B and D Semileptonic Decays on the Lattice

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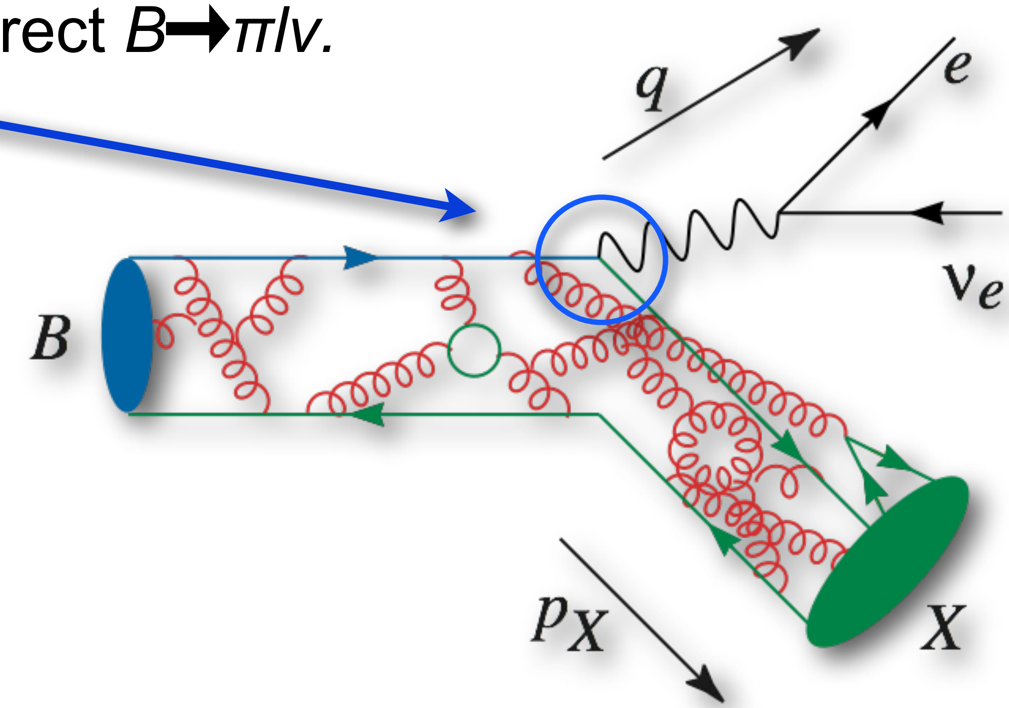
Thanks, Richard Hill, Ruth Van de Water

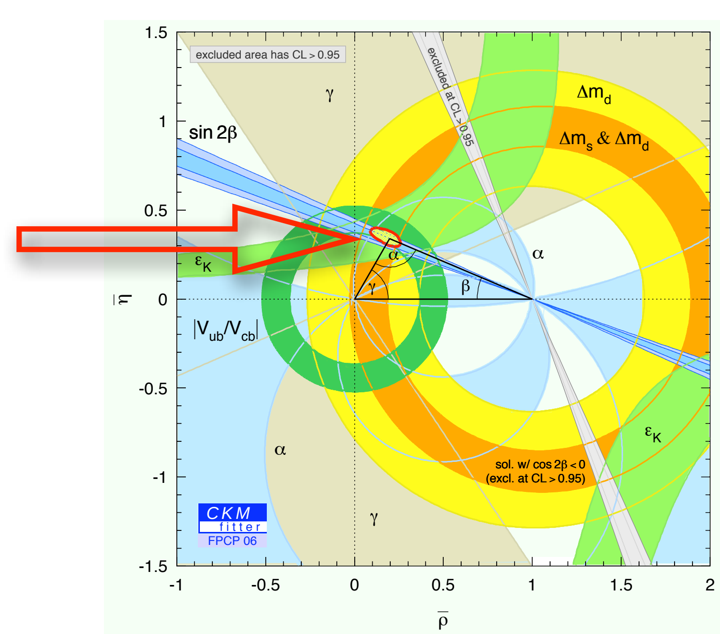
Exclusive semileptonic decays on the lattice

provide good determinations of CKM matrix elements.

Tune V_{ub} to get correct $B \to \pi l \nu$.



= {π, K, ...}



Outline

- Overview of Fermilab/MILC semileptonic program
- Constrained curve fitting
- Constrained curve fitting and the shape of semileptonic form factors.

Current Fermilab/MILC semileptonic projects and writeups.

$D \to \{\pi, K\} l\nu$	Phys. Rev. Lett. **94:**011601, 2005	
$B \to D l\nu$	Okamoto, Lattice 05	
$B \to D^* l\nu$	Laiho, Lattice 06	
$B \to \pi l\nu$	Masataka Okamoto, Lattice 05 Van de Water, Lattice 06	
$B \to K l^+ l^-$	Jain, Lattice 06	

Long-term plan is to analyze all of these on the MILC lattices with *a*=0.15, 0.125, and 0.09 fm.

	a(fm)	L	m_l	m_s	m_π(MeV)	approx. # configs.
	0.09	40	0.0031	0.031	—	600
Fine	0.09	28	0.0062	0.031	336	600
	0.09	28	0.0124	0.031	467	600
	0.125	24	0.005	0.05	254	600
	0.125	20	0.007	0.05	300	800
	0.125	20	0.01	0.05	357	800
Coarse	0.125	20	0.02	0.05	494	600
	0.125	20	0.03	0.05	600	600
	0.125	20	0.04	0.05	–	600
	0.125	20	0.05	0.05	–	600
	0.15	20	0.00484	0.0484	212	600
	0.15	16	0.0097	0.0484	327	600
	0.15	16	0.0194	0.0484	453	600
	0.15	16	0.0290	0.0484	550	600
	0.15	16	0.0484	0.0484	700	600

B→D{*}lν

Laiho, Lattice 06

B->D* RUN PARAMETERS:

Fine Lattices:

Coming soon

Coarse Lattices:

t_sink - t_source = 12
t_source = 0, 16, 32, 48
smeared heavy clover daughter quark
local heavy clover parent quark
local staggered spectator quark
full QCD only
heavy kappas = ($\underbrace{0.074, 0.086, 0.093}_{b}$, $\underbrace{0.119, 0.114, 0.122}_{c}$)

Medium-Coarse Lattices:

t_sink = 10?
t_source = 0, 24 (also 12, 36?)
full QCD only

AVAILABLE 3pt DATA:

Coarse Lattices:

0.02/0.05 ensemble -- t_source = 0, 16, 32, 48
0.01/0.05 ensemble -- t_source = 0, 16, 32, 48
0.007/0.05 ensemble -- t_source = 0, 16, 32, 48

Medium-Coarse Lattices:

0.0194/0.0484 ensemble (In progress)
0.0290/0.0484 ensemble (In progress)

Staggered chiral PT,
Laiho and Van de Water,
Phys. Rev. **D73**:054501, 2006

Currently working on a run at *a*=0.15 fm, to obtain an estimate of discretization errors, before moving on to *a*=0.09 fm.

$B \rightarrow \pi l \nu$

Van de Water, Lattice 06

B->pi RUN PARAMETERS:

Coarse Lattices:

t_sink = 12
t_source = 0, 32 (also 16, 48?)
local pion
smeared B
full QCD only

Medium-Coarse Lattices:

t_sink = 10?
t_source = 0, 24 (also 12, 36?)
local pion
smeared B
full QCD only

AVAILABLE 3pt DATA:

Coarse Lattices:

0.02/0.05 ensemble -- t_source = 0, 16, 32, 48
0.01/0.05 ensemble -- t_source = 0, 32
0.007/0.05 ensemble -- t_source = 0, 32

Medium-Coarse Lattices:

0.0194/0.0484 ensemble -- t_source = 0,32 @ t_sink = 12; t_source = 0 @ t_sink = 8,10
0.0290/0.0484 ensemble -- t_source = 0 @ t_sink = 8,10,12

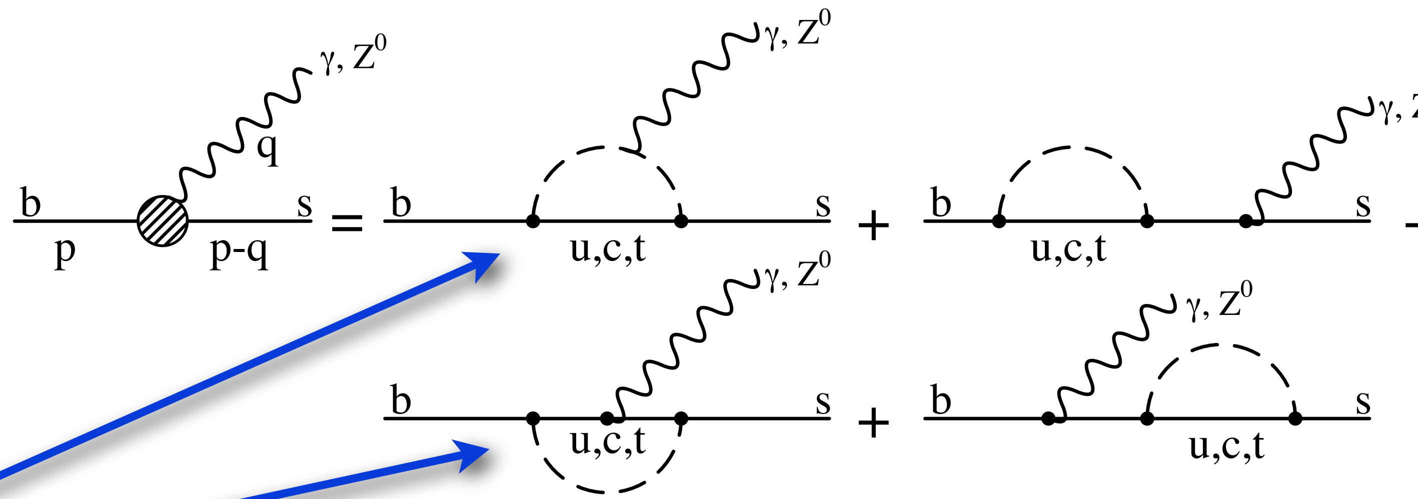
Currently working on a run at a=0.15 fm, to obtain an estimate of discretization errors, before moving on to a=0.09 fm;
studying optimal ways of performing unitarity-based fits (see later).

$B \rightarrow K l^+ l^-$

$$\langle B(p)|\bar{b}\gamma^\mu s|K(k)\rangle = f_+(p^\mu + k^\mu - \frac{m_B^2 - m_K^2}{q^2}q^\mu) + f_0 \frac{m_B^2 - m_K^2}{q^2}q^\mu$$

$$\langle B(p)|\boxed{\bar{b}\sigma^{\mu\nu}q_\nu s}|K(k)\rangle = i\frac{f_T}{m_B + m_K}\{(p+k)^\mu q^2 - q^\mu(m_B^2 - m_K^2)\}$$

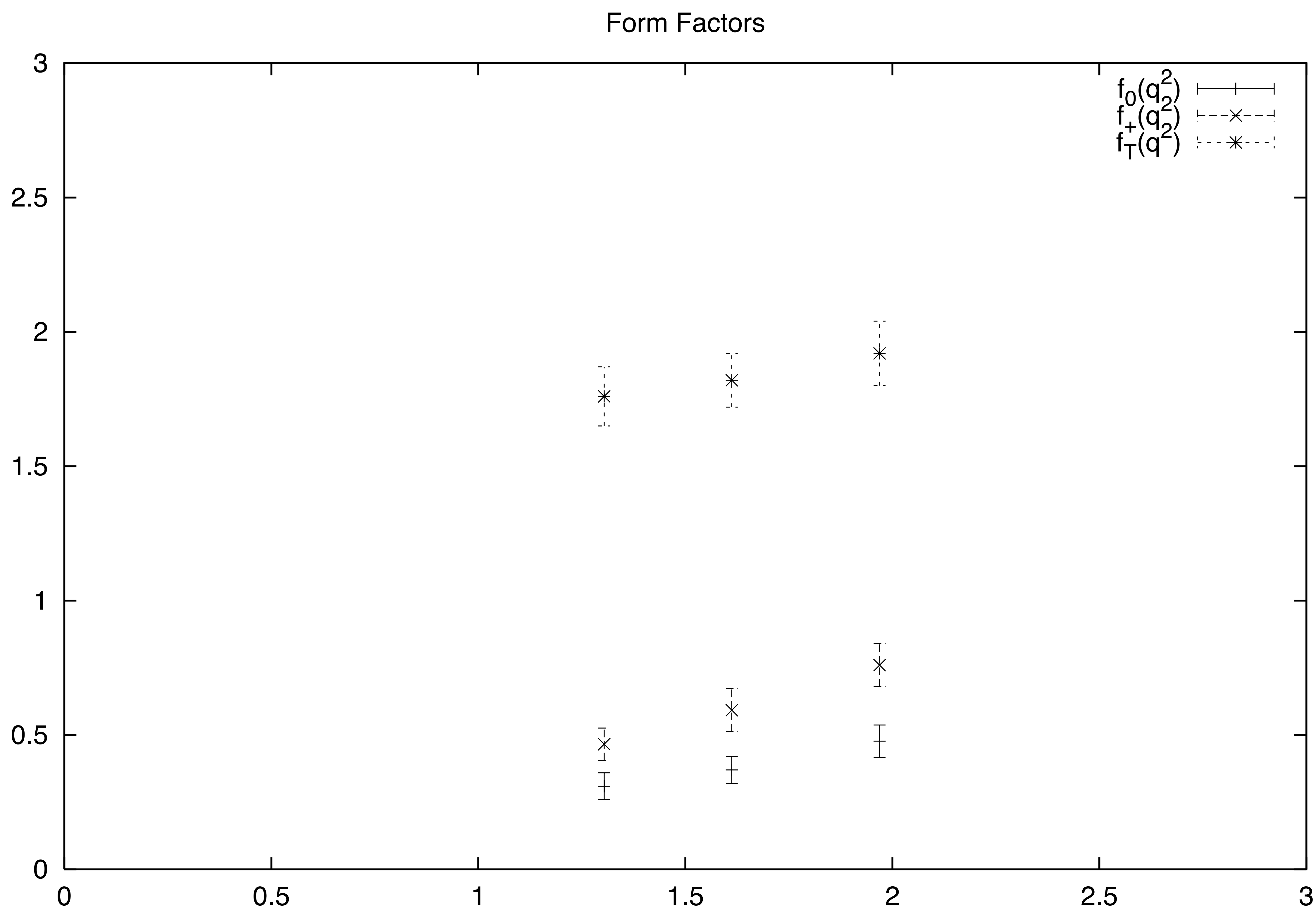
New operators and form factors in the Standard Model.



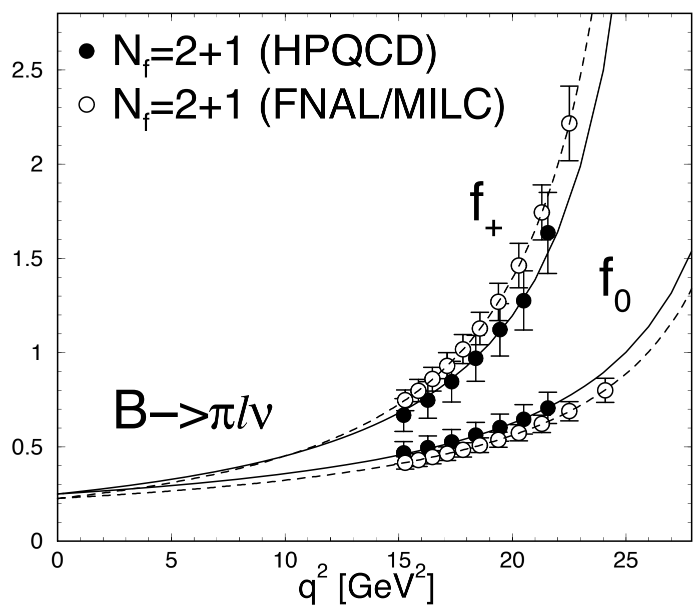
Penguin diagrams contributing to $B \rightarrow K\ell^+\ell^-$

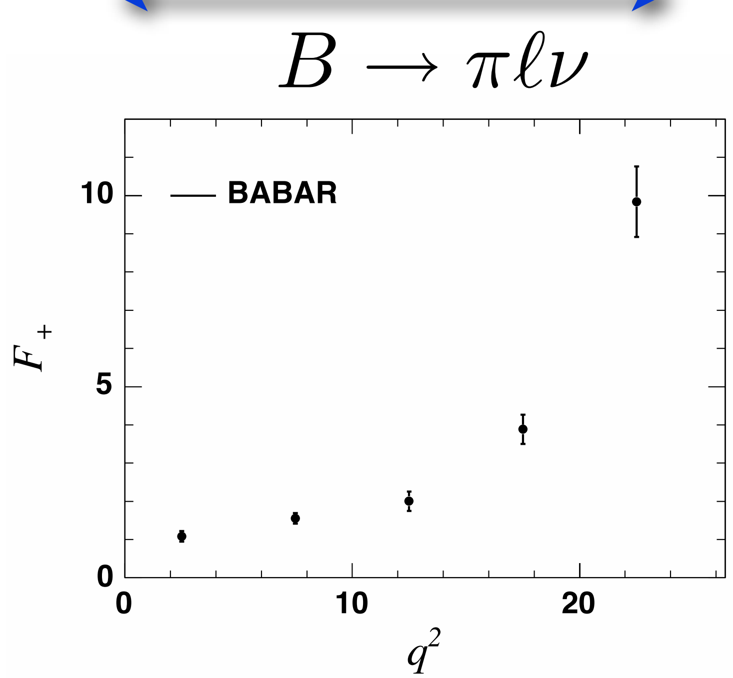
Standard Model effects are small.
(Proceed through penguins.)
Possibility of seeing Beyond-the-Standard-Model effects?

Very preliminary.



$B \rightarrow \pi l \nu$, finite range of q^2





Lattice data extend over only a fraction of the q^2 range on the **physical** $B \rightarrow \pi l \nu$ decay.

With standard methods, discretization errors go like $O(ap)^2$, signal goes like $\exp(-E_\pi t)$.

Uncertainties in lattice and experiment both highly q^2 dependent. Harder and more important to understand shape.

Proposals to address:

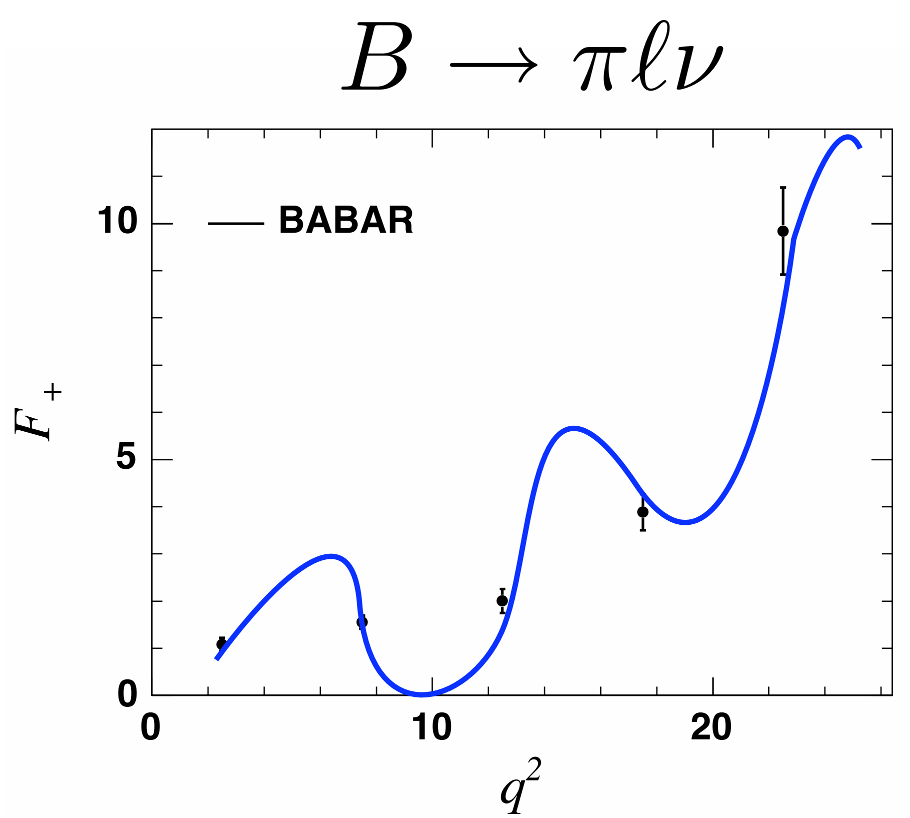
*) Moving NRQCD (Davies, Lepage, et al.)

*) Calculate in charm region, extrapolate to bottom (Abada et al.)

*) Gibbons: global simultaneous fit of all experimental and lattice data.

*) Unitarity and analyticity (Lellouch, Fukunaga-Onogi, Arnesen et al., ...)

$B \to \pi l\nu$



Always have to make some assumptions about shape to obtain any fit.

If any crazy shape were allowed, we could never fit anything.

Constrained curve fitting allows the possibility of an infinite number of free parameters in fits, while formalizing assumptions about their values.

Constrained curve fitting

Add an infinite number of parameters to the fit function, but constrain them to their plausible ranges.

$$\chi^2 \quad \to \quad \chi^2_{\mathrm{aug}} \equiv \chi^2 + \chi^2_{\mathrm{prior}}$$

Use “augmented” chi squared:

$$\chi^2(A_n, E_n) \equiv \sum_{t,t'} \Delta G(t)\ \sigma^{-2}_{t,t'}\ \Delta G(t')$$

$$\chi^2_{\mathrm{prior}} \equiv \sum_n \frac{(A_n - \tilde{A}_n)^2}{\tilde{\sigma}^2_{A_n}} + \sum_n \frac{(E_n - \tilde{E}_n)^2}{\tilde{\sigma}^2_{E_n}}$$

Bayes formula:

$$P(\rho|\overline{G}) = \frac{P(\overline{G}|\rho)\, P(\rho)}{P(\overline{G})} \propto P(\overline{G}|\rho)\, P(\rho)$$

Likelyhood of fit parameters before doing the calculation.

$$P(\rho) = \mathrm{e}^{-\chi^2_{\mathrm{prior}}(\rho)/2}$$

Probability of your data given the fit parameters.

$$P(\overline{G}|\rho) \propto \mathrm{e}^{-\chi^2(\rho)/2}$$

Likelyhood of fit parameters given your data.

What do we know in advance about the fit function for form factors?

A nice ansatz, the Becirevic-Kaidalov parameterization:

$$f_+(q^2) = \frac{F}{\boxed{(1-\tilde{q}^2)}\boxed{(1-\alpha\tilde{q}^2)}}, \quad f_0(q^2) = \frac{F}{1-\tilde{q}^2/\beta}$$

Correct effects of the B* pole that we know are there.

Additional pole parameterizes higher mass states.

$$F_+(q^2) = \frac{F_+(0)/(1-\alpha)}{1-\frac{q^2}{m_{B*}^2}} + \boxed{\frac{1}{\pi}\int_{t_+}^{\infty} dt \, \frac{\mathrm{Im} F_+(t)}{t-q^2-i\epsilon}}$$

Real life higher mass states for B→πlv: a cut.

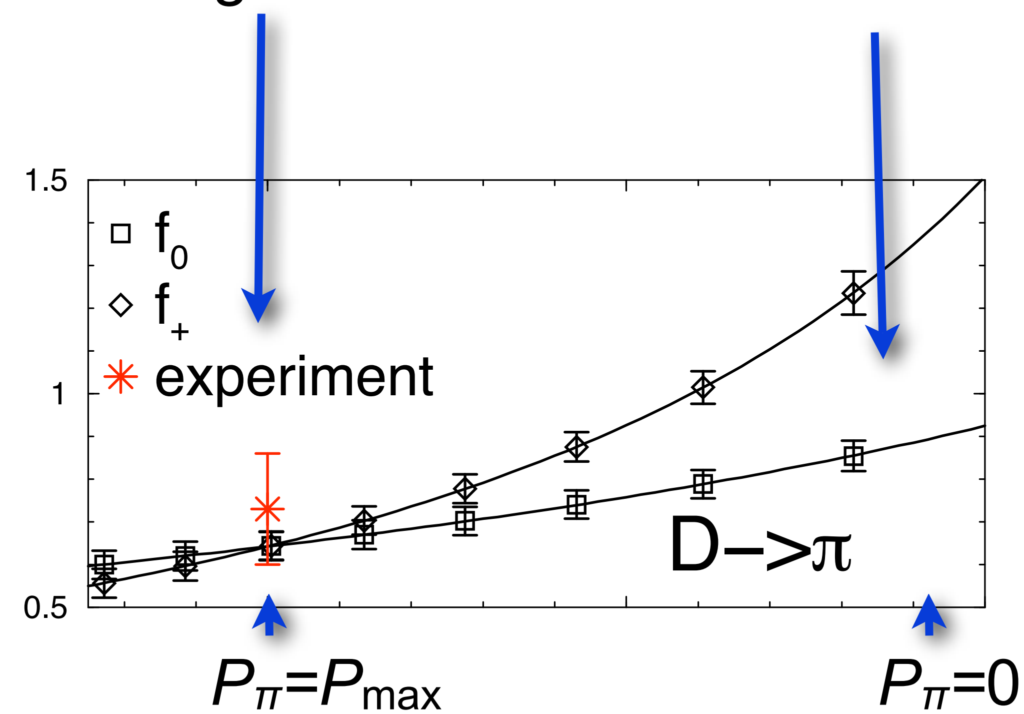
BK could be extended with increasing accuracy by adding more and more poles. (Hill.)

$$F_+(q^2) = \frac{F_+(0)/(1-\alpha)}{1-\frac{q^2}{m_{B*}^2}} + \sum_{k=1}^{N} \frac{\rho_k}{1-\frac{1}{\gamma_k}\frac{q^2}{m_{B*}^2}}$$

Current limitations of BK

1) In lattice data fit with BK, statistical errors are smaller at high momentum than at low.

An effect of the model not present in the raw data.



2) Richard Hill: experimental data better fit with an extended BK with an extra parameter:

$$F_+(q^2) = \frac{F_+(0)/(1-\alpha)}{1-q^2/m_{B*}^2} + \frac{c}{1-q^2/M'^2} + \cdots$$

$$= \frac{F_+(0)(1-\delta q^2/m_{B*}^2)}{(1-q^2/m_{B*}^2)(1-[\alpha+\delta(1-\alpha)]q^2/m_{B*}^2)}$$

What do we know in advance about the fit function for form factors?

Not easy to fix prior uncertainties to parameters in BK extensions.

Analyticity and unitarity have long been used to constrain shapes of form factors.

Lellouch, Fukunaga-Onogi, Arnesen et al., ...

A particularly simple form has recently been emphasized by Arnesen et al.

Consider a remapping of the semileptonic decay variable $t=q^2$ into a new variable z in the complex plane:

$$\boxed{z(t, t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}}$$

z maps $q^2=t>t_+$ onto $|z|=1.$, and
$t<t_+$ onto [-1,1] in the complex plane.

($t = (p_H-p_L)^2$, $t_+ = (m_H+m_L)^2$, $t_- = (m_H-m_L)^2$).

t_0, taken as 0.65 t_ here, is a fudge factor adjusted to center the physical region on z~0.

A power series expansion of the form factors in *z* can be written in the form:

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

Function that has unit norm at $z=1$., and that vanishes at the poles of f, e.g., at the B^* pole.

Function calculated in perturbation theory to produce a simple form for the a_k.

P and φ contain most of the complexity of the form factors.

By calculating the current-current correlation function in perturbation theory and using the $J^\mu B\pi$ amplitude,

$$\mathrm{Im}\,\Pi^{\mu\nu} = \int [\mathrm{p.s.}]\,\delta(q - p_{B\pi})\langle 0|J^{\dagger\nu}|\bar{B}\pi\rangle\langle\bar{B}\pi|J^\mu|0\rangle + \ldots$$

with crossing symmetry and analyticity, one obtains a simple constraint on the a_ks in the equation

$$f(t) = \frac{1}{P(t)\phi(t,t_0)}\sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

It is simply $\displaystyle\sum_{k=0}^{n_A} a_k^2 \le 1$!

The allowed range of z in physical semileptonic decay is small

B->π l ν: -0.34<z<0.22,

D->π l ν : -0.17<z<0.16,

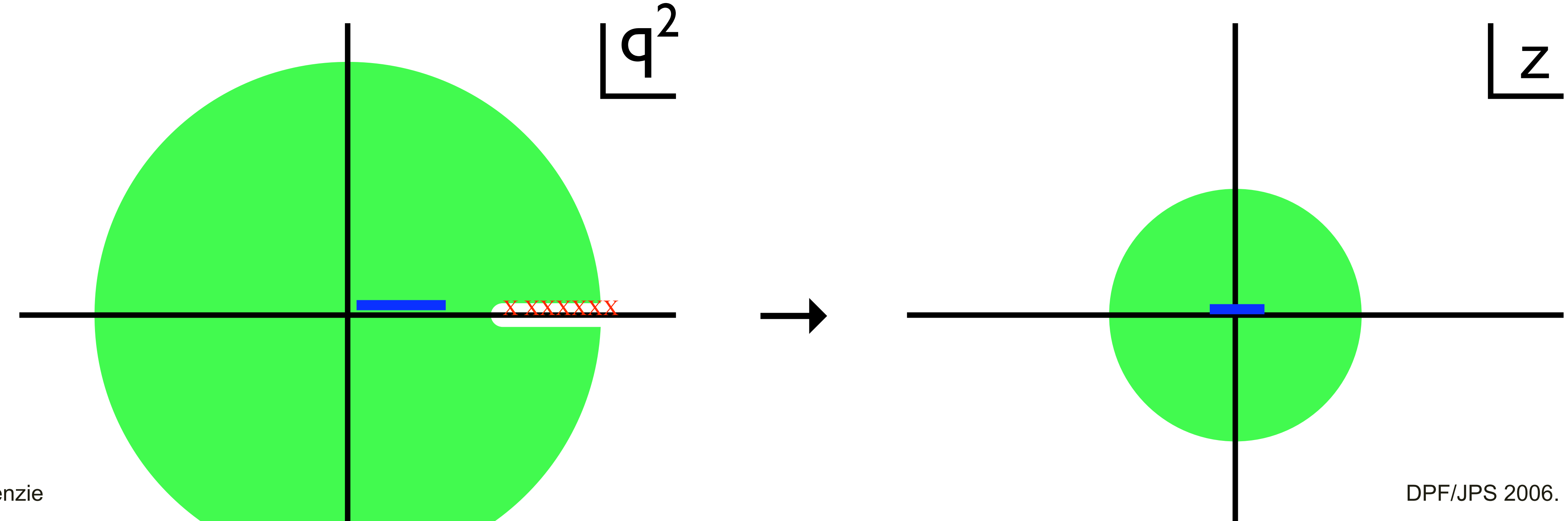
D->K l ν : -0.04<z<0.06,

B->D l ν : -0.02<z<0.04.

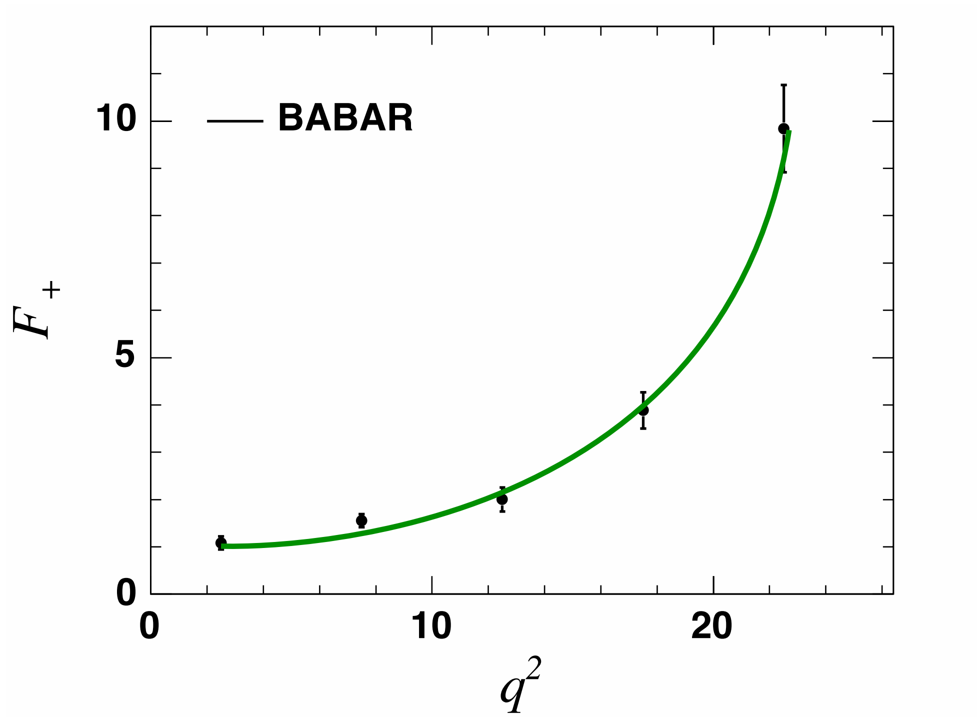
$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

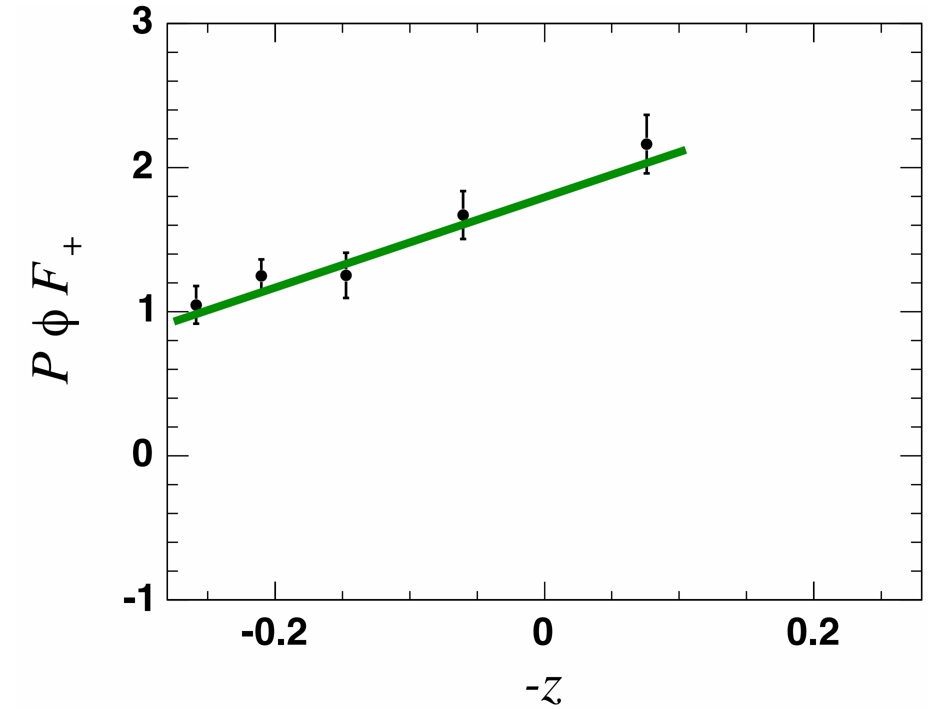
Since $\sum_{k=0}^{n_A} a_k^2 \leq 1$,

to obtain the form factors to high accuracy, say 1%, only a small number of parameters is needed, only 5 or 6 even in the case of B->π l ν.



$B \to \pi l \nu$, unitarity fits





$$P(t)\ \phi(t,t_0)\ f(t) = \sum_{k=0}^{\infty} a_k(t_0)\, z(t,t_0)^k$$

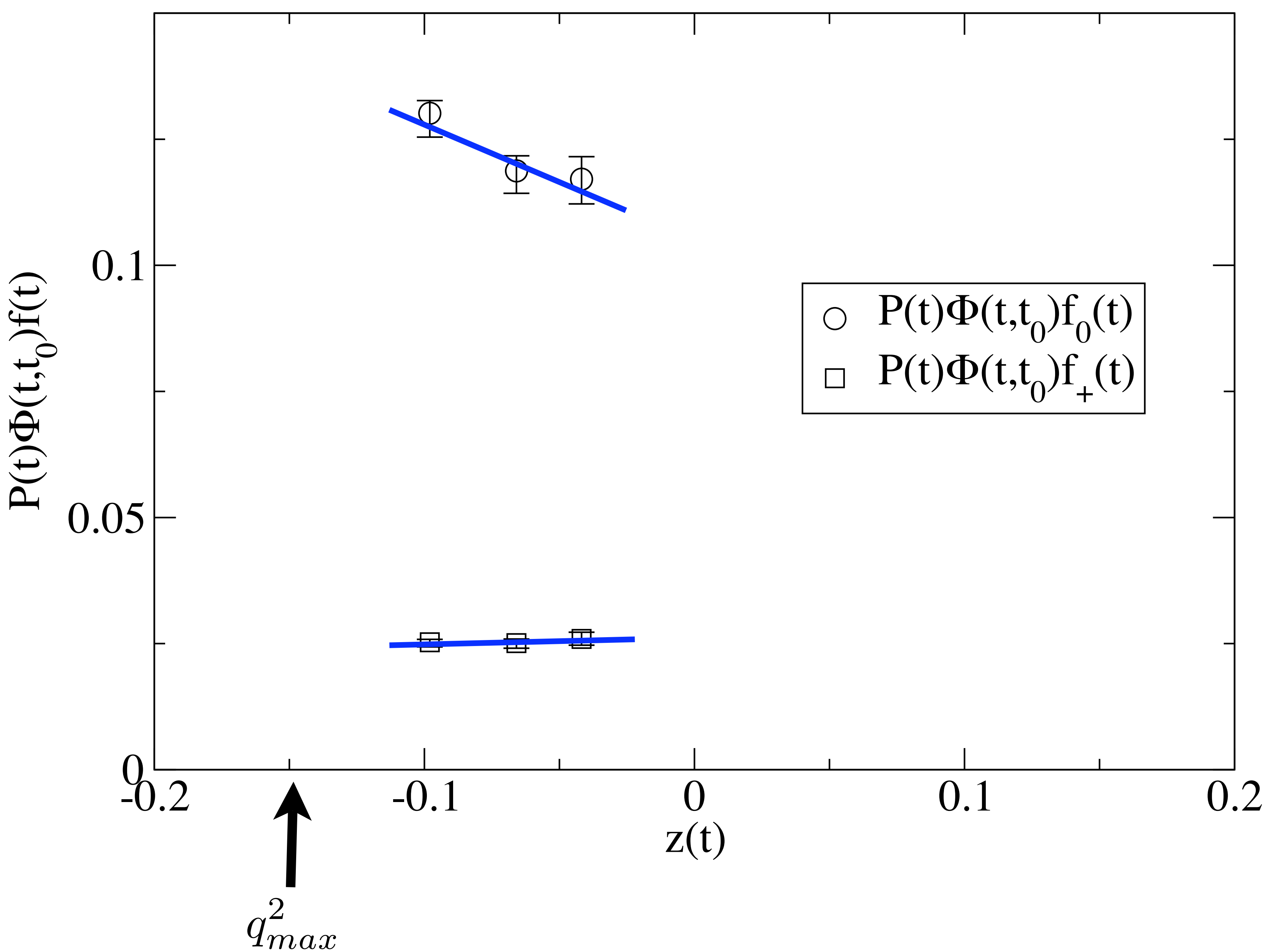
Vanishes at subthreshold (e.g. B*) poles

"Arbitrary" analytic function -- choice only affects particular values of coefficients (a's)

Strong q^2 dependence in form factor is due to calculable effects. When those are factored out, two parameters suffice to describe the current experimental data. (Just like *$B \to D l \nu$, $K \to \pi l \nu$?!!)*

$B \rightarrow \pi l \nu$, unitarity fits

B->π form factor data normalized by P(t) x Φ(t,t$_0$) vs. z(t)

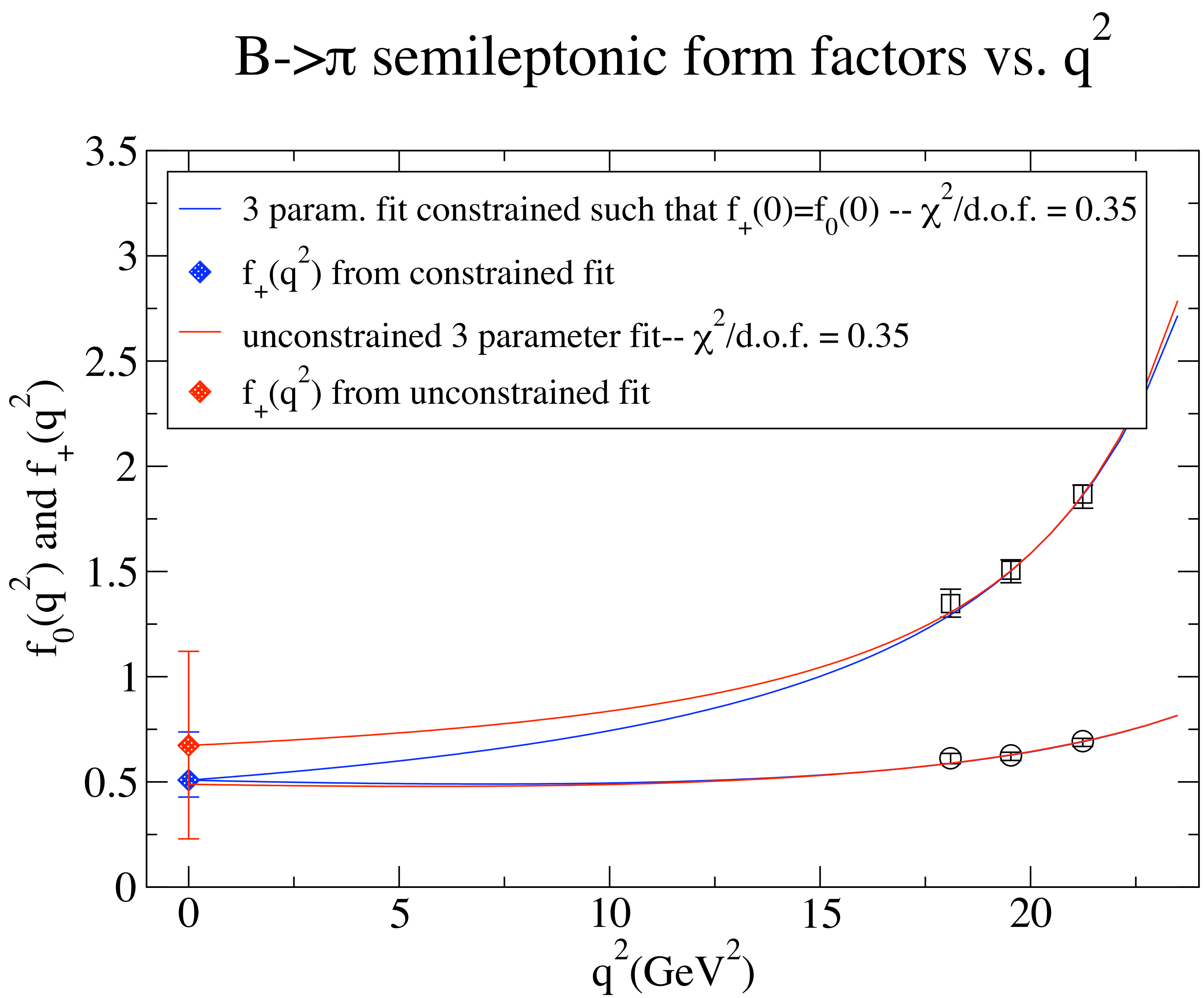


Coefficients in z expansion are compatible with experiment.

a0: 0.0257228 +- 0.003
a1: 0.020256 +- 0.068
a2: 0.152234 +- 0.41

<<1: see Becher, Hill.

$B \to \pi l \nu$, unitarity fits



Combined fits of f+ and f0 may give surprisingly good prediction for form factors well beyond the range of lattice data.

- Raw lattice data,
- Not extrapolated in *m* or *a*,
- Momentum dependent discretization errors not yet included.

How can the results of such fits best be compared with experiment?

Summary

- Fermilab and MILC are calculating an extensive set of semileptonic form factors at several lattice spacings.
  - $D \rightarrow \{\pi, K\} l \nu$
  - $B \rightarrow D\{^*\} l \nu$
  - $B \rightarrow \pi l \nu$
  - $B \rightarrow K l^+ l^-$
- The analyticity-based *z* expansion limits the number of parameters needed to describe form factor data, without introducing model dependence.
- In terms of the *z* expansion, all semileptonic form factor data, both lattice and experiment are consistent with straight lines: normalization and slope.
  - Even $B \rightarrow \pi l \nu$.