Wilson Line Correlators in Noncommutative Gauge Theory

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Introduction

Noncommutative (NC) gauge theories emerge by considering classical background (higher dimensional D-brane) in type IIB matrix model.

Our universe may be constructed on D-brane.

We consider correlation functions between Wilson lines in 4 dimensional euclidean NC gauge theory.

constructed by [Ishibashi-Iso-Kawai-Kitazawa ’99]

The couplings between Wilson line operators and closed string modes are read from vertex operators in IIB matrix model.

We focus on graviton modes.

There are dual descriptions in NC gauge theory.

NC gauge theory with maximal SUSY ⟺ SUGRA on dual background

proposed by [Hashimoto-Itzhaki '99, Maldacena-Russo '99]

UV/IR mixing will be seen in both descriptions.

Plan of Talk

1. Introduction
2. Wilson line correlators in NC gauge theory
3. Dual supergravity description
4. Conclusion

Vertex operators

[Kitazawa '02, Iso-Terachi-Umetsu '04]

- Supergravity multiplet

Dilaton vertex operator $Trexp(ikA)\Phi$

generated by SUSY transf.

⋮

Graviton vertex operator

$$Strexp(ik{\cdot}A)([A^\rho,A^\mu][A^\rho,A_\nu]+\frac{1}{2}\bar{\psi}\Gamma^{(\nu}[A^{\mu)},\psi])h_{\nu\mu}+\cdots$$

In analogy with these operators, we introduce Wilson line operators on fuzzy G/H space as

$$V^{grav.}_{G/H}(k)\equiv Str\boxed{\mathcal{Y}_k(A)}([A_\rho,A_\mu][A_\rho,A_\nu]+\frac{1}{2}\bar{\psi}\Gamma^{(\nu}[A^{\mu)},\psi])$$

Spherical harmonics

We calculate the amplitude by choosing the representation of G/H. For example, on CP(2),

$$\langle V^{grav.}_{CP(2)}(k) V^{grav.}_{CP(2)}(k)^* \rangle \sim \frac{N}{k^2}\sqrt{\pi}\zeta(3/2)$$

where we omit Lorentz indices.

We have obtained the $1/k^2$ dependence.[hep-th/0512204]

This result is universal with respect to the choice of G/H.

In ordinary (commutative) gauge theory, this type of correlator gives $k^4 \log l^2/k^2$ from CFT.

We can confirm that this behavior is seen in IR behavior in NC gauge theory.

Dual supergravity description

Supergravity background dual to NCYM:

$$ds^2 = (\frac{R}{r})^2(\frac{d\vec{x}^2}{1+(\frac{R}{r})^4} + dr^2 + r^2 d\Omega_5^2) \ ,$$

Since fuzzy G/H spaces approach flat space in large N limit, we use this solution for the analysis.

Equation of motion for gravity mode (scalar mode) is described as Mathieu equation:

$$\frac{1}{r^5}\partial_r(r^5\partial_r\phi(\vec{k},r)) - R^4k^2(\frac{1}{r^4}+\frac{1}{R^4})\phi(\vec{k},r) = 0 \ ,$$

Two independent solutions [Gubser-Hashimoto '98]

$\frac{1}{r^2}H^{(1)}(\nu, z)$: regular in the region 0<r<R

$\frac{1}{r^2}H^{(2)}(\nu, -z)$: regular in the region R<r

We define Green function in the region $0<r<R$.

We look for the prescription which

i) is smoothly connected with that in the ordinary AdS/CFT correspondence with respect to IR contribution.

ii) reproduces UV behavior which is seen in NC gauge theory due to UV/IR mixing effect.

Neumann boundary condition

For the Green function,

$$G(r,r',k)=\left(\frac{x}{r^2}H^{(1)}(\nu,z)+\frac{1}{r^2}H^{(2)}(\nu,-z)\right)\frac{1}{r'^2}H^{(1)}(\nu,z')\ ,\qquad r>r'\ ,$$

$$G(r,r',k)=\left(\frac{x}{r'^2}H^{(1)}(\nu,z')+\frac{1}{r'^2}H^{(2)}(\nu,-z')\right)\frac{1}{r^2}H^{(1)}(\nu,z)\ ,\qquad r'>r\ ,$$

we impose the boundary condition as

$$\underline{\partial_r G(r,r',k)|_{r=R}=0}\ ,\qquad G(r,r',k)|_{r'\to 0}=0$$

Neumann b. c. at r=R

Small momentum expansion gives

$$G(r,r',k)\sim\frac{3}{2k^2R^6}+\mathcal{O}(1)\ .$$

Conclusion

- We have investigated the two point correlation functions of graviton vertex operators in 4D NC gauge theory.

- We have shown that it behaves as 4D massless graviton.

- In the dual supergravity description, Neumann boundary condition at r=R also gives the behavior of Green function of graviton mode to be $1/k^2$.

- It may suggest that brane is located at r=R, which comes from the minimum length scale in NC gauge theory.