Presence and Absence of Degeneracies in the LBL NuOscExp

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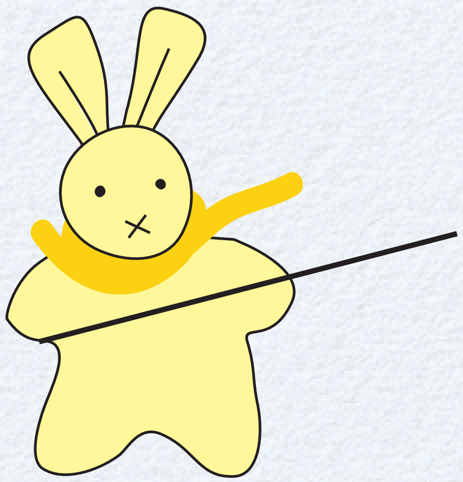


JPS + APS-DPF
November 1, 2006, Honolulu

Neutrino Oscillation

- Evolution equation of neutrinos

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\nu_\mathrm{e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix} = \frac{1}{2E}\left[U\begin{pmatrix}0 & & \\ & \delta m_{21}^2 & \\ & & \delta m_{31}^2\end{pmatrix}U^\dagger + \begin{pmatrix}a & & \\ & 0 & \\ & & 0\end{pmatrix}\right]\begin{pmatrix}\nu_\mathrm{e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix} \equiv H\begin{pmatrix}\nu_\mathrm{e}\\ \nu_\mu\\ \nu_\tau\end{pmatrix}$$

  - Quadratic mass differences

$$\delta m_{ij}^2 \equiv m_i^2 - m_j^2$$

$$\delta m_{21}^2 \simeq 8\times 10^{-5}\,\mathrm{eV}^2\,,$$
$$\left|\delta m_{31}^2\right| \simeq (2-2.5)\times 10^{-3}\,\mathrm{eV}^2$$

(Fogli *et al.* 2005)

$$\begin{cases}\delta m_{31}^2 > 0 & \text{“Normal” hierarchy}\\ \delta m_{31}^2 < 0 & \text{“Inverted” hierarchy}\end{cases}$$

  - Mixing matrix, mixing angles and CP phase(s)

$$U = \begin{pmatrix}1 & 0 & 0\\ 0 & c_{23} & s_{23}\\ 0 & -s_{23} & c_{23}\end{pmatrix}\begin{pmatrix}c_{13} & 0 & s_{13}\mathrm{e}^{-\mathrm{i}\delta_\mathrm{CP}}\\ 0 & 1 & 0\\ -s_{13}\mathrm{e}^{\mathrm{i}\delta_\mathrm{CP}} & 0 & c_{13}\end{pmatrix}\begin{pmatrix}c_{12} & s_{12} & 0\\ -s_{12} & c_{12} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & & \\ & \mathrm{e}^{\mathrm{i}\phi_1} & \\ & & \mathrm{e}^{\mathrm{i}\phi_2}\end{pmatrix}$$

$$c_{ij} \equiv \cos\theta_{ij} \qquad s_{ij} \equiv \sin\theta_{ij}$$

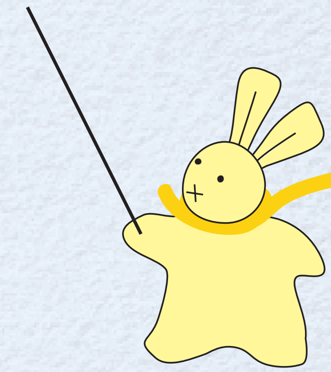
$$\sin^2\theta_{12} \sim 0.3\,, \quad \sin^2\theta_{23} \sim 0.45\,, \quad \sin^2\theta_{13} \lesssim 0.04$$

(Fogli *et al.* 2005)

  - Matter effect

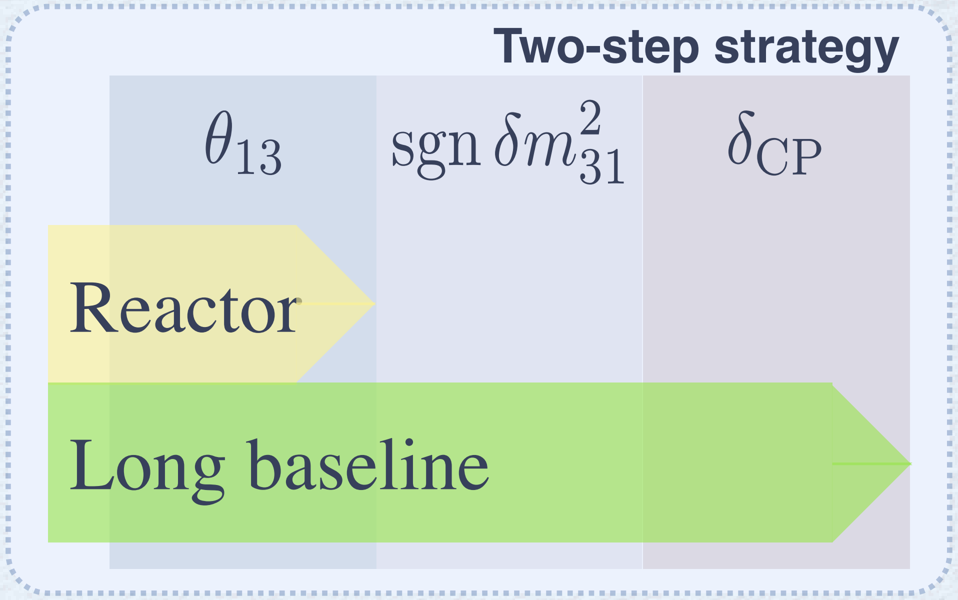
$$a = 2\sqrt{2}G_\mathrm{F}n_\mathrm{e}E = 7.63\times 10^{-5}\,\mathrm{eV}^2\,\frac{\rho}{\mathrm{g\,cm}^3}\,\frac{E}{\mathrm{GeV}}$$

  - Unknowns to date: $\theta_{13}\,,\ \mathrm{sgn}\,\delta m_{31}^2\,,\ \delta_\mathrm{CP}\,,\ \phi_1, \phi_2$

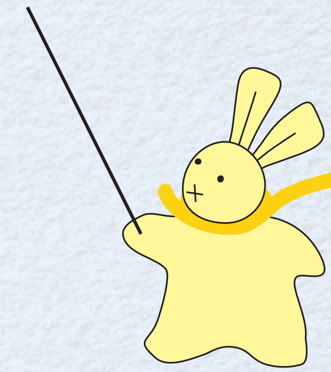


Scope of this Talk

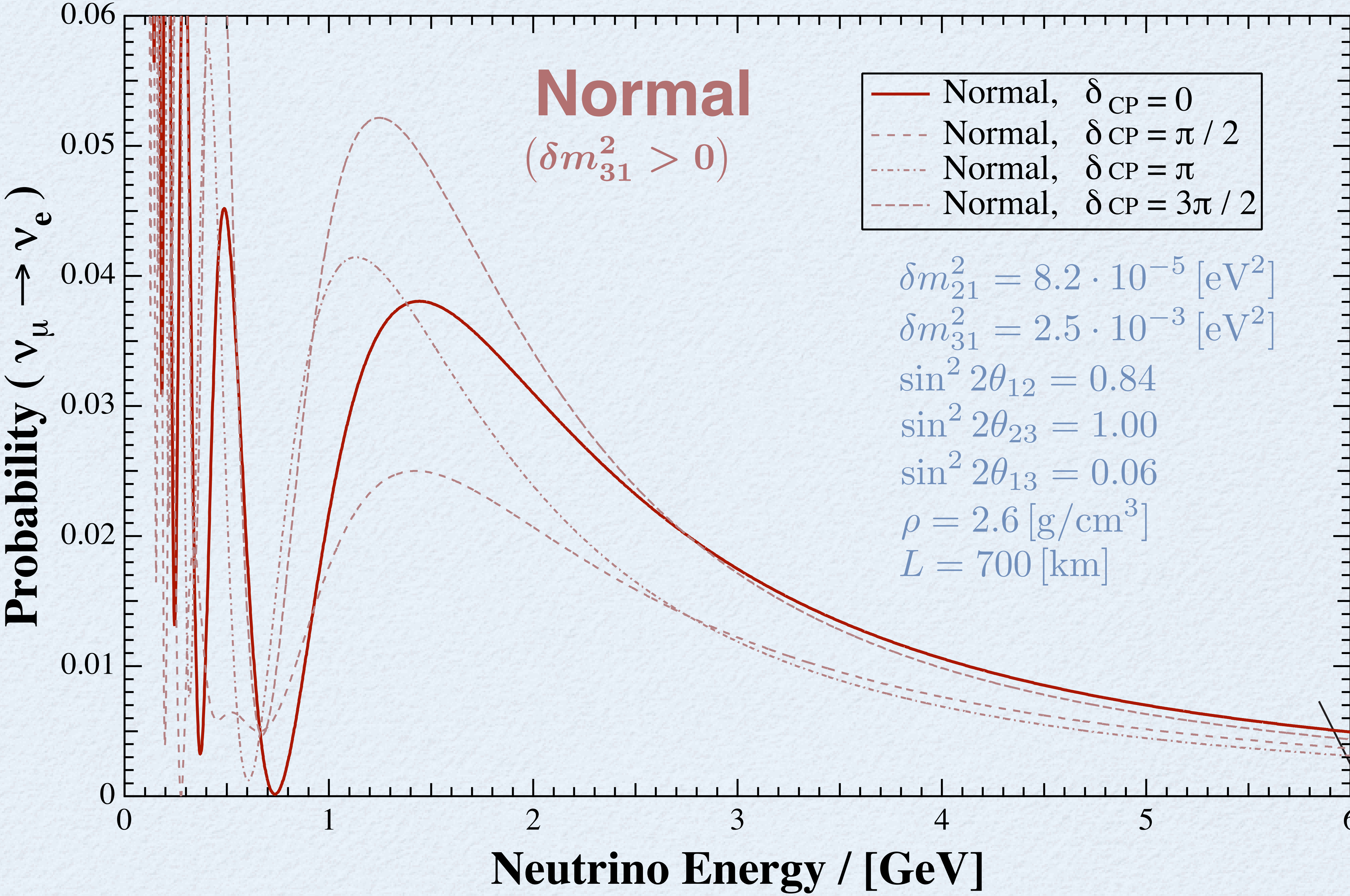
- Situations considered
  - θ_{13} is constrained in advance of the future generation of long-baseline experiment.
  - The value of θ_{13} is not too small $(\sin^2 2\theta_{13} \gtrsim 0.01)$ so that the *CP*-violation effect is accessible.



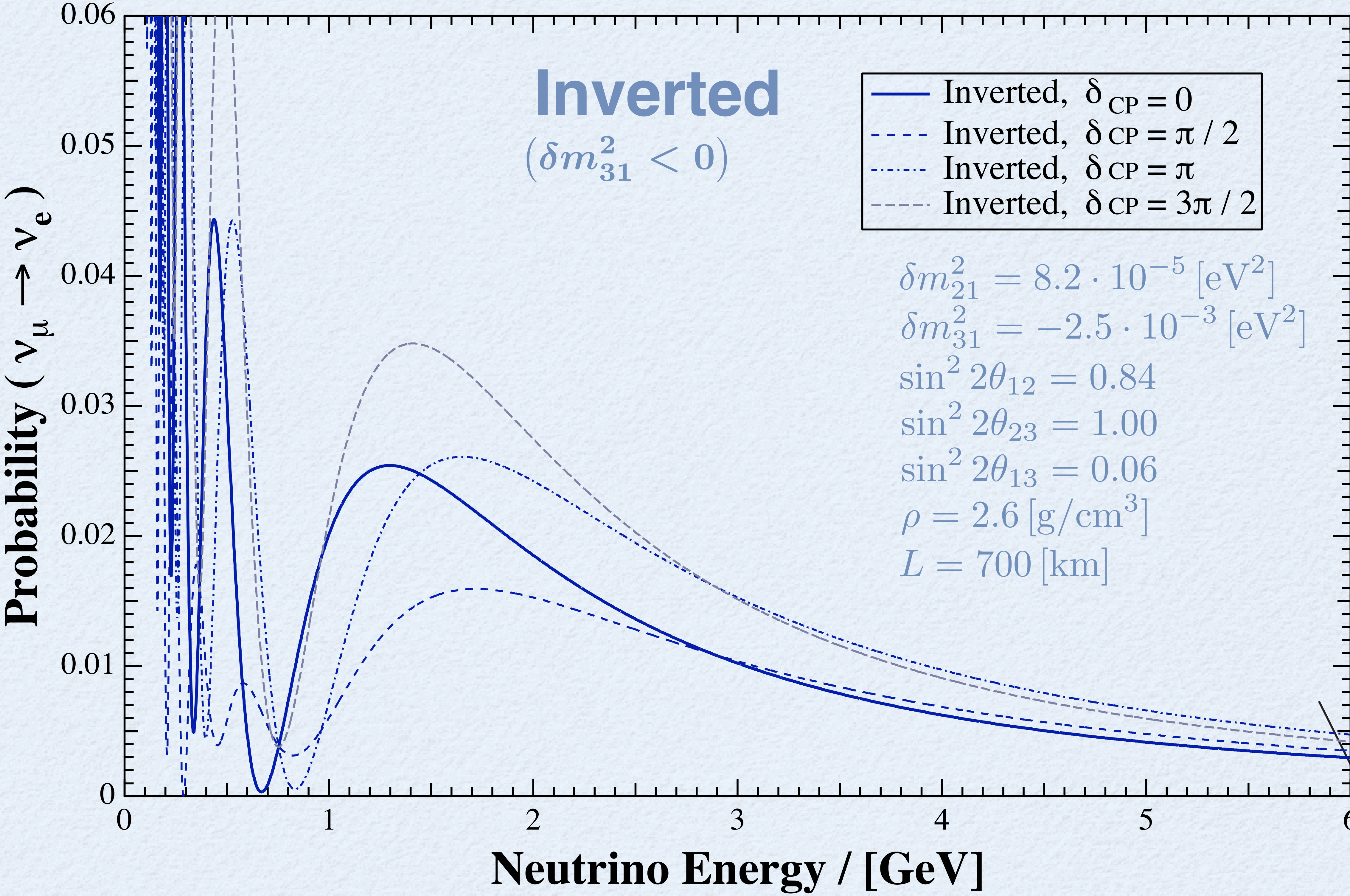
- Purpose
  - Provide a perspective of the presence and absence of the degeneracies regarding hierarchy $(\mathrm{sgn}\,\delta m^2_{31})$ and δ_{CP} .
  - Discuss the way to get out of this degeneracy.

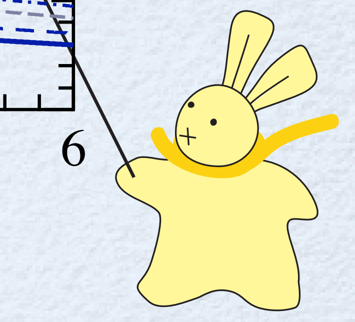


Oscillation Probabilities

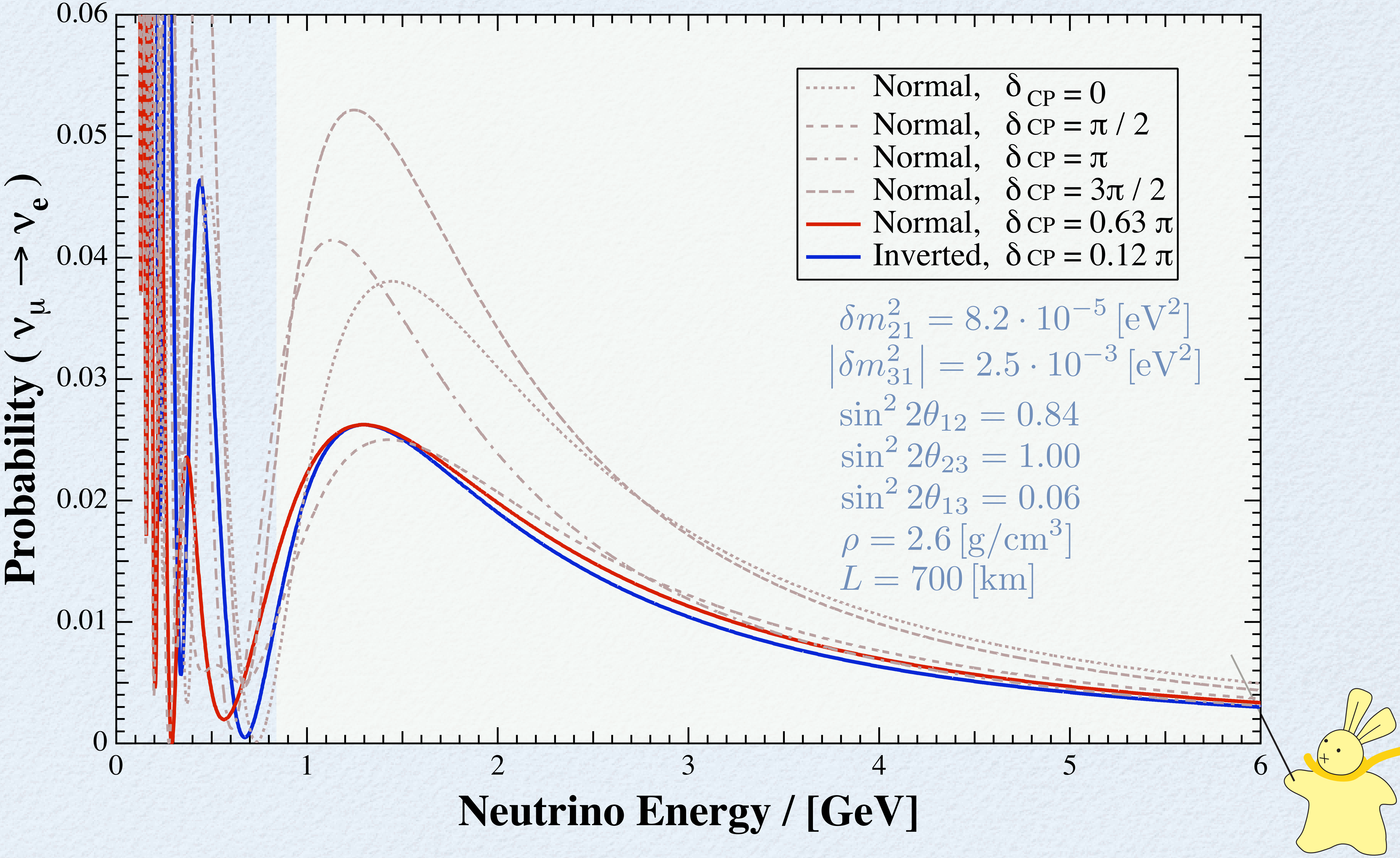


Oscillation Probabilities

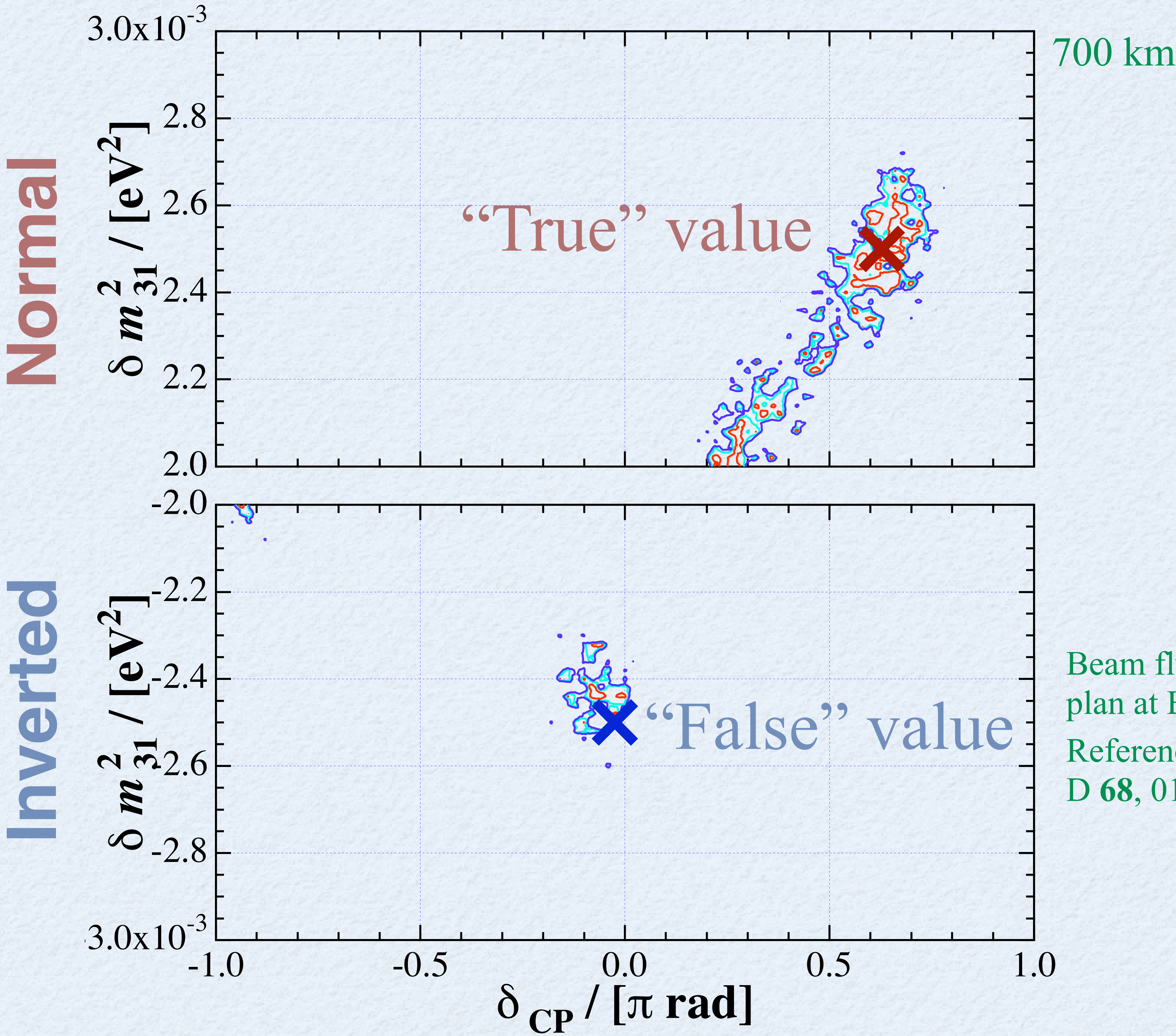




Oscillation Probabilities

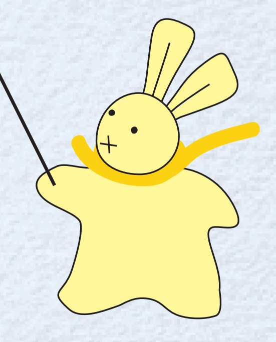


Distinguishability of Hierarchies

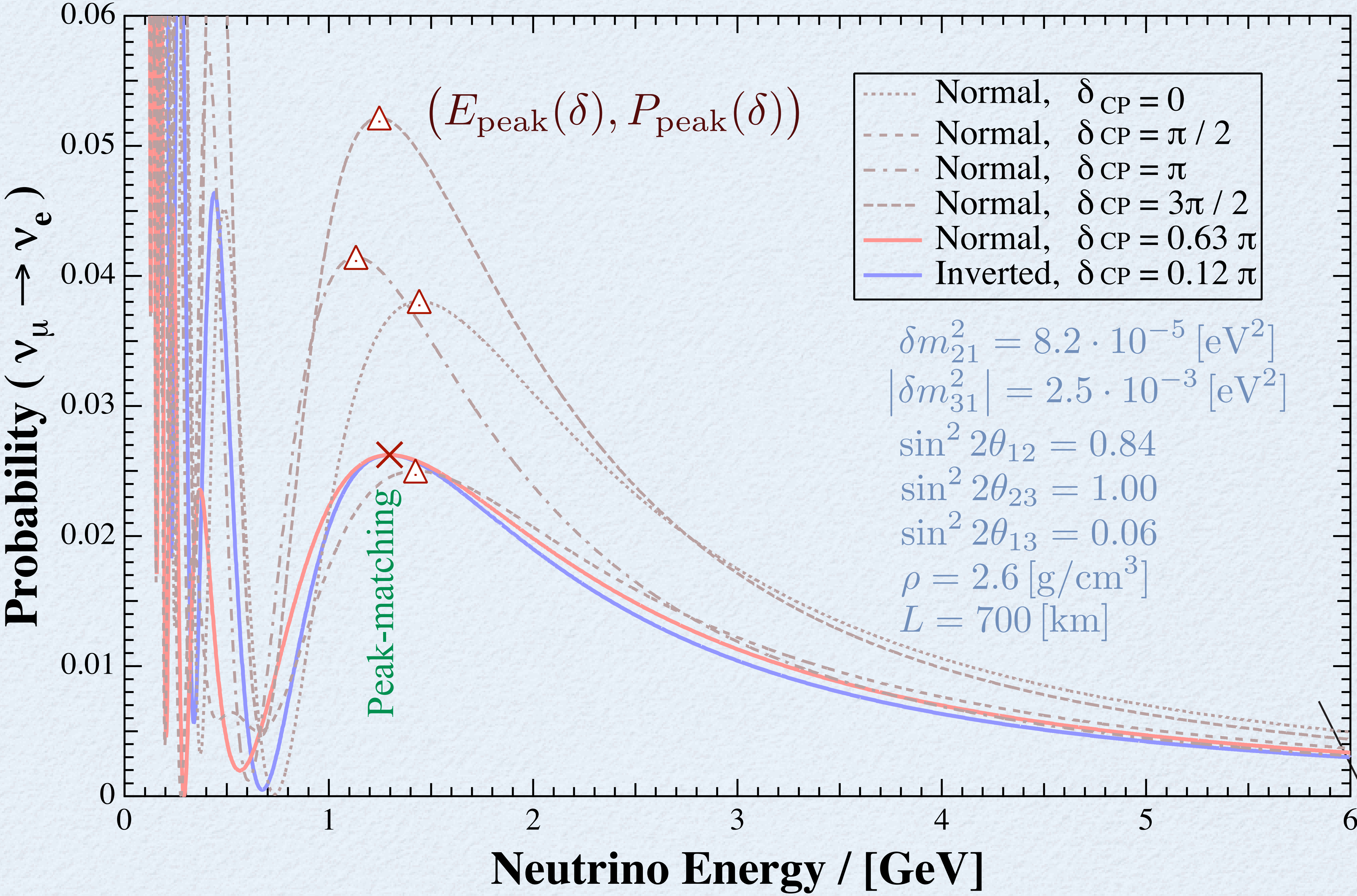


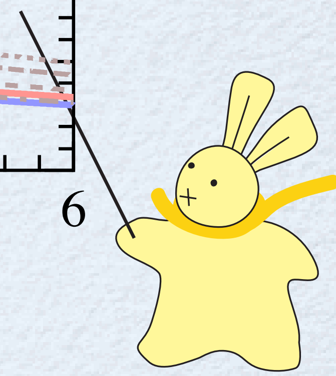
Beam flux taken from the AGS upgrade plan at BNL.

Reference: M. V. Diwan *et al*., Phys. Rev. D **68**, 012002 (2003).

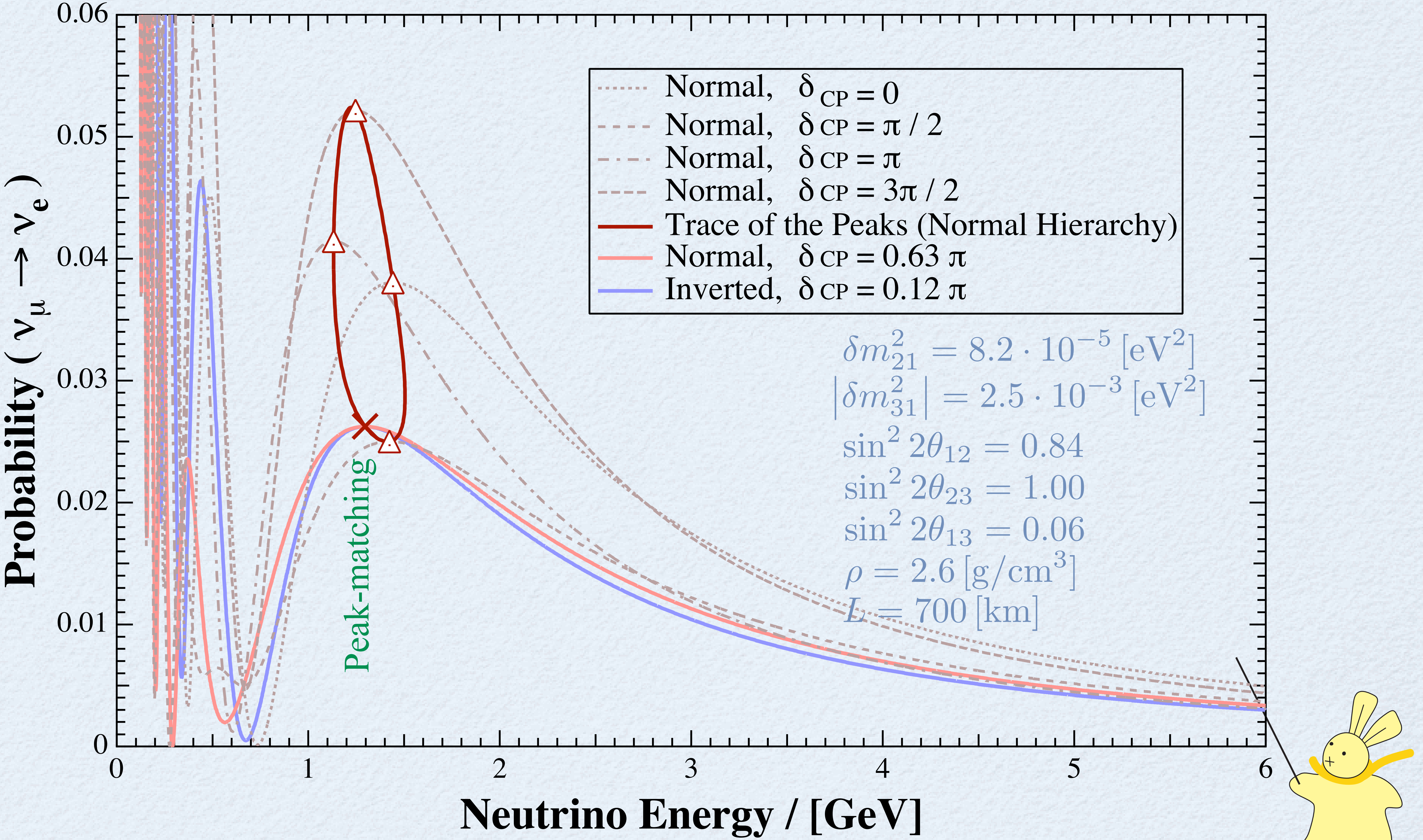


Oscillation Peaks

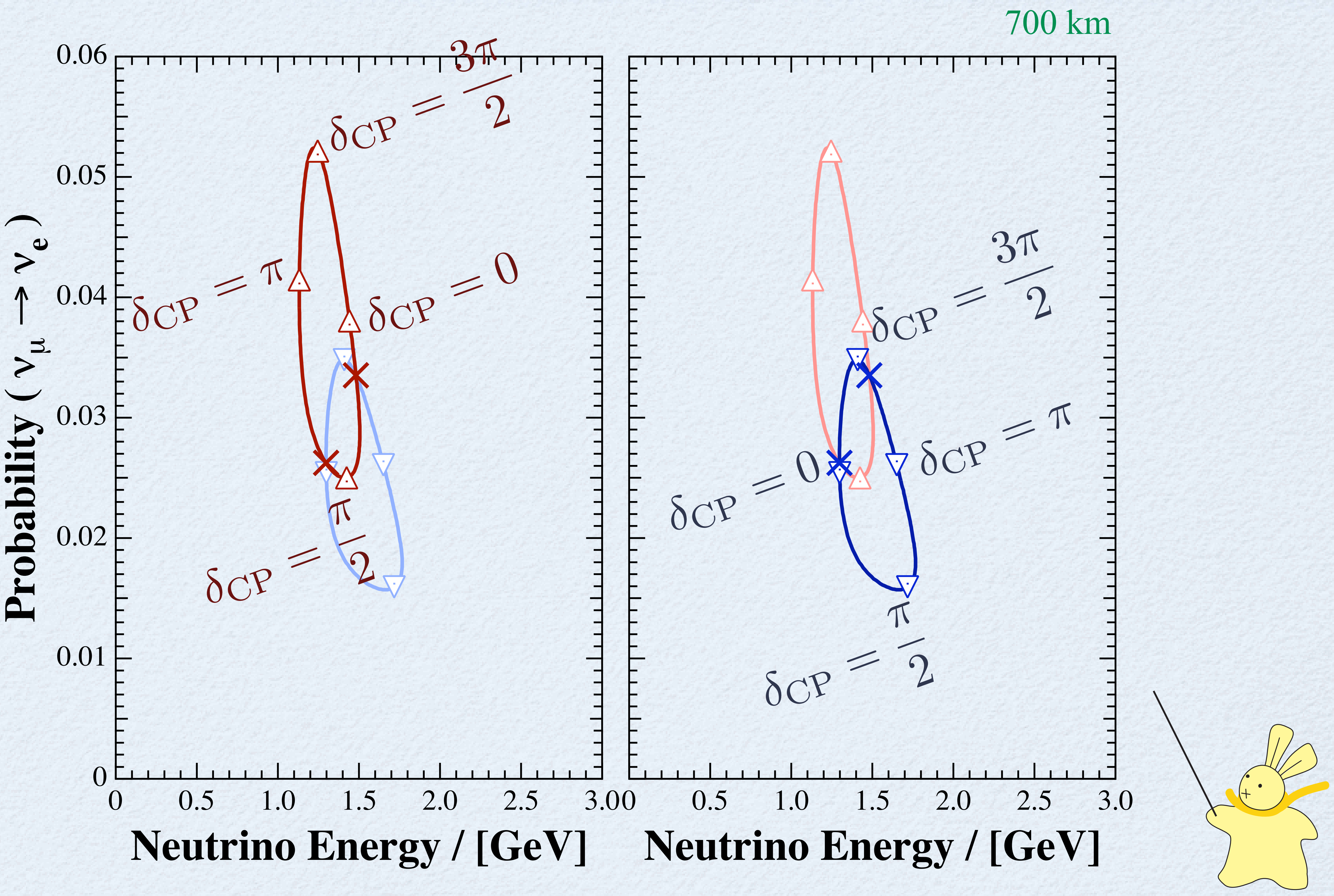




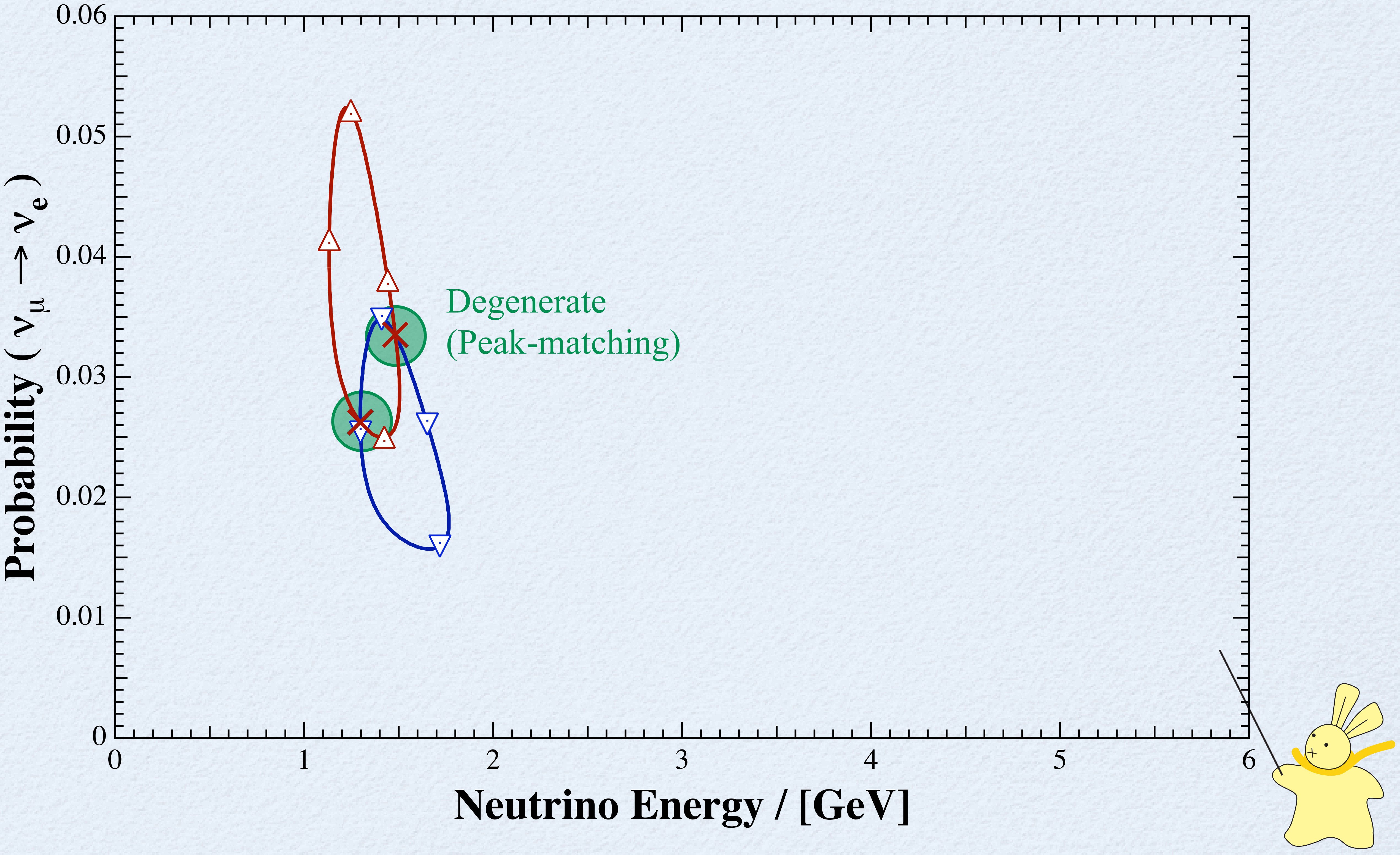
Ring of the Peaks



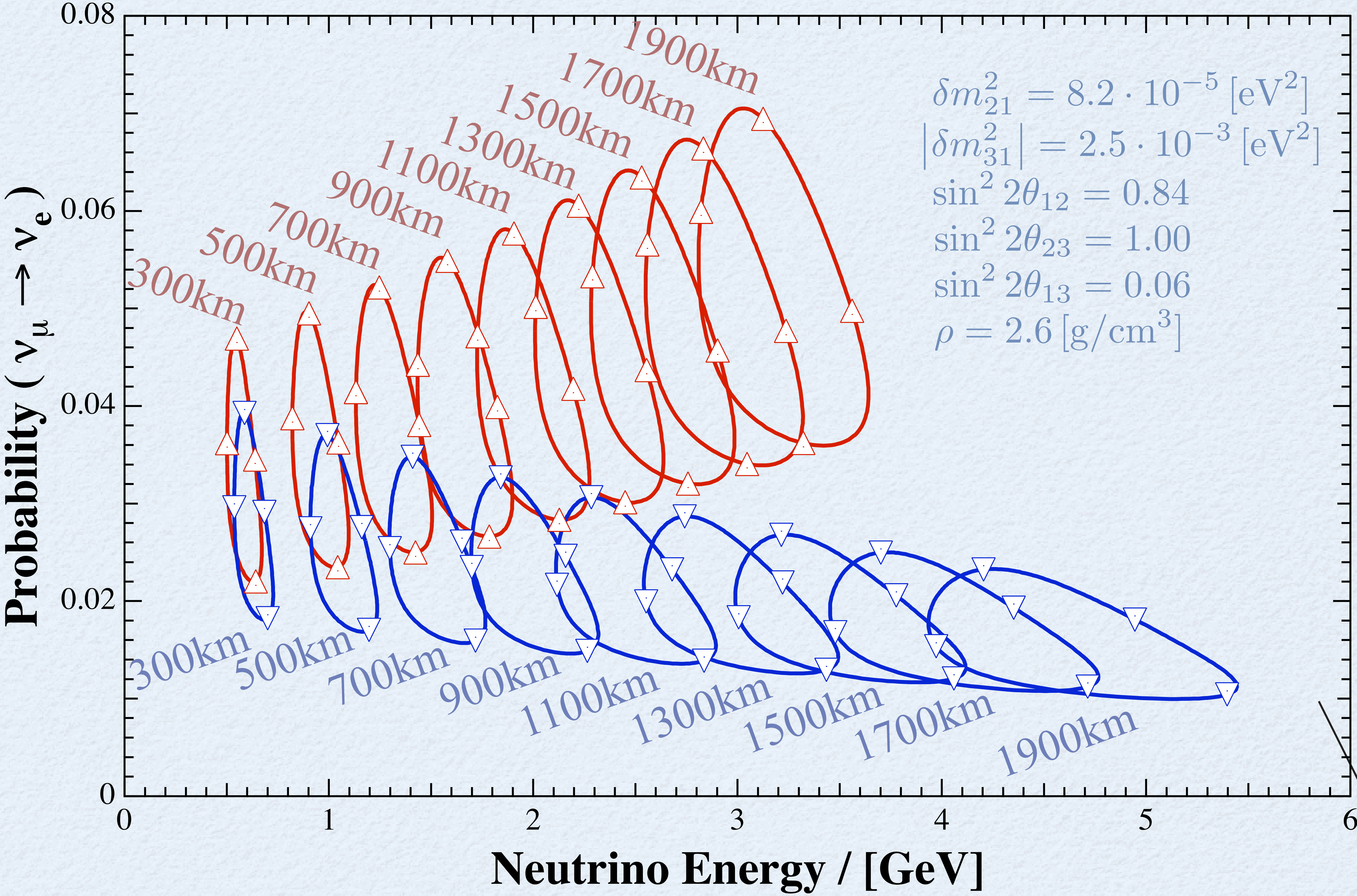
Movement on the Rings

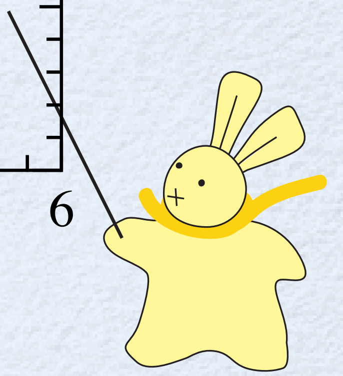


Crosspoints of Rings



The Road of the Rings





Analytic Expressions

- AKS (Arafune-Koike-Sato) approximation

J. Arafune, MK, J. Sato (1997)

$$\mathrm{i}\frac{\mathrm{d}\nu}{\mathrm{d}t} = H\nu = (H_0 + H_1)\nu$$

$$\nu(x) = S(x)\nu(0)$$

$$S(x) = \mathrm{e}^{-\mathrm{i}H_0 x}\,\mathrm{Texp}\left[-\mathrm{i}\int_0^x \mathrm{d}s\, H_1^{(\mathrm{I})}(s)\right] \qquad \left(H_1^{(\mathrm{I})}(x) \equiv \mathrm{e}^{\mathrm{i}H_0 x} H_1 \mathrm{e}^{-\mathrm{i}H_0 x}\right)$$

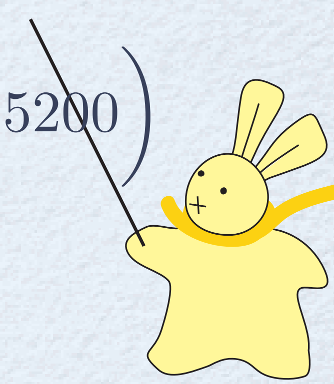
$$= \mathrm{e}^{-\mathrm{i}H_0 x} - \mathrm{e}^{-\mathrm{i}H_0 x}\,\mathrm{i}\int_0^x \mathrm{d}s\, H_1^{(\mathrm{I})}(s) + \cdots$$

$$P(\nu_\alpha \to \nu_\beta) = \left|S_{\beta\alpha}\right|^2 \quad (\{\alpha,\beta\} \in \{\mathrm{e},\mu,\tau\})$$

- Conditions of applicability

(i) $\delta m^2_{21} \ll \delta m^2_{31}$, (ii) $a \ll \delta m^2_{31}$, (iii) $\frac{aL}{2E} \ll 1$

$$\left(\frac{\rho}{[\mathrm{g\,cm^3}]}\frac{E}{[\mathrm{GeV}]} \ll \frac{\delta m^2_{31}}{7.56\cdot 10^{-5}\,[\mathrm{eV^2}]}\right) \quad \left(\frac{\rho}{[\mathrm{g\,cm^3}]}\frac{L}{[\mathrm{km}]} \ll 5200\right)$$



→ Short baseline approximation

Oscillation Probability

- AKS (Arafune-Koike-Sato) second-order approximation (omitted in part)

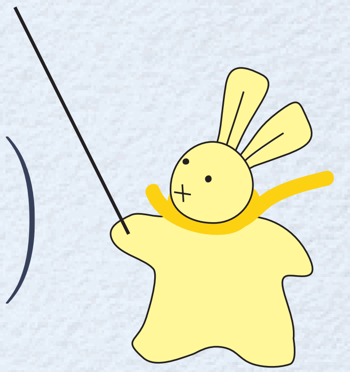
$$P(\nu_\mu \to \nu_{\mathrm{e}}; E) = 4l\Big[C(E)\sin^2\Theta(E) + D(E)\Big],$$

$$\begin{aligned} C(E) &= 1 + 2\frac{\Delta_{\mathrm{m}}}{\Delta_{31}}(1-2s_{13}^2) - \Delta_{21}\frac{j}{l}\sin\delta - \Delta_{21}\frac{\Delta_{\mathrm{m}}}{\Delta_{31}}\frac{j}{l}\left(\sin\delta + \frac{\Delta_{31}}{2}\cos\delta\right) \\ &+ \frac{\Delta_{21}^2}{2}\left[\frac{j}{l}\cos\delta + (1-2s_{12}^2)\right]\frac{j}{l}\cos\delta + 3\frac{\Delta_{\mathrm{m}}^2}{\Delta_{31}^2} \end{aligned}$$

$$\begin{aligned} \Theta(E) &= \frac{\Delta_{31}}{2} - \frac{\Delta_{\mathrm{m}}}{2}(1-2s_{13}^2) + \frac{\Delta_{21}}{2}\left(\frac{j}{l}\cos\delta - s_{12}^2\right) \\ &- \frac{\Delta_{21}}{2}\frac{\Delta_{\mathrm{m}}}{\Delta_{31}}\frac{j}{l}\left(\cos\delta + \frac{\Delta_{31}}{2}\sin\delta\right) + \frac{\Delta_{21}^2}{2}\left[\frac{j}{l}\cos\delta + \frac{1}{2}(1-2s_{12}^2)\right]\frac{j}{l}\sin\delta \end{aligned}$$

$$D(E) = \frac{\Delta_{21}^2}{4}\frac{j^2}{l^2}\sin^2\delta$$

$$\left(\ \Delta_{ij} = \frac{\delta m_{ij}^2 L}{2E}, \quad \Delta_{\mathrm{m}} = \frac{aL}{2E} \qquad l = c_{13}^2 s_{13}^2 s_{23}^2\,, \quad j = c_{13}^2 s_{13} c_{23} s_{23} c_{12} s_{12}\ \right)$$



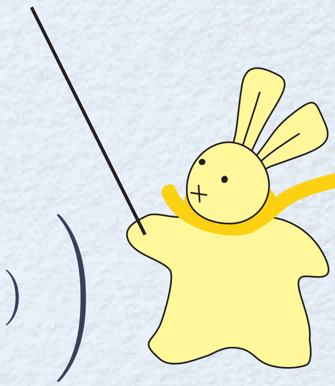
Peak of the Oscillation Probability

- AKS (Arafune-Koike-Sato) second-order approximation (omitted in part)

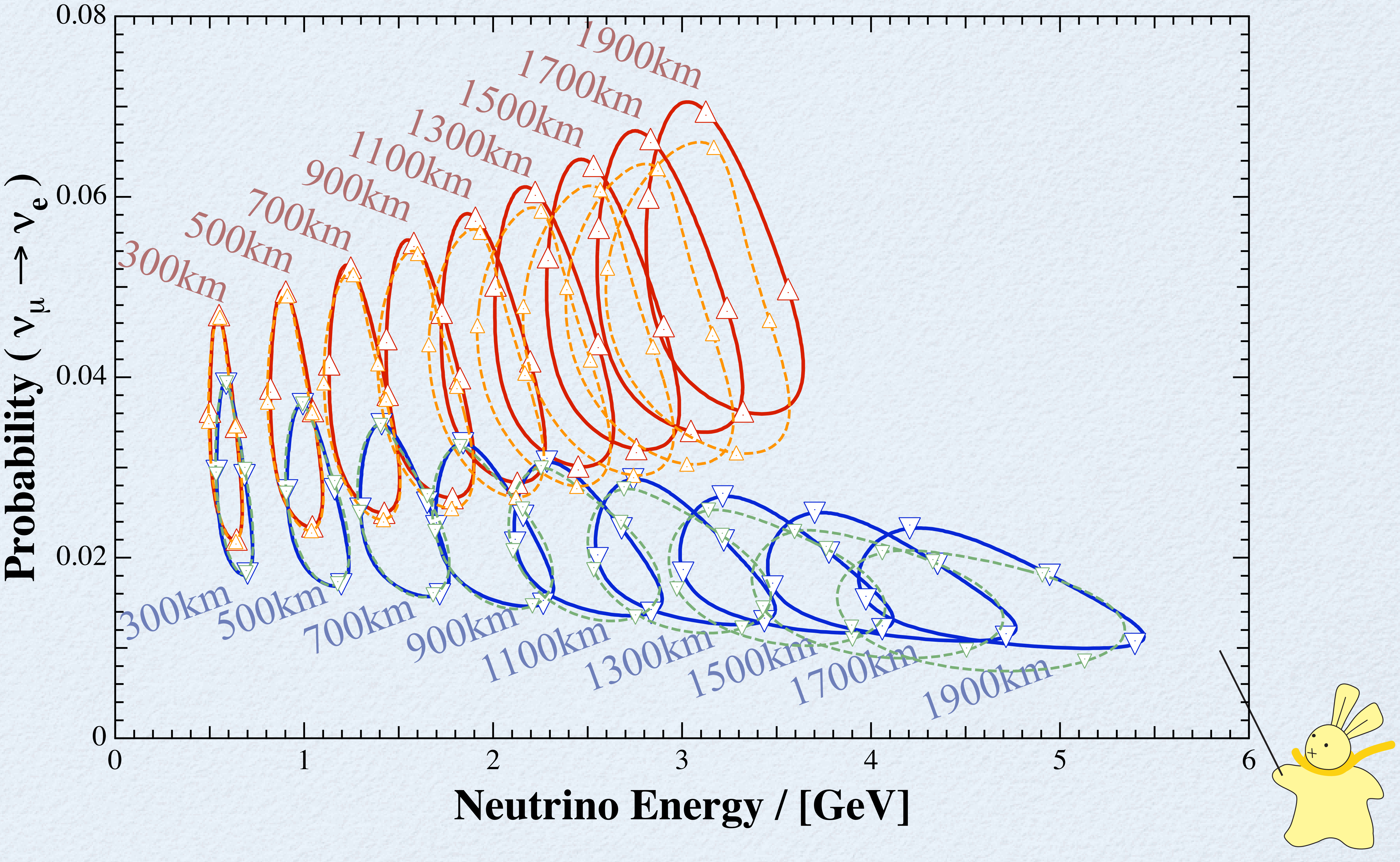
$$
\begin{aligned}
E_{\mathrm{peak},n} = \frac{|\delta m_{31}^2|L}{2\Pi} \Bigg\{ & 1 \mp \frac{\Delta_{\mathrm{m}}}{\Pi}(1-2s_{13}^2)\left(1-\frac{4}{\Pi^2}\right) \mp R s_{12}^2 \pm R\frac{j}{l}\left(\cos\delta \pm \frac{2}{\Pi}\sin\delta\right) \\
& \mp \frac{\Delta_{\mathrm{m}}}{2}R\frac{j}{l}\left[\left(1+\frac{8}{\Pi^2}-\frac{64}{\Pi^4}\right)\sin\delta \pm \frac{2}{\Pi}\left(1-\frac{8}{\Pi^2}\right)\cos\delta\right] \\
& \pm R^2\frac{\Pi}{2}\frac{j}{l}(1-2s_{12}^2)\left(\sin\delta \mp \frac{4}{\Pi}\cos\delta\right) + \frac{\Delta_{\mathrm{m}}^2}{\Pi^2}\left(1-\frac{12}{\Pi^2}+\frac{48}{\Pi^4}\right) \\
& + R^2\frac{j^2}{l^2}\left(\pm\Pi\cos\delta\sin\delta + 1 - 3\cos^2\delta + \frac{4}{\Pi^2}\sin^2\delta\right)\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
P_{\mathrm{peak},n} &\equiv P(\nu_\mu \to \nu_{\mathrm{e}}, E_{\mathrm{peak},n}) \\
&= 4l\,\Bigg\{1 \pm 2\frac{\Delta_{\mathrm{m}}}{\Pi}(1-2s_{13}^2) - R\Pi\frac{j}{l}\sin\delta - \frac{\Pi}{2}\Delta_{\mathrm{m}}R\frac{j}{l}\left[\left(1-\frac{4}{\Pi^2}\right)\cos\delta \mp \frac{4}{\Pi}\left(1-\frac{2}{\Pi^2}\right)\sin\delta\right] \\
&\quad + R^2\frac{\Pi^2}{2}\frac{j}{l}\left[(1-2s_{12}^2)\cos\delta \mp \frac{2}{\Pi}s_{12}^2\sin\delta\right] + \frac{\Delta_{\mathrm{m}}^2}{\Pi^2}\left(1+\frac{4}{\Pi^2}\right) \\
&\quad + \frac{1}{4}R^2\Pi^2\frac{j^2}{l^2}\left(1+\cos^2\delta \pm \frac{4}{\Pi}\cos\delta\sin\delta + \frac{4}{\Pi^2}\sin^2\delta\right)\Bigg\}
\end{aligned}
$$

$$
\left(R \equiv \frac{\delta m_{21}^2}{|\delta m_{31}^2|}\ , \quad \Pi \equiv (2n+1)\pi \ \ (n = 0, 1, 2, \cdots) \right)
$$



Numerical vs. Analytic



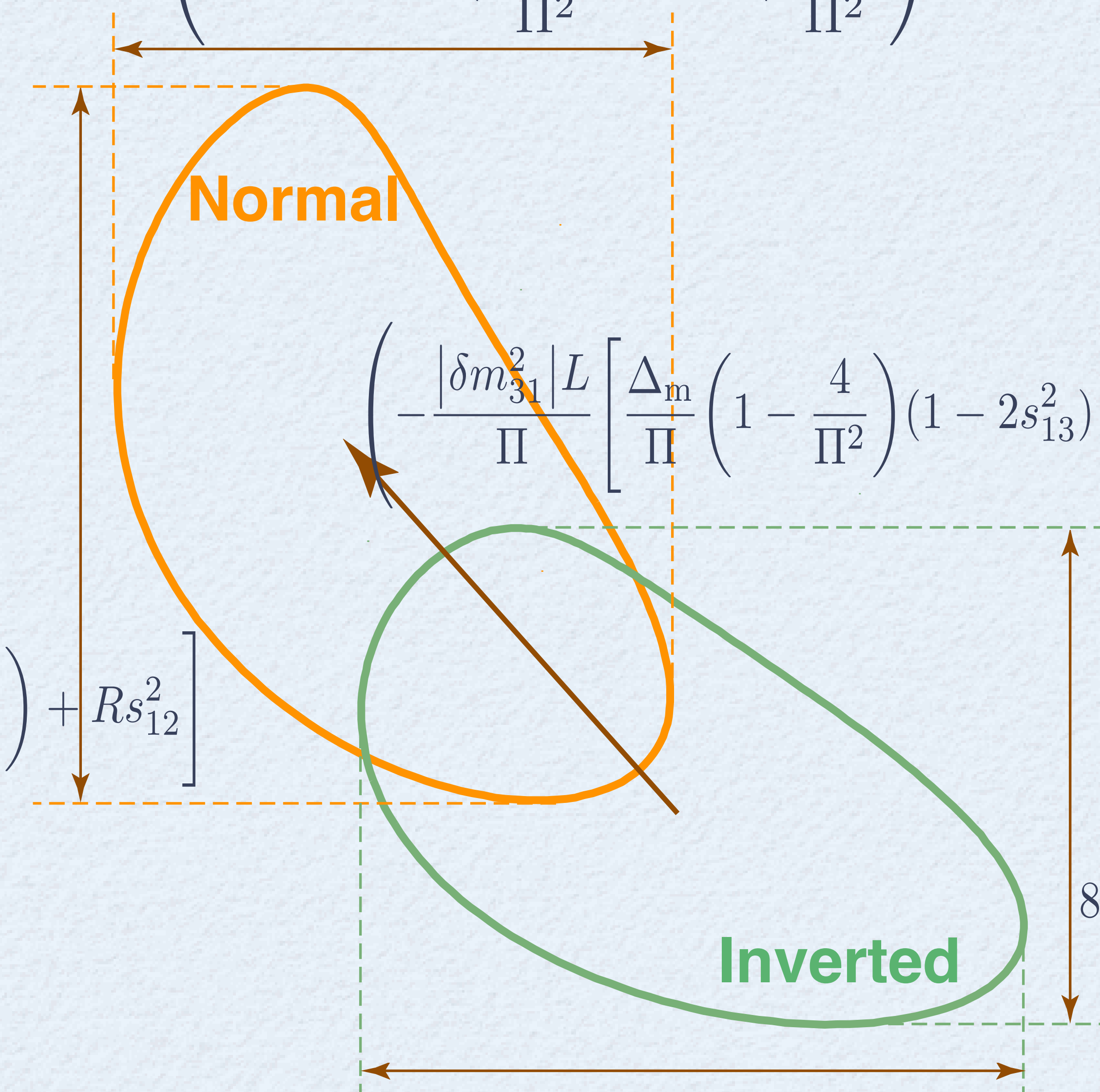
Size of Rings

$$\left(R \equiv \frac{\delta m^2_{21}}{|\delta m^2_{31}|}\right.$$
$$\left.\Pi \equiv (2n+1)\pi \quad (n = 0, 1, 2, \cdots)\right)$$

$$\delta m^2_{21} L \,\frac{j}{l}\left(1 - 2\frac{\Delta_{\rm m}}{\Pi}\,\frac{1-\frac{32}{\Pi^4}}{1+\frac{4}{\Pi^2}} - R\frac{1-2s^2_{12}}{1+\frac{4}{\Pi^2}}\right)$$

$$\left(-\frac{|\delta m^2_{31}|L}{\Pi}\left[\frac{\Delta_{\rm m}}{\Pi}\left(1-\frac{4}{\Pi^2}\right)(1-2s^2_{13}) + Rs^2_{12}\right], \frac{16 l \Delta_{\rm m}}{\Pi}(1-2s^2_{13})\right)$$

$$8Rj\sqrt{1+\frac{4}{\Pi^2}}\left[1+2\frac{\Delta_{\rm m}}{\Pi}\left(1-\frac{2}{\Pi^2}\right)+Rs^2_{12}\right]$$

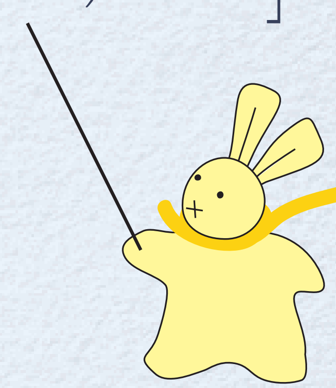


$$8Rj\sqrt{1+\frac{4}{\Pi^2}}\left[1-2\frac{\Delta_{\rm m}}{\Pi}\left(1-\frac{2}{\Pi^2}\right)-Rs^2_{12}\right]$$

$$\delta m^2_{21} L \,\frac{j}{l}\left(1 + 2\frac{\Delta_{\rm m}}{\Pi}\,\frac{1-\frac{32}{\Pi^4}}{1+\frac{4}{\Pi^2}} + R\frac{1-2s^2_{12}}{1+\frac{4}{\Pi^2}}\right)$$

Recall:

$\Delta_{\rm m} \propto L$



Baseline Length of Ring Separation

- Rings, and thus hierarchies, separate at a long baseline $L > L_{\rm crit}$

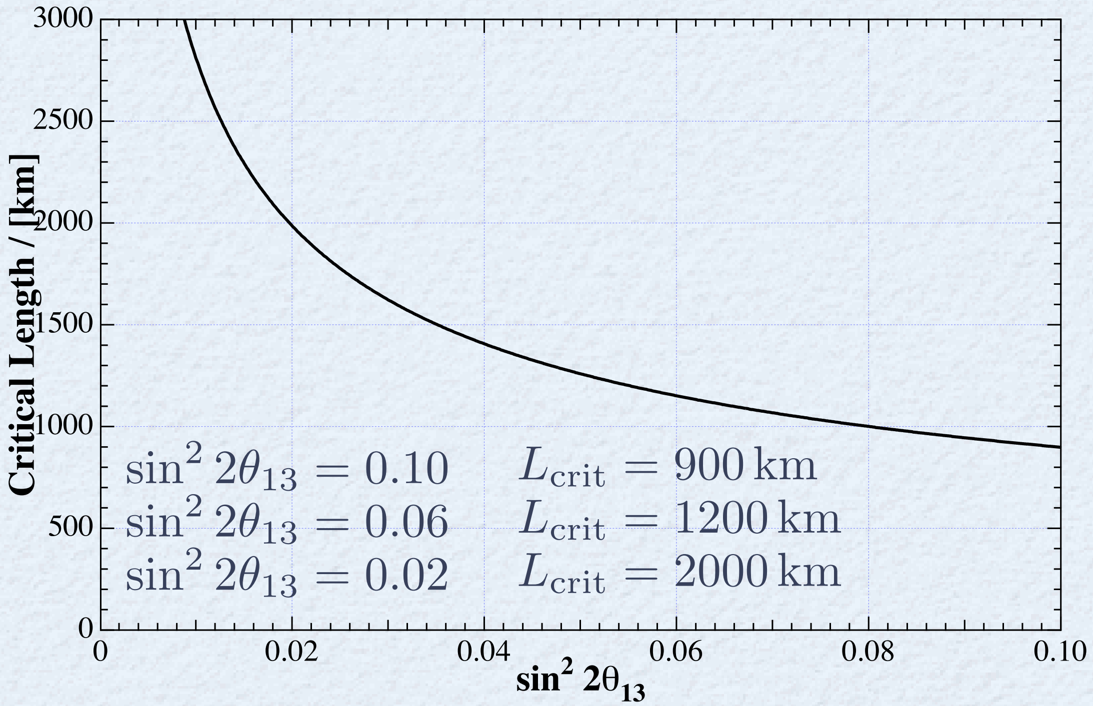
$$L_{\rm crit} = \frac{1}{a'}\frac{\delta m^2_{21}}{|\delta m^2_{31}|}\frac{1}{1-2s^2_{13}}\frac{\Pi}{1-\frac{12}{\Pi^2}+\frac{64}{\Pi^4}}\left[-\left(1-\frac{8}{\Pi^2}\right)s^2_{12}+\sqrt{\left(1-\frac{12}{\Pi^2}+\frac{64}{\Pi^4}\right)\frac{c^2_{23}c^2_{12}s^2_{12}}{s^2_{23}s^2_{13}}-\frac{4}{\Pi^2}s^4_{12}}\right]$$

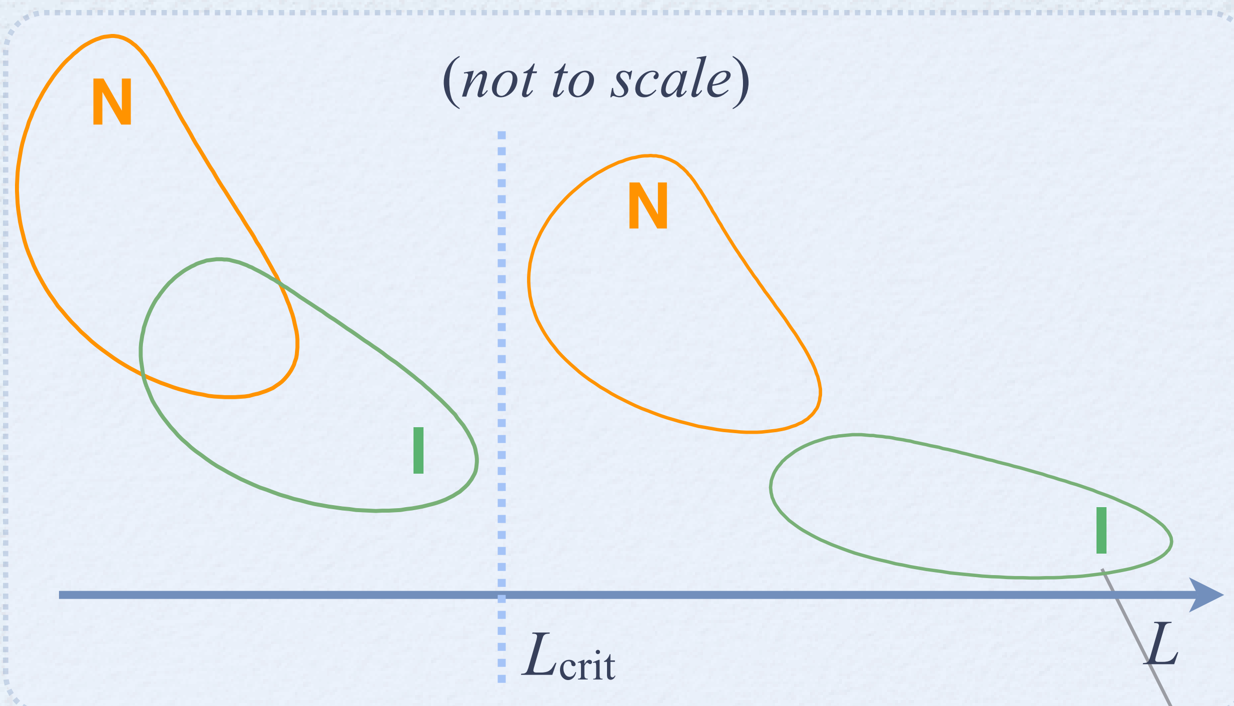
(*up to first order*)

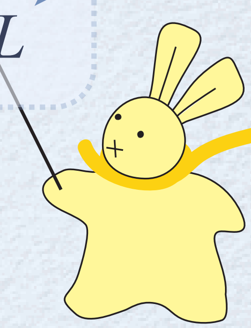
$$\simeq \frac{1}{a'}\frac{\delta m^2_{21}}{|\delta m^2_{31}|}\frac{\Pi}{\sqrt{1-\frac{12}{\Pi^2}+\frac{64}{\Pi^4}}}\frac{c_{23}c_{12}s_{12}}{s_{23}s_{13}(1-2s^2_{13})} \sim \frac{1}{s_{13}}$$

$$\left(\frac{1}{a'} \equiv \frac{1}{\sqrt{2}G_{\rm F}n_{\rm e}} = \frac{5.17\cdot 10^3\,[{\rm km}]}{\frac{\rho}{[{\rm g\,cm^{-3}}]}}\right)$$

For our example parameter set,

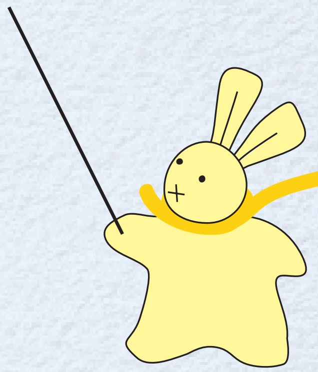




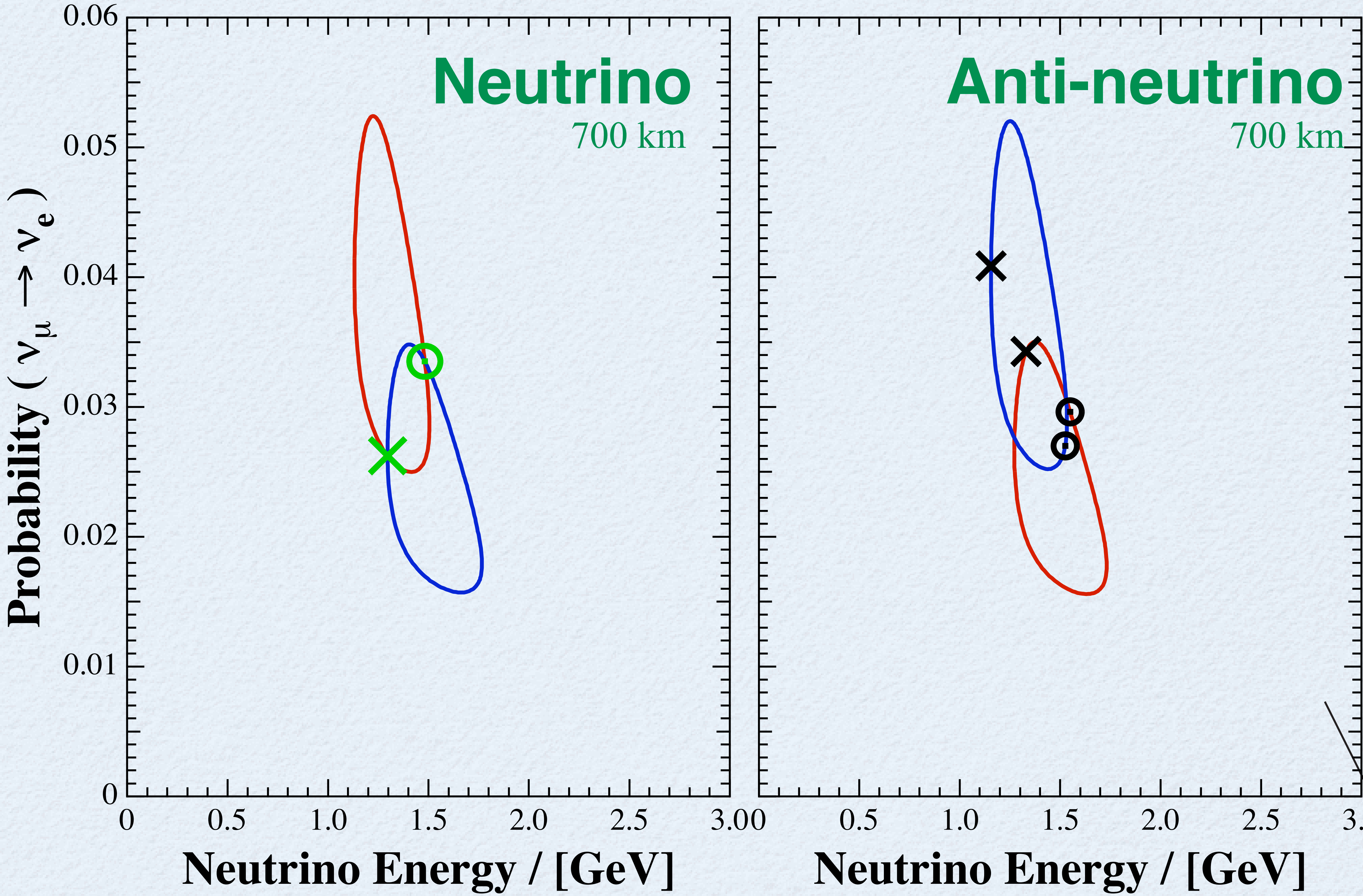


Resolving the Degeneracy

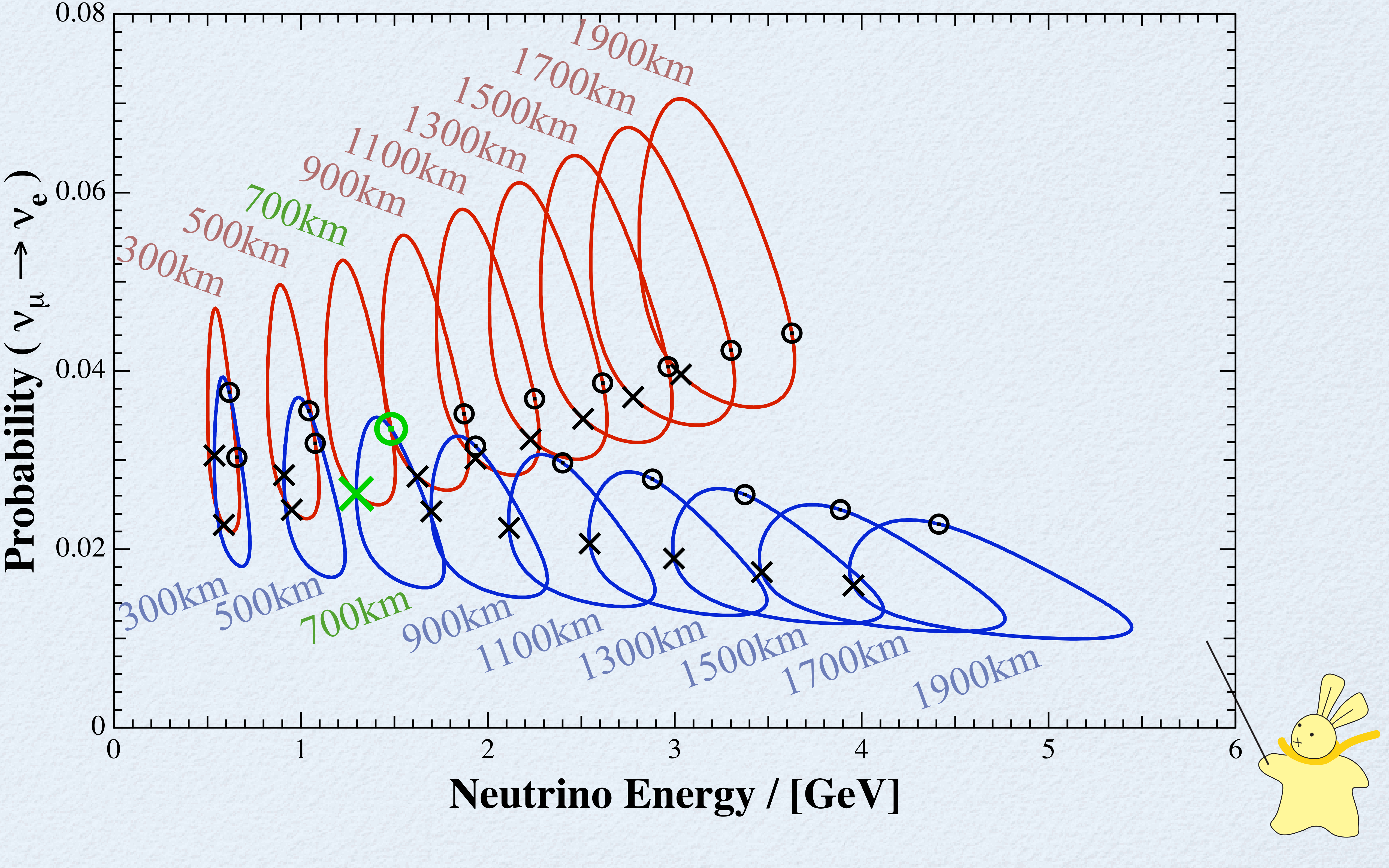
- The way out from the degeneracy (we are discussing)?
  - Go for a long-length baseline, $L > L_{\mathrm{crit}}$.
  - Employ anti-neutrino beams together with neutrino beams.
  - Combine two different baseline lengths. (“The Two Towers”)
  - Push the detection to the lower-energy neutrinos.



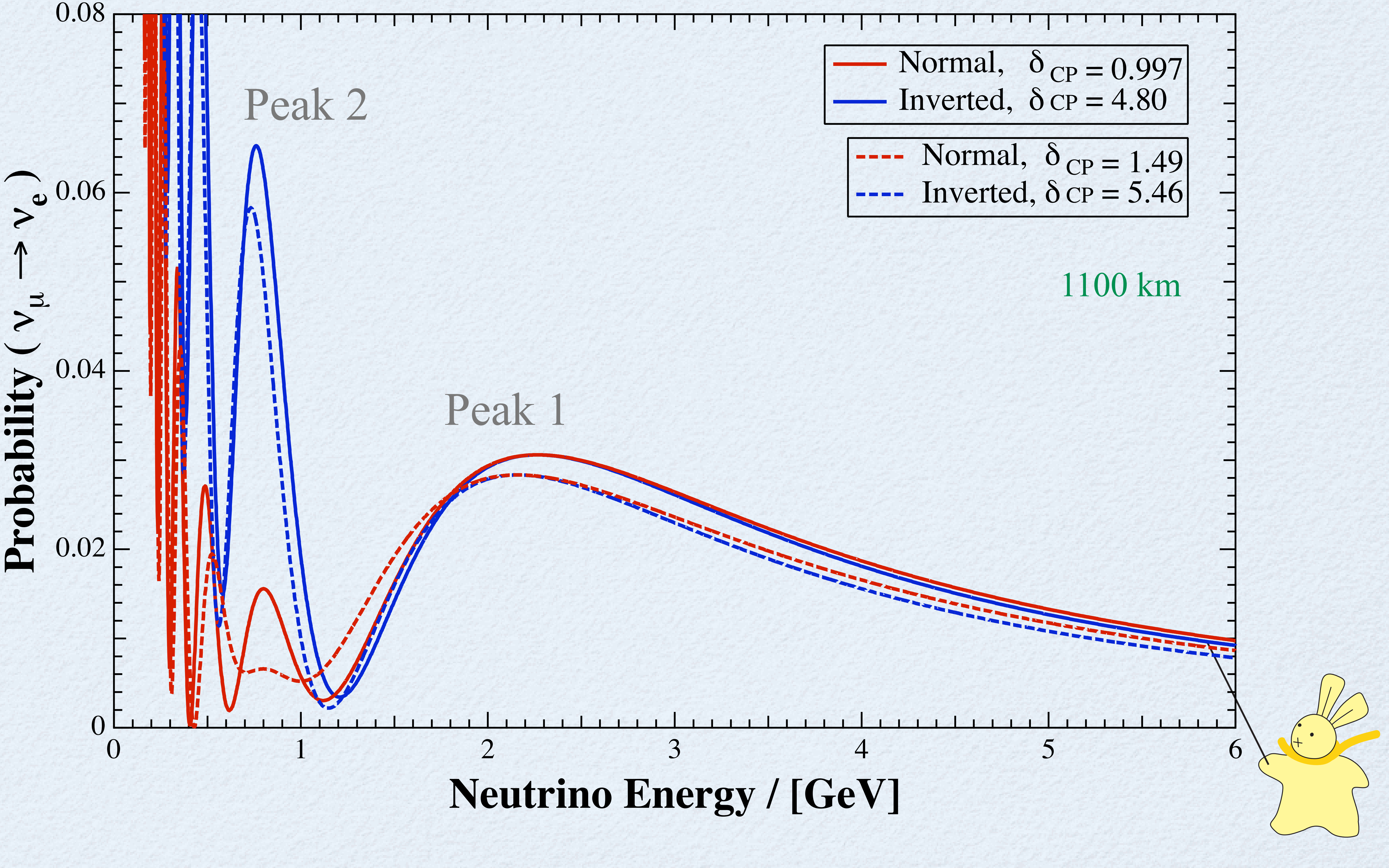
Use of Anti-neutrinos



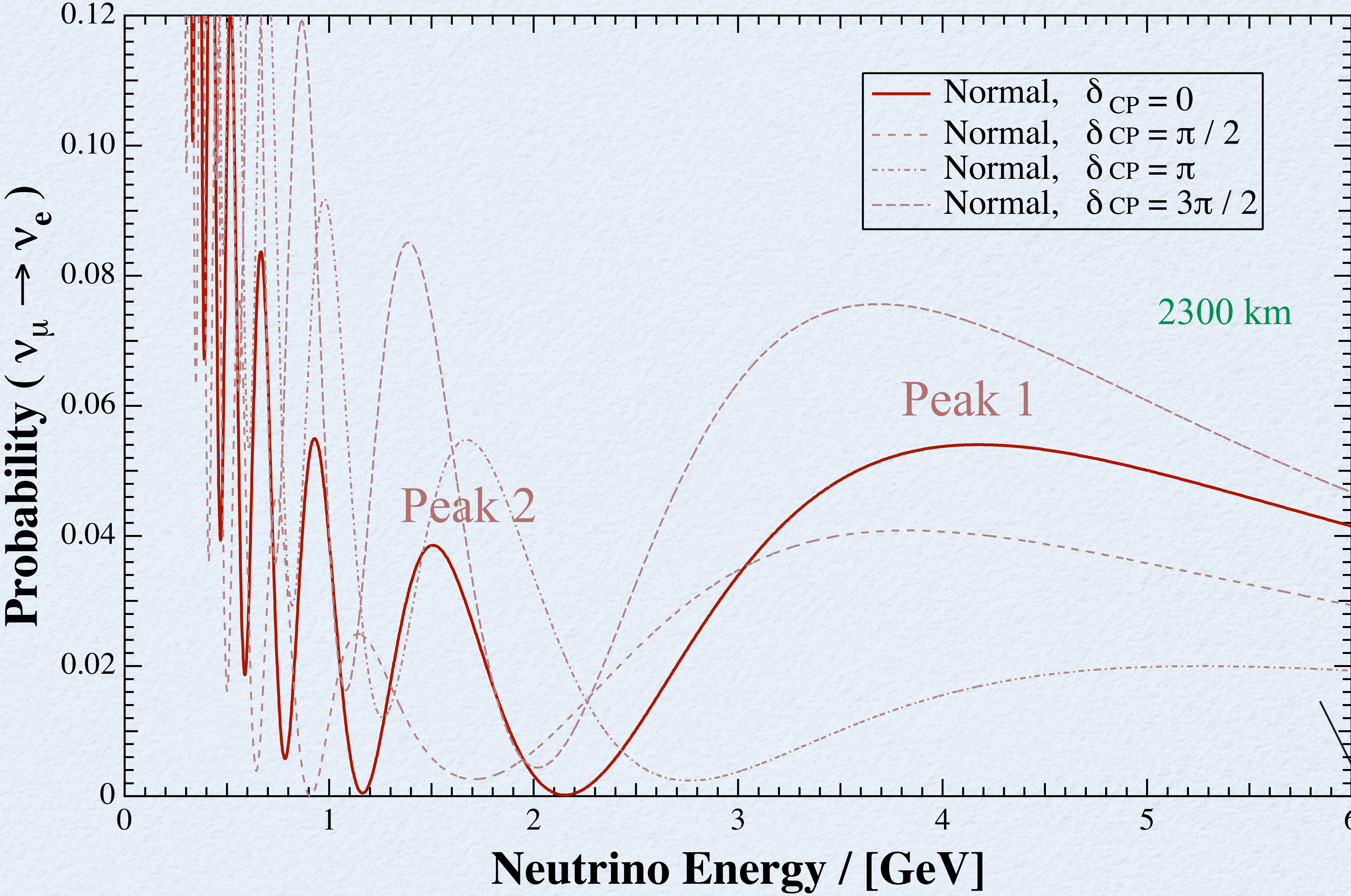
Use of Another Distance

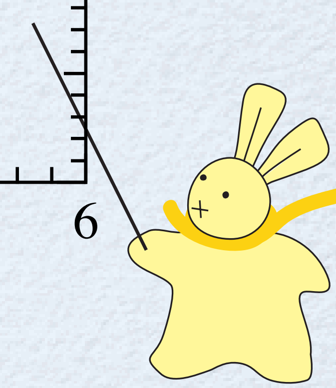


Use of Another Peak

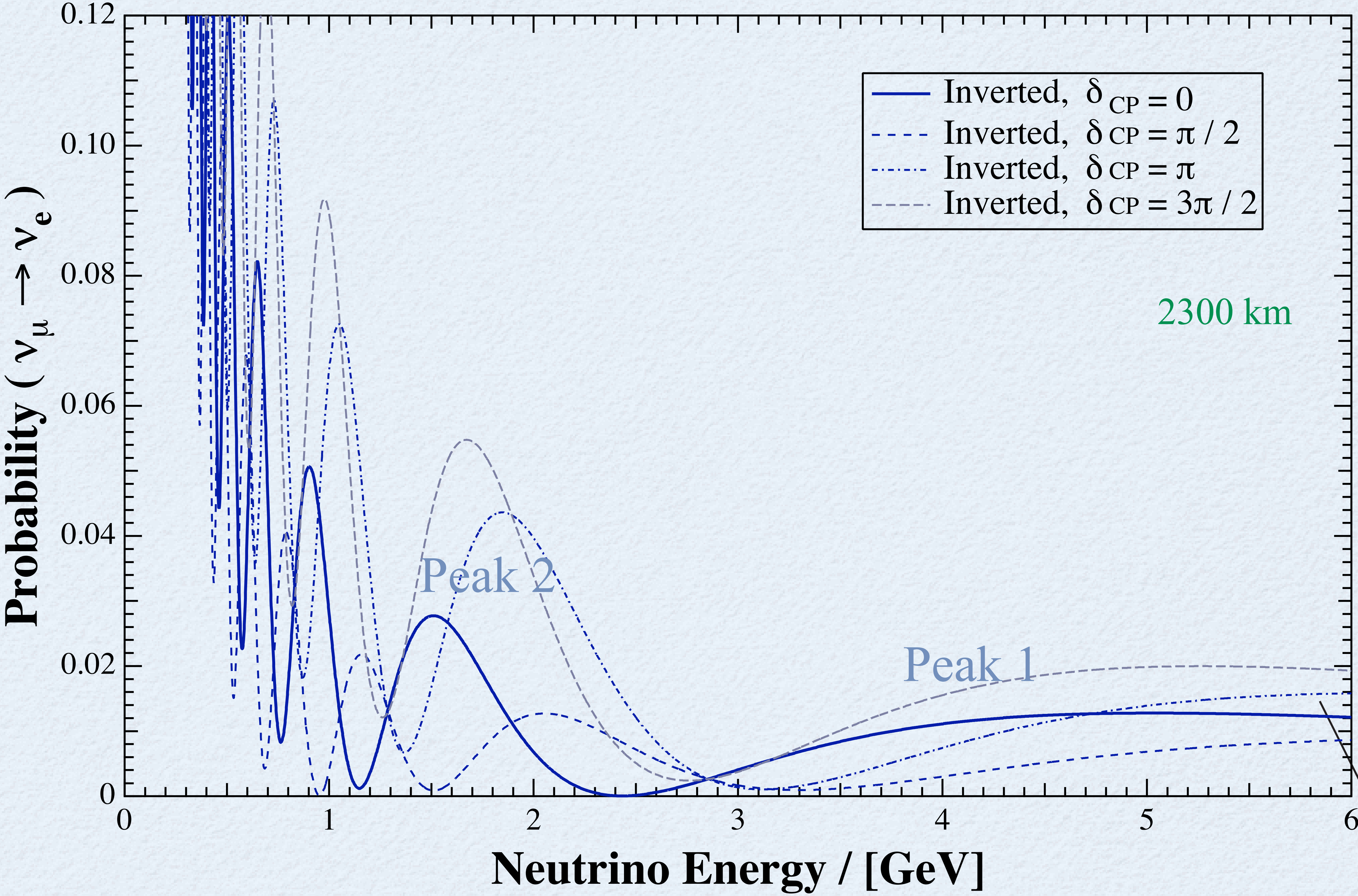


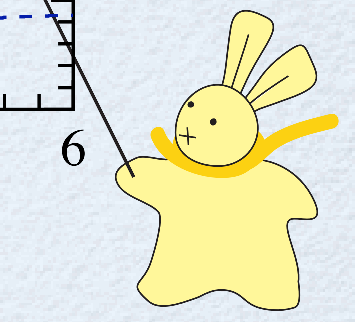
Use of Another Peak





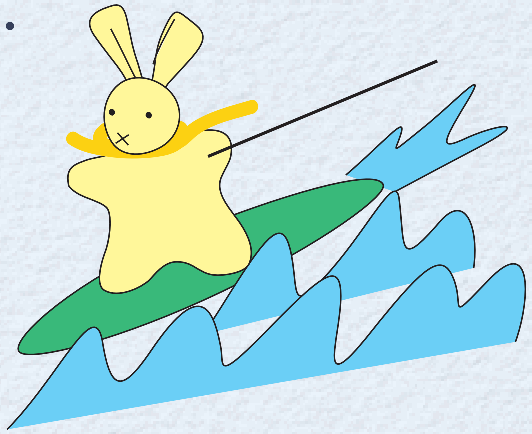
Use of Another Peak





Conclusions, Outlooks

- Determination of neutrino parameters may be complicated due to the degeneracy.
- The peak of the oscillation is a good representative of the whole spectra.
  - Peak-matching leads to the mutually "similar " oscillation spectra.
- The analysis of the peak position provides a perspective of the presence and absence of parameter degeneracies in the long baseline experiments.



- The parameter-searching power can be systematically analyzed.
  - The road of the rings: Various baseline length
  - Blurred rings (*to be done*): Ambiguities of the oscillation parameters
  - Another road of the rings (*to be done*): Combination of neutrinos and anti-neutrinos