Kaon B-parameters

for Generic $\Delta S = 2$ Four-Quark Operators in Quenched Domain Wall QCD

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Plan of talk

- **Introduction**
  - **Generic** $\Delta S = 2$ **four quark operators in the physics Beyond Standard Model**
- **Calculation of perturbative renormalization factors**
- **Simulation Results**
  - **Measurement of**
    $R_i \equiv \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{\langle \overline{K^0}|O_1(\mu)|K^0\rangle}, B_i \equiv \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{C_i\langle \overline{K^0}|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle}, i = 2, \ldots, 5$
- **Summary**

Introduction

- **Purpose**
  - **Focus on the indirect CP violation in neutral kaon physics determine parameter ϵ using first principle of QCD $\Rightarrow$ examination of SM, constraints on the physics BSM**
- **Indirect CP violation parameter ϵ**

$$\epsilon \sim \frac{e^{\frac{i}{4}\pi}}{\sqrt{2}\Delta M_K} \mathrm{Im}\left(\frac{1}{2m_K} \langle \overline{K^0} | H_{eff}^{\Delta S=2} | K^0 \rangle \right)$$

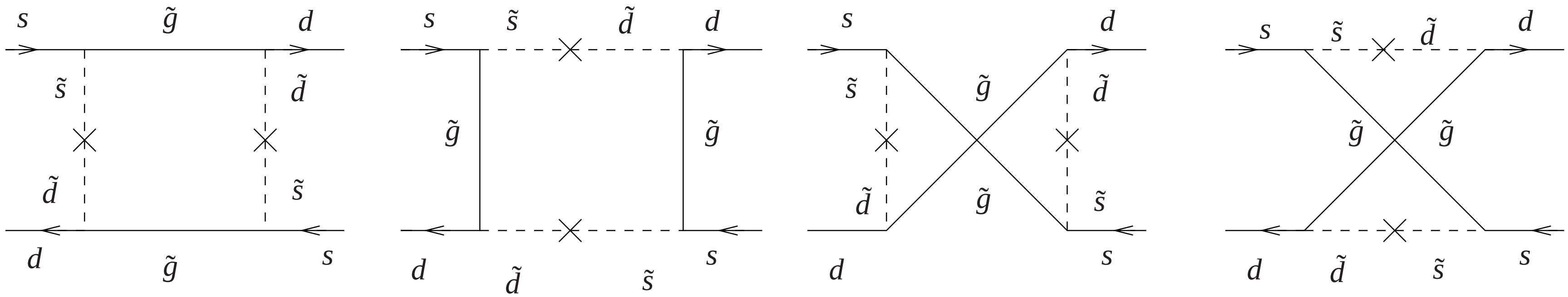
$$H_{eff}^{\Delta S=2} = \frac{G_F^2}{16\pi^2} M_W^2 F_{\mathrm{Inami-Lin}} C(\mu) O_{\mathrm{LL}}(\mu)$$

- **Bag parameter for SM**

$$\langle \overline{K^0} | \bar{s}\gamma^\mu(1-\gamma_5)d\bar{s}\gamma^\mu(1-\gamma_5)d | K^0 \rangle = \frac{8}{3} B_K F_K^2 m_K^2$$

$$B_K \equiv \frac{\langle \overline{K^0} | \bar{s}\gamma^\mu(1-\gamma_5)d\bar{s}\gamma^\mu(1-\gamma_5)d | K^0 \rangle}{\frac{8}{3}\langle \overline{K^0} | \bar{s}\gamma^\mu(1-\gamma_5)d | 0 \rangle \langle 0 | \bar{s}\gamma^\mu(1-\gamma_5)d | K^0 \rangle}$$

$\Delta S = 2$ transition in super symmetric model



- **Effective Hamiltonian** M. Ciuchini et al.,JHEP 9810(1998)008

$$H_{eff}^{\Delta S=2} = \sum_{i=1}^{5} C_i O_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{O}_i \qquad C_i, \tilde{C}_i : \text{Wilson coefficients}$$

$$O_1 = \bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha \bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta , O_2 = \bar{s}^\alpha (1-\gamma_5) d^\alpha \bar{s}^\beta (1-\gamma_5) d^\beta$$

$$O_3 = \bar{s}^\alpha (1-\gamma_5) d^\beta \bar{s}^\beta (1-\gamma_5) d^\alpha , O_4 = \bar{s}^\alpha (1-\gamma_5) d^\alpha \bar{s}^\beta (1+\gamma_5) d^\beta$$

$$O_5 = \bar{s}^\alpha (1-\gamma_5) d^\beta \bar{s}^\beta (1+\gamma_5) d^\alpha$$

Operator$\tilde{O}_{1,2,3}$ are obtained from the $O_{1,2,3}$ by exchange $L \leftrightarrow R$

the physics beyond the SM involves the four quark operators with more general chiral structures than the SM

Lattice QCD calculations

- **The ratio of BSM- to SM- matrix elements**

$$R_i(\mu) \equiv \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{\langle \overline{K^0}|O_1(\mu)|K^0\rangle}, \qquad i = 2, \ldots, 5$$

- **B-parameter for SM**

$$B_1(\mu) = B_K(\mu) \equiv \frac{\langle \overline{K^0}|O_1(\mu)|K^0\rangle}{\frac{8}{3}\langle \overline{K^0}|\bar{s}\gamma_\mu\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_\mu\gamma_5 d|K^0\rangle}$$

- **B-parameters for BSM**

$$B_i(\mu) \equiv \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{C_i\langle \overline{K^0}|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle}, \qquad i = 2, \ldots, 5$$

convention factors :$C_i \equiv \{\frac{5}{3}, -\frac{1}{3}, -2, -\frac{2}{3}\}$

Some systematic errors are expected to cancel out in the R_i **and** B_i

Perturbative renormalization factors

Operators on the lattice $\overset{\textit{Matching}}{\longleftrightarrow}$ **$\overline{\text{MS}}$ in continuum**

Z **factor**

Chiral symmetry in the Domain Wall fermion
$\Rightarrow$ **The renormalization patterns of four quark operators are same as in the continuum**

- O_1 **is renormalized multiplicatively**
- **mixing between O_2 and O_3**
- **mixing between O_4 and O_5**

One loop renormalization factors with mean field improvement

Y.N&Y.Kuramashi PRD73(2006)094502

$$O_i^{\overline{\text{MS}}}(\mu) = \frac{1}{(1-w_0)^2 Z_w^2} Z_{ij}(\mu a) O_j(1/a)^{\text{lat}}$$

Where matching scale is $\mu = 1/a$

Numerical values of Z_{ij} with mean field improvement

$$Z_{ij}^{\mathrm{Plaquette}}(\mu = 1/a) = \begin{pmatrix} 0.7287 & 0 & 0 & 0 & 0 \\ 0 & 0.6845 & -0.00156 & 0 & 0 \\ 0 & -0.0352 & 0.8682 & 0 & 0 \\ 0 & 0 & 0 & 0.6325 & -0.0414 \\ 0 & 0 & 0 & -0.0689 & 0.7564 \end{pmatrix}$$

$$Z_{ij}^{\mathrm{Iwasaki}}(\mu = 1/a) = \begin{pmatrix} 0.8062 & 0 & 0 & 0 & 0 \\ 0 & 0.8124 & -0.00679 & 0 & 0 \\ 0 & -0.0156 & 0.9241 & 0 & 0 \\ 0 & 0 & 0 & 0.7847 & -0.0427 \\ 0 & 0 & 0 & -0.0477 & 0.8425 \end{pmatrix}$$

- **Plaquette gauge action :** $\beta = 6.0$, $a^{-1} = 2.12$**GeV,** $\tilde{M} = 1.311$
- **Iwasaki gauge action :** $\beta = 2.6$, $a^{-1} = 2.00$**GeV,** $\tilde{M} = 1.420$
- O_1 **is renormalized multiplicatively,**
  mixing between O_2 **and** O_3**,** O_4 **and** O_5
- **The Iwasaki gauge action shows the smaller 1-loop correction than the Plaquette gauge action.**

Simulation parameters

Same parameters as in previous CP-PACS calculation of ϵ'/ϵ

PhysRevD68(2003)014501

- **Domain Wall fermion with wall height** $M = 1.8$
  - $m_f a = 0.02, 0.03, 0.04, 0.05, 0.06$
  - **Degenerate quark mass for** d **and** s
- **Plaquette gauge action at** $\beta = 6.0$
  - $a^{-1} \approx 2.12$**GeV from Sommer scale** r_0
  - **lattice size:** $N_\sigma^3 \times N_t \times N_5 = 16^3 \times 32 \times 16$
  - 180 **configurations**
- **Iwasaki gauge action at** $\beta = 2.6$
  - $a^{-1} \approx 2.00$**GeV from Sommer scale** r_0
  - **lattice size:** $N_\sigma^3 \times N_t \times N_5 = 16^3 \times 32 \times 16$ **and** $24^3 \times 32 \times 16$
  - 400 **configurations for** $N_\sigma^3 = 16^3$
    400 **or** 200 **configurations for** $N_\sigma^3 = 24^3$

Simulation Results for R_i

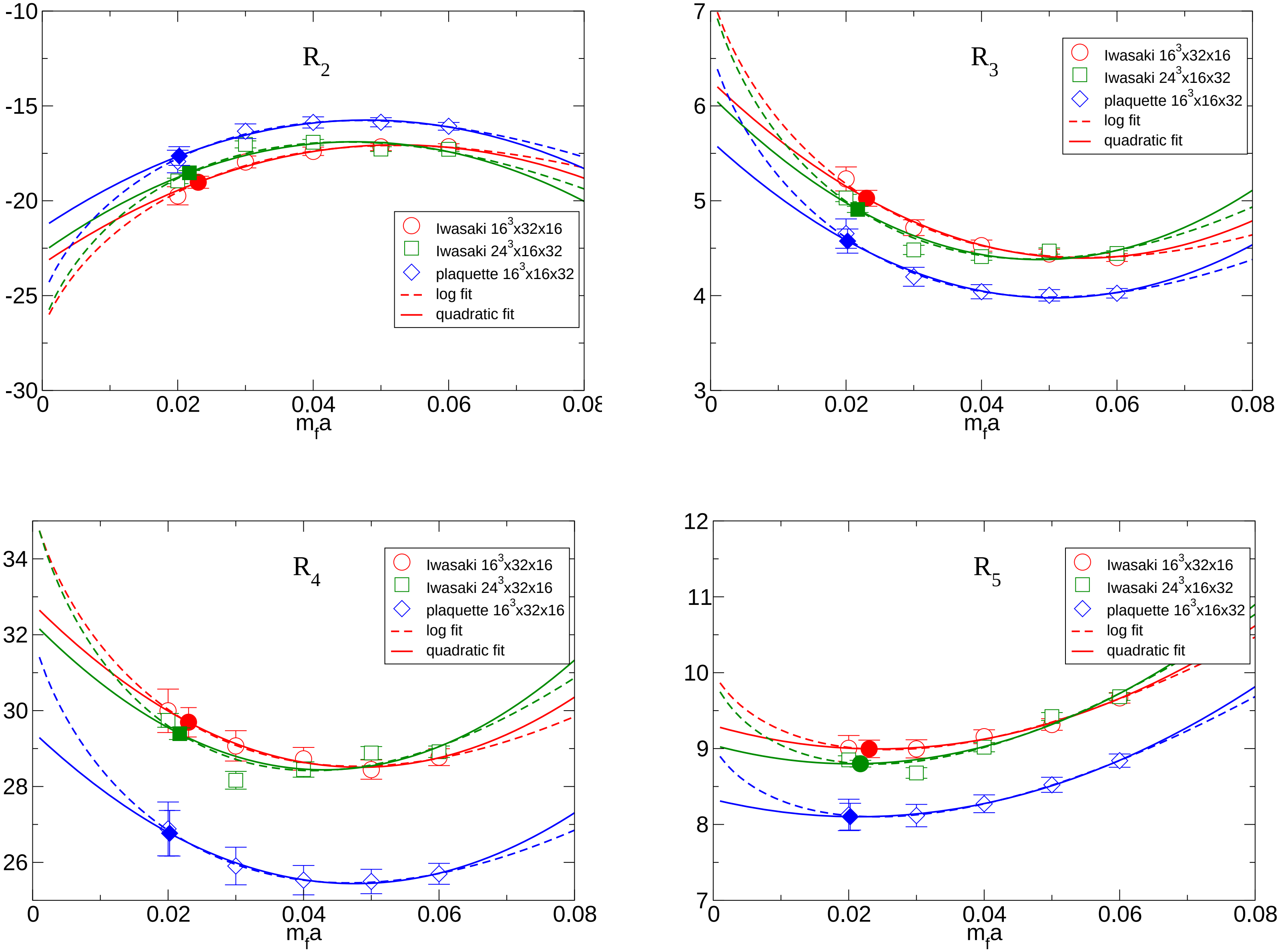
- **The ratio of BSM- to SM- matrix elements**

$$R_i(\mu) \equiv \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{\langle \overline{K^0}|O_1(\mu)|K^0\rangle} \Rightarrow \left[\frac{1}{M_K^2}\right]_{\rm exp} \left[m_M^2 \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{\langle \overline{K^0}|O_1(\mu)|K^0\rangle}\right]_{\rm lat}, \qquad i = 2, \ldots, 5$$

  - $\langle \overline{K^0}|O_1|K^0\rangle \propto m_K^2$
    $\Rightarrow$ **the ratio diverges in the chiral limit**
  - **The factor of m_M^2 is to keep the ratio finite in the chiral limit**
    - m_M **is the pseudo scalar mass on the lattice**
    - $[M_K]_{\rm exp}$ **is the experimental value of kaon mass**

- **Fitting functions**
  - **quadratic form :** $R_i = b_0 + b_1 m_f a + b_2 (m_f a)^2$
  - **logarithmic form :** $R_i = b_0 + b_1 m_f a + b_2 m_f a \log(m_f a)$

Quark mass dependence of $R_i = \left[\frac{1}{M_K^2}\right]_{\rm exp}\left[m_M^2 \frac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{\langle \overline{K^0}|O_1(\mu)|K^0\rangle}\right]_{\rm lat}$



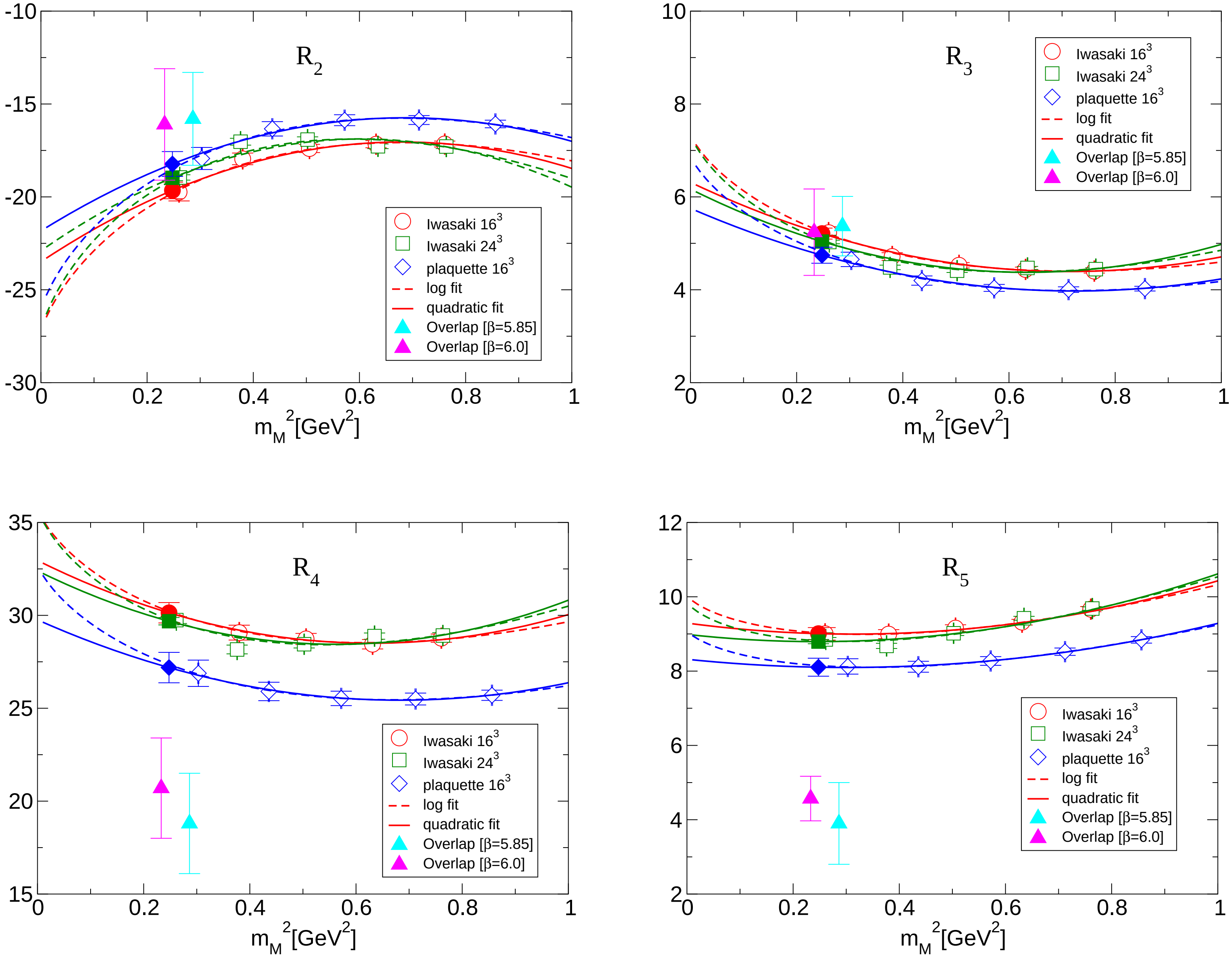
- R_i **at the physical point with the quadratic fit**

	Plaquette	**Iwasaki**	
	$16^3 \times 32 \times 16$	$16^3 \times 32 \times 16$	$24^3 \times 32 \times 16$
R_1	1	1	1
R_2	$-17.64(49)(12)$	$-19.03(32)(2)$	$-18.53(12)(1)$
R_3	4.57(13)(3)	5.03(08)(0)	4.91(03)(0)
R_4	26.77(60)(6)	29.69(39)(2)	29.39(15)(0)
R_5	8.10(18)(2)	9.00(12)(1)	8.80(05)(0)

- **Small volume dependence**
- 10% **disrepancies for** R_4 **and** R_5 **between Iwasaki and Plaquette could be** $O(a^2)$ **effects**

$|\langle \overline{K^0}|O_i(\mu)|K^0\rangle|$ $(i = 2, \ldots, 5)$ **are much larger than** $|\langle \overline{K^0}|O_1(\mu)|K^0\rangle|$

Comparison with the overlap results R.Babich et.al. hep-lat/0605016



- **Consistent results for R_2 and R_3**
- **Large discrepancies for R_4 and R_5**
- **Possible source for the discrepancies**
  - **Renormalization method**
    - **1-loop perturbation vs Non-perturbative RI/MOM scheme**

Future work

- **Non-Perturbative renormalization for DWF using Schödinger functional method**

  Y.Taniguchi hep-lat/0604002

Simulation Results for B_i

- **B-parameter for SM**

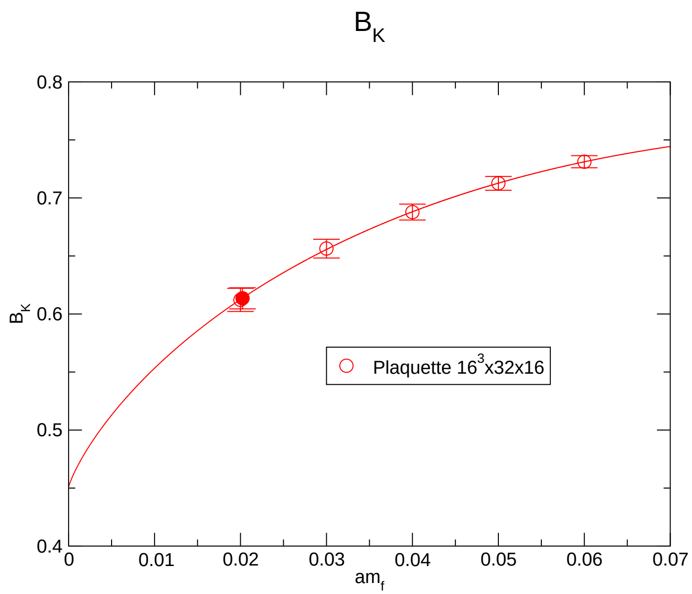
$$B_1(\mu) = B_K(\mu) \equiv \frac{\langle \overline{K^0} | O_1(\mu) | K^0 \rangle}{\frac{8}{3}\langle \overline{K^0} | \bar{s}\gamma_\mu\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_\mu\gamma_5 d | K^0 \rangle}$$

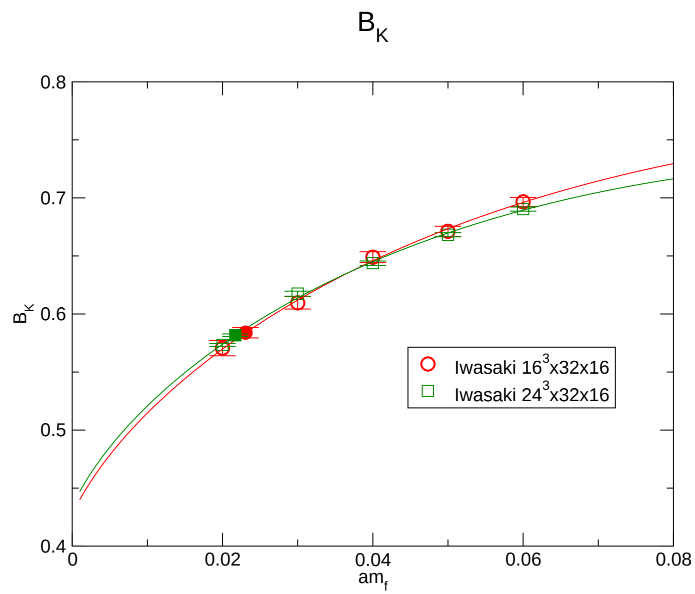
- **B-parameters for BSM**

$$B_i(\mu) \equiv \frac{\langle \overline{K^0} | O_i(\mu) | K^0 \rangle}{C_i \langle \overline{K^0} | \bar{s}\gamma_5 d | 0 \rangle \langle 0 | \bar{s}\gamma_5 d | K^0 \rangle}, \qquad i = 2, \ldots, 5$$

convention factors : $C_i \equiv \{\frac{5}{3}, -\frac{1}{3}, -2, -\frac{2}{3}\}$

Quark mass dependence of B_K



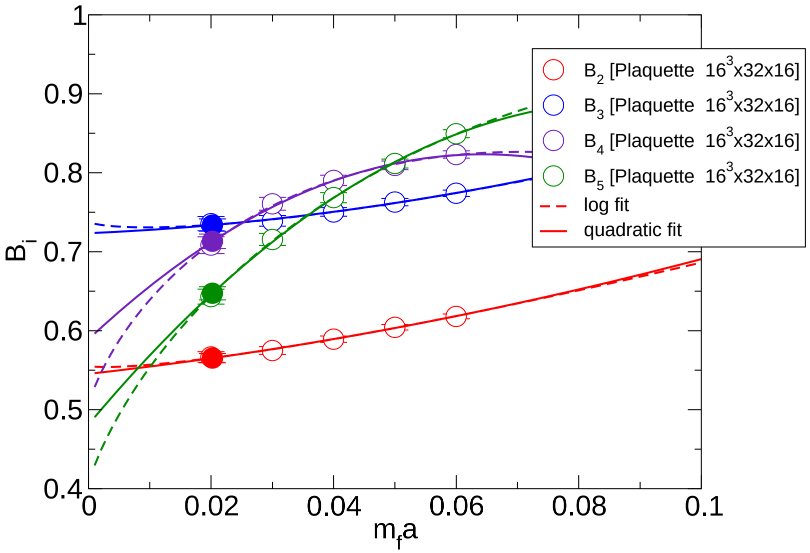


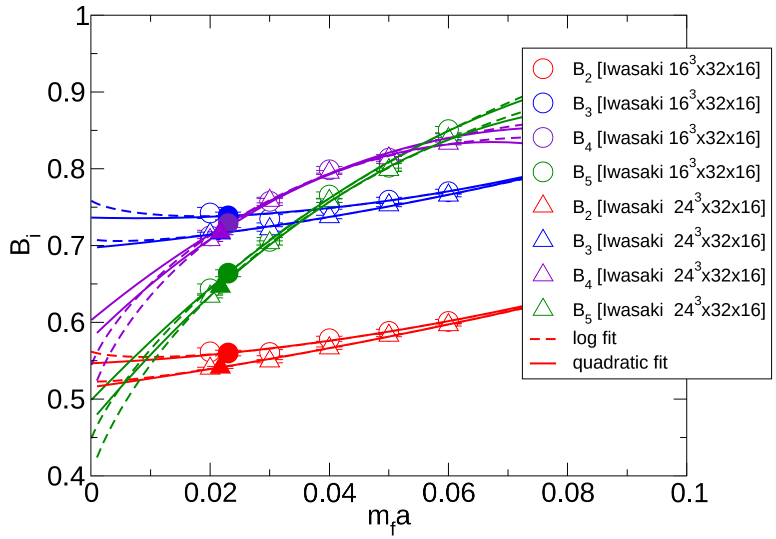
Fitting function :

$$B_K = B(1 - 3cm_f a \log(m_f a) + bm_f a)$$

The solid symbols denote the interpolated results at $m_s a/2$

Quark mass dependence of $B_i = \dfrac{\langle \overline{K^0}|O_i(\mu)|K^0\rangle}{C_i\langle \overline{K^0}|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle}$





Fitting function

- **quadratic fit :** $B_i = b_0 + b_1 m_f a + b_2 (m_f a)^2$
- **logarithmic fit :** $B_i = b_0 + b_1 m_f a + b_2 m_f a \log(m_f a)$

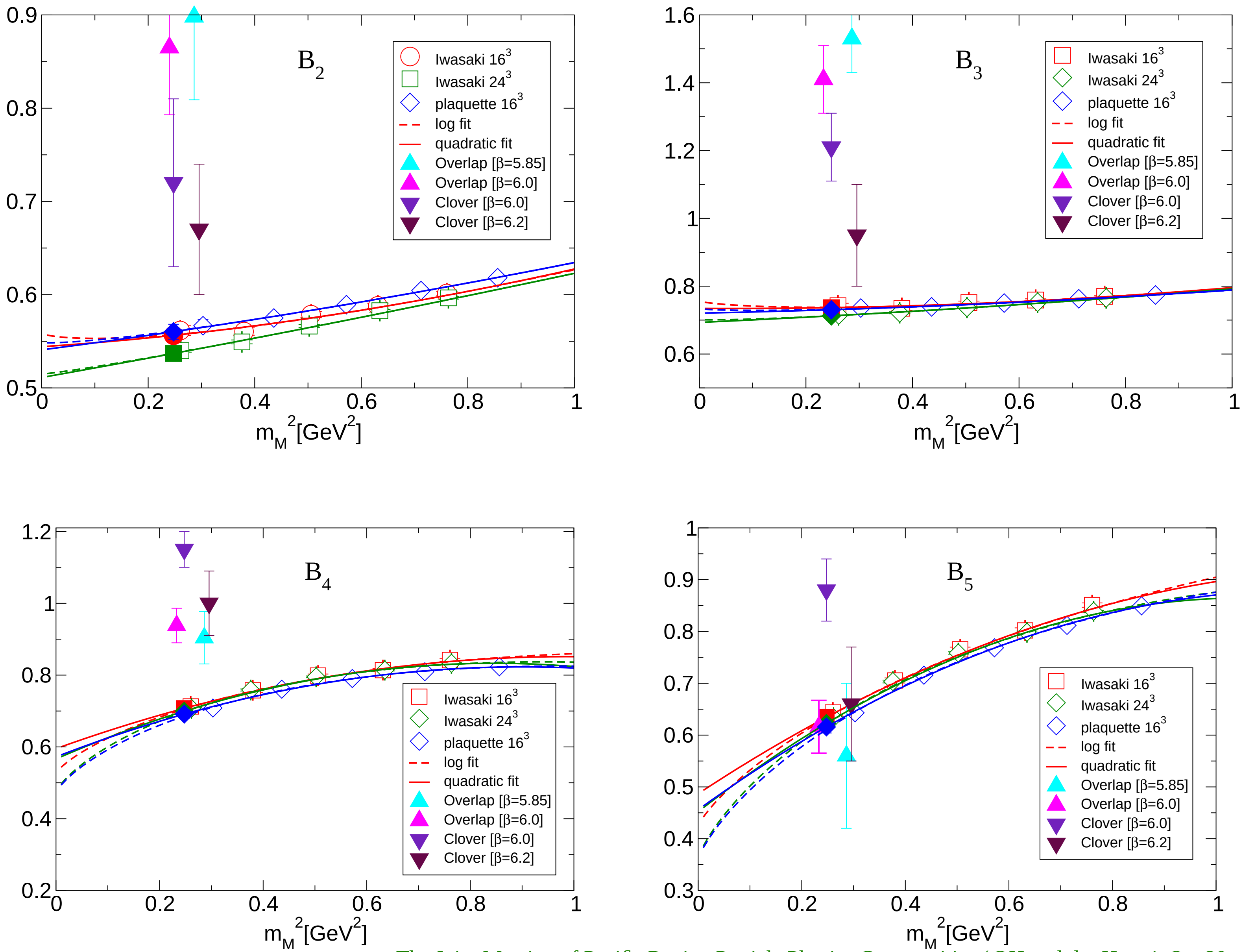
B_i at the physical point with quadratic fit

	Plaquette	Iwasaki	
	$16^3 \times 32 \times 16$	$16^3 \times 32 \times 16$	$24^3 \times 32 \times 16$
B_1	0.6135(91)	0.5839(45)	0.5817(11)
B_2	0.5653(58)(4)	0.5600(36)(2)	0.5413(13)(0)
B_3	0.7337(78)(5)	0.7386(50)(3)	0.7160(19)(0)
B_4	0.7132(78)(19)	0.7285(50)(2)	0.7172(17)(7)
B_5	0.6474(83)(15)	0.6640(45)(3)	0.6466(15)(6)

- **Volume and gauge action dependences are small**

Comparison with the other fermion results

Overlap:R.Babich et.al. hep-lat/0605016 , Wilson:A.Donini et.al. PLB470(1999)233



- **Large discrepancies for B_2, B_3 and B_4 between our results and the other fermion results.**

- **Consistency for B_5**

- **Possible sources of discrepancies**
  - **Renormalization method**
    - **1-loop perturbation vs Non-perturbative RI/MOM scheme**

  - $\langle \overline{K^0} | \bar{s} \gamma_5 d | 0 \rangle$ **itself could be different**

Summary

- **Measurements of matrix elements for generic $\Delta S = 2$ four quark operator**
  - **DWF + Plaquette on $16^3 \times 32 \times 16$ lattices at $a^{-1} \approx 2$GeV**
  - **DWF +Iwasaki on $16^3 \times 32 \times 16$ and $24^3 \times 32 \times 16$ lattices at $a^{-1} \approx 2$GeV**
- $|\langle \overline{K^0}|O_i(\mu)|K^0\rangle|$ $(i = 2, \ldots, 5)$ **much larger than** $|\langle \overline{K^0}|O_1(\mu)|K^0\rangle|$
  - **Small volume dependences**
- **Comparison with the previous works using different fermion actions**
  - **Large discrepancies for some matrix elements**
  - **Difference of renormalization methods could be a large source of discrepancies**
    - **1-loop perturbation vs Non-perturbative RI/MOM scheme**

Future work

Non-Perturbative renormalization for DWF using Schödinger functional method Y.Taniguchi hep-lat/0604002