Minimal Lepton Flavor Violation

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DPF, Hawaii, Oct 2006

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Rationale for MFV (Minimal Flavor Violation)

- Buchmuller & Wyler: order $1/\Lambda^2$ terms in $H_{\text{eff}}$
- If $\Lambda$ is related to EW-breaking, or higgs hierarchy problem, expect $\Lambda \sim 10^{3-4}$ GeV
- All fine, except processes with FCNC
- To avoid FCNC need small coefficients $C \sim 10^{-(2-3)}$, or large scale $\Lambda \sim 10^{6-7}$ GeV
- If $\Lambda \sim 10^{6-7}$ GeV then no relation to hierarchy
- Can we make $C$ naturally small?
Minimal Flavor Violation (MFV)

- Premise: Unique source of flavor braking
- Quark sector in SM, in absence of masses has large flavor (global) symmetry: $G_F = SU(3)^3 \times U(1)^2$
- Symmetry is only broken by Yukawa interactions, parametrized by couplings $\lambda_U$ and $\lambda_D$
- MFV: all breaking of $G_F$ must transform as these
- When going to mass eigenstate basis, all mixing is parametrized by CKM and GIM is automatic
How does this work?
Consider \( K_L \to \pi \nu \bar{\nu} \)

Recall, \( G_F \) breaking from:
\[
\mathcal{L}_{\text{Yuk}} = H \bar{q}_L \lambda_U u_R + \tilde{H} \bar{q}_L \lambda_D d_R
\]

\( G_F \), using spurion method:
\[
\begin{align*}
q_L & \to V_L q_L \\
u_R & \to V_u u_R \\
d_R & \to V_d d_R
\end{align*}
\]

Effective lagrangian
\[
\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda^2} \sum C_i O_i
\]

where the operator is, for example
\[
O = \bar{q}_L (\lambda_U \lambda_{U}^\dagger) \gamma_\mu q_L \bar{\nu}_L \gamma^\mu \nu_L
\]

In mass basis \( \Rightarrow \)
\[
\left( \sum_{q=u,c,t} V_{qs}^* V_{qd} \frac{m_q^2}{v^2} \right) \bar{s}_L \gamma_\mu d_L \bar{\nu}_L \gamma^\mu \nu_L
\]

Since \( |V_{ts} V_{td}| m_t^2 / v^2 \approx A^2 \lambda^5 \approx 5 \times 10^{-4} \) so effectively \( C \sim 10^{-3} \)
Minimal Lepton Flavor Violation (MLFV)

- Rationale: elevate MFV to a principle
- If leptons acquire Dirac masses (mimic quark sector) copy from above. But uninteresting: flavor violation proportional to tiny neutrino masses
- More attractive/interesting: Small neutrino masses from see-saw mechanism.
- What are the restrictions from MLFV in see-saw models?

Two cases. Field content (below LNV scale):
- Minimal: three $L_i$ and $e_{Ri}$
- Extended: three $L_i$, $e_{Ri}$, and $\nu_{Ri}$
A Note on: LN vs LF

• Distinguish Lepton Number (LN) violating interactions from Lepton Flavor (LF) violating interactions

• LN is a $U(1)$ symmetry, assigning unit charge to all leptons.
  • Neutrino Majorana mass breaks LN

• LF is an $SU(3)$ symmetry, mixing different flavors
  • It commutes with $U(1)_{LN}$, ie, preserves the LN charge
MLFV: Minimal Field Content

Assumptions:

1. The breaking of the $U(1)_{LN}$ is independent from the breaking of lepton flavor $G_{LF}$, with large $\Lambda_{LN}$ (associated with see-saw)

2. There are only two irreducible sources of $G_{LF}$ breaking, $\lambda _e$ and $g_\nu$, defined by

$$L_{Sym.\text{Br.}} = -\lambda _e^{ij} \bar{e}_R^i (H^\dagger L^j_L) - \frac{1}{2\Lambda _{LN}} g_\nu^{ij} (\bar{L}_L^c i \tau_2 H)(H^T \tau_2 L^j_L) + h.c.$$ 

3. The scale of LNV is large compared to the scale of LFV, $\Lambda_{LF} \gg \Lambda_{LN}$

$$L_{LFV} = \frac{1}{\Lambda_{LFV}^2} \sum C_i O_i$$
Implementation of MLFV in Minimal Field Content Case

• Add all possible terms to the lagrangian consistent with assumptions (also: Lorentz invariance, gauge symmetry, locality, ...)

• Characterize allowed operators by spurion method

\[
\begin{align*}
L_L &\to V_L L_L & e_R &\to V_R e_R \\
\lambda_e &\to V_R \lambda_e V_L^\dagger & g_\nu &\to V_L^* g_\nu V_L^\dagger
\end{align*}
\]

(recall: \( L_{\text{Sym.Br.}} = -\lambda_i^j \bar{e}_R^i (H^\dagger L_L^j) - \frac{1}{2\Lambda_{LN}} g_{ij}^\nu (\bar{L}^{ci}_L \tau_2 H) (H^T \tau_2 L_L^j) + \text{h.c.} \) )

• All ops for \( \mu \to e\gamma, \mu + N \to e + N', \) have at least one factor of \( \Delta \equiv g_\nu^\dagger g_\nu \) (neglect \( \Delta^2 \)); \( m_\nu \sim v^2 g_\nu / \Lambda_{LN} \)
- Amplitudes are given in terms of
- $\Lambda_{LN}$ and $\Lambda_{LFV}$ (actually only ratio $\Lambda_{LN}/\Lambda_{LFV}$)
- Coefficients, C, of order 1
- Low energy measured (or measurable) masses and mixing angles

- In particular, the following two combinations appear in the operators ($\delta$ appears in 4L ops, eg, $\mu \to eee\bar{e}$):

  $\Delta = \frac{\Lambda_{LN}^2}{v^4} U m^2_\nu U^\dagger$
  $\delta = \delta^T = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger$

  $U =$ PMNS mixing matrix, $m_\nu$ diagonal
MLFV: Extended Field Content

Recall, now we include RH neutrinos, flavor group has additional $SU(3)_{VR}$ factor

Assumptions:

1. The right handed neutrino mass is flavor neutral, i.e., it breaks $SU(3)_{VR}$ to $O(3)_{VR}$. Denote $M^{ij}_\nu = M_\nu \delta^{ij}$

2. The right handed neutrino mass is the only source of LN breaking and $M_\nu \gg \Lambda_{LFV}$

3. Remaining LF-symmetry broken only by $\lambda_e$ and $\lambda_\nu$ defined by

$$\mathcal{L}_{\text{Sym.Br.}} = -\lambda^{ij}_e \bar{e}^i_R (H^\dagger L^j_L) + i \lambda^{ij}_\nu \bar{\nu}^i_R (H^T \tau_2 L^j_L) + \text{h.c.}$$
MLFV: Phenomenology

• Future experiments will (continue to) look for flavor changing neutral interactions in the charged lepton sector:

• MECO ... was cancelled

• PRIME at the PRISM muon facility at JPARC will measure $\mu$-to-e conversion at $10^{-18}$ sensitivity

• MEG at PSI looks for $\mu^+ \rightarrow e^+ \gamma$ at $10^{-13}$ single event sensitivity
Sample results, minimal field content

\[ B_{\ell_i \rightarrow \ell_j}(\gamma) = 10^{-50} \left( \frac{\Lambda_{LN}}{\Lambda_{LFV}} \right)^4 R_{\ell_i \rightarrow \ell_j}(\gamma)(s_{13}, \delta; c^{(i)}) \]

- $s_{13}$ and $\delta$ unknown PMNS parameters (scan on $\delta$)
- choose $c^{(i)}$ of order one for the estimate
- ratio of scales can be large
  
  perturbative $g_\nu \Rightarrow \Lambda_{LN} \lesssim 3 \times 10^{13}(1 \text{ eV}/m_\nu)$ GeV

so $\Lambda_{LFV} \sim 1 \text{ TeV} \Rightarrow \Lambda_{LN}/\Lambda_{LFV} \lesssim 10^{10}$

Predictive:
patterns are independent of unknown input parameters

If $s_{13}$ is small, look at tau modes.
Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c_{RL}^{(1)} - c_{RL}^{(2)} = 1$
quick example (probably out of time by now):

\[ \tau \rightarrow \mu \gamma, \ \tau \rightarrow e \gamma \ \& \ \mu \rightarrow e \gamma \]

\[ \Delta L_{\text{eff}} = \frac{v}{\Lambda^2} \bar{e}_R \left[ c_1 \lambda_e \lambda_{1} \lambda_1 + c_2 \lambda_u \lambda_{u} \lambda_e + c_3 \lambda_u \lambda_{u} \lambda_d \lambda_{T} \right] \sigma^{\mu \nu} e_L F_{\mu \nu} \]

just like MLFV above

Generalizes Barbieri & Hall
New mixing structures
Independent of \( M_\nu \)
Hierarchical
Large: for \( \Lambda = 10 \text{TeV} \)

\[ \text{Br}(\mu \rightarrow e \gamma) \sim 10^{-12} \]
**MFV/MLFV and GUTs**

- Quarks/leptons in same reps of $G_{\text{GUT}}$, eg., $\psi \sim \bar{5}, \chi \sim 10$

- Flavor symmetry smaller: $\text{SU}(3)^2$

- Broken by Yukawas, including trans-PL term for light quark/lepton mass relations $(H \sim 5, \Sigma \sim 24)$

\[
\mathcal{L}_{\text{sym.br.}} = \psi^T \lambda_5 \chi H^* + \chi^T \lambda_{10} \chi H + \frac{1}{M} \psi^T \lambda'_5 \Sigma \chi H^*
\]

- Right handed neutrino for see-saw (not mandatory); extends $G_F$ to $\text{SU}(3)^3$, and

\[
\Delta \mathcal{L}_{\text{sym.br.}} = N^T \lambda_1 \psi H + N^T M_R N
\]

- MFV/MLFV: GF broken only by

\[
\lambda_5, \lambda'_5, \lambda_{10}, \lambda_1, M_R
\]
Implementation similar,

\[
\begin{align*}
    Q_L & \rightarrow V_{10} \, Q_L \\
    u_R & \rightarrow V_{10}^* \, u_R \\
    d_R & \rightarrow V_{\bar{5}}^* \, d_R \\
    L_L & \rightarrow V_{\bar{5}} \, L_L \\
    e_R & \rightarrow V_{10}^* \, e_R \\
    \lambda_{10} & \rightarrow V_{10}^* \, \lambda_{10} \, V_{10}^\dagger \\
    \lambda_5 & \rightarrow V_{\bar{5}}^* \, \lambda_5 \, V_{10}^\dagger \\
    \lambda_1 & \rightarrow V_1^* \, \lambda_1 \, V_{\bar{5}}^\dagger \\
    M_R & \rightarrow V_1^* \, M_R \, V_1^\dagger
\end{align*}
\]

but trade parameters for low energy ones, eg.,

\[
\begin{align*}
    \lambda_u & = a_u \left[ \lambda_{10} + \epsilon_{1u} \lambda_{10} \lambda_{10}^\dagger \lambda_{10} + \epsilon_{2u}^{(n)} \lambda_{10} (\lambda_5^{(l)})^\dagger \lambda_5^{(l)} + \ldots \right] \\
    \lambda_d & = a_d \left[ \left( \lambda_5 + \lambda_5' \right) + \epsilon_{d1}^{(l)} \lambda_5' + \epsilon_{d2}^{(l)} \lambda_5^{(l)} \lambda_{10}^\dagger \lambda_{10} + \epsilon_{d3}^{(l)} \lambda_1^T \lambda_1^* \lambda_5^{(l)} + \ldots \right] \\
    \lambda_e^T & = a_e \left[ \left( \lambda_5 - \frac{3}{2} \lambda_5' \right) + \epsilon_{e1}^{(l)} \lambda_5' + \epsilon_{e2}^{(l)} \lambda_5^{(l)} \lambda_{10}^\dagger \lambda_{10} + \epsilon_{e3}^{(l)} \lambda_1^T \lambda_1^* \lambda_5^{(l)} + \ldots \right]
\end{align*}
\]
FCNCs: Bilinears as building blocks of operators

**Old**

quarks: \( \bar{Q}_L \lambda^\dagger_u \lambda_u Q_L \), \( \bar{d}_R \lambda_d \lambda^\dagger_u \lambda_u Q_L \)

leptons: \( \bar{L}_L \lambda^\dagger_1 \lambda_1 L_L \), \( \bar{e}_R \lambda_e \lambda^\dagger_1 \lambda_1 L_L \)

**New** (plus replace anywhere \( \lambda_e \leftrightarrow \lambda^T_d \))

quarks: \( \bar{Q}_L (\lambda_e \lambda^\dagger_e)^T Q_L \)
\( \bar{d}_R (\lambda_e \lambda^\dagger_e)^T Q_L \), \( \bar{d}_R (\lambda_e \lambda^\dagger_1 \lambda_1)^T Q_L \)
\( \bar{d}_R (\lambda^\dagger_e \lambda_e)^T d_R \), \( \bar{d}_R (\lambda^\dagger_1 \lambda_1)^T d_R \)

leptons: \( \bar{L}_L (\lambda_d \lambda^\dagger_d)^T L_L \)
\( \bar{e}_R (\lambda_d \lambda^\dagger_d \lambda_d)^T L_L \), \( \bar{e}_R \lambda_u \lambda^\dagger_u \lambda_d L_L \)
\( \bar{e}_R \lambda_u \lambda^\dagger_u e_R \), \( \bar{e}_R (\lambda^\dagger_d \lambda_d)^T e_R \)
Summary and Conclusions

- Notion of MLFV for see-saw models formulated
- Two approaches: minimal and extended field content
- MFV in GUTs formulated
- Predictive (in extended f.c. case need CP limit and M-diagonal assumptions)
- All or nothing: effects observable in near future rare process experiments iff LFV scale low (LHC) and LNV scale as large as possible (perturbative couplings)
- A lot to study: mixing, non-CP limit and leptogenesis, LHC physics, other versions...
The End
backup material
Write all operators of dimension 5, 6, ... consistent with assumptions.

For \( \mu \to e\gamma, \; \mu + N \to e + N' \), need two lepton field ops:

<table>
<thead>
<tr>
<th>Ops with LL</th>
<th>Ops with RL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( O^{(1)}_{LL} = \bar{L}<em>L \gamma^\mu \Delta L_L H^\dagger i D</em>\mu H )</td>
<td>( O^{(1)}_{RL} = g' H^\dagger \bar{e}<em>R \sigma^{\mu\nu} \lambda_e \Delta L_L B</em>{\mu\nu} )</td>
</tr>
<tr>
<td>( O^{(2)}_{LL} = \bar{L}<em>L \gamma^\mu \tau^a \Delta L_L H^\dagger \tau^a i D</em>\mu H )</td>
<td>( O^{(2)}_{RL} = g H^\dagger \bar{e}<em>R \sigma^{\mu\nu} \tau^a \lambda_e \Delta L_L W^a</em>{\mu\nu} )</td>
</tr>
<tr>
<td>( O^{(3)}_{LL} = \bar{L}_L \gamma^\mu \Delta L_L \bar{Q}<em>L \gamma</em>\mu Q_L )</td>
<td>( O^{(3)}<em>{RL} = (D</em>\mu H)^\dagger \bar{e}<em>R \lambda_e \Delta D</em>\mu L_L )</td>
</tr>
<tr>
<td>( O^{(4d)}_{LL} = \bar{L}_L \gamma^\mu \Delta L_L \bar{d}<em>R \gamma</em>\mu d_R )</td>
<td>( O^{(4)}_{RL} = \bar{e}_R \lambda_e \Delta L_L \bar{Q}_L \lambda_D d_R )</td>
</tr>
<tr>
<td>( O^{(4u)}_{LL} = \bar{L}_L \gamma^\mu \Delta L_L \bar{u}<em>R \gamma</em>\mu u_R )</td>
<td>( O^{(5)}_{RL} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \bar{Q}<em>L \sigma</em>{\mu\nu} \lambda_D d_R )</td>
</tr>
<tr>
<td>( O^{(5)}_{LL} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \bar{Q}<em>L \gamma</em>\mu \tau^a Q_L )</td>
<td>( O^{(6)}_{RL} = \bar{e}_R \lambda_e \Delta L_L \bar{u}_R \lambda_U^\dagger i \tau^2 Q_L )</td>
</tr>
<tr>
<td>( O^{(5)}_{LL} = \bar{L}_L \gamma^\mu \tau^a \Delta L_L \bar{Q}<em>L \gamma</em>\mu \tau^a Q_L )</td>
<td>( O^{(7)}_{RL} = \bar{e}_R \sigma^{\mu\nu} \lambda_e \Delta L_L \bar{u}<em>R \sigma</em>{\mu\nu} \lambda_U^\dagger i \tau^2 Q_L )</td>
</tr>
</tbody>
</table>

Have used \( \Delta \equiv g^\dagger g \) with transformation \( \Delta \to V_L \Delta V_L^\dagger \)

Also neglected \( \Delta^2 \)

We have neglected \( \sim (\lambda_e)^2 \), hence no RR operators
For $\mu \rightarrow eee$ need four lepton operators

\[ O_{4L}^{(1)} = \bar{L}_L \gamma^\mu \Delta L_L \bar{L}_L \gamma_\mu L_L \]
\[ O_{4L}^{(2)} = \bar{L}_L \gamma^\mu \tau^a L_L \bar{L}_L \gamma_\mu \tau^a L_L \]
\[ O_{4L}^{(3)} = \bar{L}_L \gamma^\mu \Delta L_L \bar{e}_R \gamma_\mu e_R \]
\[ O_{4L}^{(4)} = \delta_{nj} \delta^*_{mi} \bar{L}_L^i \gamma_\mu L_L^j \bar{L}_L^m \gamma_\mu L_L^n \]
\[ O_{4L}^{(5)} = \delta_{nj} \delta^*_{mi} \bar{L}_L^i \gamma^\mu \tau^a L_L^j \bar{L}_L^m \gamma_\mu \tau^a L_L^n \]

where we used $\delta = g_\nu$ (so we can use same expressions for extended field content case)
Up to dimension 6 operators, the new interactions are

\[ \mathcal{L}_{\text{eff}} = \frac{1}{\Lambda_{\text{LFV}}^2} \sum_{i=1}^{5} \left( c_{LL}^{(i)} O_{LL}^{(i)} + c_{4L}^{(i)} O_{4L}^{(i)} \right) + \frac{1}{\Lambda_{\text{LFV}}^2} \left( \sum_{j=1}^{2} c_{RL}^{(j)} O_{RL}^{(j)} + \text{h.c.} \right) \]

with coefficients naively \( c \sim 1 \)

We can now study the phenomenology of MLFV with minimal field content.

Useful to look at parameters first
Use $G_{LF}$ symmetry to rotate to the mass eigenstate basis ($\nu = \text{Higgs vev}$)

$$\lambda_e = \frac{m_\ell}{v} = \frac{1}{v} \text{diag}(m_e, m_\mu, m_\tau)$$

$$g_\nu = \frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger = \frac{\Lambda_{LN}}{v^2} U^* \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) U^\dagger$$

$U$ is the PMNS matrix. It is determined from neutrino mixing:

$$U \approx \begin{pmatrix} 
    c e^{i\alpha_1/2} & s e^{i\alpha_2/2} & s_{13} e^{-i\delta} \\
    -s e^{i\alpha_1/2}/\sqrt{2} & c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2} \\
    s e^{i\alpha_1/2}/\sqrt{2} & -c e^{i\alpha_2/2}/\sqrt{2} & 1/\sqrt{2}
\end{pmatrix}$$

Here $c \equiv \cos \theta_{\text{sol}}$ $s \equiv \sin \theta_{\text{sol}}$ $\theta_{\text{sol}} \simeq 32.5^\circ$

$s_{13}$ is poorly known, $s_{13} < 0.3$
Implementation of MLFV in Extended Field Content Case

\[ L_{\text{Sym.Br.}} = -\lambda_{e}^{ij} \bar{e}_{R}^{i} (H_{L}^{\dagger} L_{L}^{j}) + i \lambda_{\nu}^{ij} \bar{\nu}_{R}^{i} (H_{L}^{T} \tau_{2} L_{L}^{j}) + \text{h.c.} \]

Same as before, but now transformations are:

\[
\begin{align*}
L_{L} & \rightarrow V_{L} L_{L} \\
e_{R} & \rightarrow V_{R} e_{R} \\
\nu_{R} & \rightarrow O_{\nu} \nu_{R} \\
\lambda_{e} & \rightarrow V_{R} \lambda_{e} V_{L}^{\dagger} \\
\lambda_{\nu} & \rightarrow O_{\nu} \lambda_{\nu} V_{L}^{\dagger}
\end{align*}
\]

As before \[ \Delta = \lambda_{\nu}^{\dagger} \lambda_{\nu} \quad \Delta \rightarrow V_{L} \Delta V_{L}^{\dagger} \]

but now not directly related to mass matrix \[ m_{\nu} = \frac{v^{2}}{M_{\nu}} \lambda_{\nu}^{T} \lambda_{\nu} \]

However \[ \delta = \lambda_{\nu}^{T} \lambda_{\nu} \quad \delta \rightarrow V_{L}^{*} \delta V_{L}^{\dagger} \]

In CP limit \[ \lambda_{\nu}^{\dagger} = \lambda_{\nu} \quad \text{and} \quad \Delta = \lambda_{\nu}^{T} \lambda_{\nu} \]
• Same operator basis as before (chose $\Delta$ and $\delta$ by transformation properties)

• Same effective lagrangian, but with $\Lambda_{NL} \rightarrow M_\nu$

• Summary: In mass eigenstate basis

\[
\Delta = \begin{cases} 
\frac{\Lambda_{LN}^2}{v^4} U m_\nu^2 U^\dagger & \text{minimal field content} \\
\frac{M_\nu}{v^2} U m_\nu U^\dagger & \text{extended field content, CP limit}
\end{cases}
\]

\[
\delta = \delta^T = \begin{cases} 
\frac{\Lambda_{LN}}{v^2} U^* m_\nu U^\dagger & \text{minimal field content} \\
\frac{M_\nu}{v^2} U^* m_\nu U^\dagger & \text{extended field content}
\end{cases}
\]
MFV by example: consider $K_L \rightarrow \pi \nu \nu$

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} C^\ell \bar{s}_L \gamma_\mu d_L \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.}$$

$$C^\ell = \frac{\alpha X \left( \frac{m_t}{M_W} \right)}{2\pi \sin^2 \theta_W} V_{ts}^* V_{td}$$

1 loop factor, $V_{\text{CKM}} = \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}$

$$V_{\text{CKM}} \approx \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A\lambda^3(\bar{\rho} - i\bar{\eta}) \\
-\lambda(1 + iA^2\lambda^4\bar{\eta}) & 1 - \frac{1}{2} \lambda^2 & A\lambda^2 \\
A\lambda^3(1 - \bar{\rho} - i\bar{\eta}) & -A\lambda^2(1 + i\lambda^2\bar{\eta}) & 1
\end{pmatrix} + \mathcal{O}(\lambda^6).$$

$$\lambda \approx 0.22 \quad |V_{ts}V_{td}| \sim A^2\lambda^5$$
\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \sum_{\ell=e,\mu,\tau} C^\ell \bar{s}_L \gamma_\mu d_L \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.} \]

New physics

\[ \mathcal{H}_{\text{eff}} = \frac{1}{\Lambda^2_F} \sum_{\ell=e,\mu,\tau} C^\ell_{\text{new}} \bar{s}_L \gamma_\mu d_L \bar{\nu}_L^\ell \gamma^\mu \nu_L^\ell + \text{h.c.} \]

with \[ C_{\text{new}}^\ell \sim 1 \]

Assume sensitivity to fractional deviation \( r \) from SM rate:

\[ 1 + r \sim \left| 1 + \frac{(M_W/\Lambda)^2}{A^2 \lambda^5/(16\pi^2)} \right|^2 \]

For example, \( r = 4\% \) gives sensitivity to \( \Lambda_F \sim 10^6 \text{ GeV} \)

(\( r=4\% \) is KOPIO’s would be sensitivity)
• The small factor comes from CKM

• Generalized GIM mechanism

  • old GIM: smallness of \( V_{cs} V_{cd}^* (m_c^2 - m_u^2) / M_W^2 \)

  • new GIM: smallness of 1-to-3 generation jump

• If new physics respects this then the same small CKM factor appears. New estimate

\[
1 + r \sim \left| 1 + \frac{(M_W / \Lambda)^2}{1/(16\pi^2)} \right|^2
\]

And now \( r = 4\% \) gives sensitivity to \( \Lambda_F \sim 10^{3-4} \text{ GeV} \)
\( \mu \rightarrow e\gamma, \mu\text{-to-}e \) conversion and their relatives I: minimal field content

\[
B_{\ell_i \rightarrow \ell_j(\gamma)} = 10^{-50} \left( \frac{\Lambda_{LN}}{\Lambda_{LFV}} \right)^4 R_{\ell_i \rightarrow \ell_j(\gamma)}(s_{13}, \delta; c^{(i)})
\]

- since \( \Delta \propto U(m_\nu)^2 U^\dagger \), only differences of \( m^2 \) enter; these are measured!

- \( s_{13} \) and \( \delta \) unknown PMNS parameters (scan on \( \delta \))

- choose \( c^{(i)} \) of order one for the estimate

- ratio of scales can be large:
  
  perturbative \( g_\nu \Rightarrow \Lambda_{LN} \lesssim 3 \times 10^{13} (1 \text{ eV}/m_\nu) \) GeV

  so \( \Lambda_{LFV} \sim 1 \text{ TeV} \Rightarrow \Lambda_{LN}/\Lambda_{LFV} \lesssim 10^{10} \)
Predictive: \( l \rightarrow l' \gamma \) patterns are independent of unknown input parameters (scales cancel in ratios, in this case \( c^{(i)} \)'s cancel too, and all other parameters are from long distance)
If $s_{13}$ is small, look at tau modes.

Here $\Lambda_{LN}/\Lambda_{LFV} = 10^{10}$ and $c_{RL}^{(1)} - c_{RL}^{(2)} = 1$

Belle and BaBar have recent bounds (summer ‘05) of a few $\times 10^{-7}$ for $\text{Br}(\tau \to l\gamma)$ and $\text{Br}(\tau \to lll)$
$\mu \rightarrow e\gamma$, $\mu$-to-$e$ conversion and their relatives II: extended field content

- Replace $\Lambda^2_{LN}/\Lambda^2_{LFV}$ by $vM_\nu/\Lambda^2_{LFV}$
- Now $\Delta \propto U m_\nu U^\dagger$ so amplitudes depend on overall neutrino mass scale (ie, lightest neutrino mass)

$$\text{lightest } m_\nu = 0$$

$$\text{lightest } m_\nu = 0.2 \text{ eV}$$

$$B_{\ell_i \rightarrow \ell_j(\gamma)} = 10^{-25} \left( \frac{vM_\nu}{\Lambda^2_{LFV}} \right)^2 \hat{R}_{\ell_i \rightarrow \ell_j(\gamma)}(s_{13}, m_\nu^{\text{lightest}}; c^{(i)})$$

perturbative $\lambda_\nu \Rightarrow M_\nu \lesssim 10^{13} \text{ GeV}$; with $\Lambda_{LFV} \geq 1 \text{ TeV}$, $\frac{vM_\nu}{\Lambda^2_{LFV}} \leq 10^9$
One final note: results depend on hierarchy of neutrino masses, 
*normal* \( (m_{\nu_1} \sim m_{\nu_2} \ll m_{\nu_3}) \) vs. *inverted* \( (m_{\nu_1} \ll m_{\nu_2} \sim m_{\nu_3}) \)

\[
(v M_\nu) / \Lambda_{LFV}^2 = 5 \times 10^7 \]
\[
c^{(1)}_{RL} - c^{(2)}_{RL} = 1
\]

shading: \( 0 \leq m_{\nu_{\text{lightest}}} \leq 0.02 \text{ eV} \)
\[ \Gamma_{\mu \to 3e} / \Gamma_{\mu \to e\nu\bar{\nu}} = \left| a_+ \right|^2 + 2 \left| a_- \right|^2 - 8 \text{Re}(a_0^* a_-) - 4 \text{Re}(a_0^* a_+) + 6 \left| a_0 \right|^2 \right] \left\{ \left( \frac{\Lambda_{\text{LN}}}{\Lambda_{\text{LFV}}} \right)^4 |a_{e\mu}|^2 \text{ minimal} \right. \\
\left. \left( \frac{\nu M_{\nu}}{\Lambda_{\text{LFV}}^2} \right)^2 |b_{e\mu}|^2 \text{ extended} \right. \\
\]

\[ a_+ = \sin^2 \theta_w (c^{(1)}_{LL} + c^{(2)}_{LL}) + c^{(3)}_{4L} \]

\[ a_- = (\sin^2 \theta_w - \frac{1}{2})(c^{(1)}_{LL} + c^{(2)}_{LL}) + c^{(1)}_{4L} + c^{(2)}_{4L} + \frac{2\delta_{e\mu}\delta_{ee}}{\Delta_{e\mu}} (c^{(4)}_{4L} + c^{(5)}_{4L}) \]

\[ a_0 = 2e^2 (c^{(1)}_{RL} - c^{(2)}_{RL})^* \]
\[ \Gamma_{\tau \to e\mu\bar{\mu}} = \Gamma_{\tau \to e\nu\bar{\nu}} \frac{v^4|\Delta e_\tau|^2}{\Lambda_{LFV}^4} \left[ |a_+|^2 + |\tilde{a}_-|^2 - 4\text{Re}[a_0^*(a_+ + \tilde{a}_-)] + 12\tilde{I}|a_0|^2 \right] \]

\[ \Gamma_{\tau \to \mu\mu e} = \Gamma_{\tau \to e\nu\bar{\nu}} \frac{v^4|2\delta e_\tau \delta_{\mu\mu}|^2}{\Lambda_{LFV}^4} \left| c_L^{(4)} + c_L^{(5)} \right|^2 \]
• We have also explored the effects of deleting a class of operators.
• For example: assume 4L operators are not present
• Can we get 3l decays? Yes, through loops
• Need care in loops of light quarks: chiral lagrangian does the job
• Result: amplitude is ~0.1 of 4L ops (large logs)
• Equivalently, this give a ~20% correction to rate
• Patterns are similar to those from 4L
Decays of/to hadrons

Hopelessly small!

\[
\begin{align*}
\pi^0 &\rightarrow \mu^+ e^- & \text{Br} &\quad 10^{-25} \\
\Upsilon &\rightarrow \tau \mu & \quad & 10^{-20} \\
\tau &\rightarrow \pi \mu & \quad & 10^{-15}
\end{align*}
\]