Search for decoherence of $B\bar{B}$ quantum entanglement at Belle II

Update: Unbinned decoherence fitter using individual decay times

Short Overview

- We want to develop a fitter able to search for the decoherence strength parameter λ, by doing a 2D unbinned fit of the individual decay times t_{sig} and t_{tag}
- Therefore, generate multiple toy MC data sets to test low and high λ values (low: $\lambda \leq 0.01\ \mathrm{ps}^{-1}$; high: $\lambda > 0.01\ \mathrm{ps}^{-1}$)
- For each λ value generate 100 toy experiments

  ➡ Each toy experiment consists of 72k events

  $$\left(72k/\ \mathrm{toy\ exp.} \sim \mathscr{L}_{\mathrm{int}} = 385\ \mathrm{fb}^{-1}/\ \mathrm{toy\ exp.}\right)$$

  ➡ Use random exponential to create t_1 and t_2 pairs
- To make the toy MC study semi-realistic introduce two major effects

  (1) Wrong tagging ✓

  (2) Resolution effects ← Working on!

Next plans: Resolution and convolution

$$f = a\left[\exp\left(-\frac{x}{\tau}\right)\exp\left(-\frac{y}{\tau}\right) + q\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)x\right)\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)y\right)\exp\left(\frac{\lambda}{2}|x-y|\right)\cos\left(\Delta m\,(x-y)\right)\right]$$

1. 2.

Our two resolution functions for x and y look the following:

$$R_1 = \frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{\left(u-x-\mu_1\right)^2}{2\sigma_1^2}\right) \qquad R_2 = \frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{\left(v-y-\mu_2\right)^2}{2\sigma_2^2}\right)$$

with $u = x + \delta x$ and $v = y + \delta y$. → Convolution integral to solve:

$$\mathscr{C} = f \otimes R_1 \otimes R_2 = a\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y \; 1.\cdot R_1R_2 + 2.\cdot R_1R_2$$

I. II.

Problem with analytical convolution

Integral I. is comparably easy to determine analytically with regards to II.

II. is more complicated as shown below:

$$\mathrm{II.} = \frac{q}{2\pi\sigma_1\sigma_2}\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y \exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)x\right)\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)y\right)\exp\left(\frac{\lambda}{2}|x-y|\right)\cos\left(\Delta m\left(x-y\right)\right)\exp\left(-\frac{\left(u-x-\mu_1\right)^2}{2\sigma_1^2}\right)\exp\left(-\frac{\left(v-y-\mu_2\right)^2}{2\sigma_2^2}\right)$$

Due to the absolute value we have to consider two cases:

$$\text{Case 1} \rightarrow x \geq y \Rightarrow |x-y| = x-y \Rightarrow \exp\left(\frac{\lambda}{2}|x-y|\right) = \exp\left(\frac{\lambda}{2}x\right)\exp\left(-\frac{\lambda}{2}y\right)$$

$$x\epsilon\left[0,\infty\right] \ \& \ y\epsilon\left[0,x\right]$$

$$\text{Case 2} \rightarrow x < y \Rightarrow |x-y| = y-x \Rightarrow \exp\left(\frac{\lambda}{2}|x-y|\right) = \exp\left(-\frac{\lambda}{2}x\right)\exp\left(\frac{\lambda}{2}y\right)$$

$$x\epsilon\left[0,\infty\right] \ \& \ y\epsilon\left[x,\infty\right]$$

New Idea: Use histograms

- Solving these integrals analytically is not possible, but how about numerical?
  ➡ Problem with “continuous” numerical approach: We still need to now the parametrized pdf for our resolution function to solve the integral numerically

  ➡ Solution: Use discrete resolution histogram and do the convolution

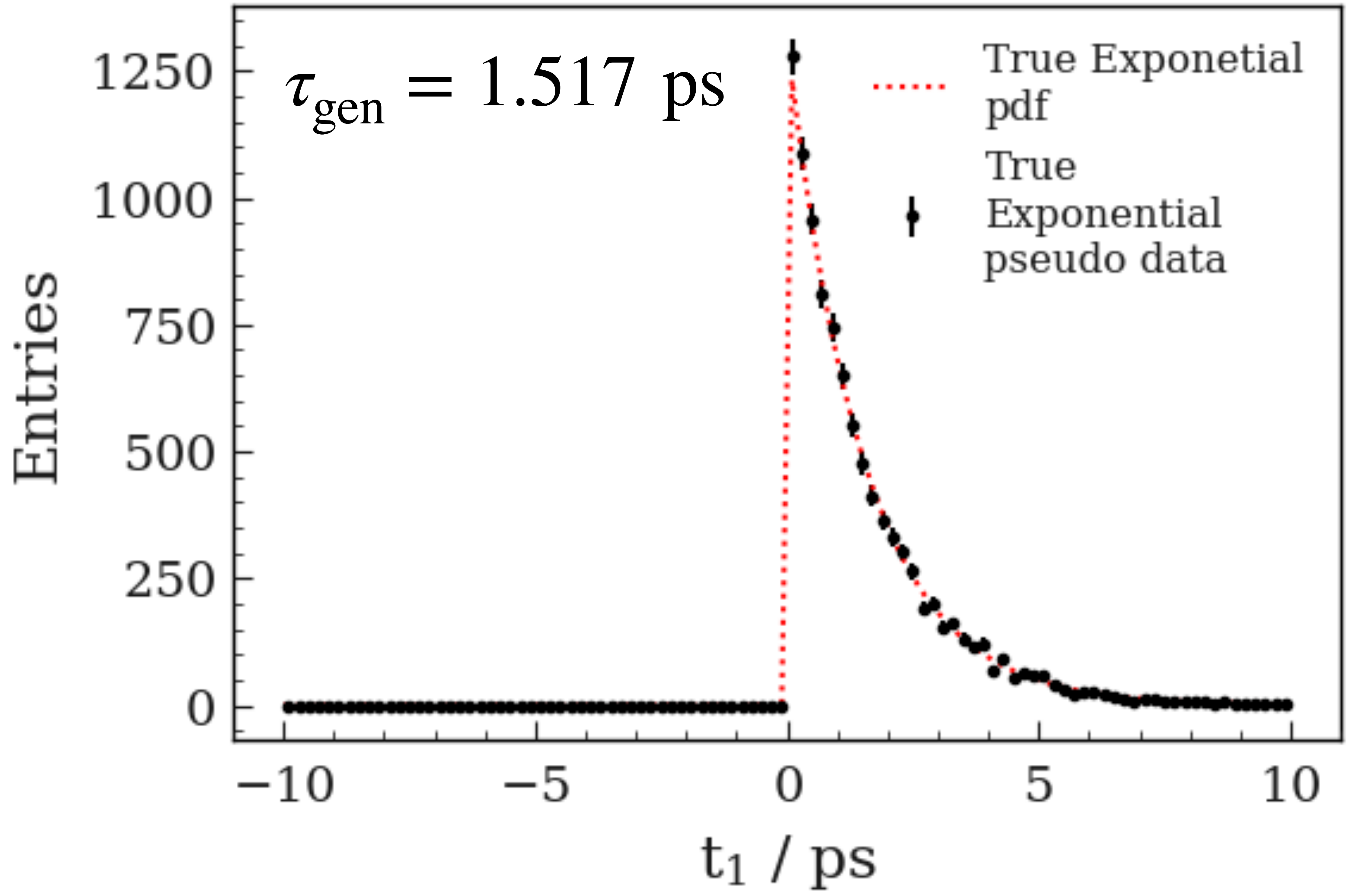
$$f \otimes g = \sum_{x'} f(x - x')\, g(x')$$

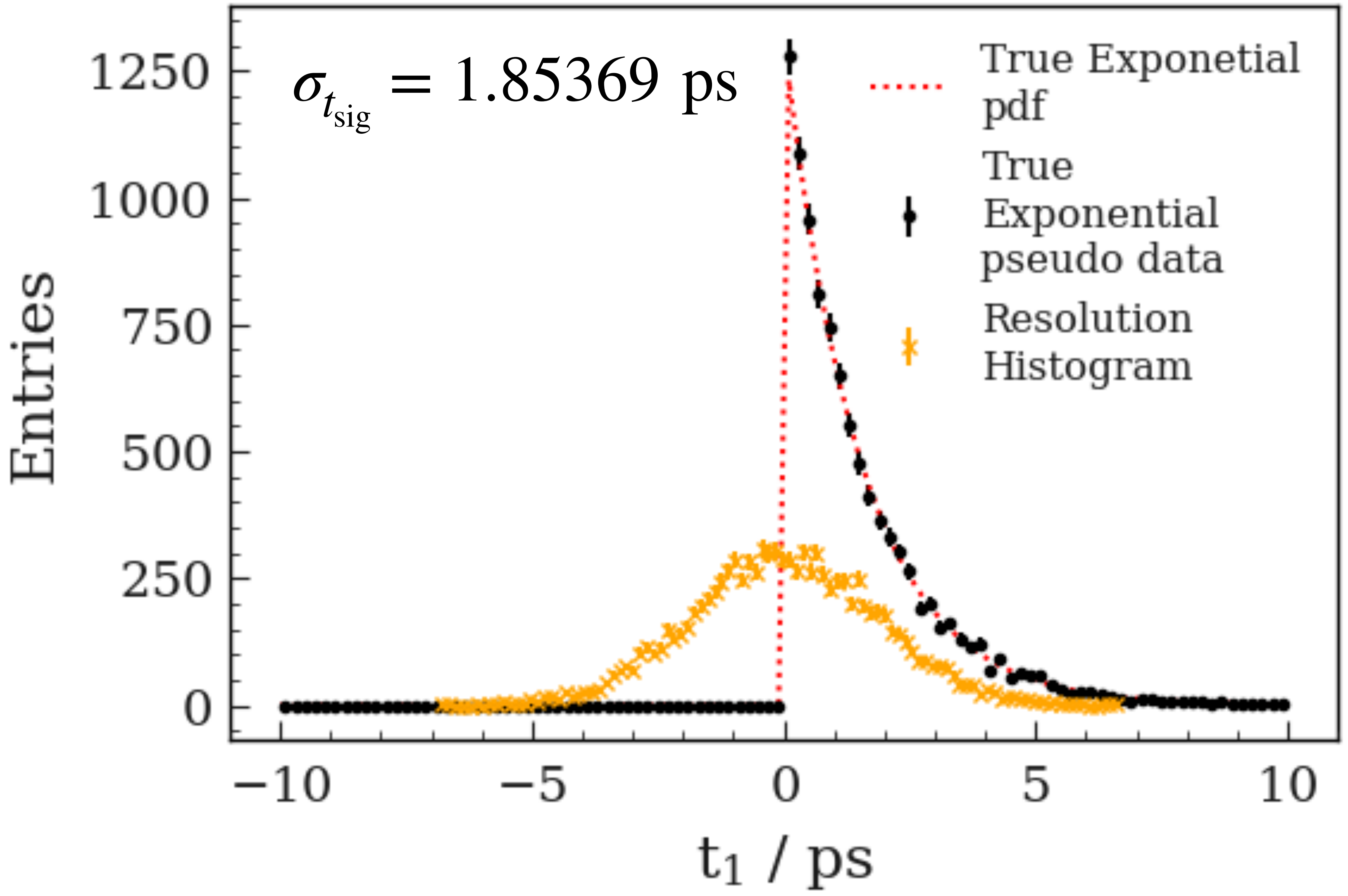
➡ $g(x') := \dfrac{\text{\# Entries in Resolution bin i}}{\text{\# Entries in Resolution Hist}},$

$x' :=$ Hist centers of Resolution hist, $f :=$ pdf

Simple example of convolution (10k events gen)

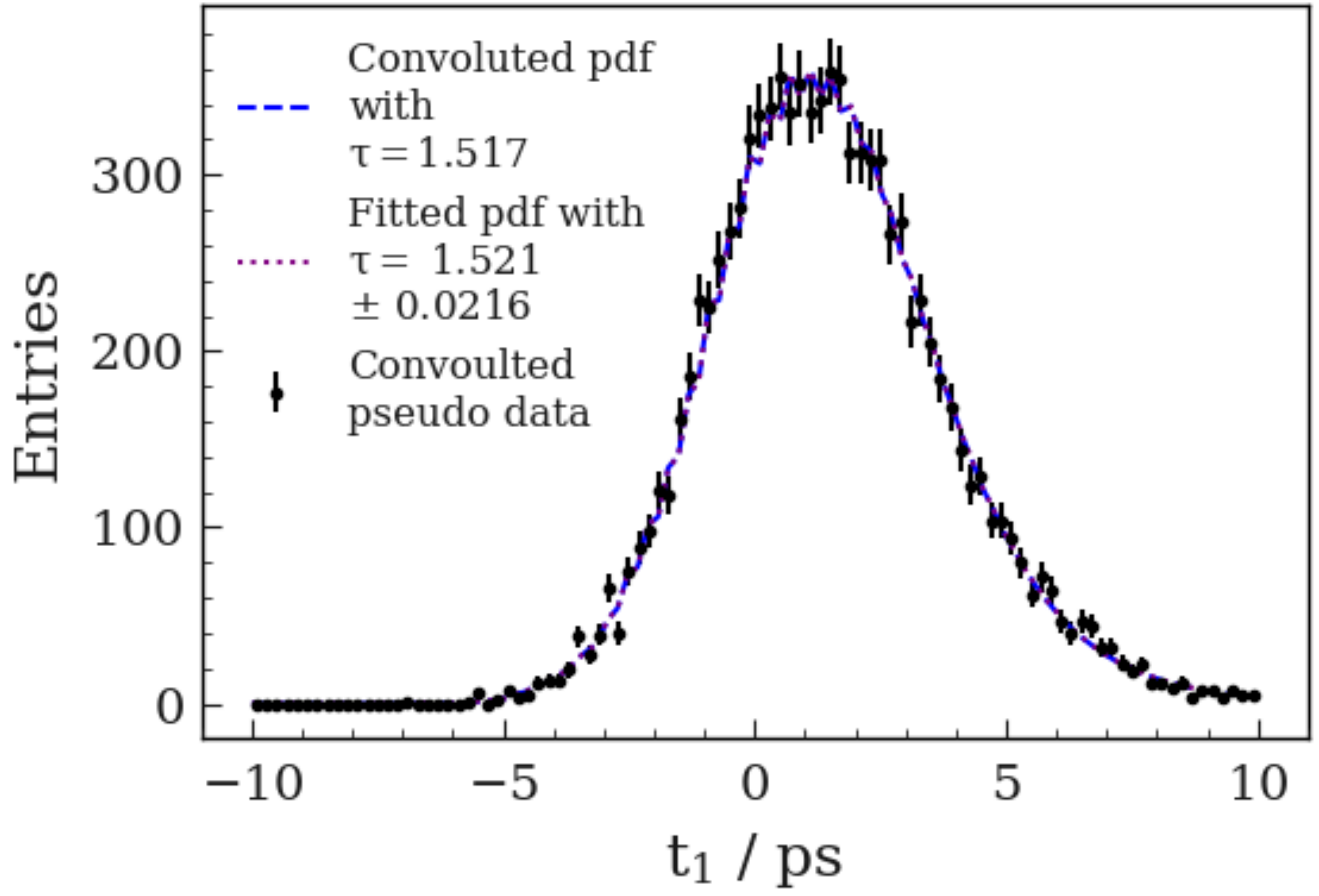
- Use $f \otimes g = \sum_{x'} f(x - x')\, g(x')$ for $f := \dfrac{\exp\left(-\frac{x}{\tau}\right)}{\tau}$ and $g :=$ Gaussian



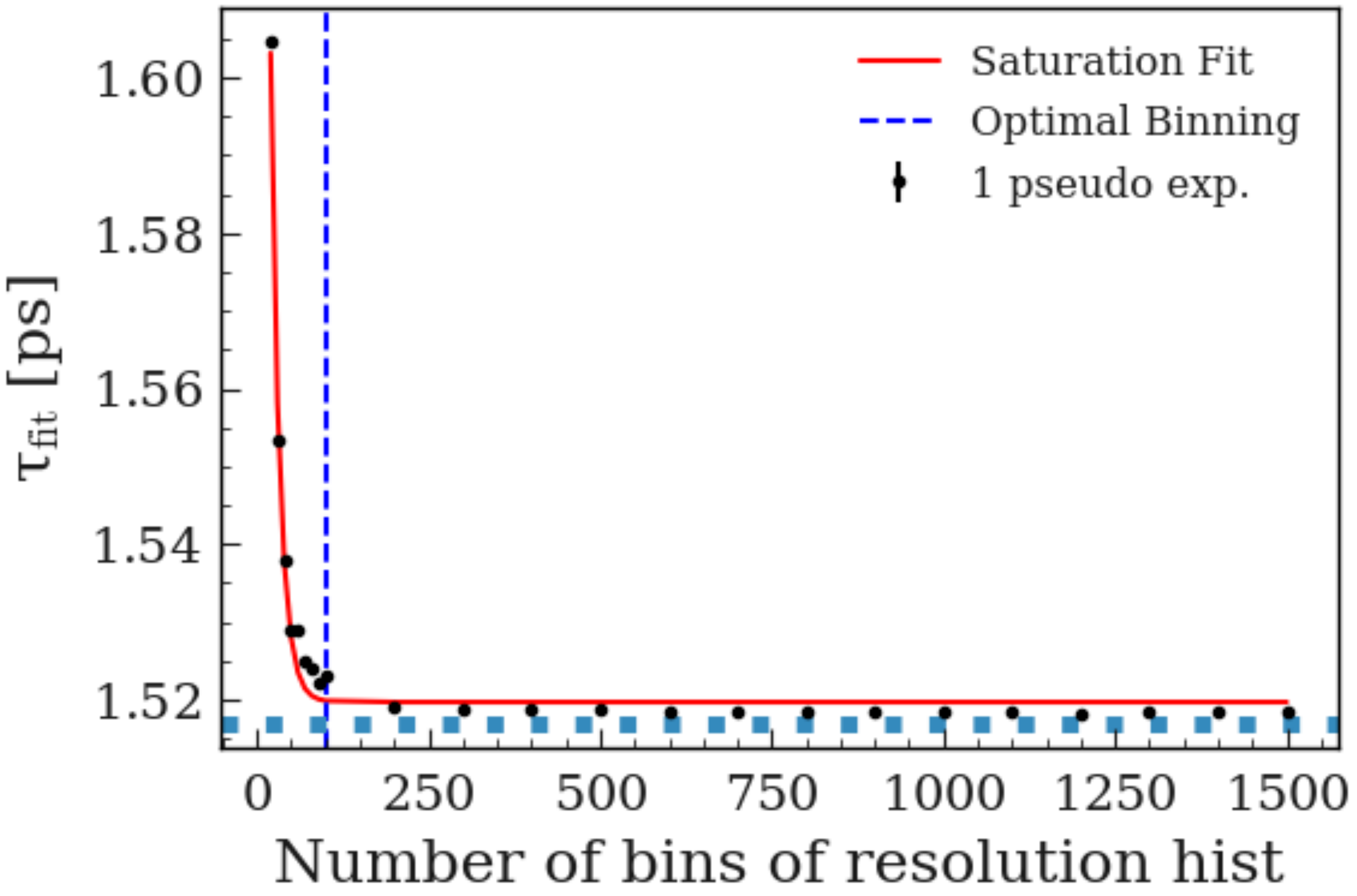


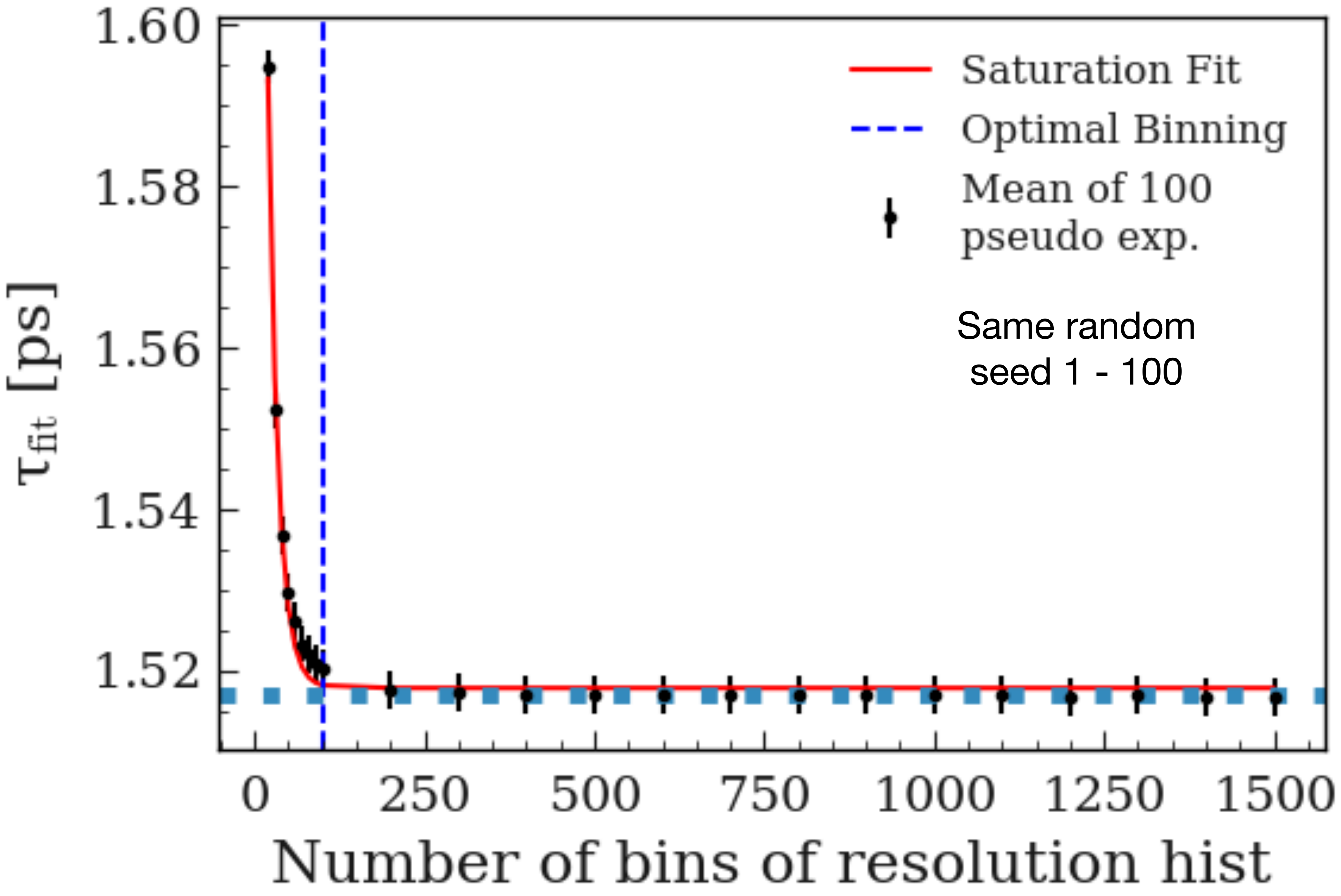
Fit for parameter τ while doing the convolution

- Do an Unbinned fit where
  $$f \otimes g = \sum_{x'} f(x - x')\, g(x')$$ is
  the fit function (x is you t_1 data)
- Fit works very well
- But how does the binning of the resolution histogram effect the fit parameter?

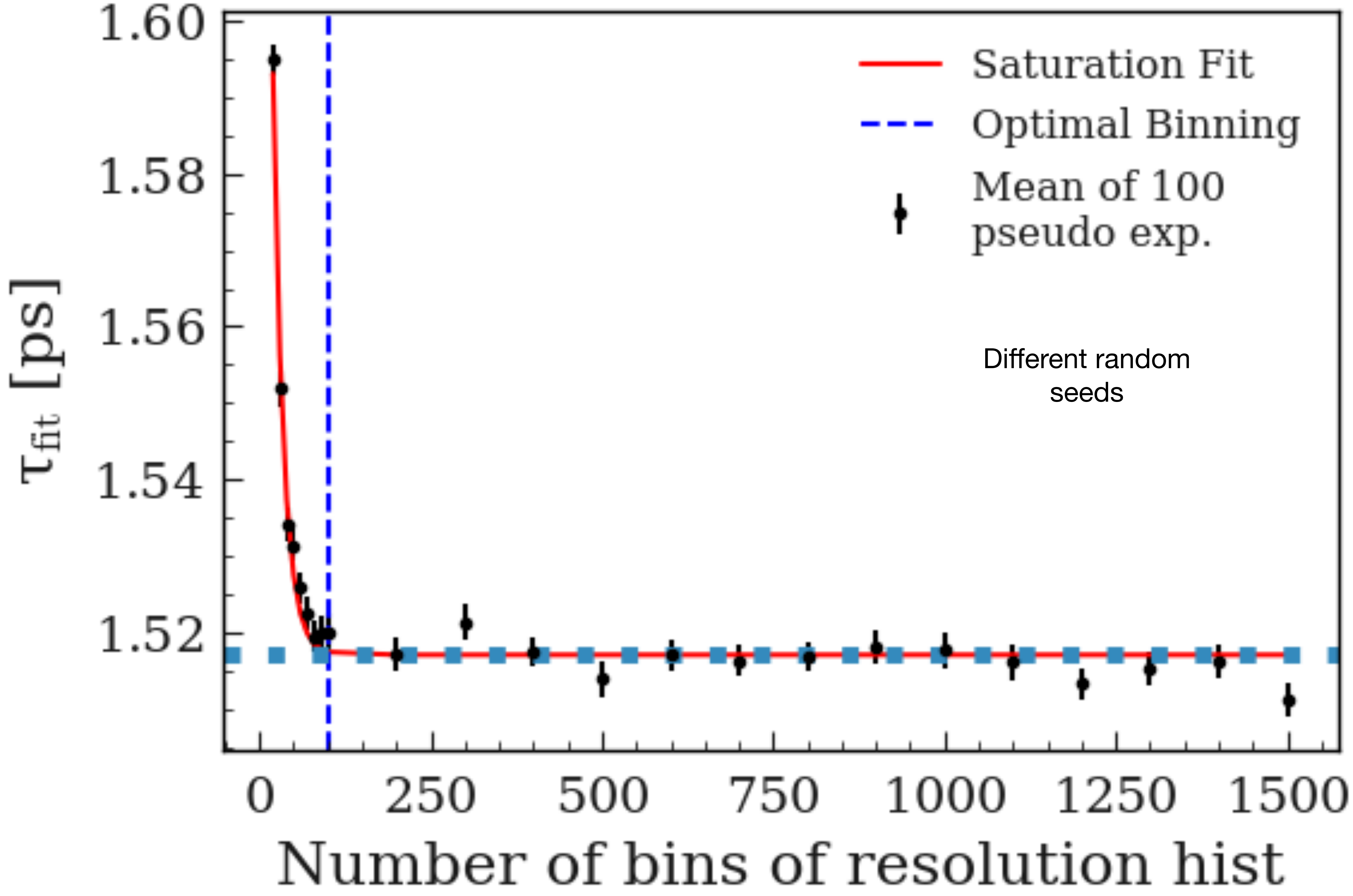


Resolution pdf binning influence

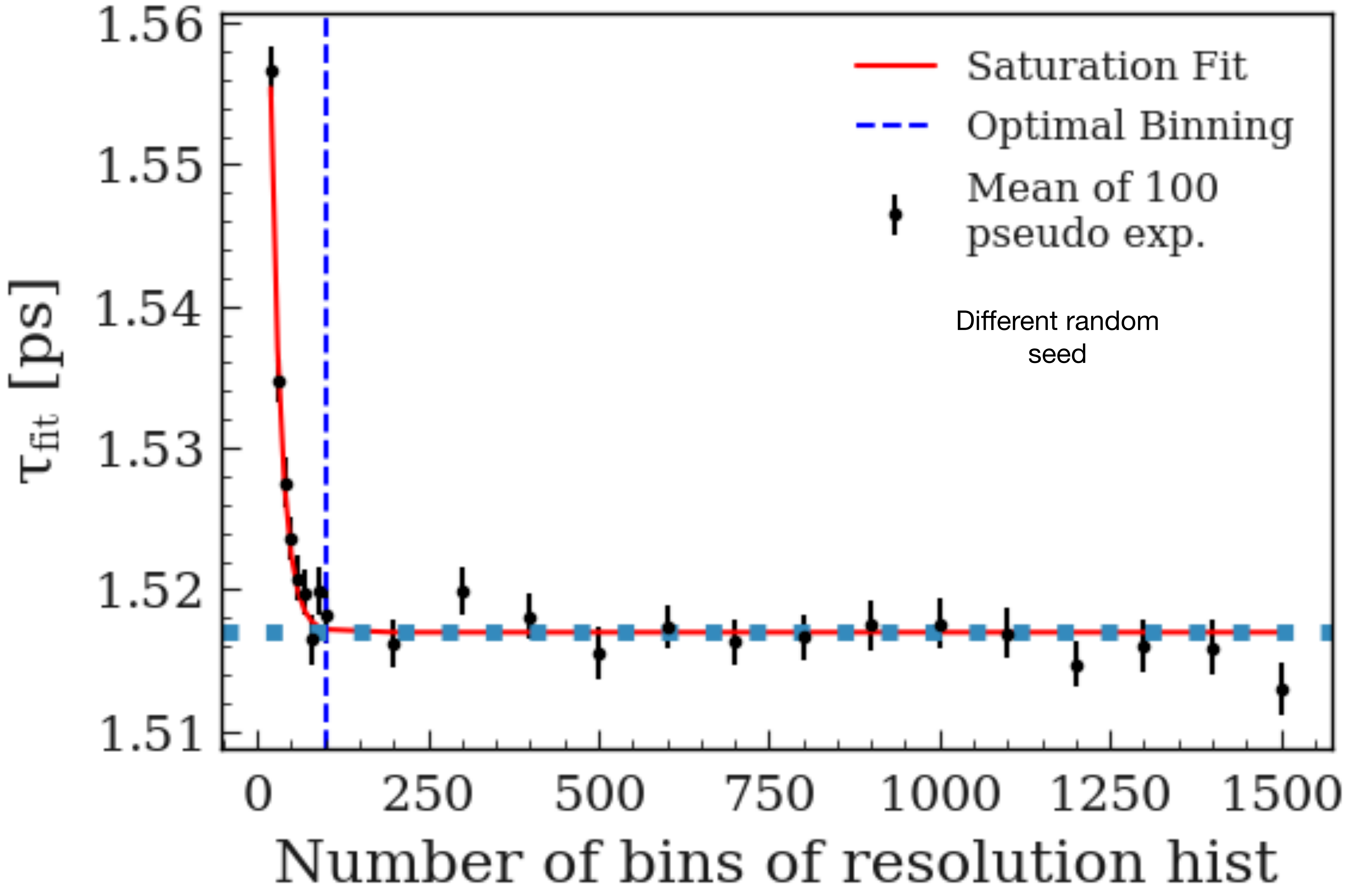




Resolution pdf binning influence



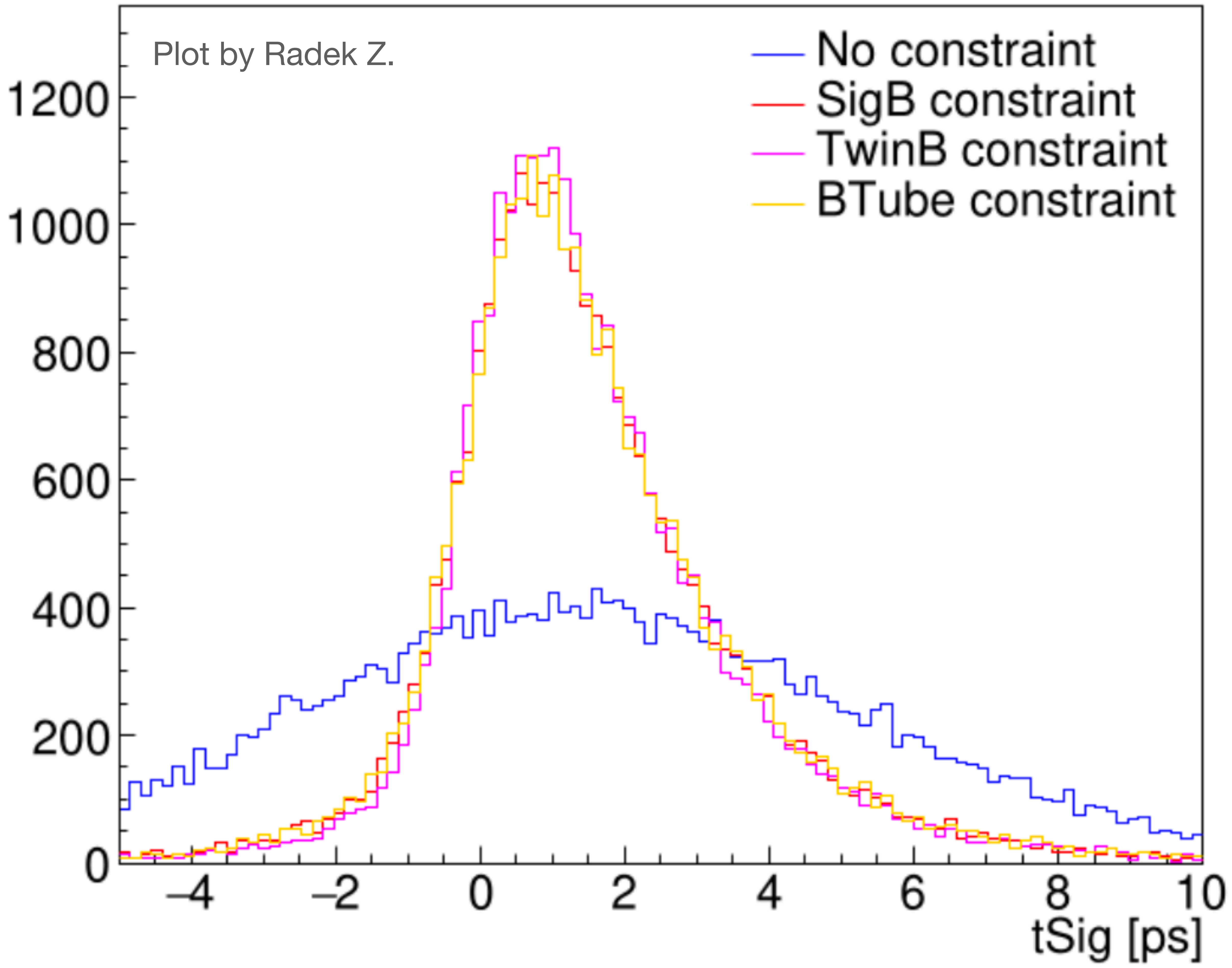
Resolution histogram binning influence



Different random seed

How does it work in experimental data?

- Everything below 0 ps is due to resolution
  ➡ Model resolution histogram based on negative tail in experimental data
  ➡ Similar for t_{tag}



Simple two dimensional convolution

- $f := \frac{\exp\left(-\frac{(x+y)}{\tau}\right)}{\tau^2}$, $g := \text{Gaussian}\,(\delta x)$ and $h := \text{Gaussian}\left(\delta y\right)$
- In two dimension with variables x and y the convolution look like:
$$f \otimes g \otimes h = \sum_{x'} \sum_{y'} f\left(x - x', y - y'\right) \cdot g\,(x') \cdot h\left(y'\right)$$
- $x := t_{\text{sig}}$ and $y := t_{\text{tag}}$, $\delta x := \delta t_{\text{sig}} = 1.85369\ \text{ps}$ and $\delta y := \delta t_{\text{tag}} = 2.19106\ \text{ps}$
- Result (direct pdf binning): $\tau_{\text{fit}} = 1.508 \pm 0.016\ \text{ps}$
- Result (resolution histogram): $\tau_{\text{fit}} = 1.512 \pm 0.016\ \text{ps}$
- Next step: Fit decoherence function for λ