Non-Relativistic AdS Branes

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Collaboration with Kentaroh Yoshida (KEK)

“Non-Relativistic AdS Branes and Newton-Hooke Superalgebra,”
§1 Introduction

IIB superstrings in $AdS_5 \times S^5 \Leftrightarrow \mathcal{N} = 4$ super-Yang-Mills \[\text{[Maldacena'9711]}\]

\[PSU(2, 2|4)\]

- Penrose limit $\rightarrow$ pp-wave algebra
  pp-wave string \[\text{[Metsaev'0112, Berenstein-Maldacena-Nastase'0202]}\]

- Non-relativistic limit $\rightarrow$ Newton-Hooke algebra (bosonic\[\text{[Brugués-Gomis-Kamimura'0603]}\])
  non-relativistic AdS string \[\text{[Gomis-Gomis-Kamimura'0507]}\]
  - non-relativistic dispersion relation
  - minimal surface dual to Wilson line (loop)
    \[\text{[Maldacena, Rey-Yee'9803]}\]
  - in flat limit,
    non-relativistic closed string (NRCS)\[\text{[Gomis-Ooguri'0009]}\]
    non-commutative open string (NCOS)\[\text{[Seiberg-Strominger-Toumbas, Gopakumar-Maldacena-Minwalla-Strominger'0005]}\]
    Galilean algebra (contraction of Poincare algebra)
In this talk

§2. Newton-Hooke superalgebras of AdS branes as IW contraction of $\mathfrak{psu}(2,2|4)$

⊃ superalgebra of the brane worldvolume isometry and transverse Lorentz symmetry.

§4. Non-relativistic AdS brane actions ← §3. AdS brane actions

cancellation of the volume of the brane worldvolume and the constant flux

We have also examined branes in 11-dimensional AdS spaces in the paper.
§2 Newton-Hooke superalgebra

bosonic part \[ \mathfrak{so}(4, 2) + \mathfrak{so}(6) \subset \mathfrak{psu}(2, 2|4) \]

\[ \{P_a, J_{ab}; P_{a'}, J_{a'b'}\} \]

IW contraction

\[ \tilde{A} = A_0, \cdots, A_p, \quad \tilde{A} = A_{p+1}, \cdots, A_9, \]

scale \[ P_A \rightarrow \frac{1}{\Omega} P_A, \quad J_{AB} \rightarrow \frac{1}{\Omega} J_{\tilde{A}\tilde{B}} \quad \text{and take} \quad \Omega \rightarrow 0 \]

NH algebra of an AdS brane [Brugues-Gomis-Kamimura’0603]

\[ \lambda = 1/R \]

\[ [P_{\tilde{a}}, P_{\tilde{b}}] = \lambda^2 J_{\tilde{a}\tilde{b}}, \quad [P_{\tilde{a}}, P_{\tilde{b}}] = \lambda^2 J_{\tilde{a}\tilde{b}}, \quad [P_{\tilde{a}'}, P_{\tilde{b}'}] = -\lambda^2 J_{\tilde{a}'\tilde{b}'}, \quad [P_{\tilde{a}'}, P_{\tilde{b}'}] = -\lambda^2 J_{\tilde{a}'\tilde{b}'} \]

\[ [P_{\tilde{A}}, J_{\tilde{B}\tilde{C}}] = \eta_{\tilde{A}\tilde{B}} P_{\tilde{C}} - \eta_{\tilde{A}\tilde{C}} P_{\tilde{B}}, \quad [P_{\tilde{A}}, J_{\tilde{B}\tilde{C}}] = \eta_{\tilde{A}\tilde{B}} P_{\tilde{C}} - \eta_{\tilde{A}\tilde{C}} P_{\tilde{B}} \]

\[ [P_{\tilde{A}}, J_{\tilde{B}\tilde{C}}] = \eta_{\tilde{A}\tilde{B}} P_{\tilde{C}}, \quad [P_{\tilde{A}}, J_{\tilde{B}\tilde{C}}] = \eta_{\tilde{A}\tilde{B}} P_{\tilde{C}} \]

\[ [J_{\tilde{A}\tilde{B}}, J_{\tilde{C}\tilde{D}}] = \eta_{\tilde{A}\tilde{D}} J_{\tilde{B}\tilde{C}} + 3\text{-terms}, \quad [J_{\tilde{A}\tilde{B}}, J_{\tilde{C}\tilde{D}}] = \eta_{\tilde{A}\tilde{D}} J_{\tilde{B}\tilde{C}} + 3\text{-terms} \]

\[ [J_{\tilde{A}\tilde{B}}, J_{\tilde{C}\tilde{D}}] = \eta_{\tilde{B}\tilde{C}} J_{\tilde{A}\tilde{D}} - \eta_{\tilde{A}\tilde{C}} J_{\tilde{B}\tilde{D}}, \quad [J_{\tilde{A}\tilde{B}}, J_{\tilde{C}\tilde{D}}] = \eta_{\tilde{B}\tilde{D}} J_{\tilde{C}\tilde{A}} - \eta_{\tilde{A}\tilde{D}} J_{\tilde{C}\tilde{B}} \]

It has two subalgebras:

\[ \text{AdS}_m \times S^n (H^m \times S^n) \quad \text{and} \quad E^{5-m} \times E^{5-n} (E^{4-m,1} \times E^{5-n}) \]

\[ \mathfrak{so}(m-1,2) \times \mathfrak{so}(n+1) \quad \text{and} \quad \mathfrak{iso}(5-m) \times \mathfrak{iso}(5-n) \]

where \((m,n)\) are \# of the worldvolume directions extending along \(\text{AdS}_5\) and \(S^5\).
fermionic part

We require that the isometry of AdS-brane worldvolume and transverse Lorentz symmetry naturally extends to superisometry, that is

\[ \{Q_+, Q_+\} \sim P_A + J_{\bar{A}B} + J_{AB} \]

\[ Q_{\pm} P_{\pm} = Q_{\pm}, \quad P_{\pm} = \frac{1}{2}(1 + M), \quad M = \ell \Gamma^{\bar{A}_0 \cdots \bar{A}_p} \otimes \rho, \quad \rho_{IJ} : 2 \times 2 \text{ matrix} \]

<table>
<thead>
<tr>
<th>( \rho )</th>
<th>1-brane</th>
<th>3-brane</th>
<th>5-brane</th>
<th>7-brane</th>
<th>9-brane</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_1, \sigma_3 )</td>
<td>(2,0), (0,2)</td>
<td>(3,1), (1,3)</td>
<td>(4,2), (2,4)</td>
<td>(5,3), (3,5)</td>
<td>(5,5)</td>
</tr>
<tr>
<td>( i\sigma_2 )</td>
<td>(1,1)</td>
<td></td>
<td>(5,1), (3,3), (1,5)</td>
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1/2 BPS or not 1/2 BPS

Dirichlet branes of F- and D-strings in AdS\(_5 \times S^5\) [Yoshida-MS'0310,'0403,'0408,'0604]

brane probe analysis [Skenderis-Taylor'0204]

We will find consistent non-relativistic limits for 1/2 BPS branes.
IW contraction scale \( Q_+ \rightarrow Q_+ \), \( Q_- \rightarrow \frac{1}{\Omega} Q_- \) and take \( \Omega \rightarrow 0 \)

fermionic part of NH superalgebra

\[
\begin{align*}
[P_A, Q_+] &= \frac{\lambda}{2} Q_+ \hat{\Gamma}_A i\sigma_2 , \\
[P_A, Q_+] &= \frac{\lambda}{2} Q_- \hat{\Gamma}_A i\sigma_2 , \\
[P_A, Q_-] &= \frac{\lambda}{2} Q_- \hat{\Gamma}_A i\sigma_2 , \\
[J_{\bar{A}B}, Q_{\pm}] &= \frac{1}{2} Q_{\pm} \Gamma_{\bar{A}B} , \\
[J_{AB}, Q_{\pm}] &= \frac{1}{2} Q_{\pm} \Gamma_{AB} , \\
[J_{\bar{A}B}, Q_+] &= \frac{1}{2} Q_- \Gamma_{\bar{A}B} , \\
\{Q_+, Q_+\} &= 2iC\Gamma^A h_+ \mathcal{P} + P_{\bar{A}} - i\lambda C\hat{\Gamma}_{\bar{A}B} i\sigma_2 h_+ \mathcal{P} + J_{\bar{A}B} - i\lambda C\hat{\Gamma}_{AB} i\sigma_2 h_+ \mathcal{P} + J_{AB} , \\
\{Q_+, Q_-\} &= 2iC\Gamma^A h_+ \mathcal{P} - P_A - 2i\lambda C\hat{\Gamma}_{\bar{A}B} i\sigma_2 h_+ \mathcal{P} - J_{\bar{A}B} .
\end{align*}
\]

where \( Q_I h_+ = Q_I \), \( h_+ = \frac{1}{2}(1 + \Gamma_{11}) \), \( \Gamma_{11} = \Gamma_{01\ldots9} \), and

\[
\hat{\Gamma}_A = (-\Gamma_a I, \Gamma_a I') , \quad \hat{\Gamma}_{AB} = (-\Gamma_{ab} I, \Gamma_{ab} I') , \quad I = \Gamma^{01234} , \quad J = \Gamma^{56789} .
\]

NH superalgebra contains a sub-superalgebra generated by \( \{P_\bar{A}, J_{\bar{A}B}, J_{AB}, Q_+\} \), the unbroken symmetry in presence of an AdS brane.
§3 Branes in $AdS_5 \times S^5$

**Supercurrents**

\[ g^{-1} dg = L^A P_A + \frac{1}{2} L^{AB} J_{AB} + Q_I L^I \quad \text{where} \quad g \in PSU(2, 2|4) \]

**Dp-brane action**

\[ S = S_{DBI} + S_{WZ} = T \int \Sigma [\mathcal{L}_{DBI} + \mathcal{L}_{WZ}] \]

- **DBI part**
  \[ \mathcal{L}_{DBI} = \sqrt{s} \det(g + \mathcal{F}) \, d^{p+1} \xi \]
  \[ g_{ij} = L_i^A L_j^B \eta_{AB}, \quad \mathcal{F} = F - \mathcal{B}, \quad L_i^A = \partial_i \hat{Z}^\hat{M} L_A^\hat{M}, \quad \hat{Z}^\hat{M} = (X^M, \theta^\alpha) \]

- **WZ part**
  \[ h_{p+2} = d\mathcal{L}_{WZ} = \sum_{n=0}^{\frac{p-1}{2}} \frac{1}{n!} h^{(p+2-2n)} \mathcal{F}^n, \quad \varrho = (\sigma)^{-\frac{p-3}{2}} i \sigma_2, \quad \sigma = \left\{ \begin{array}{l} \sigma_3 \\ -\sigma_1 \end{array} \right\} \]
  \[ h^{(\ell+2)} = \frac{\sqrt{s}}{\ell!} \left[ L^A_1 \cdots L^A_{\ell} \hat{L} \Gamma_{A_1 \cdots A_{\ell}} \xi L + \delta^{\ell,3} \frac{i \lambda}{5} (\epsilon_{a_1 \cdots a_5} L^{a_1} \cdots L^{a_5} - \epsilon_{a_1' \cdots a_5'} L_{a_1'} \cdots L_{a_5'}) \right] \]

\[ h_{p+2} \text{ is a non-trivial element of CE-cohomology on } psu(2, 2|4) \text{ except for } h_3 \]

\[ \mathcal{L}_{WZ} = \int_0^1 dt [\mathcal{C} \wedge e^{\hat{\mathcal{F}}}]_{p+1} + C^{(p+1)} \, , \quad h_{p+2}|_{\text{bosonic}} = dC^{(p+1)} \]

where \[ C = \bigoplus_n \frac{2\sqrt{s}}{(2n-1)!} \hat{L}^A_1 \cdots \hat{L}^A_{2n-1} \hat{L} \Gamma_{A_1 \cdots A_{2n-1}} (\sigma)^n i \sigma_2 \theta \quad \text{and} \quad \hat{E} = E(t \theta) \]

These reproduce the F1- and D3-brane actions given in [Metsaev-Tseytlin'9805,'9806].
§4 Non-Relativistic limit

Let us consider the scaling \( X^A \rightarrow \Omega X^A, \; \theta_- \rightarrow \Omega \theta_- \), \( T = \Omega^{-2} T_{NR} \)
which is equivalent to IW contraction and the scaling used in [Gomis-Gomis-Kamimura'0507].
(only light modes satisfying the non-relativistic dispersion relation are kept)

\( \text{D-string} \) For \( \rho = \sigma_1 \)

DBI part is expanded as \( T L_{DBI} = T_{NR} \Omega^{-2} L_{DBI}^{\text{div}} + T_{NR} L_{DBI}^{\text{fin}} + O(\Omega^2), \; F = \Omega F_1 + O(\Omega^3) \)

\[ L_{DBI}^{\text{div}} = \sqrt{s} \det g_0 d^2 \xi = \frac{1}{2} \epsilon_{\bar{A}B} (\bar{L}_0^A) (\bar{L}_0^B), \; \epsilon_{\bar{A}0\bar{A}_1} = 1, \; \quad (g_0)_{ij} = (L_0^\bar{A})_i (L_0^\bar{B})_j \eta_{\bar{A}\bar{B}}. \]

while \( h_3 \) is expanded as \( Th_3 = T_{NR} \Omega^{-2} h_3^{\text{div}} + T_{NR} h_3^{\text{fin}} + O(\Omega^2) \)

\[ h_3^{\text{div}} = \sqrt{s} L_0^\bar{A} L_0^{\bar{A}_0} \Gamma_{\bar{A}} \sigma_1 L_{0+}. \]

We show that \( L_{DBI}^{\text{div}} + L_{WZ}^{\text{div}} = 0 \) for 1/2 BPS branes: (2,0)- and (0,2)-branes.

- \( dL_{DBI}^{\text{div}} + h_3^{\text{div}} = 0 \) only for \( \rho = \sigma_1 \). This implies that \( \theta \)-dependent part of \( L_{DBI}^{\text{div}} + L_{WZ}^{\text{div}} \) cancels out. So (1,1)-brane (\( \rho = 1 \)) does not admit consistent limit.
• bosonic part of $\mathcal{L}_{\text{DBI}}^{\text{div}}$, $\frac{1}{2} \epsilon_{AB}(e^A_0)(e^B_0)$, where $e^A$ is the bosonic part of $L^A$, is canceled by adding $C^{(2)}_0 = -\frac{1}{2} \epsilon_{\bar{A}\bar{B}}(e^\bar{A}_0)(e^\bar{B}_0)$ which is closed $dC^{(2)}_0 = 0$. □

Thus the non-relativistic action is $S_{\text{NR}} = T_{\text{NR}} \int (\mathcal{L}_{\text{DBI}}^{\text{fin}} + \mathcal{L}_{\text{WZ}}^{\text{fin}})$.

The $\kappa$-gauge symmetry is fixed by $\theta_+ = 0$, which makes it easy to perform the $t$-integration in $\mathcal{L}_{\text{WZ}}^{\text{fin}}$. In static gauge, we obtain

$$S_{\text{NR}} = T_{\text{NR}} \int \frac{d^2 \xi}{\sqrt{s}} \det g_0 \left[ \frac{1}{2} g_0^{ij} \partial_i y^A \partial_j y_A + \frac{\lambda^2}{2} (m y^2 - n y'^2) + 2i \bar{\theta} - \gamma^i D_i \theta + \frac{1}{2} F_{ij} F^{ij} \right].$$

This is a free field action on $AdS_2(H^2)$ or $S^2$.

• Consistent non-relativistic limits are possible for all 1/2 BPS D$p$-branes listed in §1.

In flat limit $\lambda \rightarrow 0$, our non-relativistic AdS D$p$-brane actions reproduce the non-relativistic D$p$-brane actions in flat spacetime [Gomis-Passerini-Ramirez-Van Proeyen'0507].

• For F1, $AdS_2$-brane action was obtained in [Gomis-Gomis-Kaminura'0507]. The same action is derived as fluctuations on $AdS_2$-brane in [Drukker-Gross-Tseytlin'0001].
\section*{5 Summary}

- NH superalgebras of branes in $AdS_5 \times S^5$ and $AdS_{4/7} \times S^{7/4}$
  - IW contraction of $\mathfrak{psu}(2, 2|4)$ and $\mathfrak{osp}(8^{(*)}|4)$
  - sub-superalgebra as supersymmetrization of brane worldvolume isometry and transverse Lorentz symmetry.
  - classification of $1/2$ supersymmetric subspaces $\supset 1/2$ BPS branes

- Non-relativistic AdS brane actions
  - consistent limit can be performed for $1/2$ BPS AdS branes
  - in flat limit they reproduce non-relativistic brane actions in flat spacetime
comments on M-branes

- Newton-Hooke superalgebras of M-branes in $AdS_{4/7} \times S^{7/4}$
  
  IW contraction of $\mathfrak{osp}(8|4)$ or $\mathfrak{osp}(8^*|4)$

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$p$-branes with $p = 1 \mod 4$ are 1/2 BPS Dirichlet branes of open supermembrane [Yoshida-MS'0310,'0405]; brane probe analysis [Kim-Yee'0211]

- 2- and 5-branes admit consistent non-relativistic limit.
  
  - M2- and M5-brane actions are characterized as non-trivial elements of CE cohomology on $\mathfrak{osp}(8|4)$ or $\mathfrak{osp}(8^*|4)$.
  
  - M2- and M5-branes admit consistent non-relativistic limit, and derived non-relativistic AdS M-brane actions reproduce the non-relativistic M2-brane action [Gomis-Kamimura-Townsend'0409] and the linearized M5-brane action [Claus-Kallosh-Van Proeyen'9711] in flat limit.