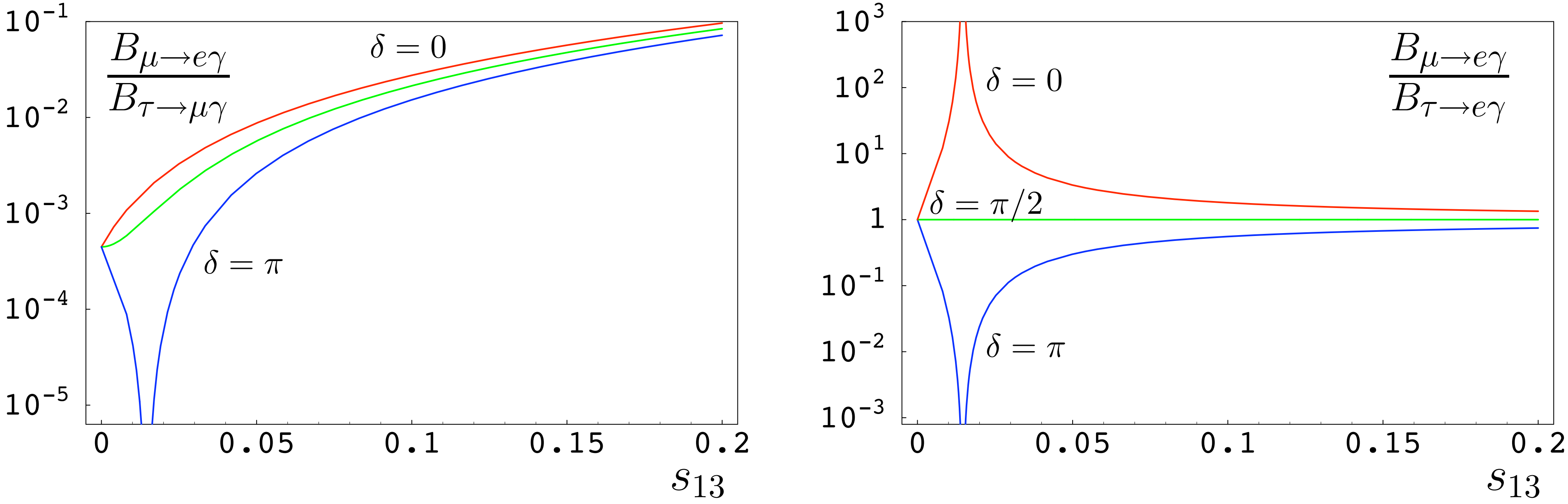
μ→eγ, μ-to-e conversion and their relatives I: minimal field content

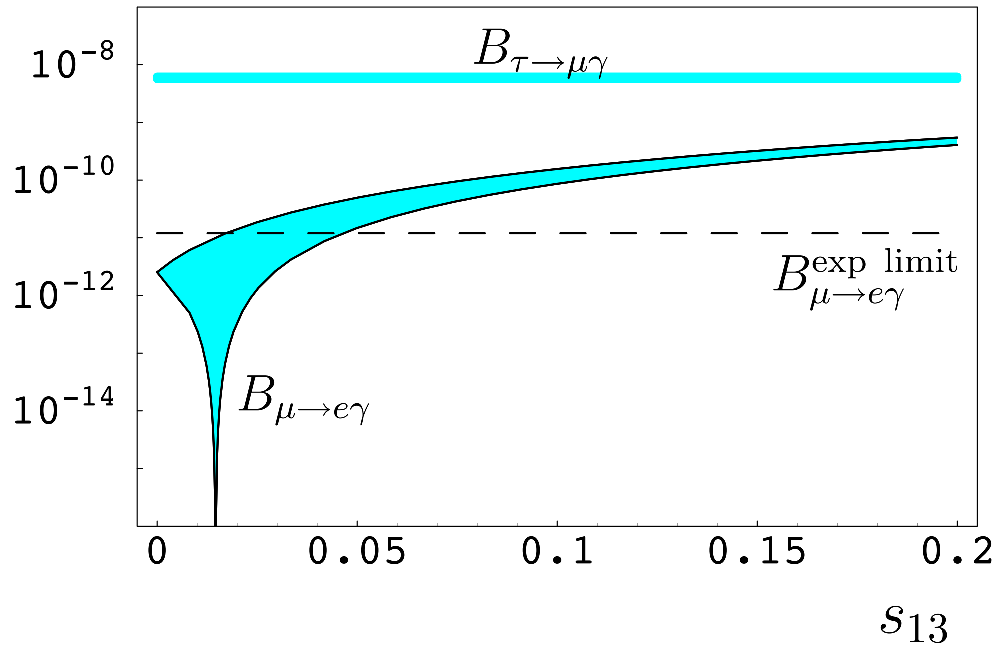
$$B_{\ell_i\to\ell_j(\gamma)} = 10^{-50}\left(\frac{\Lambda_{\rm LN}}{\Lambda_{\rm LFV}}\right)^4 R_{\ell_i\to\ell_j(\gamma)}(s_{13},\delta;c^{(i)})$$



- since $\Delta \propto U(m_\nu)^2 U^\dagger$, only differences of m^2 enter; these are measured!
- s_{13} and δ unknown PMNS parameters (scan on δ)
- choose $c^{(i)}$ of order one for the estimate
- ratio of scales can be large:
  perturbative $g_\nu \Rightarrow\ \Lambda_{\rm LN} \lesssim 3\times 10^{13}(1\ {\rm eV}/m_\nu)$ GeV
  so $\Lambda_{\rm LFV} \sim 1$ TeV $\Rightarrow \Lambda_{\rm LN}/\Lambda_{\rm LFV} \lesssim 10^{10}$

Predictive: l→l′ γ patterns are independent of unknown input parameters (scales cancel in ratios, in this case c(i)'s cancel too, and all other parameters are from long distance)



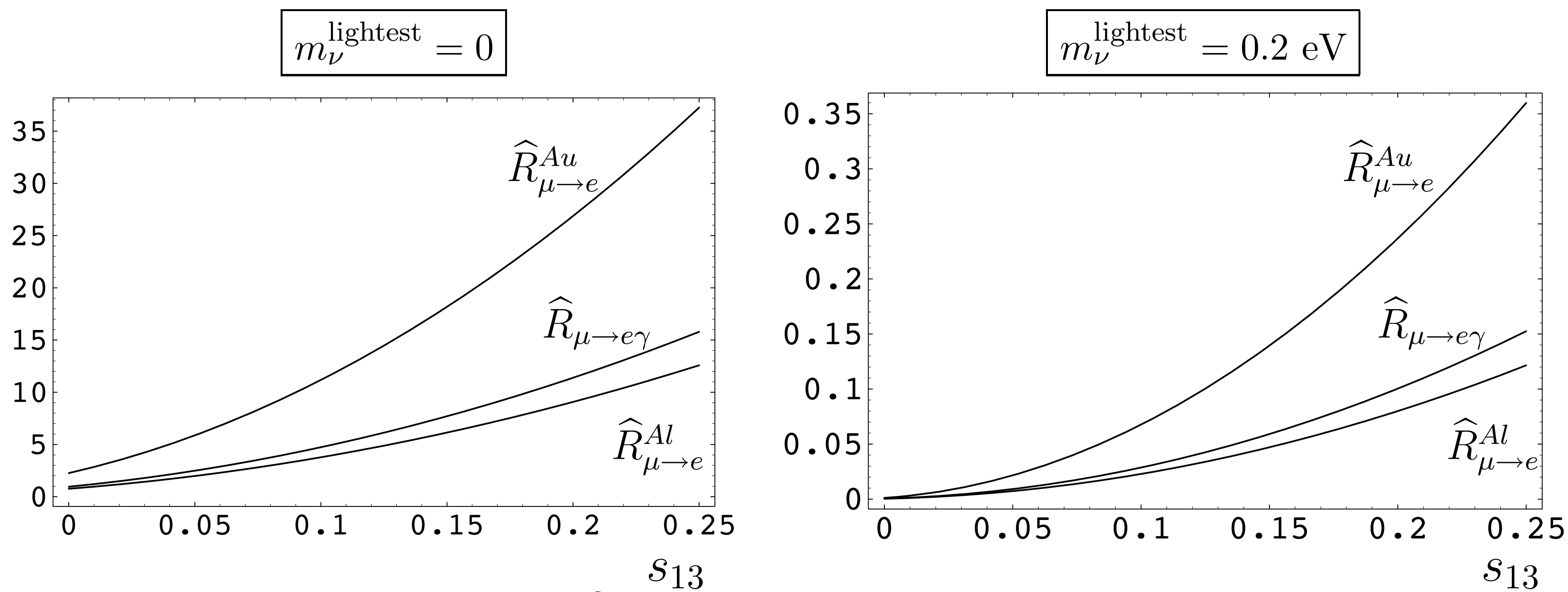


If s_{13} is small, look at tau modes.
Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c^{(1)}_{RL} - c^{(2)}_{RL} = 1$

Belle and BaBar have recent bounds (summer '05) of a few × 10^{-7} for Br(τ→lγ) and Br(τ→lll)

$\mu \to e\gamma$, μ-to-e conversion and their relatives II: extended field content

- Replace $\Lambda^2_{\rm LN}/\Lambda^2_{\rm LFV}$ by $vM_\nu/\Lambda^2_{\rm LFV}$
- Now $\Delta \propto U\, m_\nu\, U^\dagger$ so amplitudes depend on overall neutrino mass scale (ie, lightest neutrino mass)

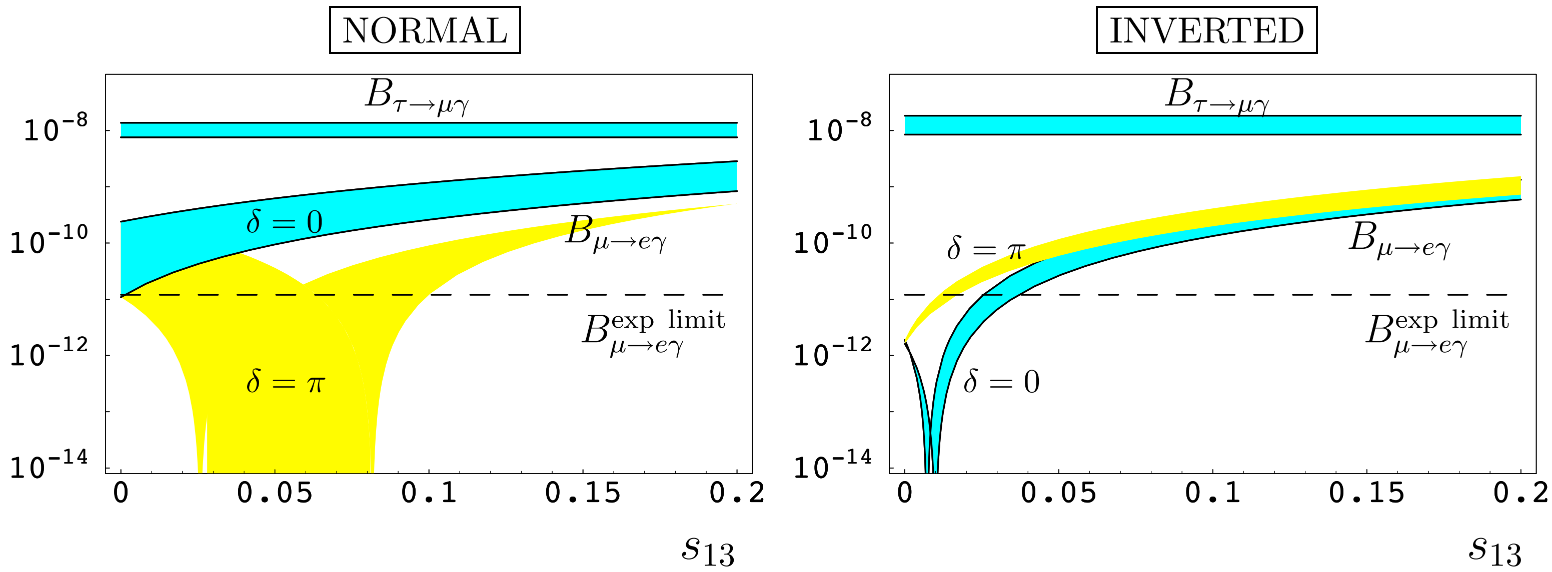


$$B_{\ell_i\to\ell_j(\gamma)} = 10^{-25}\left(\frac{vM_\nu}{\Lambda^2_{\rm LFV}}\right)^2 \widehat{R}_{\ell_i\to\ell_j(\gamma)}(s_{13}, m_\nu^{\rm lightest}; c^{(i)})$$

perturbative $\lambda_\nu \Rightarrow M_\nu \lesssim 10^{13}$ GeV; with $\Lambda_{LFV} \geq 1$ TeV, $\dfrac{vM_\nu}{\Lambda^2_{LFV}} \leq 10^9$

One final note: results depend on hierarchy of neutrino masses,

normal ($m_{\nu 1} \sim m_{\nu 2} \ll m_{\nu 3}$) vs. *inverted* ($m_{\nu 1} \ll m_{\nu 2} \sim m_{\nu 3}$)



$$(vM_\nu)/\Lambda_{\mathrm{LFV}}^2 = 5\times 10^7$$

$$c_{RL}^{(1)} - c_{RL}^{(2)} = 1$$

$$\text{shading:}\quad 0 \le m_\nu^{\mathrm{lightest}} \le 0.02\ \mathrm{eV}$$

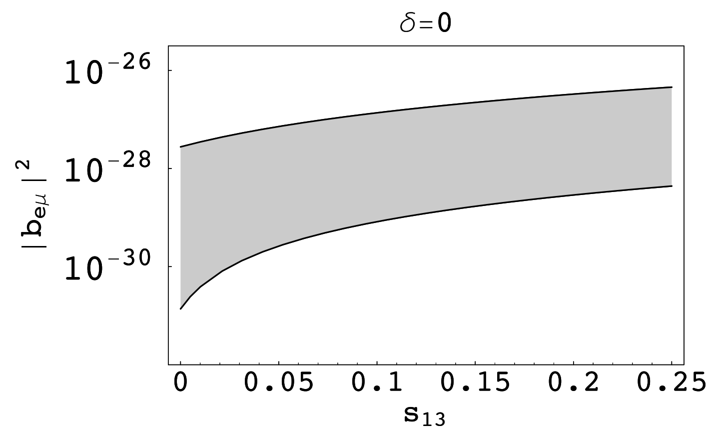
3l Decays: 4L operators

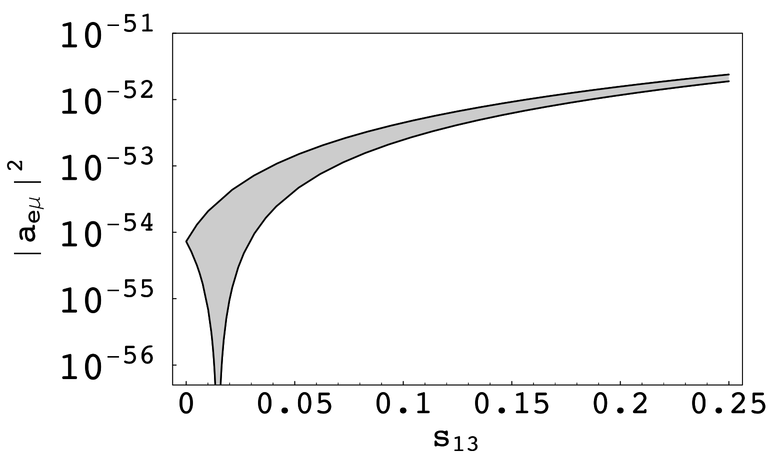
$$\Gamma_{\mu\to 3e}/\Gamma_{\mu\to e\nu\bar{\nu}} = \left[|a_+|^2+2|a_-|^2-8\mathrm{Re}(a_0^* a_-)-4\mathrm{Re}(a_0^* a_+)+6I|a_0|^2\right] \begin{cases} \left(\frac{\Lambda_{\mathrm{LN}}}{\Lambda_{\mathrm{LFV}}}\right)^4 |a_{e\mu}|^2 & \text{minimal} \\ \left(\frac{vM_\nu}{\Lambda_{\mathrm{LFV}}{}^2}\right)^2 |b_{e\mu}|^2 & \text{extended} \end{cases}$$

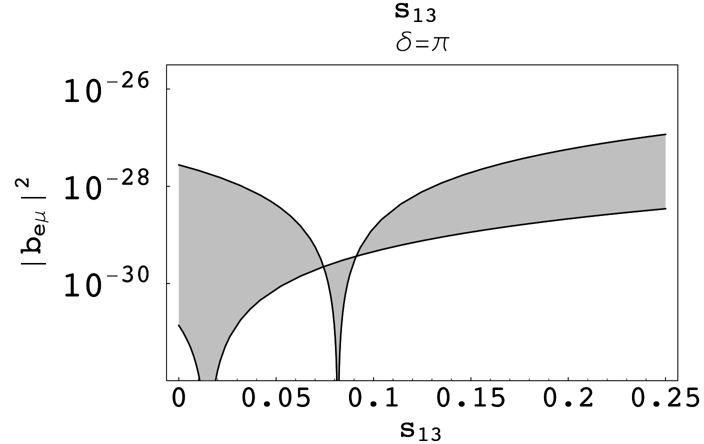
$$a_+ = \sin^2\theta_w (c_{LL}^{(1)} + c_{LL}^{(2)}) + c_{4L}^{(3)}$$

$$a_- = (\sin^2\theta_w - \frac{1}{2})(c_{LL}^{(1)} + c_{LL}^{(2)}) + c_{4L}^{(1)} + c_{4L}^{(2)} + \frac{2\delta_{e\mu}\delta_{ee}^*}{\Delta_{e\mu}}(c_{4L}^{(4)} + c_{4L}^{(5)})$$

$$a_0 = 2e^2(c_{RL}^{(1)} - c_{RL}^{(2)})^*$$

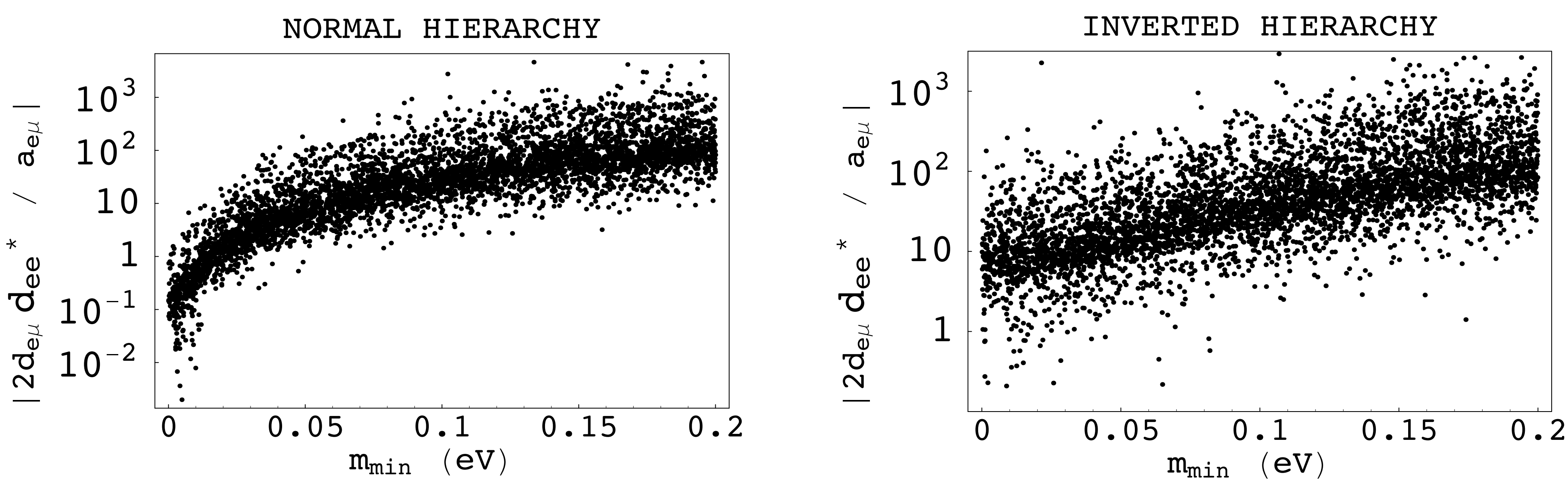




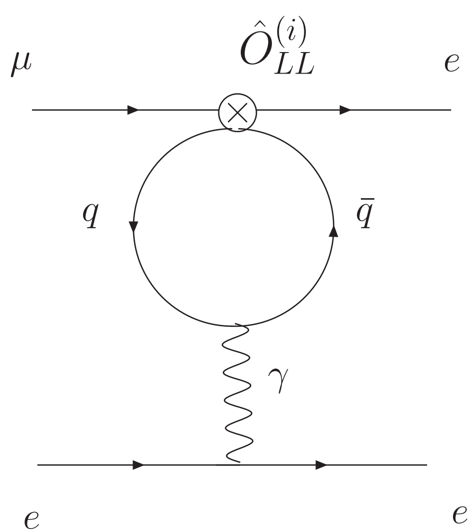


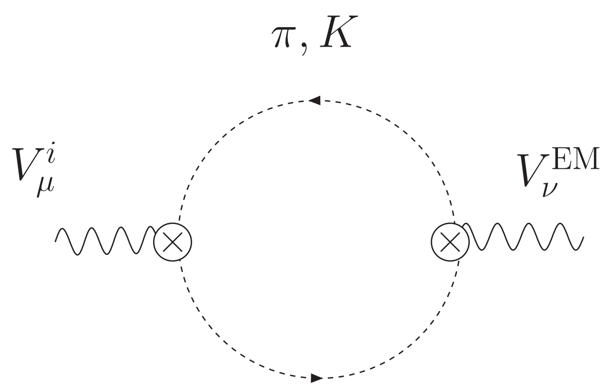
$$\Gamma_{\tau\to e\mu\bar{\mu}} = \Gamma_{\tau\to e\nu\bar{\nu}}\ \frac{v^4|\Delta_{e\tau}|^2}{\Lambda^4_{\rm LFV}}\ \left[|a_+|^2 + |\tilde{a}_-|^2 - 4{\rm Re}[a_0^*(a_+ + \tilde{a}_-)] + 12\tilde{I}|a_0|^2\right]$$

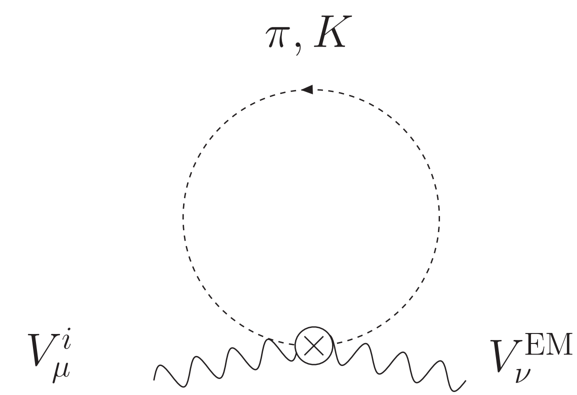
$$\Gamma_{\tau\to\mu\mu\bar{e}} = \Gamma_{\tau\to e\nu\bar{\nu}}\frac{v^4|2\delta_{e\tau}\delta_{\mu\mu}|^2}{\Lambda^4_{\rm LFV}}|c_L^{(4)} + c_L^{(5)}|^2$$



- We have also explored the effects of deleting a class of operators.
- For example: assume 4L operators are not present
- Can we get 3l decays? Yes, through loops
- Need care in loops of light quarks: chiral lagrangian does the job
- Result: amplitude is ~0.1 of 4L ops (large logs)
- Equivalently, this give a ~20% correction to rate
- Patterns are similar to those from 4L







Decays of/to hadrons

Hopelessly small!

	Br
$\pi^0 \to \mu^+ e^-$	10^{-25}
$\Upsilon \to \tau\mu$	10^{-20}
$\tau \to \pi\mu$	10^{-15}