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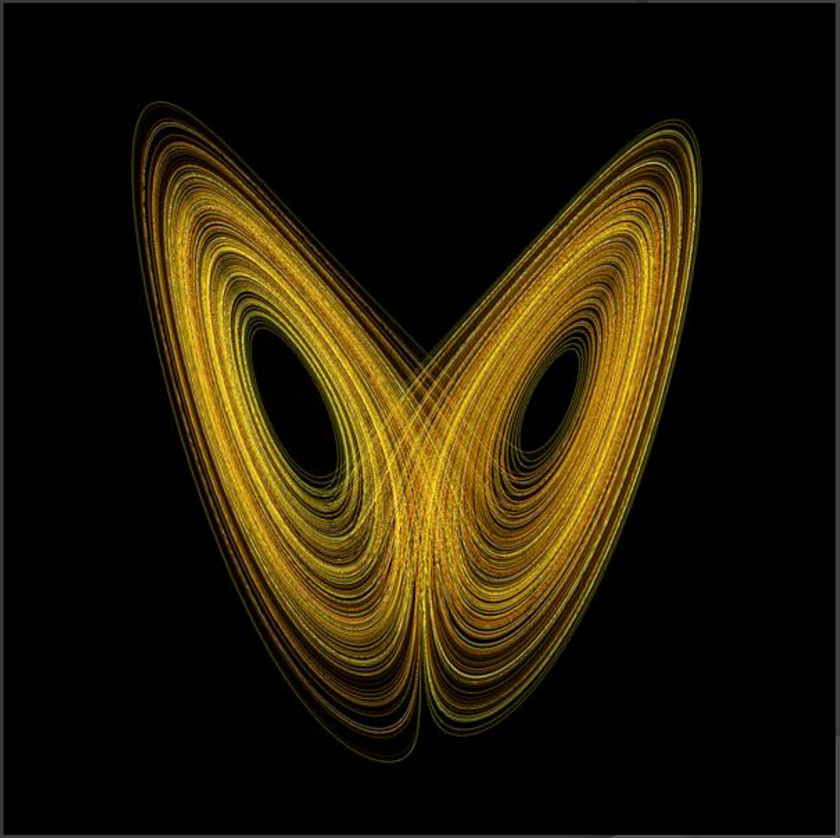
Strange Attractors

Lorenz attractor

$$dx/dt = \sigma(y - x)$$
$$dy/dt = \rho x - y - xz$$
$$dz/dt = xy - \beta z$$

- 3D differential equation created to model weathersystems.
- Ergodic (approaching every possible value) and aperiodic (never repeating).
- Highly sensitive to changes in initial conditions.
- ” *And to me this implied that if the real atmosphere behaved in this method then we simply couldn't make forecasts two months ahead. “* - Edward Lorenz

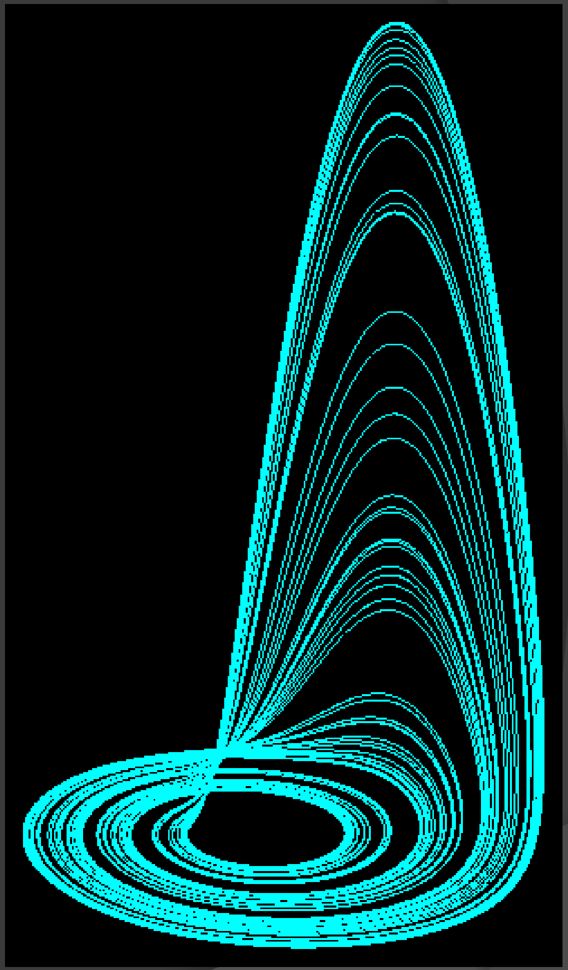


A 2D projection of a Lorenz ”butterfly”.

Rössler attractor

$$\frac{dx}{dt} = -y - z$$
$$\frac{dy}{dt} = x + ay$$
$$\frac{dz}{dt} = b + z(x-c)$$

- Wanted to simplify Lorenz attractor for quantitative analysis.
- Single manifold.
- Can be used to modeling equilibrium in chemical reactions.
- Also ergodic and aperiodic.
- Can be linearized by setting $z = 0$.



Rossler

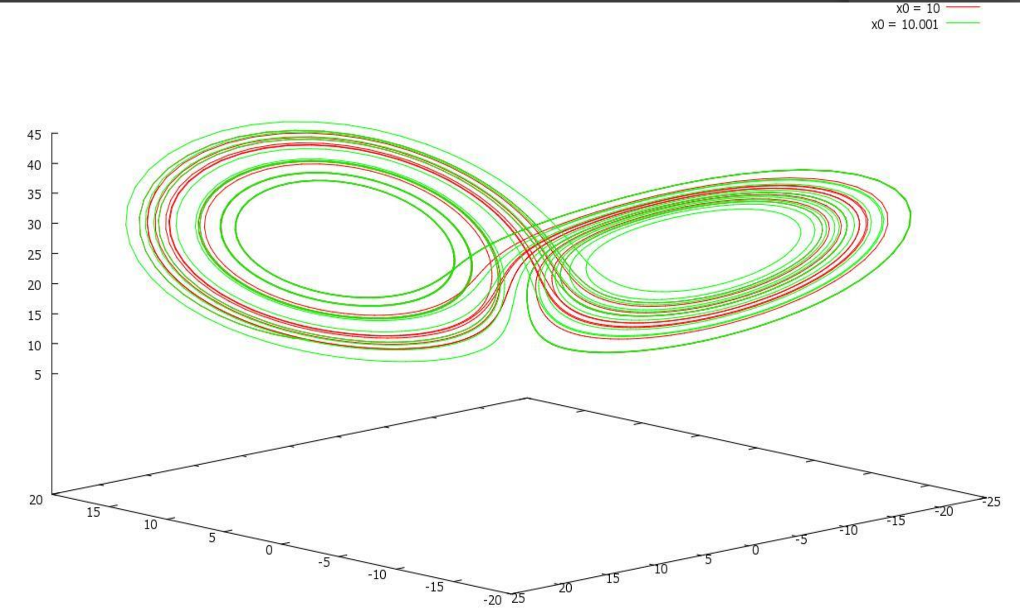
The program

- I created a program to model these strange attractors.
- 3rd order Runge Kutta.
- Investigated effects of changing initial conditions.
- As well as varying the parameters.
- All plots made with Gnuplot.

```
 void lorenz(double t, double X[], double dXdt[])
    {
      double sigma=10.0, b=1.0, r=20.0;
      dxdt = -sigma * x + sigma * y;
      dydt = -x * z + r * x - y;
      dzdt = x * y - b * z;
    }
```

Initial coordinates

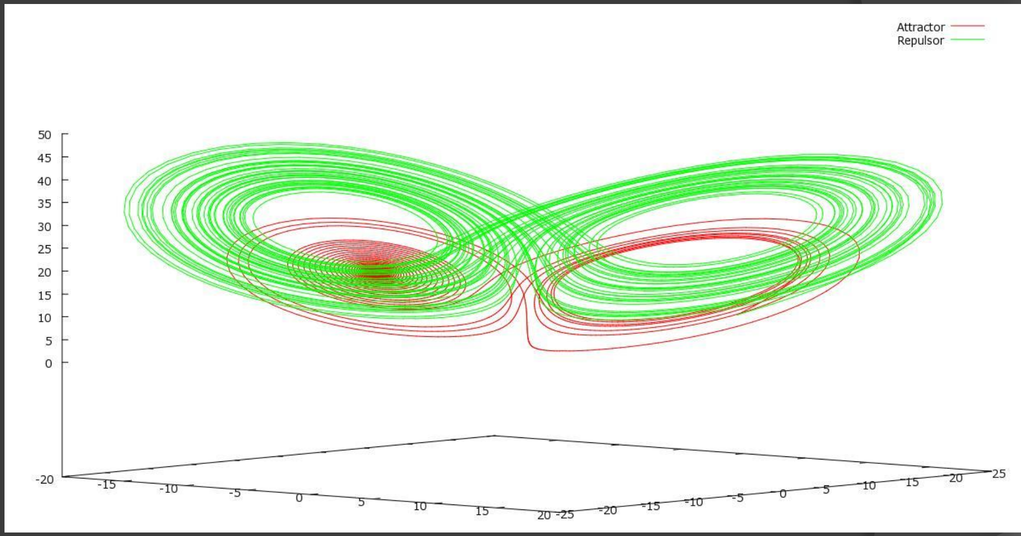
- For both Lorenz and Rössler attractor.
- Chaotic behaviour, undeterministic.
- Small changes gives large deviations.
- Found a 0.00001% sensitivity for Lorenz.
- The Rössler not as sensitive, about 0.1%.
- *”Butterfly effect”*



Varying Lorenz parameters

$$dx/dt = \sigma(y - x)$$
$$dy/dt = \rho x - y - xz$$
$$dz/dt = xy - \beta z$$

- Harder to examine.
- Chaotic when $\beta = 8/3$ and $\sigma = 10$, ρ is varied.
- Has two fixed point attractors.
- When $\rho > 24.28$ they become repulsors.
- Not very sensitive to changes in parameters, basic shape stays the same.

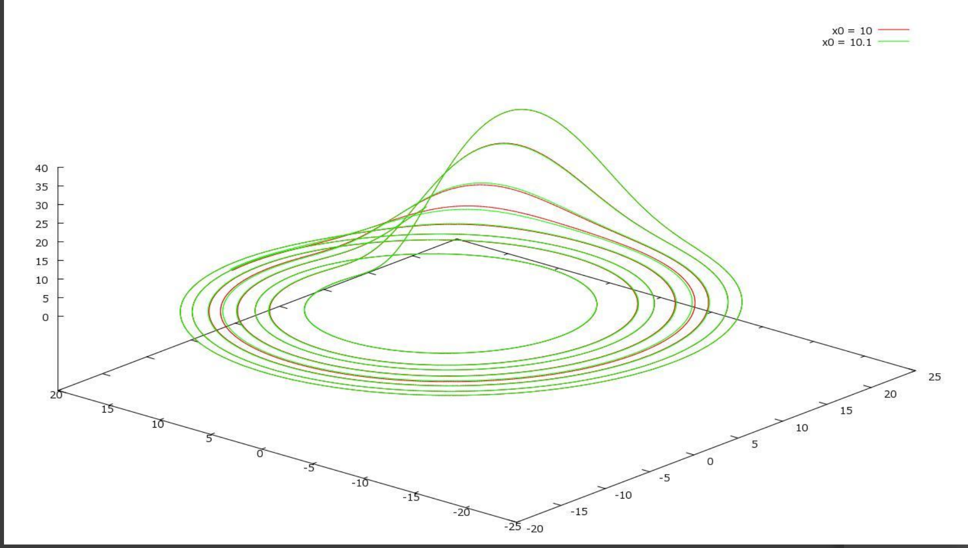


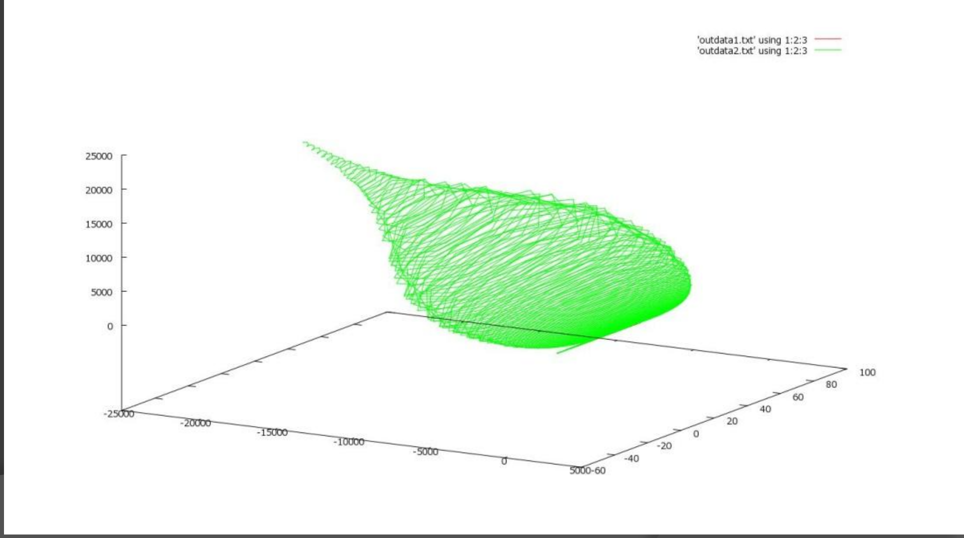
Red curve is an attractor, green is a repulsor

Varying Rössler parameters

$$dx/dt = -y - z$$
$$dy/dt = x + ay$$
$$dz/dt = b + z(x-c)$$

- Also has two attraction points.
- One close to the xy plane.
- As x gets bigger than c, z increases. Causes decrease in dx/dt.
- Varying a,b and c generates unpredictable result.
- Highly sensitive to changes in parameters. Not as sensitive to changes in initial position.





Final summary

- Lorenz attractor very sensitive to changes in initial position.
- Rössler attractor very sensitive to changes in parameters.
- Rössler attractor easier to explain quantitatively.
- Started the science of describing chaotic systems, chaos theory.