Non-Relativistic AdS Branes

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"Non-Relativistic AdS Branes and Newton-Hooke Superalgebra,"
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§1 Introduction

IIB superstrings in $AdS_5 \times S^5$ $\Leftrightarrow$ $\mathcal{N} = 4$ super-Yang-Mills [Maldacena'9711]

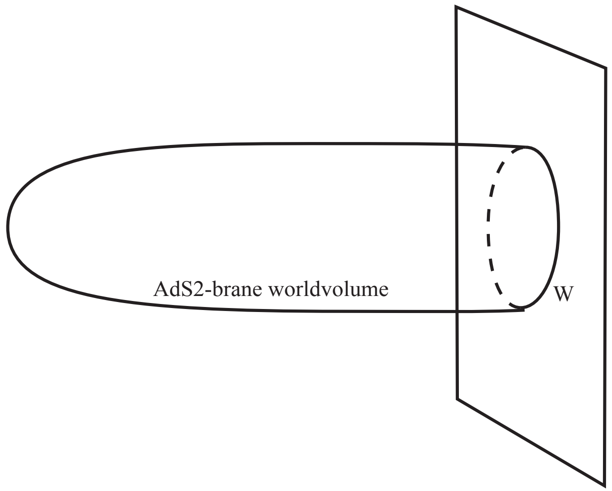
$PSU(2,2|4)$

- Penrose limit $\longrightarrow$ pp-wave algebra

  pp-wave string [Metsaev'0112, Berenstein-Maldacena-Nastase'0202]

- Non-relativistic limit $\longrightarrow$ Newton-Hooke algebra (bosonic[Brugues-Gomis-Kamimura'0603])

  non-relativistic AdS string [Gomis-Gomis-Kamimura'0507]

  - non-relativistic dispersion relation
  - minimal surface dual to Wilson line (loop)

    [Maldacena, Rey-Yee'9803]
  - in flat limit,

    non-relativistic closed string (NRCS)[Gomis-Ooguri'0009]

    non-commutative open string (NCOS)[Seiberg-Strominger-Toumbas,Gopakumar-Maldacena-Minwalla-Strominger'0005]

    Galilean algebra (contraction of Poincare algebra)

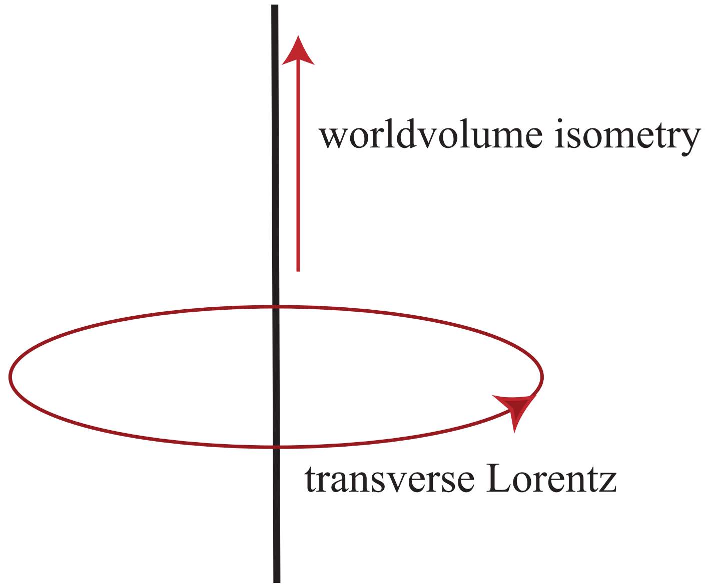
In this talk

§2. Newton-Hooke superalgebras of AdS branes as IW contraction of $\mathfrak{psu}(2,2|4)$

$\supset$ superalgebra of the brane worldvolume isometry and transverse Lorentz symmetry.

§4. Non-relativistic AdS brane actions $\longleftarrow$ §3. AdS brane actions

cancellation of the volume of the brane worldvolume and the constant flux



We have also examined branes in 11-dimensional AdS spaces in the paper.

§2 Newton-Hooke superalgebra

bosonic part $\mathfrak{so}(4,2)+\mathfrak{so}(6)\subset\mathfrak{psu}(2,2|4)$ $\{P_a, J_{ab}\,;\,P_{a'}, J_{a'b'}\}$

IW contraction $\bar{A}=A_0,\cdots,A_p\ ,\quad \underline{A}=A_{p+1},\cdots,A_9\ ,$

scale $P_{\underline{A}}\to\frac{1}{\Omega}P_{\underline{A}}\ ,\quad J_{\bar{A}\underline{B}}\to\frac{1}{\Omega}J_{\bar{A}\underline{B}}$ and take $\Omega\to 0$

NH algebra of an AdS brane [Brugues-Gomis-Kamimura'0603] $\lambda=1/R$

$$[P_{\bar{a}},P_{\bar{b}}]=\lambda^2 J_{\bar{a}\bar{b}}\ ,\quad [P_{\bar{a}},P_{\underline{b}}]=\lambda^2 J_{\bar{a}\underline{b}}\ ,\quad [P_{\bar{a}'},P_{\bar{b}'}]=-\lambda^2 J_{\bar{a}'\bar{b}'}\ ,\quad [P_{\bar{a}'},P_{\underline{b}'}]=-\lambda^2 J_{\bar{a}'\underline{b}'}\ ,$$

$$[P_{\bar{A}},J_{\bar{B}\bar{C}}]=\eta_{\bar{A}\bar{B}}P_{\bar{C}}-\eta_{\bar{A}\bar{C}}P_{\bar{B}}\ ,\quad [P_{\underline{A}},J_{\underline{BC}}]=\eta_{\underline{AB}}P_{\underline{C}}-\eta_{\underline{AC}}P_{\underline{B}}\ ,$$

$$[P_{\bar{A}},J_{\bar{B}\underline{C}}]=\eta_{\bar{A}\bar{B}}P_{\underline{C}}\ ,$$

$$[J_{\bar{A}\bar{B}},J_{\bar{C}\bar{D}}]=\eta_{\bar{A}\bar{D}}J_{\bar{B}\bar{C}}+\text{3-terms}\ ,\quad [J_{\underline{AB}},J_{\underline{CD}}]=\eta_{\underline{AD}}J_{\underline{BC}}+\text{3-terms}\ ,$$

$$[J_{\bar{A}\bar{B}},J_{\bar{C}\underline{D}}]=\eta_{\bar{B}\bar{C}}J_{\bar{A}\underline{D}}-\eta_{\bar{A}\bar{C}}J_{\bar{B}\underline{D}}\ ,\quad [J_{\underline{AB}},J_{\bar{C}\underline{D}}]=\eta_{\underline{BD}}J_{\bar{C}\underline{A}}-\eta_{\underline{AD}}J_{\bar{C}\underline{B}}\ .$$

It has two subalgebras:

$AdS_m\times S^n$ $(H^m\times S^n)$ and $\mathbb{E}^{5-m}\times\mathbb{E}^{5-n}$ $(\mathbb{E}^{4-m,1}\times\mathbb{E}^{5-n})$

$\mathfrak{so}(m-1,2)\times\mathfrak{so}(n+1)$ and $\mathfrak{iso}(5-m)\times\mathfrak{iso}(5-n)$

where (m,n) are # of the worldvolume directions extending along AdS_5 and S^5.

fermionic part

We require that the isometry of AdS-brane worldvolume and transverse Lorentz symmetry naturally extends to superisometry, that is $\{Q_+, Q_+\} \sim P_{\bar{A}} + J_{\bar{A}\bar{B}} + J_{\underline{AB}}$

$$Q_\pm \mathcal{P}_\pm = Q_\pm \ , \quad \mathcal{P}_\pm = \frac{1}{2}(1+M) \ , \quad M = \ell \Gamma^{\bar{A}_0 \cdots \bar{A}_p} \otimes \rho \ , \quad \rho_{IJ} : 2\times 2 \text{ matrix}$$

ρ	1-brane	3-brane	5-brane	7-brane	9-brane
σ_1, σ_3	(2,0), (0,2)		(4,2), (2,4)		
$i\sigma_2$		(3,1), (1,3)		(5,3), (3,5)	
1	(1,1)		(5,1), (3,3), (1,5)		(5,5)

1/2 BPS or not 1/2 BPS

Dirichlet branes of F- and D-strings in AdS$_5\times$S^5 [Yoshida-MS'0310,'0403,'0408,'0604]

brane probe analysis [Skenderis-Taylor'0204]

We will find consistent non-relativistic limits for 1/2 BPS branes.

IW contraction scale $Q_+ \to Q_+ \ , \quad Q_- \to \frac{1}{\Omega} Q_-$ and take $\Omega \to 0$

fermionic part of NH superalgebra

$$[P_{\bar{A}}, Q_+] = \frac{\lambda}{2} Q_+ \widehat{\Gamma}_{\bar{A}} i\sigma_2 \ , \quad [P_{\underline{A}}, Q_+] = \frac{\lambda}{2} Q_- \widehat{\Gamma}_{\underline{A}} i\sigma_2 \ , \quad [P_{\bar{A}}, Q_-] = \frac{\lambda}{2} Q_- \widehat{\Gamma}_{\bar{A}} i\sigma_2 \ ,$$

$$[J_{\bar{A}\bar{B}}, Q_\pm] = \frac{1}{2} Q_\pm \Gamma_{\bar{A}\bar{B}} \ , \quad [J_{\underline{AB}}, Q_\pm] = \frac{1}{2} Q_\pm \Gamma_{\underline{AB}} \ , \quad [J_{\bar{A}\underline{B}}, Q_+] = \frac{1}{2} Q_- \Gamma_{\bar{A}\underline{B}} \ ,$$

$$\{Q_+, Q_+\} = 2i\mathcal{C}\Gamma^{\bar{A}} h_+ \mathcal{P}_+ P_{\bar{A}} - i\lambda \mathcal{C} \widehat{\Gamma}^{\bar{A}\bar{B}} i\sigma_2 h_+ \mathcal{P}_+ J_{\bar{A}\bar{B}} - i\lambda \mathcal{C} \widehat{\Gamma}^{\underline{AB}} i\sigma_2 h_+ \mathcal{P}_+ J_{\underline{AB}} \ ,$$

$$\{Q_+, Q_-\} = 2i\mathcal{C}\Gamma^{\underline{A}} h_+ \mathcal{P}_- P_{\underline{A}} - 2i\lambda \mathcal{C} \widehat{\Gamma}^{\bar{A}\underline{B}} i\sigma_2 h_+ \mathcal{P}_- J_{\bar{A}\underline{B}} \ .$$

where $Q_I h_+ = Q_I \ , \quad h_+ = \frac{1}{2}(1 + \Gamma_{11}) \ , \quad \Gamma_{11} = \Gamma_{01\cdots 9}$, and

$$\widehat{\Gamma}_A = (-\Gamma_a \mathcal{I}, \Gamma_{a'} \mathcal{J}) \ , \quad \widehat{\Gamma}_{AB} = (-\Gamma_{ab} \mathcal{I}, \Gamma_{a'b'} \mathcal{J}) \ , \quad \mathcal{I} = \Gamma^{01234} \ , \quad \mathcal{J} = \Gamma^{56789} \ .$$

NH superalgebra contains a sub-superalgebra generated by $\{P_{\bar{A}}, J_{\bar{A}\bar{B}}, J_{\underline{AB}}, Q_+\}$, the unbroken symmetry in presence of an AdS brane.

§3 Branes in $AdS_5 \times S^5$

Supercurrents $\quad g^{-1}dg = \mathbf{L}^A P_A + \frac{1}{2}\mathbf{L}^{AB} J_{AB} + Q_I L^I \quad$ where $g \in PSU(2,2|4)$

Dp-brane action $\quad S = S_{\rm DBI} + S_{\rm WZ} = T\int_\Sigma [\mathcal{L}_{\rm DBI} + \mathcal{L}_{\rm WZ}]$

- DBI part $\quad \mathcal{L}_{\rm DBI} = \sqrt{s\det(g+\mathcal{F})}\, d^{p+1}\xi \qquad s = \mp 1$ for a $\begin{cases}\text{Lorentzian}\\ \text{Euclidean}\end{cases}$ brane ,

  $g_{ij} = \mathbf{L}_i^A \mathbf{L}_j^B \eta_{AB}$, $\quad \mathcal{F} = F - \mathcal{B}$, $\quad \mathbf{L}_i^A = \partial_i Z^{\hat{M}} \mathbf{L}_{\hat{M}}^A$, $\quad Z^{\hat{M}} = (X^M, \theta^\alpha)$

- WZ part $\quad h_{p+2} = d\mathcal{L}_{\rm WZ} = \sum_{n=0} \frac{1}{n!} h^{(p+2-2n)} \mathcal{F}^n$, $\quad \varrho = (\sigma)^{-\frac{p-3}{2}} i\sigma_2$, $\quad \sigma = \begin{cases} \sigma_3 & {\rm D}p \\ -\sigma_1 & F1 \end{cases}$

$$h^{(\ell+2)} = \frac{\sqrt{s}}{\ell!}\left[\mathbf{L}^{A_1}\cdots\mathbf{L}^{A_\ell}\bar{L}\Gamma_{A_1\cdots A_\ell}\varrho L + \delta^{\ell,3}\frac{i\lambda}{5}(\epsilon_{a_1\cdots a_5}\mathbf{L}^{a_1}\cdots\mathbf{L}^{a_5} - \epsilon_{a_1'\cdots a_5'}\mathbf{L}^{a_1'}\cdots\mathbf{L}^{a_5'})\right]$$

h_{p+2} is a non-trivial element of CE-cohomology on $\mathfrak{psu}(2,2|4)$ except for h_3

$$\mathcal{L}_{\rm WZ} = \int_0^1 dt\, [\mathcal{C}\wedge e^{\hat{\mathcal{F}}}]_{p+1} + C^{(p+1)} , \quad h_{p+2}\big|_{\text{bosonic}} = dC^{(p+1)}$$

where $\mathcal{C} = \bigoplus_n \frac{2\sqrt{s}}{(2n-1)!}\hat{\mathbf{L}}^{A_1}\cdots\hat{\mathbf{L}}^{A_{2n-1}}\hat{\bar{L}}\Gamma_{A_1\cdots A_{2n-1}}(\sigma)^n i\sigma_2\theta$ and $\hat{E} = E(t\theta)$

These reproduce the F1- and D3-brane actions given in [Metsaev-Tseytlin'9805,'9806].

§4 Non-Relativistic limit

Let us consider the scaling $X^{\underline{A}} \to \Omega X^{\underline{A}}$, $\theta_- \to \Omega\theta_-$, $T = \Omega^{-2} T_{\rm NR}$
which is equivalent to IW contraction and the scaling used in [Gomis-Gomis-Kamimura'0507].
(only light modes satisfying the non-relativistic dispersion relation are kept)

D-string For $\rho = \sigma_1$

DBI part is expanded as $T\mathcal{L}_{\rm DBI} = T_{\rm NR}\Omega^{-2}\mathcal{L}^{div}_{\rm DBI} + T_{\rm NR}\mathcal{L}^{fin}_{\rm DBI} + O(\Omega^2)$, $\mathcal{F} = \Omega\mathcal{F}_1 + O(\Omega^3)$

$$\mathcal{L}^{div}_{\rm DBI} = \sqrt{s\det g_0}\, d^2\xi = \frac{1}{2}\epsilon_{\bar{A}\bar{B}}(\mathbf{L}_0^{\bar{A}})(\mathbf{L}_0^{\bar{B}}) \ , \quad \epsilon_{\bar{A}_0\bar{A}_1} = 1 \ , \qquad (g_0)_{ij} = (\mathbf{L}_0^{\bar{A}})_i(\mathbf{L}_0^{\bar{B}})_j\eta_{\bar{A}\bar{B}} \ .$$

while h_3 is expanded as $Th_3 = T_{\rm NR}\Omega^{-2}h_3^{div} + T_{\rm NR}h_3^{fin} + O(\Omega^2)$

$$h_3^{div} = \sqrt{s}\mathbf{L}_0^{\bar{A}}\bar{L}_{+0}\Gamma_{\bar{A}}\sigma_1 L_{+0} \ .$$

We show that $\mathcal{L}^{div}_{\rm DBI} + \mathcal{L}^{div}_{\rm WZ} = 0$ for 1/2 BPS branes: (2,0)- and (0,2)-branes.

- $d\mathcal{L}^{div}_{\rm DBI} + h_3^{div} = 0$ only for $\rho = \sigma_1$. This implies that θ-dependent part of $\mathcal{L}^{div}_{\rm DBI} + \mathcal{L}^{div}_{\rm WZ}$ cancels out. So (1,1)-brane ($\rho = 1$) does not admit consistent limit.

- bosonic part of $\mathcal{L}^{div}_{\rm DBI}$, $\frac{1}{2}\epsilon_{\bar{A}\bar{B}}(e_0^{\bar{A}})(e_0^{\bar{B}})$, where e^A is the bosonic part of $\mathbf{L}^A$, is canceled by adding $C_0^{(2)} = -\frac{1}{2}\epsilon_{\bar{A}\bar{B}}(e_0^{\bar{A}})(e_0^{\bar{B}})$ which is closed $dC_0^{(2)} = 0$. □

Thus the non-relativistic action is $S_{\rm NR} = T_{\rm NR}\int(\mathcal{L}^{fin}_{\rm DBI} + \mathcal{L}^{fin}_{\rm WZ})$.

The κ-gauge symmetry is fixed by $\theta_+ = 0$, which makes it easy to perform the t-integration in $\mathcal{L}^{fin}_{\rm WZ}$. In static gauge, we obtain

$$S_{\rm NR} = T_{\rm NR}\int d^2\xi\sqrt{s\det g_0}\Big[\frac{1}{2}g_0^{ij}\partial_i y^{\underline{A}}\partial_j y_{\underline{A}} + \frac{\lambda^2}{2}(my^2 - ny'^2) + 2i\bar{\theta}_-\gamma^i{\rm D}_i\theta_- + \frac{1}{2}F_{ij}F^{ij}\Big] .$$

This is a free field action on $AdS_2(H^2)$ or S^2.

- Consistent non-relativistic limits are possible for all 1/2 BPS Dp-branes listed in §1. In flat limit $\lambda \to 0$, our non-relativistic AdS Dp-brane actions reproduce the non-relativistic Dp-brane actions in flat spacetime [Gomis-Passerini-Ramirez-Van Proeyen'0507].

- For F1, AdS_2-brane action was obtained in [Gomis-Gomis-Kaminura'0507]. The same action is derived as fluctuations on AdS_2-brane in [Drukker-Gross-Tseytlin'0001].

§5 Summary

- NH superalgebras of branes in $AdS_5 \times S^5$ and $AdS_{4/7} \times S^{7/4}$
  - IW contraction of $\mathfrak{psu}(2,2|4)$ and $\mathfrak{osp}(8^{(*)}|4)$
  - sub-superalgebra as supersymmetrization of brane worldvolume isometry and transverse Lorentz symmetry.
  - classification of 1/2 supersymmetric subspaces $\supset$ 1/2 BPS branes

- Non-relativistic AdS brane actions
  - consistent limit can be performed for 1/2 BPS AdS branes
  - in flat limit they reproduce non-relativistic brane actions in flat spacetime

comments on M-branes

- Newton-Hooke superalgebras of M-branes in $AdS_{4/7} \times S^{7/4}$
  IW contraction of $\mathfrak{osp}(8|4)$ or $\mathfrak{osp}(8^*|4)$

1-brane	2-brane	5-brane	6-brane	9-brane	10-brane
(1,1)	(0,3), (2,1)	(1,5), (3,3)	(0,7), (2,5), (4,3)	(3,7)	(4,7)

  p-branes with $p = 1$ mod 4 are 1/2 BPS Dirichlet branes of open supermembrane [Yoshida-MS'0310,'0405]; brane probe analysis [Kim-Yee'0211]

- 2- and 5-branes admit consistent non-relativistic limit.
  - M2- and M5-brane actions are characterized as non-trivial elements of CE cohomology on $\mathfrak{osp}(8|4)$ or $\mathfrak{osp}(8^*|4)$.
  - M2- and M5-branes admit consistent non-relativistic limit, and derived non-relativistic AdS M-brane actions reproduce the non-relativistic M2-brane action [Gomis-Kamimura-Townsend'0409] and the linearized M5-brane action [Claus-Kallosh-Van Proeyen'9711] in flat limit.