

Wilson Line Correlators in Noncommutative Gauge Theory

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Introduction

Noncommutative (NC) gauge theories emerge by considering classical background (higher dimensional D-brane) in type IIB matrix model.

Our universe may be constructed on D-brane.

We consider correlation functions between Wilson lines in 4 dimensional euclidean NC gauge theory.

constructed by [Ishibashi-Iso-Kawai-Kitazawa '99]

The couplings between Wilson line operators and closed string modes are read from vertex operators in IIB matrix model.

We focus on **graviton modes**.

There are dual descriptions in NC gauge theory.

NC gauge theory with maximal SUSY



SUGRA on dual background

proposed by [Hashimoto-Itzhaki '99, Maldacena-Russo '99]

UV/IR mixing will be seen in both descriptions.

Plan of Talk

1. Introduction
2. Wilson line correlators in NC gauge theory
3. Dual supergravity description
4. Conclusion

Vertex operators

[Kitazawa '02, Iso-Terachi-Umetsu '04]

· Supergravity multiplet

Dilaton vertex operator $Tr \exp(ikA) \Phi$

⋮

Graviton vertex operator



generated by SUSY transf.

$$Str \exp(ik \cdot A) ([A^\rho, A^\mu][A^\rho, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^{(\nu} [A^{\mu)}, \psi]) h_{\nu\mu} + \dots$$

In analogy with these operators, we introduce Wilson line operators **on fuzzy G/H space** as

$$V_{G/H}^{grav.}(k) \equiv Str \boxed{\mathcal{Y}_k(A)} ([A_\rho, A_\mu][A_\rho, A_\nu] + \frac{1}{2} \bar{\psi} \Gamma^{(\nu} [A^{\mu)}, \psi])$$

Spherical harmonics

We calculate the amplitude by choosing the representation of G/H. For example, on CP(2),

$$\langle V_{CP(2)}^{grav.}(k) V_{CP(2)}^{grav.}(k)^* \rangle \sim \frac{N}{k^2} \sqrt{\pi} \zeta(3/2)$$

where we omit Lorentz indices.

We have obtained the $1/k^2$ dependence. [[hep-th/0512204](https://arxiv.org/abs/hep-th/0512204)]

This result is universal with respect to the choice of G/H.

In ordinary (commutative) gauge theory, this type of correlator gives $k^4 \log l^2/k^2$ from CFT.

We can confirm that this behavior is seen in IR behavior in NC gauge theory.

Dual supergravity description

Supergravity background dual to NCYM:

$$ds^2 = \left(\frac{R}{r}\right)^2 \left(\frac{d\vec{x}^2}{1 + \left(\frac{R}{r}\right)^4} + dr^2 + r^2 d\Omega_5^2 \right) ,$$

Since fuzzy G/H spaces approach flat space in large N limit, we use this solution for the analysis.

Equation of motion for gravity mode (scalar mode) is described as Mathieu equation:

$$\frac{1}{r^5} \partial_r (r^5 \partial_r \phi(\vec{k}, r)) - R^4 k^2 \left(\frac{1}{r^4} + \frac{1}{R^4} \right) \phi(\vec{k}, r) = 0 ,$$

Two independent solutions [Gubser-Hashimoto '98]

$$\frac{1}{r^2} H^{(1)}(\nu, z) \quad : \text{regular in the region } 0 < r < R$$

$$\frac{1}{r^2} H^{(2)}(\nu, -z) \quad : \text{regular in the region } R < r$$

We define Green function in the region $0 < r < R$.

We look for the prescription which

- i) is smoothly connected with that in the ordinary AdS/CFT correspondence with respect to **IR contribution**.
- ii) reproduces **UV behavior** which is seen in NC gauge theory due to **UV/IR mixing effect**.

Neumann boundary condition

For the Green function,

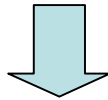
$$G(r, r', k) = \left(\frac{x}{r^2} H^{(1)}(\nu, z) + \frac{1}{r^2} H^{(2)}(\nu, -z) \right) \frac{1}{r'^2} H^{(1)}(\nu, z'), \quad r > r',$$

$$G(r, r', k) = \left(\frac{x}{r'^2} H^{(1)}(\nu, z') + \frac{1}{r'^2} H^{(2)}(\nu, -z') \right) \frac{1}{r^2} H^{(1)}(\nu, z), \quad r' > r,$$

we impose the boundary condition as

$$\underline{\partial_r G(r, r', k)|_{r=R} = 0}, \quad G(r, r', k)|_{r' \rightarrow 0} = 0$$

Neumann b. c. at $r=R$



Small momentum expansion gives

$$G(r, r', k) \sim \frac{3}{2k^2 R^6} + \mathcal{O}(1).$$

Conclusion

- We have investigated the two point correlation functions of graviton vertex operators in 4D NC gauge theory.
- We have shown that it behaves as 4D massless graviton.
- In the dual supergravity description, Neumann boundary condition at $r=R$ also gives the behavior of Green function of graviton mode to be $1/k^2$.
- It may suggest that brane is located at $r=R$, which comes from the minimum length scale in NC gauge theory.