B and D Semileptonic Decays on the Lattice

Paul Mackenzie Fermilab mackenzie@fnal.gov

> DPF/JPS 2006 Honolulu October 31, 2006

Thanks, Richard Hill, Ruth Van de Water

Exclusive semileptonic decays on the lattice

provide good determinations of CKM matrix elements.

Outline

- Overview of Fermilab/MILC semileptonic program
- Constrained curve fitting
- Constrained curve fitting and the shape of semileptonic

form factors.

Current Fermilab/MILC semileptonic projects and writeups.

Long-term plan is to analyze all of these on the y MILC lattices with *a*=0.15, 0.125, and 0.09 fm.

B->D* RUN PARAMETERS:

Fine Lattices:

Coming soon

Coarse Lattices:

t_sink - t_source = 12 t_source = $0, 16, 32, 48$ smeared heavy clover daughter quark local heavy clover parent quark local staggered spectator quark full QCD only heavy kappas = (0.074, 0.086, 0.093, 0.119, 0.114, 0.122) b c

Medium-Coarse Lattices:

t_sink = 10 ? t_source = 0, 24 (also 12, 36?) full QCD only

AVAILABLE 3pt DATA:

Coarse Lattices:

0.02/0.05 ensemble -- t_source = 0, 16, 32, 48 0.01/0.05 ensemble -- t_source = 0, 16, 32, 48 0.007/0.05 ensemble -- t_source = 0, 16, 32, 48

Medium-Coarse Lattices:

0.0194/0.0484 ensemble (In progress) 0.0290/0.0484 ensemble (In progress)

Staggered chiral PT, Laiho and Van de Water, Phys. Rev. **D73**:054501, 2006

Currently working on a run at *a*=0.15 fm, to obtain an estimate of discretization errors, before moving on to *a*=0.09 fm.

B->pi RUN PARAMETERS:

Coarse Lattices:

t_sink = 12 t source = 0, 32 (also 16, 48?) local pion smeared B full QCD only

Medium-Coarse Lattices:

t_sink = 10 ? t_source = 0, 24 (also 12, 36?) local pion smeared B full QCD only

AVAILABLE 3pt DATA:

Coarse Lattices:

 $0.02/0.05$ ensemble -- t_source = 0, 16, 32, 48 0.01/0.05 ensemble -- t source = 0, 32 0.007/0.05 ensemble -- t_source = 0, 32

Medium-Coarse Lattices:

0.0194/0.0484 ensemble -- t_source = 0,32 @ t_sink = 12; t_source = 0 @ t_sink = 8,10 0.0290/0.0484 ensemble -- t_source = 0 @ t_sink = 8,10,12

Currently working on a run at *a*=0.15 fm, to obtain an estimate of discretization errors, before moving on to *a*=0.09 fm; studying optimal ways of performing unitarity-based fits (see later).

Very preliminary.

B➙*πlν,* finite range of *q2*

Lattice data extend over only a fraction of the *q2* range on the **physical** *B*➙*πlν* decay.

methods, With standard discretization errors go like *O(ap)2,* signal goes like $\exp(-E_{\pi}t)$.

Uncertainties in lattice and experiment both highly q^2 dependent. Harder and more important to understand shape.

Proposals to address:

*) Moving NRQCD (Davies, Lepage, et al.)

*) Calculate in charm region, extrapolate to bottom (Abada et al.)

*) Gibbons: global simultaneous fit of all experimental and lattice data.

*) Unitarity and analyticity (Lellouch, Fukunaga-Onogi, Arnesen et al., ...)

Always have to make some assumptions about shape to obtain any fit.

If any crazy shape were allowed, we could never fit anything.

Constrained curve fitting allows the possibility of an infinite number of free parameters in fits, while formalizing assumptions about their values.

Constrained curve fitting The probability density density density density density density for obe-
). Then the probability density for obtaining a particular a particular a particular term and a particular term and a particular term and the origin where we assume that the energies \blacksquare der of increasing size. The change is to fit and the change is to fit an ionstrained curve fitting son beform the variety meeting $\frac{1}{2}$ fitting α $\mathcal F$ inclinity code assumes that $\mathcal F$ We need to teach physics to the fitting to the fit

Add an infinite number of parameters to the fit
function, but constrain them to their plausible ranges.
Ω αδιατική της διαφορείας της διαφορείας της διαφορείας της διαφορείας του διαφορείου του διαφορείου του διαφορεί ϵ for the constraints ϵ the entropy is maximized by the Gaussian of \mathcal{G} an infinite number of parameters to the fit and them to their probability ranged. function, but constrain them to their plausible ranges. o the fit f iusible ranges.
. $\overline{\mathbf{c}}$ T in infinite fituriber of parameters to the T $\frac{d}{dt}$ \mathbf{r}_{max} Add an infinite number of parameters to the fit

$$
\chi^2 \rightarrow \chi^2_{\text{aug}} \equiv \chi^2 + \chi^2_{\text{prior}}
$$

$$
\chi^2(A_{\pi} F_{\pi}) = \sum \Lambda G(t) \sigma^{-2}
$$

Use "augmented" chi squared: $\lambda^{(1-n, 2n)} = \sum_{t,t'} \blacksquare$

bilities: P(pG) = P(pG) P(pG) = P(pG) P(pG).
P(pG) P(pG) P(pG) P(pG).

Use "augmented" chi squared:
\n
$$
\chi^{2}(A_{n}, E_{n}) \equiv \sum_{t,t'} \Delta G(t) \sigma_{t,t'}^{-2} \Delta G(t')
$$
\n
$$
\chi^{2} = \sum_{t,t'} \frac{(A_{n} - \tilde{A}_{n})^{2}}{2} + \sum_{t} \frac{(E_{n} - \tilde{E}_{n})^{2}}{2}
$$

$$
\chi^2_{\text{prior}} \equiv \sum_n \frac{(A_n - \tilde{A}_n)^2}{\tilde{\sigma}_{A_n}^2} + \sum_n \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma}_{E_n}^2}
$$

Ω aves formula: bayes formula. $\ddot{}$ Bayes formula:

 \sum Paul Mackenzie

What do we know in advance about the fit function for form factors? max province about the For the convention of future experimental and phenomenological and phenomenological and phenomenological and p

$$
F_{+}(q^2) = \frac{F_{+}(0)/(1-\alpha)}{1-\frac{q^2}{m_{B^*}^2}} + \left(\frac{1}{\pi}\int_{t_+}^{\infty}dt\,\frac{\mathrm{Im}F_{+}(t)}{t-q^2-i\epsilon}\right) \qquad \text{Real life higher mass states for B \to \pi l v:}
$$

a cut.

by adding more and more poles. (Hill.) $1 - \frac{1}{m_{B*}^2}$ BK could be extended with increasing accuracy

BK could be extended with increasing accuracy
by adding more and more poles. (Hill.)
$$
F_{+}(q^{2}) = \frac{F_{+}(0)/(1-\alpha)}{1-\frac{q^{2}}{m_{B^{*}}^{2}}} + \sum_{k=1}^{N} \frac{\rho_{k}}{1-\frac{1}{\gamma_{k}}\frac{q^{2}}{m_{B^{*}}^{2}}}
$$

 $\frac{1}{2}$

Current limitations of BK

1) In lattice data fit with BK, statistical errors are smaller at high momentum than at low.

An effect of the model not present in the raw data.

2) Richard Hill: experimental data better fit with an extended 2) Richard Hill: experimental data better fit with an extended
BK with an extra parameter:

$$
F_{+}(q^{2}) = \frac{F_{+}(0)/(1-\alpha)}{1-q^{2}/m_{B^{*}}^{2}} + \frac{c}{1-q^{2}/M'^{2}} + \dots
$$

=
$$
\frac{F_{+}(0)(1-\delta q^{2}/m_{B^{*}}^{2})}{(1-q^{2}/m_{B^{*}}^{2})(1-[\alpha+\delta(1-\alpha)]q^{2}/m_{B^{*}}^{2})}
$$

What do we know in advance about the fit function for form factors?

Not easy to fix prior uncertainties to parameters in BK extensions.

Analyticity and unitarity have long been used to constrain shapes of form factors.

Lellouch, Fukunaga-Onogi, Arnesen et al., ...

A particularly simple form has recently been emphasized by Arnesen et al.

into a new variable z in the complex plane: Consider a remapping of the semileptonic decay variable t=q²

$$
z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}
$$

z maps $q^2=t>t_{+}$ onto $|z|=1$., and

z maps q^2 =t>t₊ onto $|z|=1$., and
t<t₊ onto [-1,1] in the complex plane.

$$
(t = (p_H - p_L)^2, t_{+} = (m_H + m_L)^2, t_{-} = (m_H - m_L)^2).
$$

t₀, taken as 0.65 t_ here, is a fudge factor adjusted to center the physical region on z~0.

A power series expansion of the form factors in z can be written in the form:

$$
f(t) = \underbrace{\sum_{P(t)\phi(t,t_0)}^1} \sum_{k=0}^{\infty} a_k(t_0) z(t,t_0)^k
$$

Function calculated in

Function that has unit norm at *z=1*., simple form for the *ak*. and that vanishes at the poles of *f,* e.g., at the *B** pole.

ulated in perturbation theory to produce a

P and *φ* contain most of the complexity of the form factors.

alculatir !
|-
| nII nII n u تا u
an the urrent-current cor melation functandier.
Trelation funct 1au011 1u11 \overline{r} culating t he curi theory and using the *J^µBπ* amplitude, ht-current correlatic
תכוול בר lculating the current-current correlation function in p The same stick same to be a
The monoin $\sum_{i=1}^{n}$ and for $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ and $\sum_{i=1}^{n}$ \sim By calculating the current-current correlation fur ρ in nerturhation derived earliers in Ref. consider the period of the such as in Ref. Eq. in Period of the Second entrier the Se calculating the current-current correlation function in perturba By calculating the current-current correlation function in perturbation <u>.</u>
87 € 1.88 By calculating the current-current correlation function in perturbation
theory and using the J^µBπ amplitude, [p.s.] ^δ(q−pBπ)&0|J†^ν [|]B¯π'&B¯π|J^µ|0' ⁺ . . . (4)

Im
$$
\Pi^{\mu\nu}
$$
= \int [p.s.] $\delta(q-p_{B\pi})\langle 0|J^{\dagger\nu}|\bar{B}\pi\rangle\langle\bar{B}\pi|J^{\mu}|0\rangle + ...$

th crossing symmetry and analyticity, o $\ddot{}$ 1.140 **as (mb)** $f(t)=\frac{1}{F}$ $\frac{1}{\rho(t)\phi(t,t)}$ $\int \phi(t,t)$, derived earlier, such as in Ref. [14], because they do not crossing symmetry and analyticity, one obtains a simple constraint $\mathcal{L}(\mathcal{L})$ $U(z, t_0)$ corresponds to the lowest moment of Indian states to the lowest moment of Indian states in the lowest moment o
The lowest moment of Indian states in the lowest moment of Indian states in the lowest moment of Indian states de aks in the equation $r(t)$ and $r(t)$ is a matter of 1 $\mathcal{L}(\nu) \psi(\nu, \nu) = k = 0$ nstraint de de river de la possibilité de la possibilité de la possibilité de la possibilité du partier de la possibili
Le possibilité du note du partier de la possibilité du note du r^{k} $\overline{}$ лυээн
e *aк*s i g symmetry and analyticity, one obtaii
∩ the equation equatioi $1 \quad \zeta$.
) 1+0.751 as (mb) as (mb
1+0.751 as (mb) as (mb $P(t)\phi(t,t_0)\nleftrightarrow_{k=0}^{\infty}$ $\overline{}$ constraint U ion directions U but in this constraint
Managem of the constraint position and the tenshing symmetry and analyticity, one obtaint on the a_k s in the equation uation $f(t) = \frac{1}{\sqrt{2\pi} \sqrt{2\pi} \$ but as a central value of $P(t)\phi(t,t_0)$ ϵ (+) ~ (+ +) k $a_k(t_0)$ $z(t,t_0)^{\sim}$ th crossing symmetry and analyticity, one obtains a simple con the $a_k s$ in the equation form for the system for $\sum_{k=1}^{\infty} a_k s^{k}$ $f(t) = \frac{1}{D(t) \phi(t+1)} \sum a_k(t_0) z(t,t_0)$ μ (*v*) φ (*v*, *v*₀) $\overline{k=0}$ with crossing symmetry and analyticity, one obtains a simple constraint Br(B¯⁰ [→] ^π+"−ν¯) ⁼ (1.³⁹ [±] ⁰.12) [×] ¹⁰−⁴ , (3) express by writing each of f+(t), f0(t) as a series which should yield |Vub| at the % 5% level. So far extrac $f(t) = \frac{1}{D(t)}$ $P(t)\phi(t,t_0)$ \sum^{∞} $k=0$ $a_k(t_0)\,z(t,t_0)^k$

 $\frac{1}{2}$ π

It is simply
$$
\sum_{k=0}^{n_A} a_k^2 \le 1
$$

 $B\rightarrow T$ allowed range of z in physical semile
| v: -0.34<z<0.22, 24U.U-
:ے 17 $\overline{}$ א<-D
א \overline{D} % 1+0.751 αs(mb) & f⁺ = 0.04°2°0.00.
2.02°2°0.04 $\frac{f'(t)}{2}$ $t) = \frac{1}{L}$ $\overline{1}$ Since $\sum_{n=1}^{n_A} a_n^2 < 1$ $\sum_{k=0}^{n} u_k \geq 1$, The allowed range of z in physical semileptonic decay is small $\sum_{k=1}^{n} \alpha_k(v_0) \times (v_0, v_0)$ $\varepsilon = 0$ $m = 70^\circ$ $m = 70^\circ$ $m = 70^\circ$)->∏
ג\ $D \rightarrow \pi$ | v : -0.17<z<0.16,
D-> π | v : -0.17<z<0.16, D->K l v : -0.04<z<0.06, $\frac{1}{4}$ $\int f(x) dx$ $-$ (as $-$ 4) 1 $\sum_{k=1}^{n} a_k^2 < 1$ $_{n_A}$ B->D I v: -0.02<z<0.04. $P(t)\phi(t,t_0)\nleftrightarrow_{k=0}^{\infty}$ rely on expanding in \mathcal{L} $\overline{}$ range $-\frac{1}{2}$ sica \sim milontor nileptor , B->π | v: -0.34<z<0.22,
D >π | ν · 0.47<z<0.16 based on the contract of the c
In the contract of the contract $J(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=1}^{t_0}$ $\sum_{k=0}^{n} \frac{w_k - 1}{k}$ $\mathbf{u} \cdot \mathbf{v} = \mathbf{u} \cdot \mathbf{v}$ \mathcal{L} $(t_0) z(t,t_0)^{\kappa}$ The allowed range of z in physical semileptonic decay is small
Decay is small Since $\sum a_k^2 \leq 1$, n_A $k=0$ from perturbative and power corrections. The experi- ∞ $\frac{1}{t_0} \sum_{k=0} a_k(t_0) z(t,t_0).$ $\kappa = 0$ $E > N \cup Y$. U.UT $\leq N \cup U$,
R $\leq N \cup V$. O O2<7<0 O4 f(B->π l ν: -0.34<z<0.22, D-2π ι ν. -υ.34\2\υ.22,
D->π ι ν : -0.17<z<0.16, B->D l ν : -0.02<z<0.04. The allowed range of z in physical semilentonic decay is small which shows
| B->D I v + -0 02<7<1 $t = 5000$ iv. -0.02 sum rules n_A and \sum_{A}^{n} in \sum_{A}^{n} $r=\sum_{k=0}^{n}a_k\geq 1,$ μ physical semiground decay is single $e^{i\theta}$ express by writing each of f+(t), f0(t) as a series by writing each of f+(t), f0(t) as a series $e^{i\theta}$ $f(t) = \frac{1}{D(t)}$ $P(t)\phi(t,t_0)$ \sum^{∞} $k=0$ $a_k(t_0)\ z(t,t_0)^k$

 κ and dispersive bound gives a constraint on the coefficients of the coefficients on the coefficients of the coeffici neters is needed, only 5 or 6 even in the case of to obtain the form factors to high accuracy, say 1%, only a small to obtain the form factors to high accuracy, say 1%, only a small w obtain the form factors to ingit accuracy, say 170, the number of parameters is needed, only 5 or 6 even in $R\rightarrow$ TT I v experimental uncertainty of 20% theory.
 experimental uncertainty dividends a small case of יר
1 \overline{a} σ is needed, only σ or σ ever
 nly a small $\mathbf i$ in the case of for any control of the cont Franko: G. paramotoro lo hoodod, only o
R->π | ν Eq. (10) are each at the ∼ 10% level even when "chirally $xy + 70$, UHJ a SHanced $x + 1$ FITTIC COST OF to obtain the form factors to high accuracy, say 1%, only a small number of parameters is needed, only 5 or 6 even in the case of B->π l ν.

B➙*πlν,* unitarity fits

Strong *q2* dependence in form factor is due to calculable effects. When those are factored out, two parameters suffice to describe the current experimental data. (Just like *B*➙*Dlν, K*➙*πlν?!!)*

B➙*πlν,* unitarity fits

B- \rightarrow form factor data normalized by P(t) x $\Phi(t,t_0)$ vs. z(t)

B➙*πlν,* unitarity fits

Combined fits of f+ and f0 may give surprisingly good prediction for form factors well beyond the range of lattice data.

- Raw lattice data,

- Not extrapolated in *m* or *a*,

- Momentum dependent discretization errors not yet included.

How can the results of such fits best be compared with experiment?

Summary

- Fermilab and MILC are calculating an extensive set of semileptonic form factors at several lattice spacings.
	- *^D*➔*{π,K}l^ν*
	- *^B*➔*D{*}l^ν*
	- *^B*➔*πl^ν*
	- *^B*➔*Kl+l -*
- The analyticity-based *z* expansion limits the number of parameters needed to describe form factor data, without introducing model dependence.
- In terms of the *z* expansion, all semileptonic form factor data, both lattice and experiment are consistent with straight lines: normalization and slope.

