Bounds on the Electroweak Chiral Lagrangian from Unitarity and Analyticity

Benjamin Grinstein (UCSD)

DPF, Hawaii, Oct 2006

J. Distler, BG, R. Porto & I. Rothstein, hep-ph/0604255

Bounds on EW Chiral Lagrangian Parameters

- Suppose LHC does not find the Higgs (either too massive or non-existent)
- Would-be-Goldstone-Boson sector strongly coupled, described by Chiral Lagrangian (an expansion in derivatives = momenta)
- We can say something about couplings of $O(p^4)$ terms from first principles
- This can be used to start understanding underlying short distance forces ("UV completion")

Bounds on What?

Higgs-less SM: EW non-linear (“sigma”) model

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} - \frac{1}{4}v^2\text{Tr}(V_\mu V^\mu) + \frac{1}{2}\alpha_1 g g' \text{Tr}\left(B_{\mu\nu} T W^{\mu\nu}\right)$$
$$+ \frac{1}{2} i\alpha_2 g' \text{Tr}\left(T[V^\mu, V^\nu]\right) B_{\mu\nu} + i\alpha_3 g \text{Tr}\left(W_{\mu\nu}[V^\mu, V^\nu]\right)$$
$$+ \alpha_4 \left(\text{Tr}(V_\mu V_\nu)\right)^2 + \alpha_5 \left(\text{Tr}(V_\mu V^\mu)\right)^2$$

where $\Sigma(x) = \exp(i\pi^a(x)\tau^a/v)$

$$T \equiv 2\Sigma T^3 \Sigma^\dagger \quad V_\mu \equiv (D_\mu\Sigma)\Sigma^\dagger \quad D_\mu\Sigma = \partial_\mu\Sigma + \frac{1}{2} i g W_\mu^a \tau^a \Sigma - \frac{1}{2} i g' B_\mu \Sigma \tau^3$$

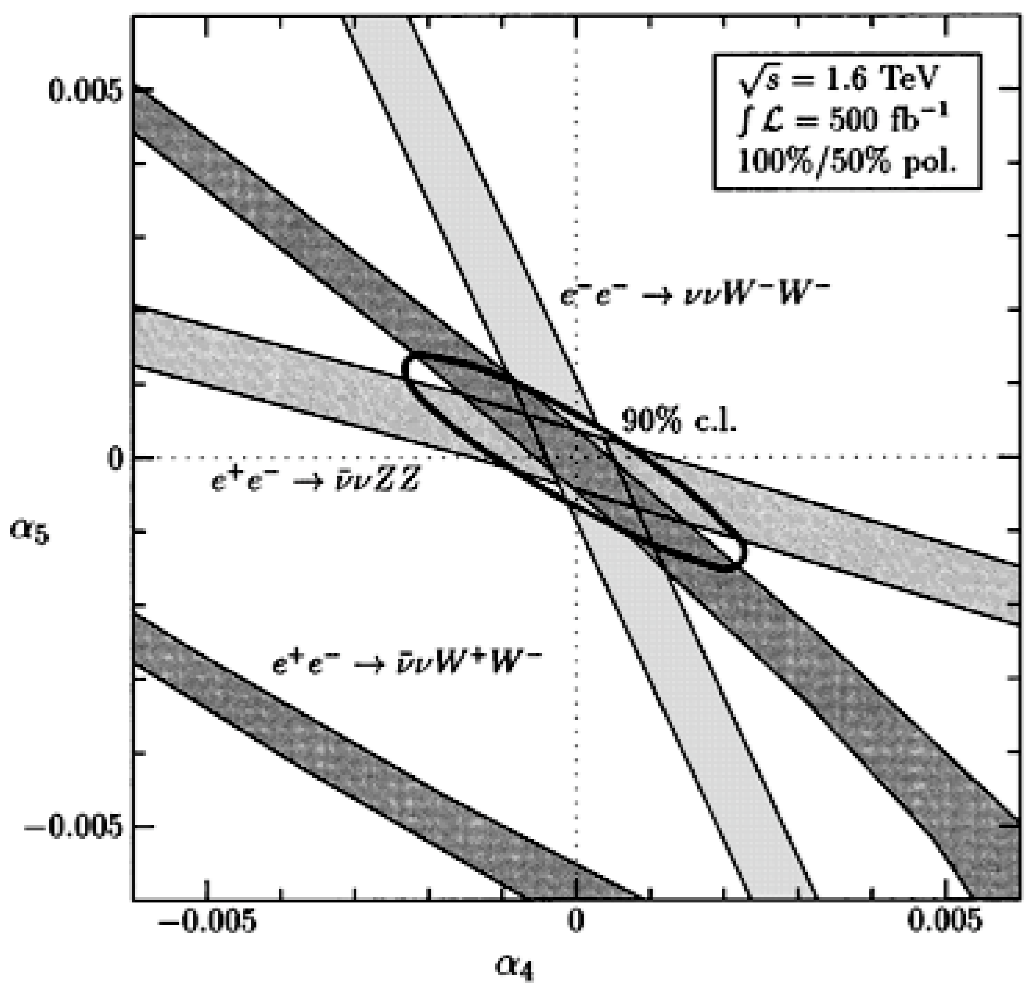
Bound α_4 and α_5

Note: other α’s already well bound by experiment

This (may not be) pie in the sky

sensitivity to parameters in gedanken linear collider

Boos et al, PRD57:1553,1998



Warm-up

Pure Chiral Lagrangian

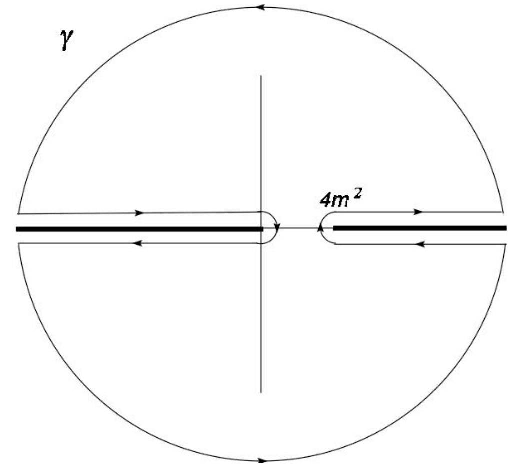
- Find bounds on parameters of SU(2)xSU(2)/SU(2) sigma model

$$\mathcal{L} = \frac{1}{4}v^2 \mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma) + \frac{1}{4}m^2 v^2 \mathrm{Tr}(\Sigma + \Sigma^\dagger) + \frac{1}{4}\ell_1 [\mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial_\mu \Sigma)]^2 + \frac{1}{4}\ell_2 [\mathrm{Tr}(\partial^\mu \Sigma^\dagger \partial^\nu \Sigma)][\mathrm{Tr}(\partial_\mu \Sigma^\dagger \partial_\nu \Sigma)]$$

- EW chiral lagrangian reduces to this at g=g'=0 (plus uncoupled free gauge sector)
- Bounds on EW parameters (α_4, α_5) are those of this model's $(\ell_2/4, \ell_1/4)$ up to corrections of order g^2, gg', g'^2

Consider $\pi\pi \longrightarrow \pi\pi$

$$\frac{\mathrm{d}^n T(s)}{\mathrm{d}s^n} = n! \int_{4m^2}^{\infty} \frac{\mathrm{d}x}{\pi} \sqrt{x(x-4m^2)} \left(\frac{\sigma(x)}{(x-s)^{n+1}} + \frac{\sigma_u(x)}{(x-4m^2+s)^{n+1}} \right)$$



Fwd Scat Ampl

Total cross section

Derivatives: convergence, kill lowest order

- Approximate *T(s)* in 0 < *s* < 4*m*^{2} from chi-lag

$$\frac{\mathrm{d}^2 T}{\mathrm{d}s^2} \sim \ell_{1,2} + f(s)$$

- Use $\sigma > 0$, get bounds on $\bar{\ell}_{1,2}$ (find *s* that minimizes *f(s)*)

$$\ell_1^{\mathrm{r}} = \frac{1}{96\pi^2} \left(\bar{\ell}_1 + \ln(m^2/\mu^2) \right)$$

$$\ell_2^{\mathrm{r}} = \frac{1}{48\pi^2} \left(\bar{\ell}_2 + \ln(m^2/\mu^2) \right)$$

Improvement: non-forward scattering (t ≠ 0)

We still use dispersion relation, but now at t ≠ 0:

$$\frac{\partial^n T}{\partial s^n}(s,t) = \frac{n!}{\pi}\int_{4m^2}^{\infty} \mathrm{d}x \left[\frac{\mathrm{Im}T(x,t)}{(x-s)^{n+1}} - \frac{\mathrm{Im}T_u(x,t)}{(u-x)^{n+1}}\right]$$

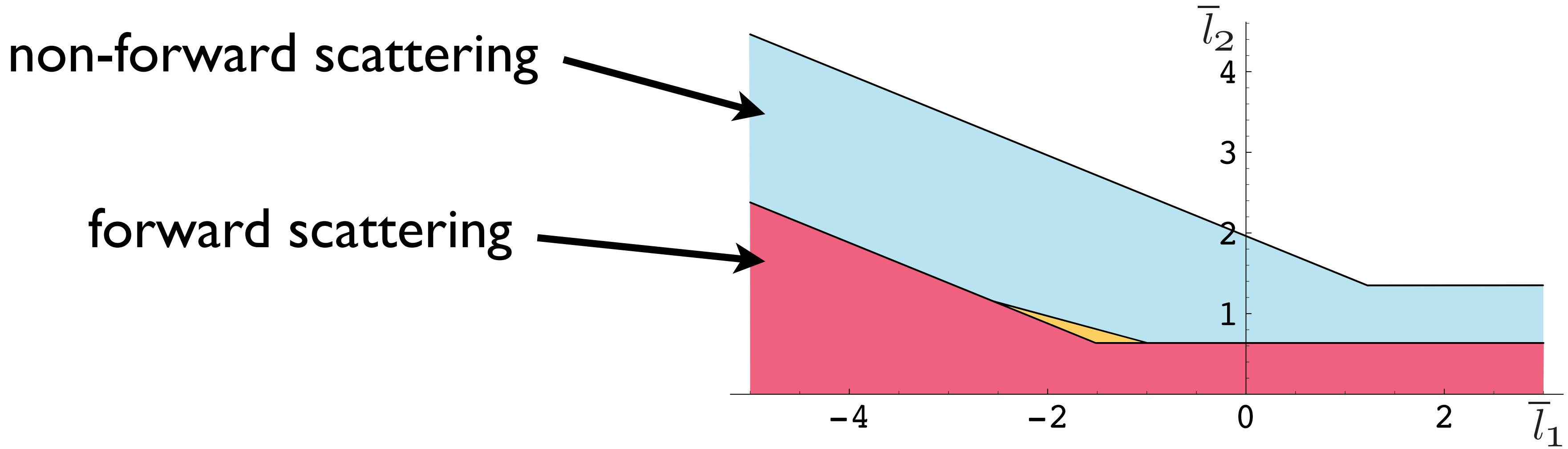
We cannot use now $\mathrm{Im}T(s,0) = \sqrt{s(s-4m^2)}\sigma(s)$

but for t>0 (cosθ >1) obtain positivity from unitarity

$$\mathrm{Im}(a_\ell(s)) = |a_\ell(s)|^2 > 0$$

and partial wave decomposition

$$\mathrm{Im}T(s,t) = \sum_\ell (2\ell+1)\mathrm{Im}(a_\ell(s))\mathrm{P}_\ell(\cos\theta)$$



- First derived by Pennington and Portolés (Phys.Lett.B344,399(1995)) by different method, and (weaker bounds) by Ananthanarayan, Toublan and Wanders (Phys.Rev.D51,1093(1995)) by similar method

- For WW scattering (as opposed to $\pi\pi$) expect corrections at order g^2

EW-Chiral Lagrangian: The real thing

- Re-do calculation using WW scattering
- Too hard to do exactly (but in progress). Instead use approximations
- Equivalence theorem (ET) gives approximation to longitudinally polarized WW scattering amplitude at leading order in m^2/s
- Problem: this needs s above threshold ($4m^2$)

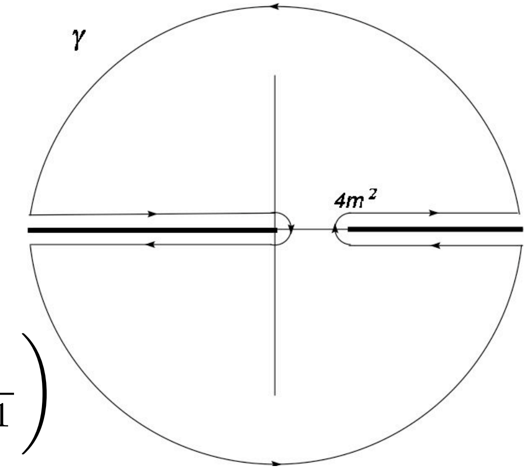
- Use real part of dispersion relation for s above threshold, $4m^2 \ll s \ll (4\pi v)^2$

- This works here because window is large
$4m^2 = 0.03\text{TeV}^2 \ll (4\pi v)^2 = 3\text{TeV}^2$
(in QCD, $4m^2 = 0.08\text{GeV}^2$, $(4\pi v)^2 = 1\text{GeV}^2$)

- Use EW chiral lagrangian to show dispersion integral over $4m^2 \ll s \ll kv^2$, with $k \sim 1$, remains positive

- check that k is small enough so chiral perturbation theory corrections remain small: order $kv^2/(4\pi v)^2 = k/16\pi^2$

few details: compute above threshold
Use as before



$$\frac{\mathrm{d}^n T(s)}{\mathrm{d}s^n} = n! \int_{4m^2}^{\infty} \frac{\mathrm{d}x}{\pi} \sqrt{x(x-4m^2)} \left(\frac{\sigma(x)}{(x-s)^{n+1}} + \frac{\sigma_u(x)}{(x-4m^2+s)^{n+1}} \right)$$

but now $s \sim v^2 \gg m^2$

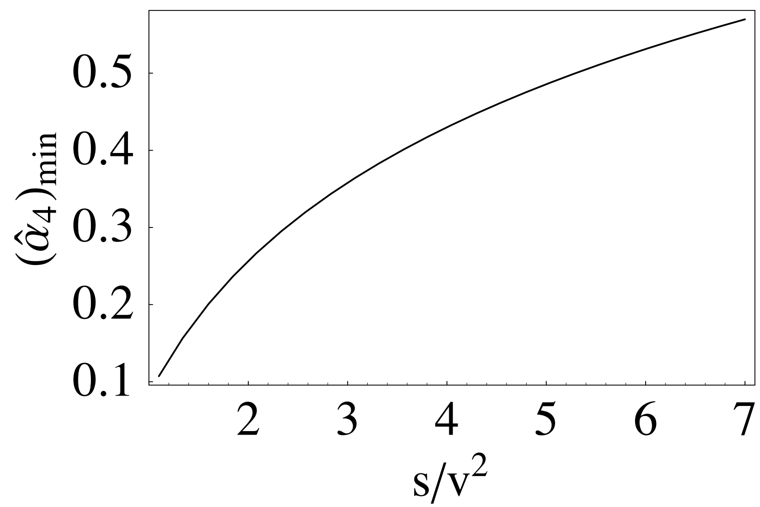
- problem: integrand negative for $x < s$
- solution: use EW chiral lagrangian to compute up to $x = kv^2$, choose $k = k(s)$ so that

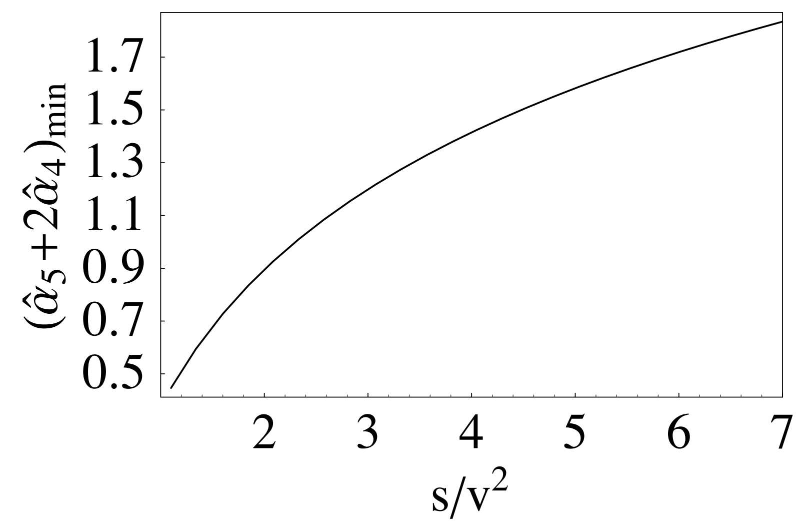
$$\mathrm{Re}\left[\int_{4m^2}^{kv^2} \frac{\mathrm{d}x}{\pi} \sqrt{x(x-4m^2)} \left(\frac{\sigma(x)}{(x-s)^3} + \frac{\sigma_u(x)}{(x-4m^2+s)^3} \right) \right] = 0$$

and use positivity of the rest. We find $k \approx 5s/v^2$

- consistency: both s and kv^2 (much) larger than m^2 but (much) smaller than $(4\pi v)^2$

we find $k \approx 5s/v^2$





choose *s*:
chiral lag corrections $\delta_\chi T \propto O\left(\frac{s^3}{(4\pi)^4 v^6}\ln(s/\mu^2)\right)$

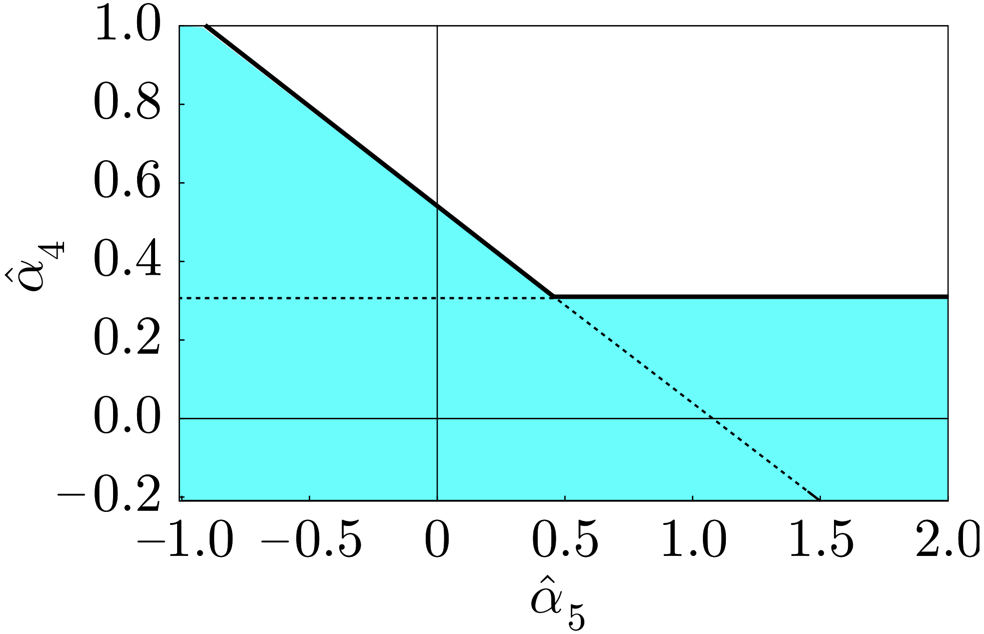
EW loop corrections $\delta_{ew} T \propto O\left(\frac{g^2 s}{(4\pi v)^2}\ln(s/\mu^2)\right)$

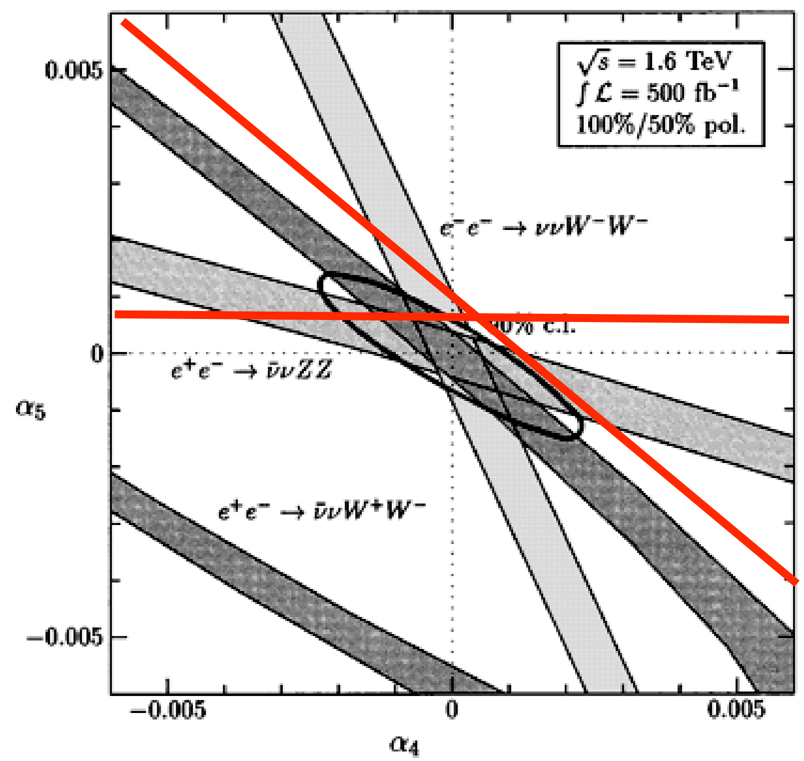
choose *s* to make these less than 20%
gives $s \approx 1.6v^2$ $\Rightarrow$ consistent! (also, ET, $m^2/s = 6\%$)

In terms of $\hat{\alpha}_i$

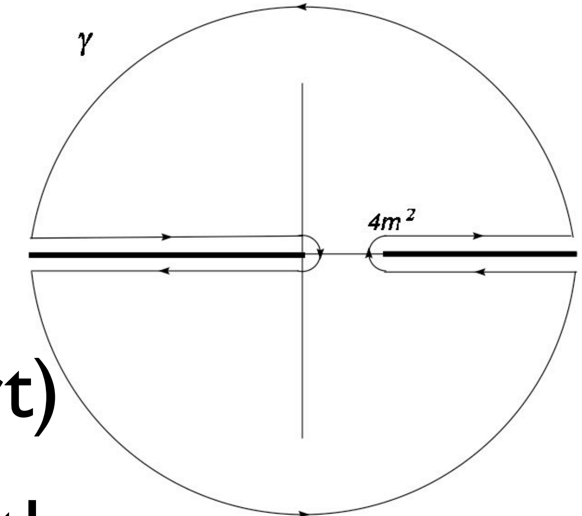
$$\alpha_i^r(\mu) = \frac{\gamma_i}{96\pi^2}\left[\hat{\alpha}_i + \frac{1}{4}\ln(v^2/\mu^2)\right] \qquad \begin{aligned}\gamma_5 &= 1 \\ \gamma_4 &= 2\end{aligned}$$

$$\begin{aligned} \hat{\alpha}_5 + 2\hat{\alpha}_4 &\geq 1.08 \\ \hat{\alpha}_4 &\geq 0.31 \end{aligned}$$





Consequences? (*ie*, what if violated?)



- Recall, assumptions are very mild: analyticity, unitarity, Lorentz invariance, crossing, temperedness (actually, Froissart)
- "Normal" QFTs and perturbative string theory satisfy assumptions
- Convergence of dispersion relation means new physics must show up at low scale (if bounds violated). To see this, choose radius of circle in contour as intermediate scale, $4\pi v \ll \sqrt{R} \ll M_{new}$
- Examples of bounds-violating physics hardly studied! One example: Higher derivative local QFT with cut-off. It produces ghost poles on first Riemann sheet (but have not studied effect on $\alpha_{4,5}$)

Superluminality?

- Our work motivated by Adams et al, "Causality, analyticity and an IR obstruction to UV completion" hep-th/0602178
- Superluminality (classical) bounds:
    - At *m*=0, Chiral-lag admits solutions $\Sigma = \exp(ic^a \sigma^a t)$
    - Small perturbations, plane waves have group velocity < light's only if $\ell_2 > 0,\ \ell_1 + \ell_2 > 0$
- Relation (classical vs quantum) unclear
    - *m* = 0 vs *m* > 0?
    - classical vs renormalized couplings?

Bounds on higher order terms in chiral lagrangian?

Again, forward scattering amplitude

$$T(s) \sim \frac{s}{v^2} + c_1 \left(\frac{s}{v^2}\right)^2 + c_2 \left(\frac{s}{v^2}\right)^3 + \cdots$$

$$\Rightarrow \frac{\mathrm{d}^3 T(s)}{\mathrm{d}s^3}\bigg|_{s\approx 0} \sim c_2 > 0 \quad ??$$

and so on.

However, lowest term in Chi-Lag gives 1-loop

$$T(s)_{1-\text{loop}} \sim \frac{1}{16\pi^2 v^4} s^2 \ln(s) \Rightarrow \frac{\mathrm{d}^3 T(s)}{\mathrm{d}s^3} \sim \frac{1}{16\pi^2} \frac{v^2}{s} + c_2$$

No useful bound on higher order terms

Future program (in lieu of summary/conclusions):

- Derive bounds directly from EW lagrangian (improve reliability by disposing of Equivalence Theorem)
- Re-do Boos et al including 1-loop, to give meaning to coupling constants
- Explore bounds-violating physics:
  - more examples that violate assumptions?
  - are bounds violated?
  - what other signals for LHC/NLC?
- Bounds on other parameters? S, T, U? (Assume new physics, related to EW breaking, at $\Lambda \sim 4\pi v$. Then effective Lagrangian has operators of dim ≥ 5, with $1/\Lambda$'s, unknown coefficients

The End