Higher Loop Bethe Ansatz for Open Spin Chains in AdS/CFT

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1. INTRODUCTION

AdS/CFT [Maldacena]

IIB string on AdS_5 x S^5 ⟷ $\mathcal{N}$=4 SYM with SU(N) (large N)

String states ⟷ Composite ops. e.g., $\mathbf{tr}(\phi^{i_1} \cdots \phi^{i_L})$

Anomalous dim. Matrix

↓ **Diagonalize**

Energy $E \overset{?}{=} \Delta$ Scaling dim.

SUGRA + stringy correction = bare dim. + **anomalous dim.**

BPS [GKP, Witten]
almost BPS [BMN]
} sectors have been well studied.

➡ Test of AdS/CFT in a far-from BPS sector

We will discuss how to obtain anomalous dimension.

How to compute anomalous dimension

1-loop planar anom. dim. matrix (SU(2) sector) ⟵ Perturbation theory

||

Heisenberg spin chain Hamiltonian ⟶

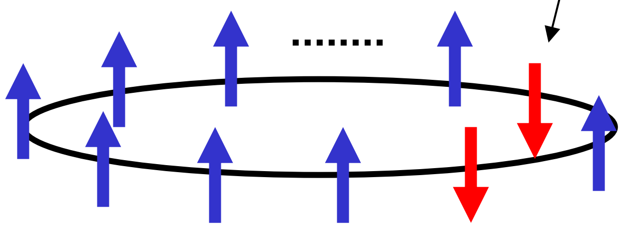
Problem: (E= anom. dim.)

$$H|\Psi\rangle = E|\Psi\rangle$$

Bethe ansatz ⟶ Energy

[Minahan-Zarembo]

Magnons



⟷

$$\mathrm{Tr}(ZWZWZ\cdots ZZZZZ)$$

nearest neighbor int. ← 1-loop

periodicity

vacuum=feromagnetic

Higher loops ?

(To compare with string)

⟶ Higher loop integrability assumption

(+ symmetry、BMN scaling)

⬇

Higher loop Hamiltonian

Higher-loop perturbation is difficult.

[Beisert-Kristjansen-Staudacher, Beisert]

A motive to consider Open Spin Chain

D-brane in AdS_5xS^5 → Open strings

e.g., AdS-brane, giant graviton

The excitations are described by open spin chains.

Hamiltonian (1-loop, perturbation)

D-brane

open spin chain = closed spin chain + **integrable boundary**

D-brane preserves the integrability (at least 1-loop level).

Our purpose: **study higher-loop open chains**

A generalization of coordinate Bethe ansatz [Staudacher]

Strategy: generalize Perturbative Asymptotic Bethe ansatz (PABA) to open chain

(assume higher-loop integrability)

apply → To fix higher-loop integrable boundary and compute energy

Plan of the Talk

1. **Introduction** ✓

2. **Generalization of PABA to open spin chains**

3. **Application to two examples**

   i) defect CFT (AdS D5-brane), ii) giant graviton

4. **Summary and Outlook**

2. Generalization of PABA to Open Spin Chains

Higher-loop open spin chain Hamiltonian: [SU(2) sector]

$$H_{\text{open}} = H_{\text{bulk}} + H_{\text{boundary}}$$

⇐ Composed of two parts

Bulk

$$H_{\text{bulk}} = \sum_{r=1}^{\infty} g^r H_r, \quad g \equiv \frac{\lambda}{16\pi^2}$$

Pauli matrices

$$Q_{l,k} \equiv \frac{1}{2}(1 - \vec{\sigma}_l \cdot \vec{\sigma}_k) = \frac{1}{2}(1 - P_{l,k})$$

$$H_1 = 2\sum_{l=1}^{L-1} Q_{l,l+1}, \quad H_2 = -8\sum_{l=1}^{L} Q_{l,l+1} + 2\sum_{l=1}^{L-2} Q_{l,l+2},$$

$$H_3 = 60\sum_{l=1}^{L-1} Q_{l,l+1} - 24\sum_{l=1}^{L-2} Q_{l,l+2} + 4\sum_{l=1}^{L-3} Q_{l,l+3}$$

$$+4\sum_{l=1}^{L-3} \underline{Q_{l,l+2}Q_{l+1,l+3}} - 4\sum_{l=1}^{L-3} \underline{Q_{l,l+3}Q_{l+1,l+2}}$$

⇐ multi-spin int.

⋮

L : length of the chain

[Minahan-Zarembo] [Beisert-Kristjansen-Staudacher] [Beisert]

[perturvative computation: Eden-Jarczak-Sokatochev]

Consider a single magnon case below

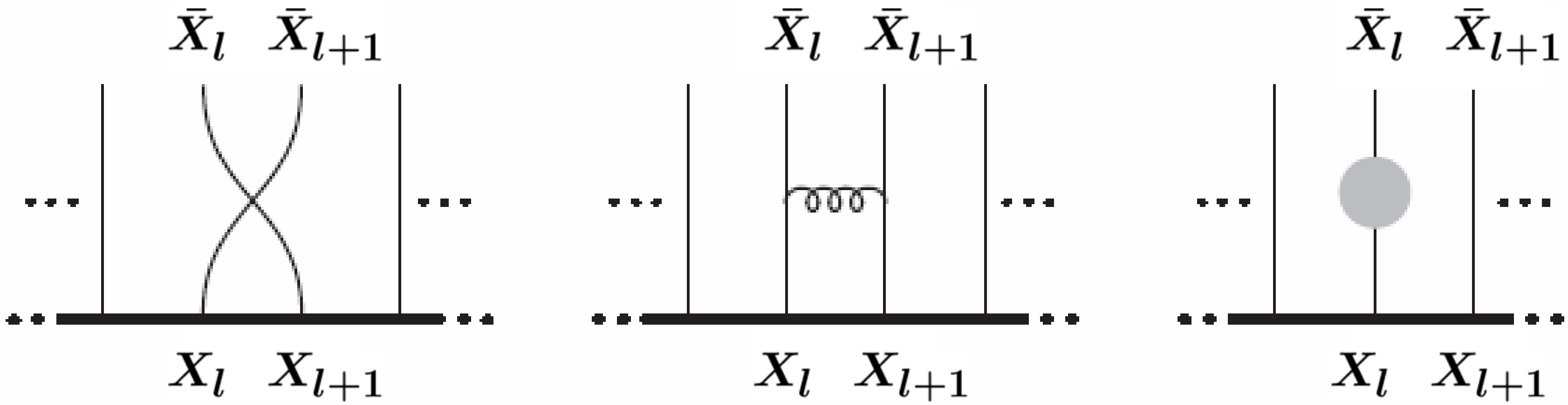
(1-magnon problem)

Note: multi-spin int. is irrelevant.

The 3 diagrams that contribute to the SU(2) spin chain Hamiltonian:

X = Z or W

(1-loop level)



(a) 4-pt. scalar int.

(b) Gluon exchange

(c) Self-energy

flavor blind

I (unit), P (permutation)

I (unit)

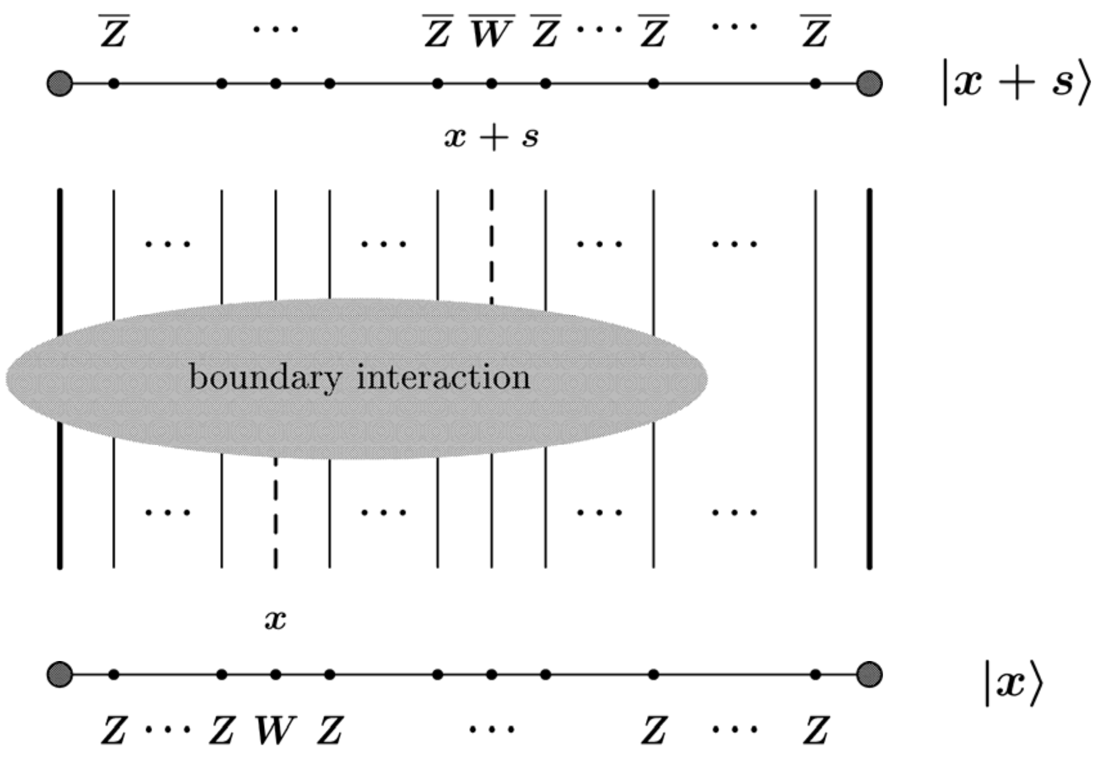
Boundary

From the diagrams connected to boundaries

*) depending on the system

$$H_{\text{boundary}} = \sum_{k=1}^{\infty} g^k \widehat{C}^{(k)}_{x,x+s}$$
$$= g(\widehat{C}^{(1)}_{1,1} + \widehat{C}^{(1)}_{L,L}) + g^2(\widehat{C}^{(2)}_{1,1} + \widehat{C}^{(2)}_{1,2} + \widehat{C}^{(2)}_{2,1} + \widehat{C}^{(2)}_{2,2}$$
$$+\widehat{C}^{(2)}_{L,L} + \widehat{C}^{(2)}_{L,L-1} + \widehat{C}^{(2)}_{L-1,L} + \widehat{C}^{(2)}_{L-1,L-1}) + \mathcal{O}(g^3)$$

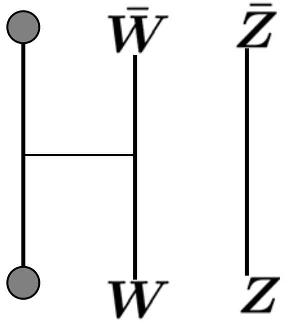
The diagrams that contribute to $\widehat{C}_{x,x+s}$



EX:

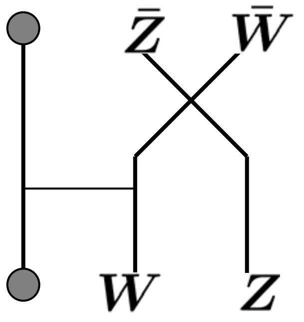
$\widehat{C}^{(1)}_{1,1}$

1-loop



$\widehat{C}^{(2)}_{1,2}$

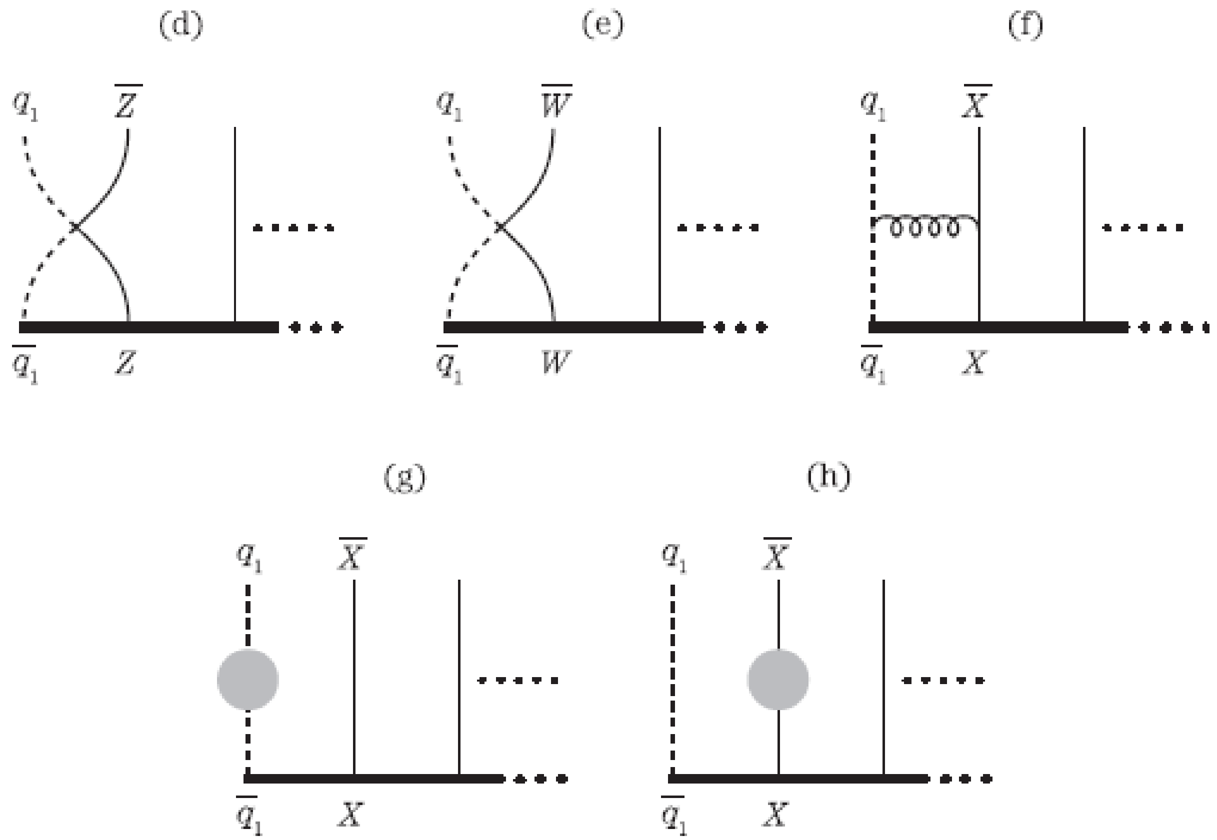
2-loop



Examples of the diagrams that contribute to the boundary:

- defect CFT case -

The 1-loop diagrams that contribute to the boundary Hamiltonian



Open Spin Chain States:

1-magnon state:

x-th site; 1; L

$$|x\rangle \equiv B_{\rm L} Z \cdots Z W Z \cdots Z B_{\rm R}$$

0 or 1/2 ($B_{\rm L}$) L+1 or L+1/2 ($B_{\rm R}$)

Depend on the dim. of the field

$$B_{L,R} = \begin{cases} q & \text{: a fundamental field (dCFT case) located at 1/2-th and L+1/2 th sites} \\ Z & \text{(giant graviton (GG) case) located at 0-th and L+1 th sites} \end{cases}$$

Note: The magnon is assumed to satisfy the Dirichlet b.c. ➡ Non-trivial boundary

Otherwise, free boundary

Operation of Hamiltonian:

x -th site x+1 -th site

$Q_{x,x+1}$:
$$Q_{x,x+1}|BZ\cdots ZWZZ\cdots ZB\rangle = |BZ\cdots ZZWZ\cdots ZB\rangle$$

a constant to be fixed

$\widehat{C}_{x,x+s}$:
$$\widehat{C}_{1,2}|BWZZ\cdots ZB\rangle = C_{1,2}|BZWZ\cdots ZB\rangle$$

Note: The boundary fields are fixed (i.e., the unit operation at both boundaries).

dCFT: q is fixed in the SU(2) sector, GG: the replacement of Z with W is subleading.

PABA for Higher Loop Open Spin Chains

Solve the eigenvalue problem: $H_{\text{open}}|\Psi\rangle = E|\Psi\rangle$

1-magnon state for open Heisenberg (1-loop):

x-th site

$$|\Psi(p)\rangle = \sum_{x=1}^{L} \psi(x)|Z\cdots ZWZ\cdots Z\rangle,$$

$$\psi(x) = A(p)\mathrm{e}^{ipx} - \tilde{A}(-p)\mathrm{e}^{-ipx}$$

➡ The ratio of A's = Boundary S-matrix

1-magnon state for higher-loop open spin chain: [Okamura-K.Y.]

$$\begin{aligned}\psi(x) \;=\; & (1 + f(|x-1|, p; g) + f(|L-x|, p; g))\, A(p; g)\mathrm{e}^{ipx} \\ & - \left(1 + \tilde{f}(|x-1|, -p; g) + \tilde{f}(|L-x|, -p; g)\right) \tilde{A}(-p; g)\mathrm{e}^{-ipx}\end{aligned}$$

Correction functions

Boundary S-matrix also depends on g ⬅ g-dep.

Energy dispersion relation (n-loop level)

$$E(p,g) = \sum_{k=1}^{n} \epsilon_k(p) g^k + \mathcal{O}(g^{n+1}) \qquad (1+n \le x \le L-n)$$

$$\epsilon_1(p) = 8\sin^2\left(\frac{p}{2}\right), \quad \epsilon_2(p) = -32\sin^4\left(\frac{p}{2}\right), \cdots .$$

For $x = 1, \cdots, n$ or $x = L-n+1, \cdots, L$ it may be changed.

Demand: boundary int. should not change the energy dispersion relation

" Compatibility conditions "

By solving the conditions

Boundary S-matrix : $B(p;g) \equiv \dfrac{A(-p;g)}{\widetilde{A}(p;g)}$ can be fixed perturbatively.

$$B(p;g) \equiv \sum_{k=0}^{n-1} B^{(k)}(p) g^k + \mathcal{O}(g^n)$$

(perturbative boundary S-matrix)

Ex: Compatibility conditions at 2-loop (n=2)

$x=1$

$$2\psi_0(0)-(2-C^{(1)}_{1,1})\psi_0(1)=g\Big[(-2+\epsilon_1(p)-C^{(1)}_{1,1})e^{ip}f_1(p)B(-p;g)$$
$$+(2-\epsilon_1(p)+C^{(1)}_{1,1})e^{-ip}\boxed{\tilde f_1(-p)}$$
$$-2\psi_0(-1)+8\psi_0(0)-(6+C^{(2)}_{1,1})\psi_0(1)-C^{(2)}_{1,2}\psi_0(2)\Big]$$

$x=2$

$$2B(-p;g)e^{ip}f_1(p)-2e^{-ip}\boxed{\tilde f_1(-p)}-2\psi_0(0)$$
$$-C^{(2)}_{2,1}\psi_0(1)-(-2+C^{(2)}_{2,2})\psi_0(2)=0$$

eliminate ➡ B is written by C

Expand B w.r.t g

➡ $B^{(0)}$, $B^{(1)}$ are fixed as functions of p and C

where

$$\psi_0(x)\equiv A(p;g)e^{ipx}+\widetilde{A}(-p;g)e^{-ipx}$$
$$f(|d|,p;g)=\sum_{k=1}^{\infty}f_k(|d|,p)g^{|d|+k}+\mathcal{O}(g^{n+1})$$

have been utilized.

3. Application to Examples

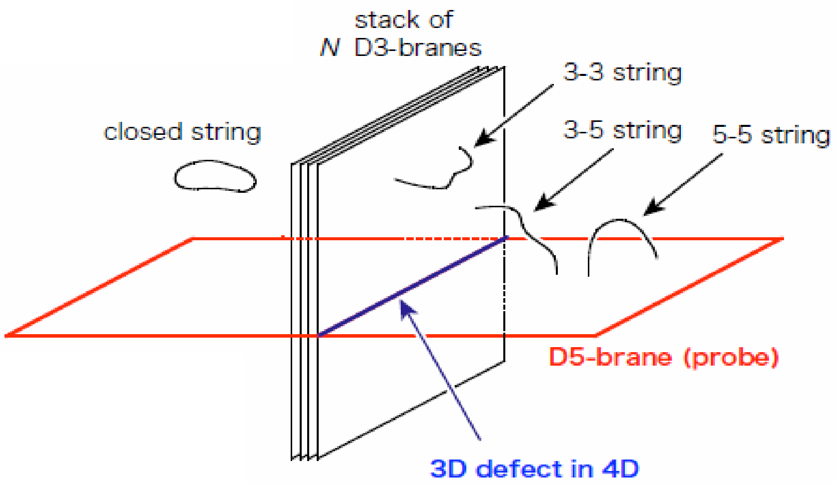
(2-loop level)

i) defect CFT (dCFT)

ii) Giant graviton (GG)

i) dCFT case （AdS D5-brane）

[DeWolfe-Freedman-Ooguri]



1-magnon state：

$$|x\rangle \equiv \bar{q}_1 \underbrace{Z \cdots Z W Z \cdots Z}_{L} q_2$$

1-loop integrability [DeWolfe-Mann]

Boundary S-matrix:

$$B^{(0)} = \frac{2e^{ip} - (2 - C^{(1)}_{1,1})}{2 - e^{ip}(2 - C^{(1)}_{1,1})}$$

[Okamura-Takayama-K.Y.]

1-loop perturbation result: $C^{(1)}_{1,1} = 4$ ➡ $B^{(0)} = 1$ [DeWolfe-Mann]

$$B^{(1)} = \frac{-i}{2\cos\left(\frac{p^{(0)}}{2}\right)} \left[(-2 + C^{(2)}_{2,2}) \sin\left(\frac{5p^{(0)}}{2}\right) + (-6 + C^{(2)}_{1,2} + C^{(2)}_{2,1} + C^{(2)}_{2,2}) \sin\left(\frac{3p^{(0)}}{2}\right) + (12 + C^{(2)}_{1,1} + C^{(2)}_{2,2}) \sin\left(\frac{p^{(0)}}{2}\right) \right]$$

(2-loop perturbation result is not known yet.)

Higher-loop boundary S-matrix should be zero: B.C. of W is Dirichlet

$$B^{(n)} = 0\,, \quad (n = 1, 2, \ldots,)$$

（consistency to the D-brane interpretation）

Coeffs.: [Okamura-K.Y.]

$$C^{(2)}_{1,1} = -14\,, \quad C^{(2)}_{1,2} = C^{(2)}_{2,1} = 2\,, \quad C^{(2)}_{2,2} = 2$$

Quasi-momemtum: $p = \dfrac{n\pi}{L}$ g-independent !

Energy:

$$E(n;\lambda,L) = \frac{n^2\lambda}{8L^2}\left(1 - \frac{n^2\pi^2}{12L^2} + \frac{n^4\pi^4}{360L^4} + \cdots\right) - \frac{n^4\lambda^2}{L^4}\left(1 - \frac{n^2\pi^2}{6L^2} + \cdots\right) + \mathcal{O}(\lambda^3)$$

➡ BMN scaling OK!

Note: Except for the above coeffs., the momentum depends on g, and BMN scaling is broken down.

ii) Giant graviton (GG) case (maximal spherical D3-brane in S^5)

Maximal GG op. (1/2 BPS) : $\epsilon^{j_1\cdots j_N}_{i_1\cdots i_N} Y^{i_1}_{j_1}\cdots Y^{i_N}_{j_N}$ [Balasubramanian et.al.]

GG with a string : $\epsilon^{j_1\cdots j_N}_{i_1\cdots i_N} \underbrace{Y^{i_1}_{j_1}\cdots Y^{i_{N-1}}_{j_{N-1}}}_{\text{GG}} \underbrace{(Z^{L+2})^{i_N}_{j_N}}$ [Balasubramanian et.al.] [Takayanagi[2]]

The ground state of string

Impurities in Z^{L+2} = Excitations of a string ending on GG

Consider 1-magnon state : $|x\rangle = Z[Z\cdots ZYZ\cdots Z]Z$ [Berenstein-Vazquez]

(Y at the x-th site)

1-loop integrability has been shown. [Berenstein-Vazquez]

The coefficients of the boundary : (the same as the bulk part due to the structure of op.)

$$C^{(1)}_{1,1}=2\ ;\quad C^{(2)}_{1,1}=-8\,,\quad C^{(2)}_{1,2}=0\,,\quad C^{(2)}_{2,1}=0\,,\quad C^{(2)}_{2,2}=2\,,$$

Quasi-momentum: $$p(g) = \frac{n\pi}{L+1} + \frac{4g}{L+1}\sin\left(\frac{n\pi}{L+1}\right) + \mathcal{O}(g^2)$$

g -dependence

Energy: $$E(n;\lambda,L) = \frac{n^2\lambda}{8L^2}\left[1 - \frac{2}{L} + \frac{36 - n^2\pi^2}{12L^2}\right] - \frac{n^4\lambda^2}{128L^4}\left[1 - \cdots\right.$$
$$\left. - \frac{n^2\lambda^2}{16\pi^2 L^3}\left[1 - \frac{3}{L} - \frac{18 - n^2\pi^2}{3L^2} - \cdots\right] + \mathcal{O}(\lambda^2)\right.$$

Break down of BMN scaling at 2-loop level !

Why?: *g* -dep. of quasi-momentum, planar level analysis

BMN limit: N/L^2 fixed, $L \to \infty$, $N \to \infty$

For GG case $L^2 \sim N$ ➡ non-planar contribution should be included.

[Balasubramanian et.al]

Moving GG drag a string (?)

BMN scaling should be recovered (?)

c.f. For single trace op. $L^2 \ll N$

[Berenstein-Correa-Vazquez]

4. Summary and Outlook

Summary

- Generalization of PABA techniques to open spin chains

- Application to two examples： (2-loop level)

i) dCFT (AdS D5-brane)

2-loop integrable boundary (prediction) ➡ BMN scaling OK

Confirmation via perturbative computation (work in progress)

ii) Giant graviton

BMN scaling is broken at 2-loop level (if the PABA is naively applied)

2-magnon problem at 2-loop level ⇒ integrability is broken [Agarwal]

Bethe ansatz cannot be applied to GG (?) [Berenstein-Correa-Vazquez]

Outlook

Relation between D-branes and integrabilities