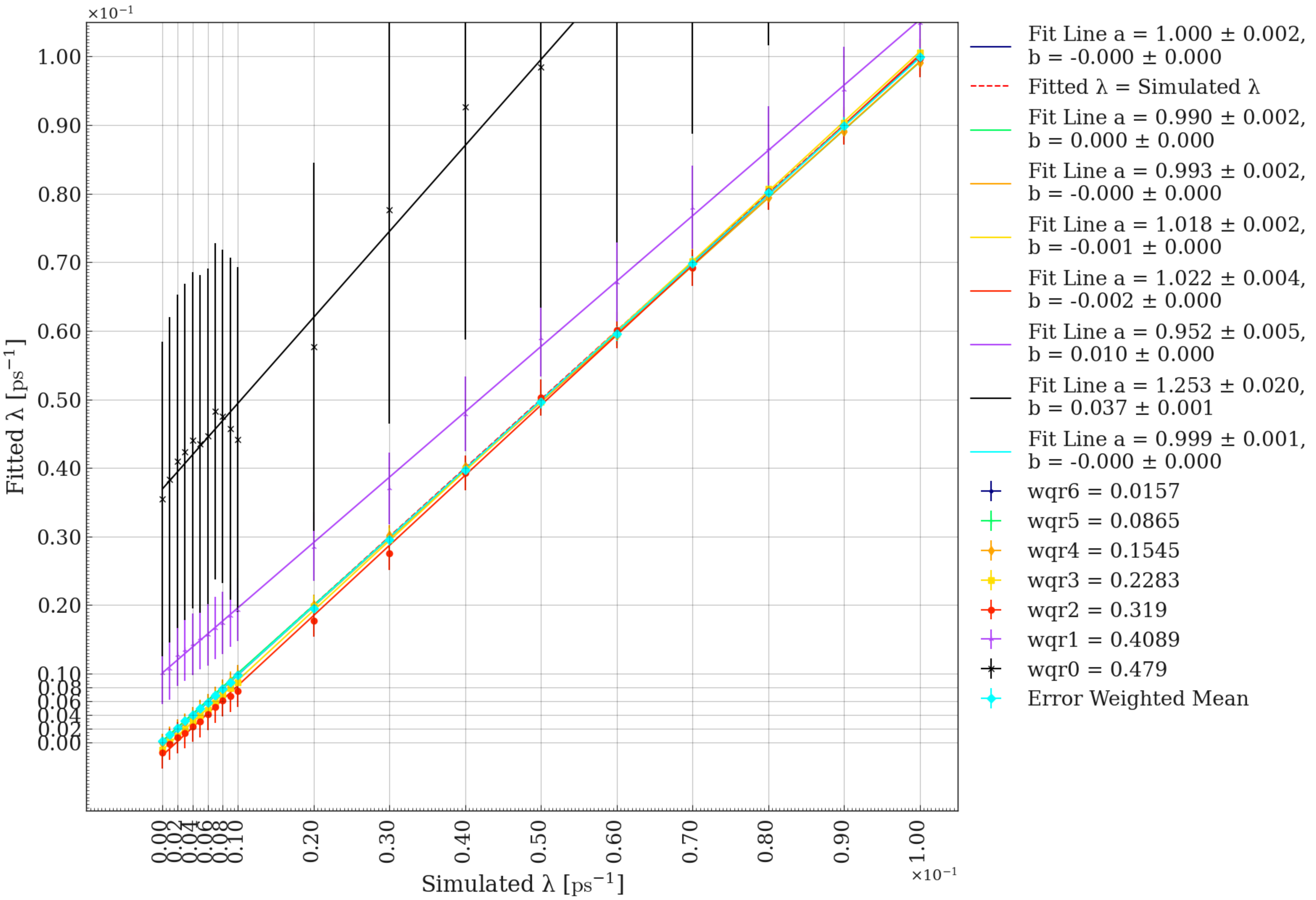


$\times10^{-1}$
Fitted λ [ps^{-1}]
Simulated λ [ps^{-1}]
$\times10^{-1}$
Fit Line a = 1.000 ± 0.002, b = -0.000 ± 0.000
Fitted λ = Simulated λ
Fit Line a = 0.990 ± 0.002, b = 0.000 ± 0.000
Fit Line a = 0.993 ± 0.002, b = -0.000 ± 0.000
Fit Line a = 1.018 ± 0.002, b = -0.001 ± 0.000
Fit Line a = 1.022 ± 0.004, b = -0.002 ± 0.000
Fit Line a = 0.952 ± 0.005, b = 0.010 ± 0.000
Fit Line a = 1.253 ± 0.020, b = 0.037 ± 0.001
Fit Line a = 0.999 ± 0.001, b = -0.000 ± 0.000
wqr6 = 0.0157
wqr5 = 0.0865
wqr4 = 0.1545
wqr3 = 0.2283
wqr2 = 0.319
wqr1 = 0.4089
wqr0 = 0.479
Error Weighted Mean

The Problem

- Generate toy MC for 19 different λ values
- For each λ generate 100 pseudo experiments of random t_1 and t_2 pairs

➡ Problem: I used the same 100 random seeds over and over again

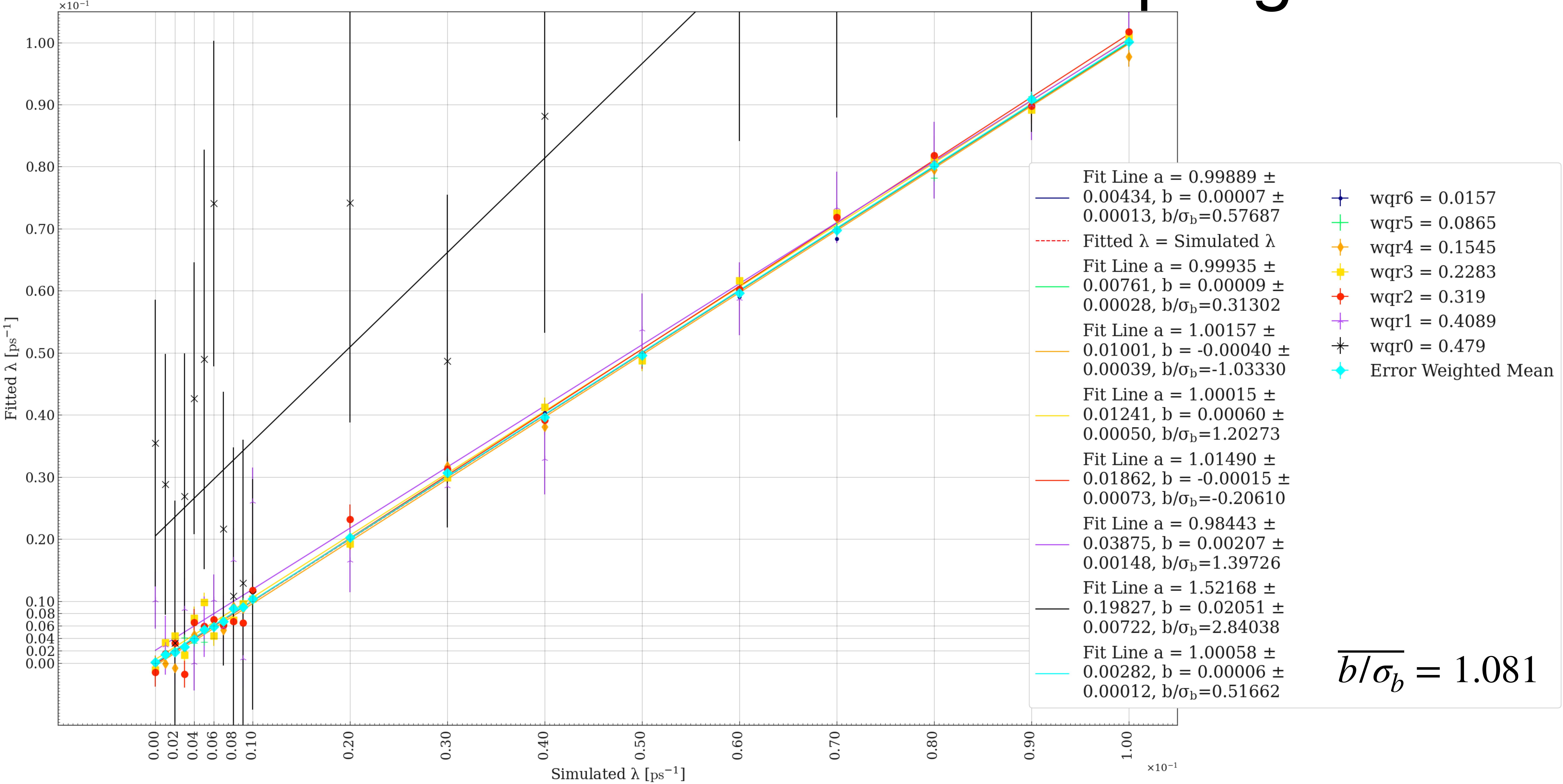
➡ Each experiment number is correlated

	Exp1	**Exp2**	**Exp3**	**…**	**Exp100**
Lam=0.0	1	2	3	…	100
Lam=0.001	1	2	3	…	100
…	…	…	…	…	…
Lam=0.1	1	2	3	…	100

➡ Solution: Use unique random seeds for each experiment for each λ value

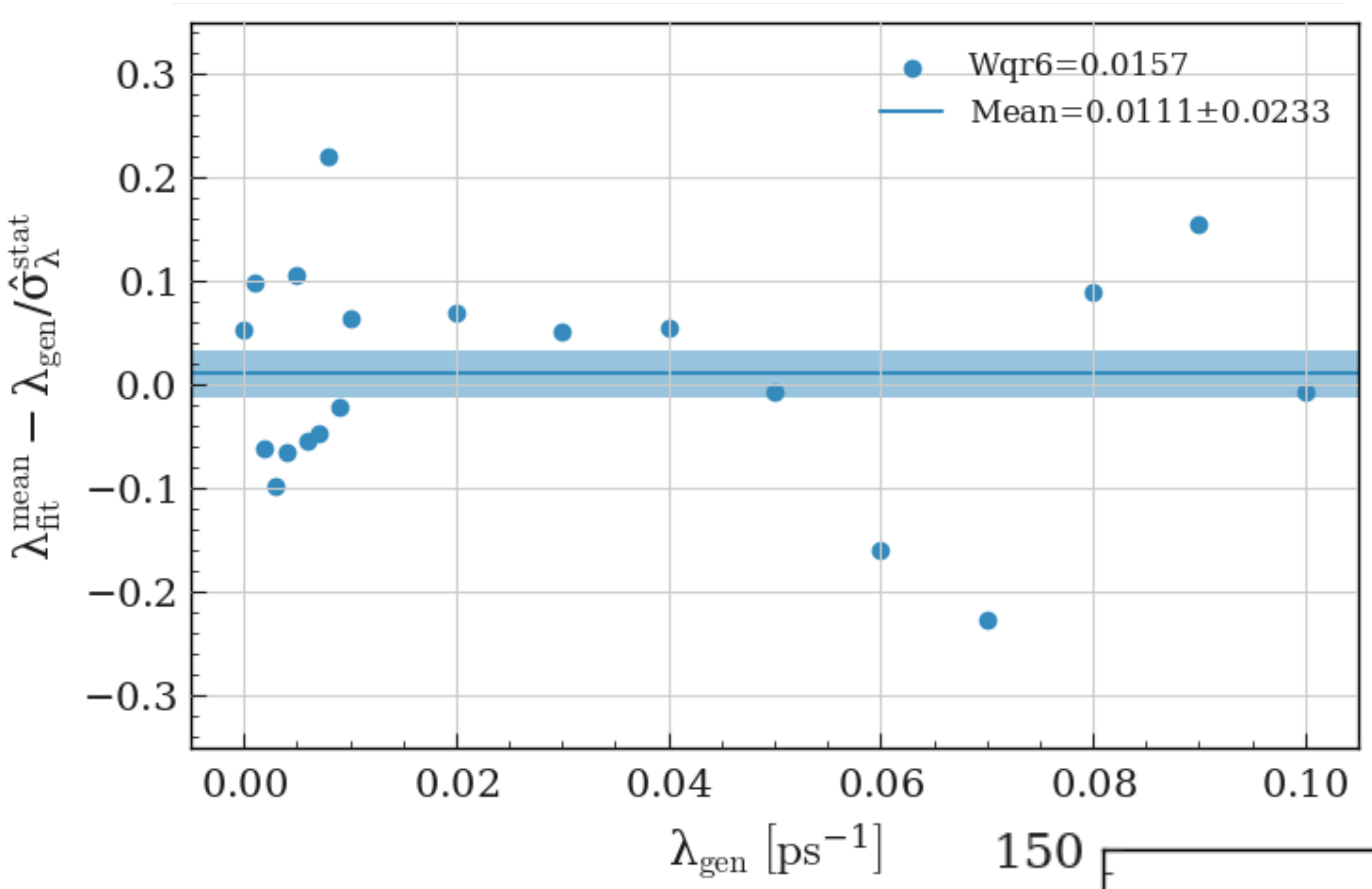
	Exp1	**Exp2**	**Exp3**	**…**	**Exp100**
Lam=0.0	1	2	3	…	100
Lam=0.001	101	102	103	…	200
…	…	…	…	…	…
Lam=0.1	1901	1902	1903	…	2000

Fix amount of events that flip sign

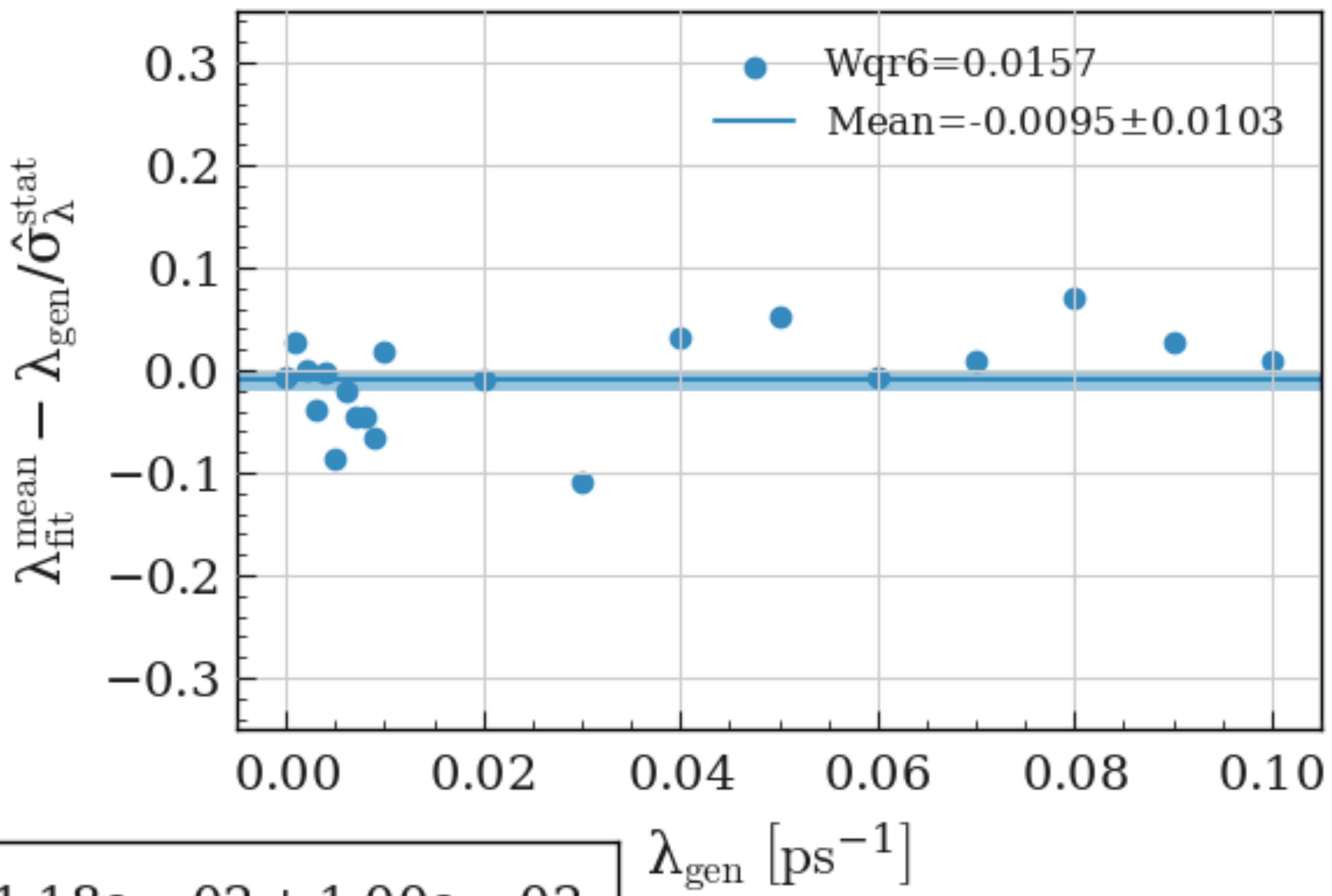


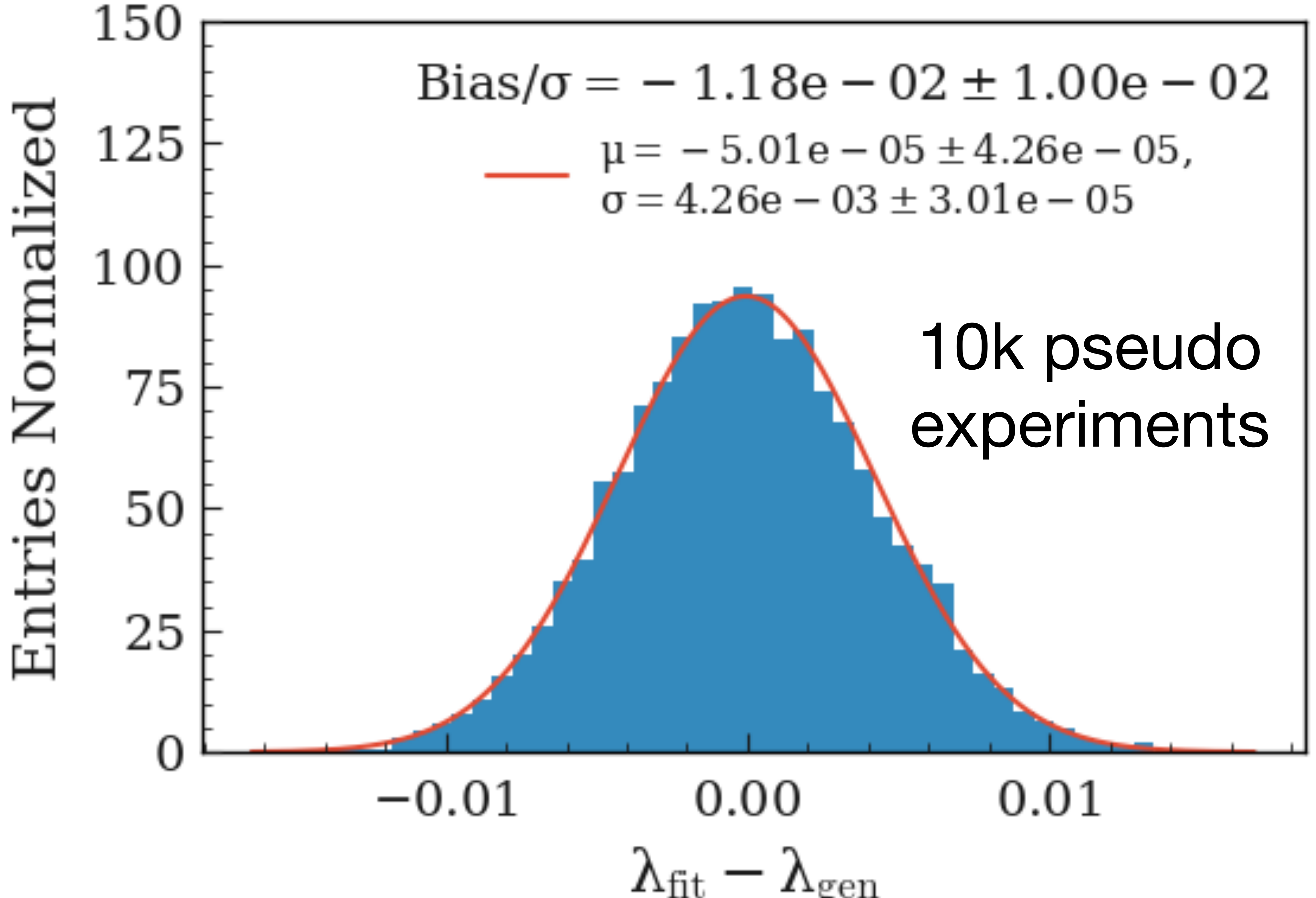
$$\overline{b/\sigma_b} = 1.081$$

100 pseudo experiments
100 pseudo experiments



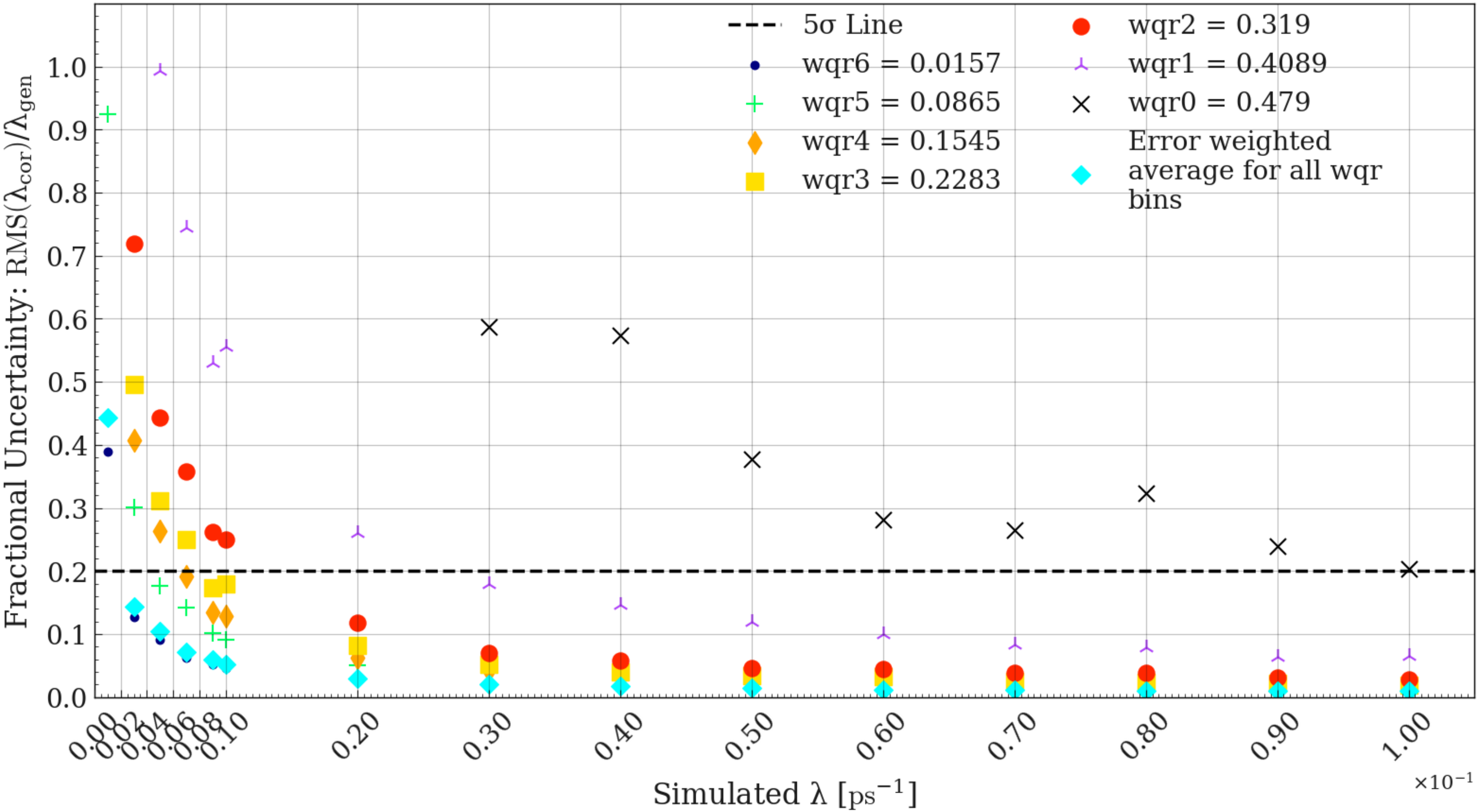
500 pseudo experiments
500 pseudo experiments





10k pseudo experiments

- No evidence for significant bias
- Bias ist statistical dominated and on the order of $\leq 1\,\%$



5σ Line
wqr6 = 0.0157
wqr5 = 0.0865
wqr4 = 0.1545
wqr3 = 0.2283
wqr2 = 0.319
wqr1 = 0.4089
wqr0 = 0.479
Error weighted average for all wqr bins
Fractional Uncertainty: RMS(λcor)/λgen
Simulated λ [ps−1]
×10−1
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
0.00
0.02
0.04
0.06
0.08
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00

Make simulation of wrong tagging realistic

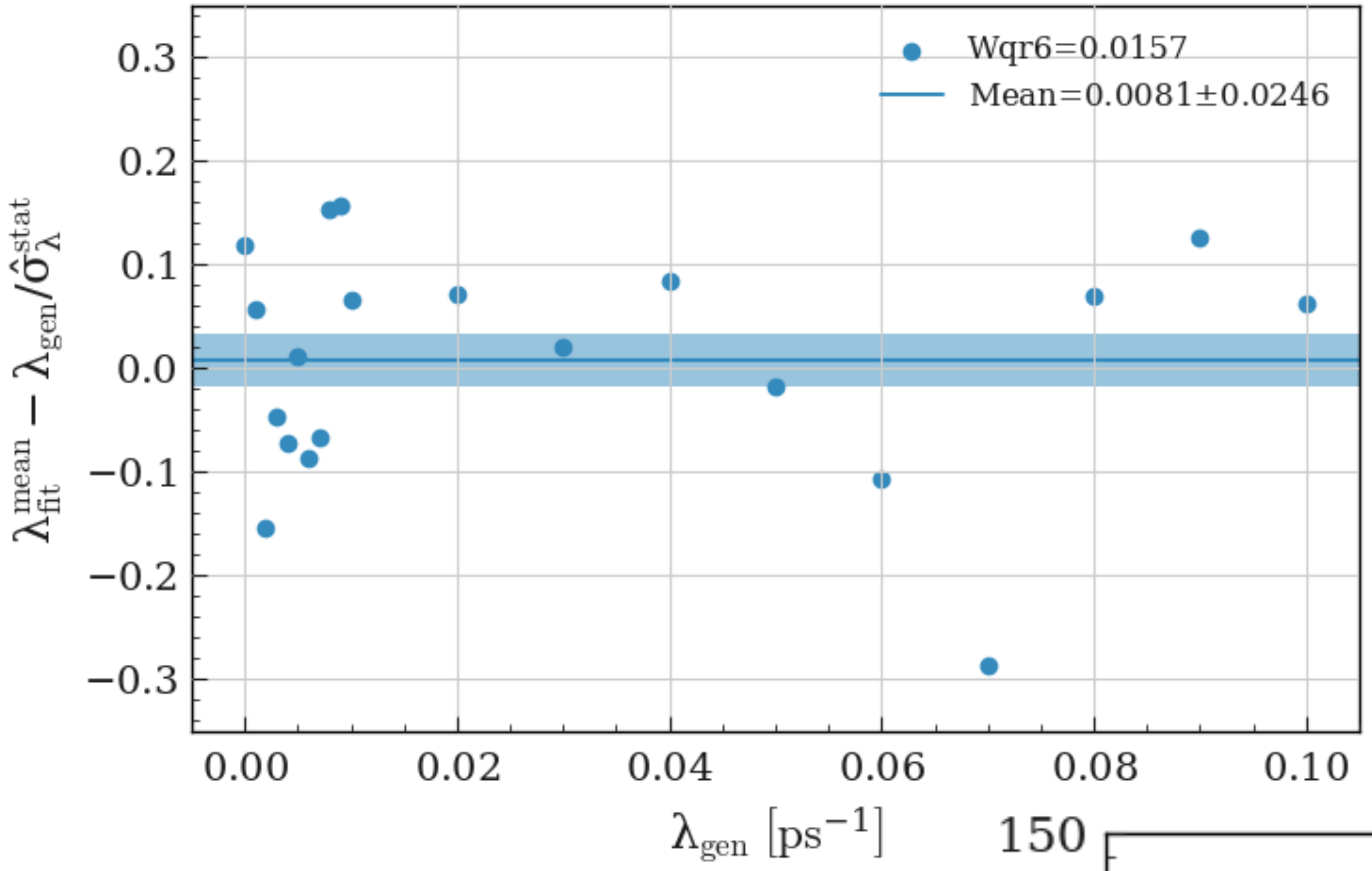
- So far: For example in best wrong tag bin we have $16.2\,\%$ of the data $\left(72k * 0.162 = 11664\right)$ with a wrong tag ration of $1.57\,\%$
  - ➡ Of these 11664 I flipped the sign (SF to OF or vice versa) of exactly $1.57\,\%$ events $\left(11664 * 0.0157 \approx 183\right)$, randomly
- The Problem: This is not correct as can be seen on coin tosses, each toss has a prob of $50\,\%$ but after 50 tosses we do not necessarily get 25 Heads-25 Tails
- Solution: Treat each event like a coin toss! Each event in the best wrong tag bin has a $1.57\,\%$ chance to flip its sign!

Every Event has percentage chance to flip sign

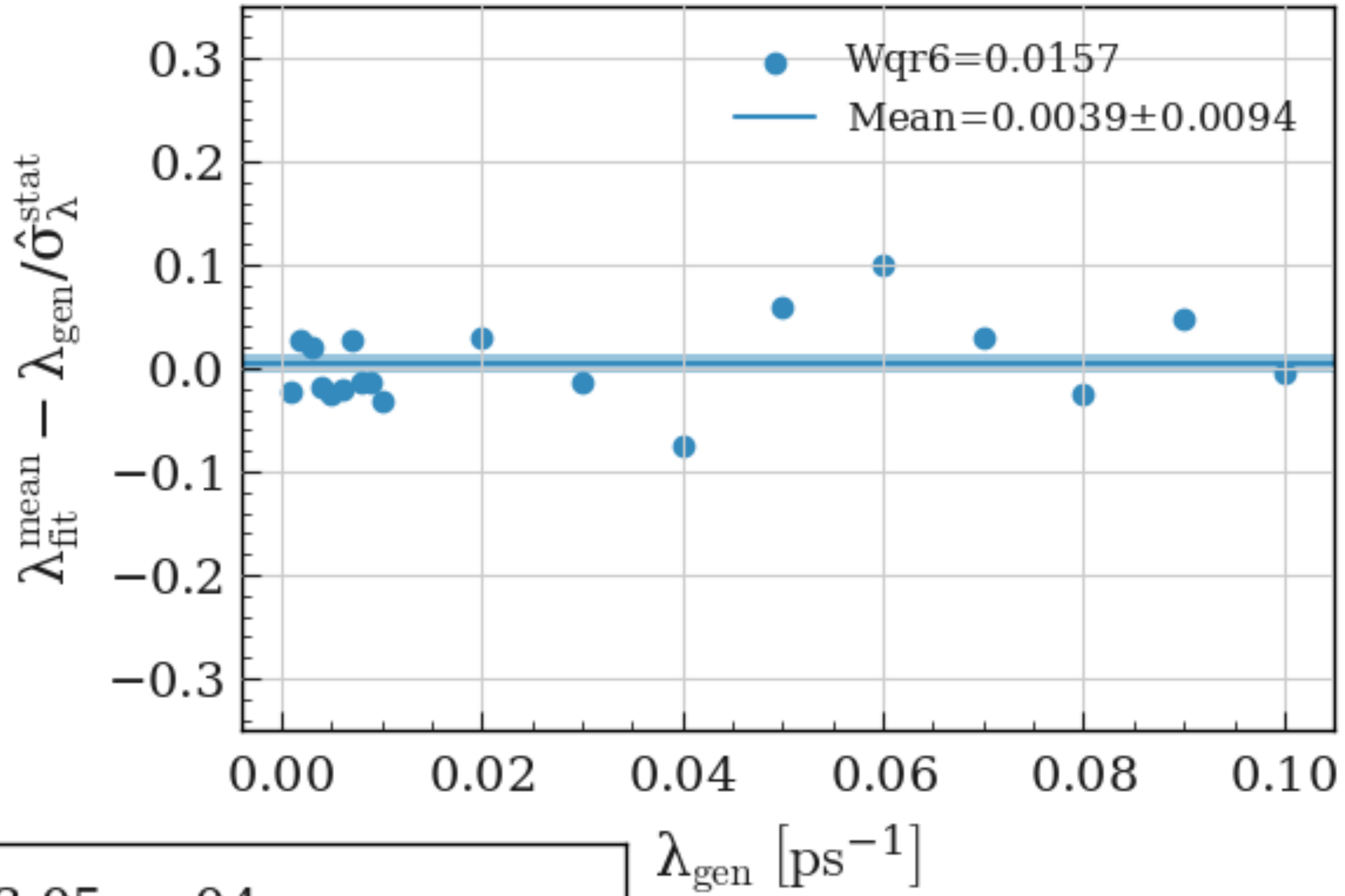
Fix wrong tag fraction in fit

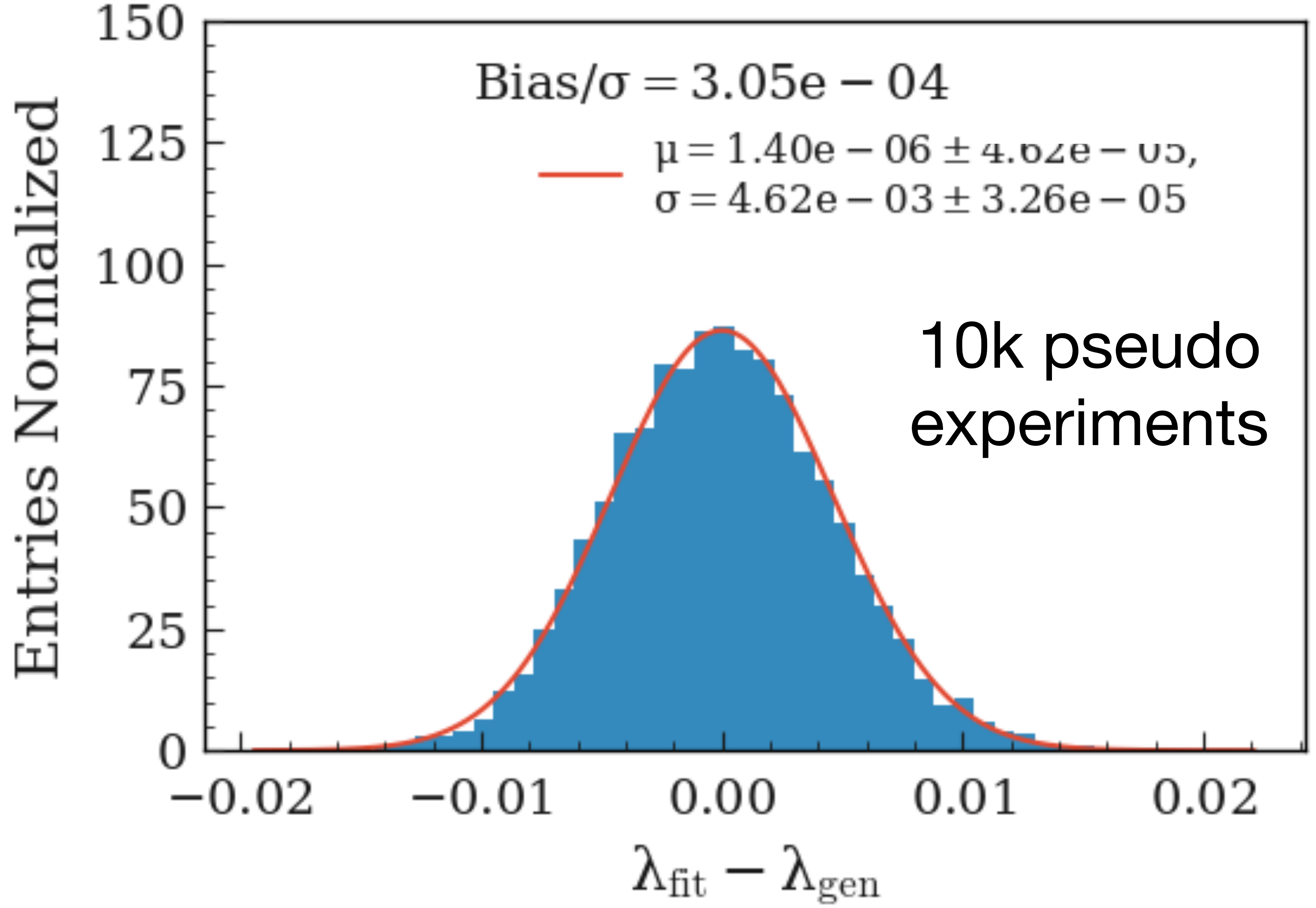
$$\overline{b/\sigma_b} = 1.5581$$

100 pseudo experiments

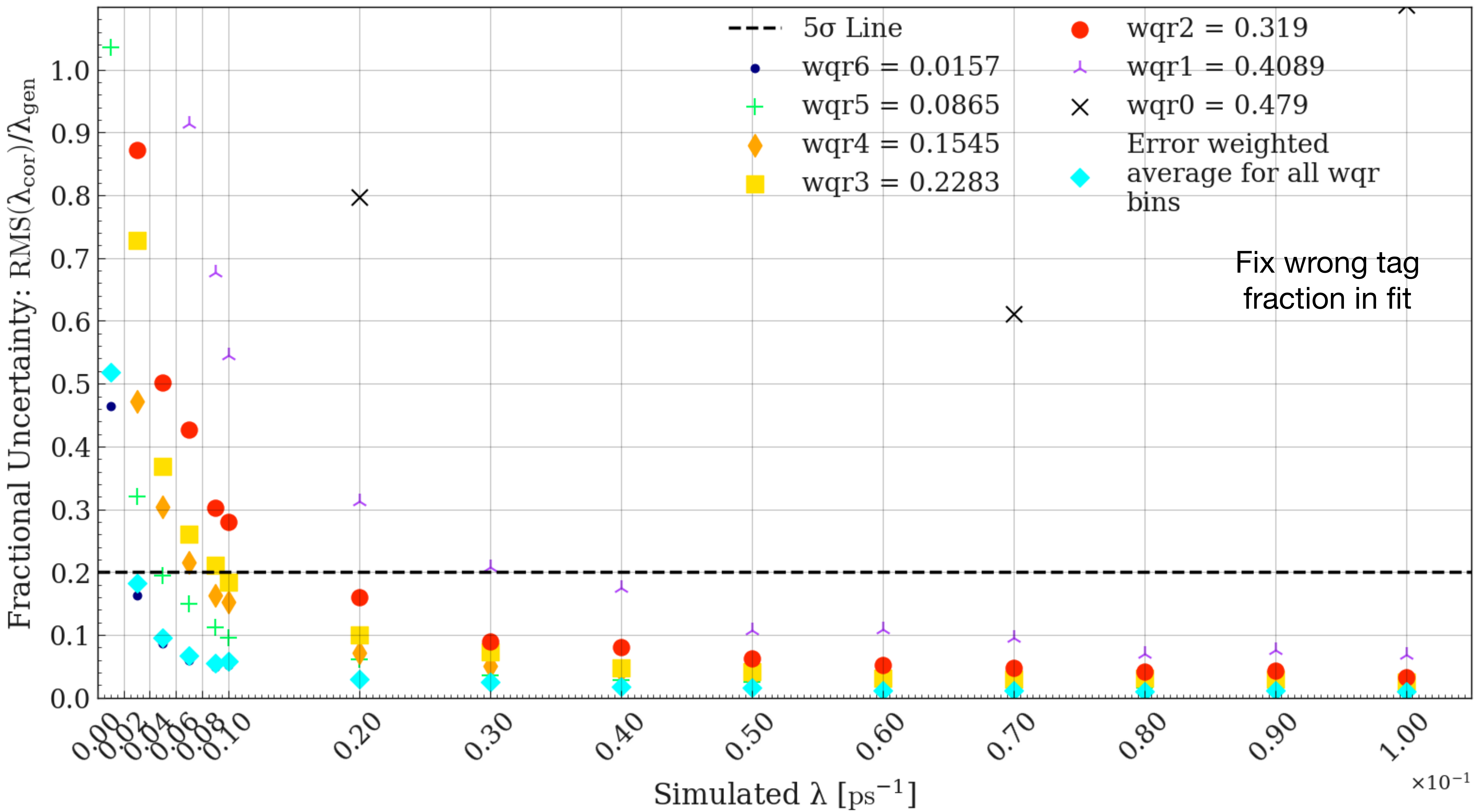


500 pseudo experiments





- No evidence for significant bias
- Bias ist statistical dominated and on the order of $\leq 1\,\%$



5σ Line
wqr6 = 0.0157
wqr5 = 0.0865
wqr4 = 0.1545
wqr3 = 0.2283
wqr2 = 0.319
wqr1 = 0.4089
wqr0 = 0.479
Error weighted average for all wqr bins
Fix wrong tag fraction in fit
Fractional Uncertainty: RMS(λ_{cor})/λ_{gen}
Simulated λ [ps^{-1}]
$\times 10^{-1}$

Every Event has percentage chance to flip sign

Float wrong tag fraction in fit

Fitting 100 pseudo experiments of $\lambda_{\text{gen}} = 0.008\ \text{ps}^{-1}$:

$$w_{qr6}^{\text{gen}} = 0.0157 \rightarrow w_{qr6}^{\text{fit}} = 0.0154 \pm 0.0003 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = \left(0.0085 \pm 0.0007\right)\ \text{ps}^{-1}$$

$$w_{qr5}^{\text{gen}} = 0.0865 \rightarrow w_{qr5}^{\text{fit}} = 0.0861 \pm 0.0006 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = \left(0.0074 \pm 0.0016\right)\ \text{ps}^{-1}$$

$$w_{qr4}^{\text{gen}} = 0.1545 \rightarrow w_{qr5}^{\text{fit}} = 0.1544 \pm 0.0007 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = (0.0083 \pm 0.0020)\ \text{ps}^{-1}$$

$$w_{qr3}^{\text{gen}} = 0.2283 \rightarrow w_{qr5}^{\text{fit}} = 0.2275 \pm 0.0008 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = \left(0.0084 \pm 0.0026\right)\ \text{ps}^{-1}$$

$$w_{qr2}^{\text{gen}} = 0.3190 \rightarrow w_{qr5}^{\text{fit}} = 0.3194 \pm 0.0008 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = \left(0.0052 \pm 0.0041\right)\ \text{ps}^{-1}$$

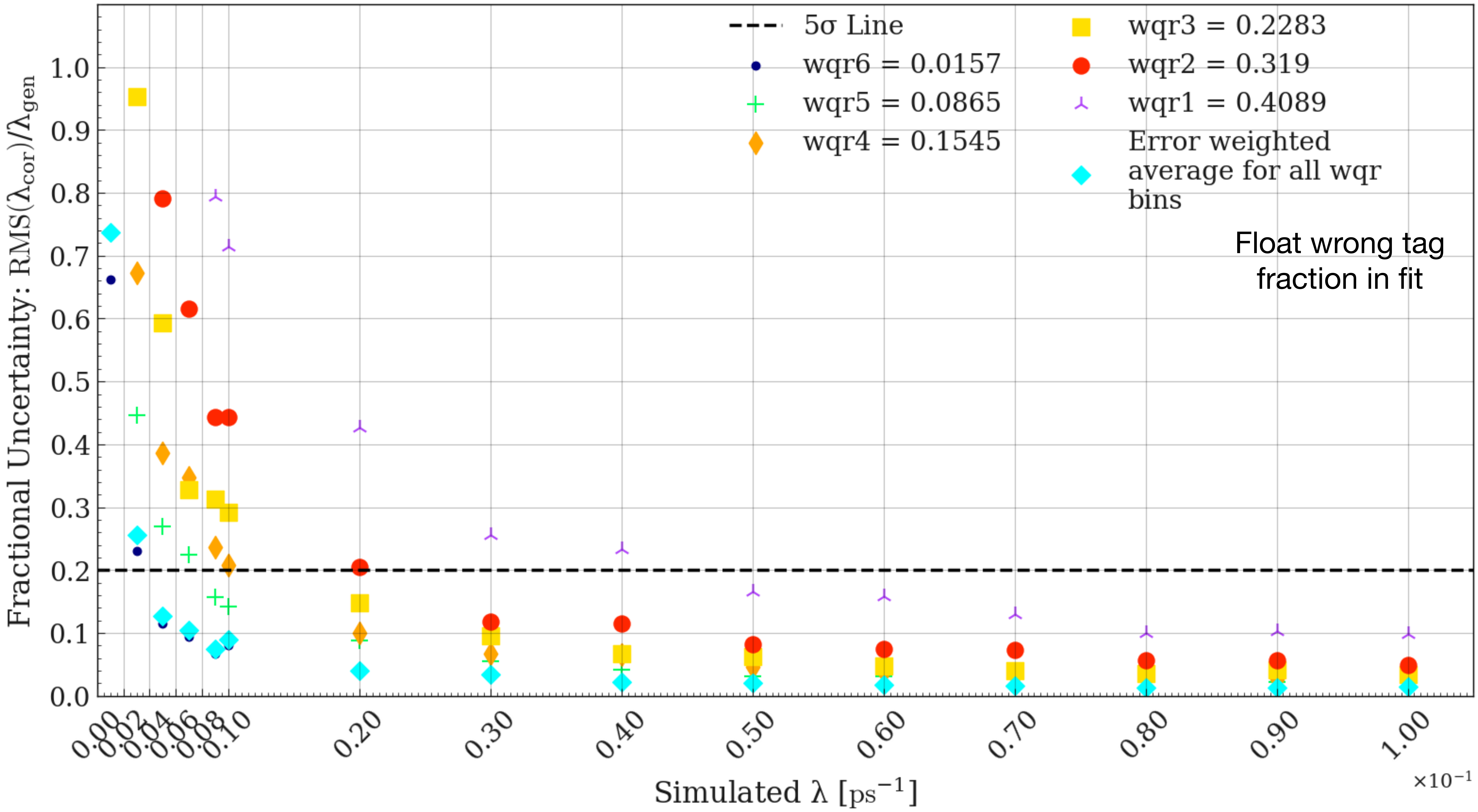
$$w_{qr1}^{\text{gen}} = 0.4089 \rightarrow w_{qr5}^{\text{fit}} = 0.4096 \pm 0.0008 \rightarrow \lambda_{\text{fit}}^{\text{mean}} = (0.0082 \pm 0.0093)\ \text{ps}^{-1}$$

$$w_{qr0}^{\text{gen}} = 0.4790 \rightarrow w_{qr5}^{\text{fit}} = ??? \pm ??? \rightarrow \lambda_{\text{fit}}^{\text{mean}} = (??? \pm ???)\ \text{ps}^{-1}$$

Every Event has percentage chance to flip sign

Float wrong tag fraction in fit

$$\overline{b/\sigma_b} = 1.0104$$



5σ Line
wqr6 = 0.0157
wqr5 = 0.0865
wqr4 = 0.1545
wqr3 = 0.2283
wqr2 = 0.319
wqr1 = 0.4089
Error weighted average for all wqr bins
Float wrong tag fraction in fit
Fractional Uncertainty: RMS(λcor)/λgen
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.00
0.02
0.04
0.06
0.08
0.10
0.20
0.30
0.40
0.50
0.60
0.70
0.80
0.90
1.00
×10⁻¹
Simulated λ [ps⁻¹]

Every Event has percentage chance to flip sign

Float wrong tag fraction in fit for all except wqr0

$$\overline{b/\sigma_b} = 1.6347$$

5σ Line
wqr6 = 0.0157
wqr5 = 0.0865
wqr4 = 0.1545
wqr3 = 0.2283
wqr2 = 0.319
wqr1 = 0.4089
wqr0 = 0.479
Error weighted average for all wqr bins
Float wrong tag fraction in fit for all except wqr0
Fractional Uncertainty: RMS(λ_{cor})/λ_{gen}
Simulated λ [ps^{-1}]
$\times 10^{-1}$

Summary

- Creating individual random numbers for the pseudo experiment generation fixed the “bias” problem
- Bias is of the order of $\leq 1\,\%$
  ➡ Therefore, no evidence for significant bias
- Incorporated more realistic wrong tag simulation, where every event has a percent chance to flip sign
- Observe good sensitivity for letting λ and wrong tag fraction float (except w_{qr0})

  $\Rightarrow$ Reach 5σ for $\lambda > 0.003\ \mathrm{ps}^{-1}$

How to do convolution?

$$f = a\left[\underbrace{\exp\left(-\frac{x}{\tau}\right)\exp\left(-\frac{y}{\tau}\right)}_{1.} + \underbrace{q\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)x\right)\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)y\right)\exp\left(\frac{\lambda}{2}|x-y|\right)\cos\left(\Delta m\,(x-y)\right)}_{2.}\right]$$

Our two resolution functions for x and y look the following:

$$R_1 = \frac{1}{\sigma_1\sqrt{2\pi}}\exp\left(-\frac{\left(u-x-\mu_1\right)^2}{2\sigma_1^2}\right) \qquad R_2 = \frac{1}{\sigma_2\sqrt{2\pi}}\exp\left(-\frac{\left(v-y-\mu_2\right)^2}{2\sigma_2^2}\right)$$

with $u = x + \delta x$ and $v = y + \delta y$. → Convolution integral to solve:

$$\mathscr{C} = f \otimes R_1 \otimes R_2 = a\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y \; \underbrace{1.\cdot R_1 R_2}_{\mathrm{I.}} + \underbrace{2.\cdot R_1 R_2}_{\mathrm{II.}}$$

How to do convolution?

Integral $\mathrm{I}.$ is comparably easy to determine analytically with regards to $\mathrm{II}.$
$\mathrm{II}.$ is more complicated as shown below:

$$\mathrm{II.} = \frac{q}{2\pi\sigma_1\sigma_2}\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y \exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)x\right)\exp\left(-\left(\frac{1}{\tau}+\frac{\lambda}{2}\right)y\right)\boxed{\exp\left(\frac{\lambda}{2}|x-y|\right)}\cos\left(\Delta m\left(x-y\right)\right)\exp\left(-\frac{\left(u-x-\mu_1\right)^2}{2\sigma_1^2}\right)\exp\left(-\frac{\left(v-y-\mu_2\right)^2}{2\sigma_2^2}\right)$$

Due to the absolute value we have to consider two cases:

$$\text{Case } 1 \rightarrow x \geq y \Rightarrow |x-y| = x-y \Rightarrow \exp\left(\frac{\lambda}{2}|x-y|\right) = \exp\left(\frac{\lambda}{2}x\right)\exp\left(-\frac{\lambda}{2}y\right)$$

$$\boxed{x\epsilon\left[0,\infty\right]\ \ \&\ y\epsilon\left[0,x\right]}$$

$$\text{Case } 2 \rightarrow x < y \Rightarrow |x-y| = y-x \Rightarrow \exp\left(\frac{\lambda}{2}|x-y|\right) = \exp\left(-\frac{\lambda}{2}x\right)\exp\left(\frac{\lambda}{2}y\right)$$

$$\boxed{x\epsilon\left[0,\infty\right]\ \ \&\ y\epsilon\left[x,\infty\right]}$$

How to do convolution?

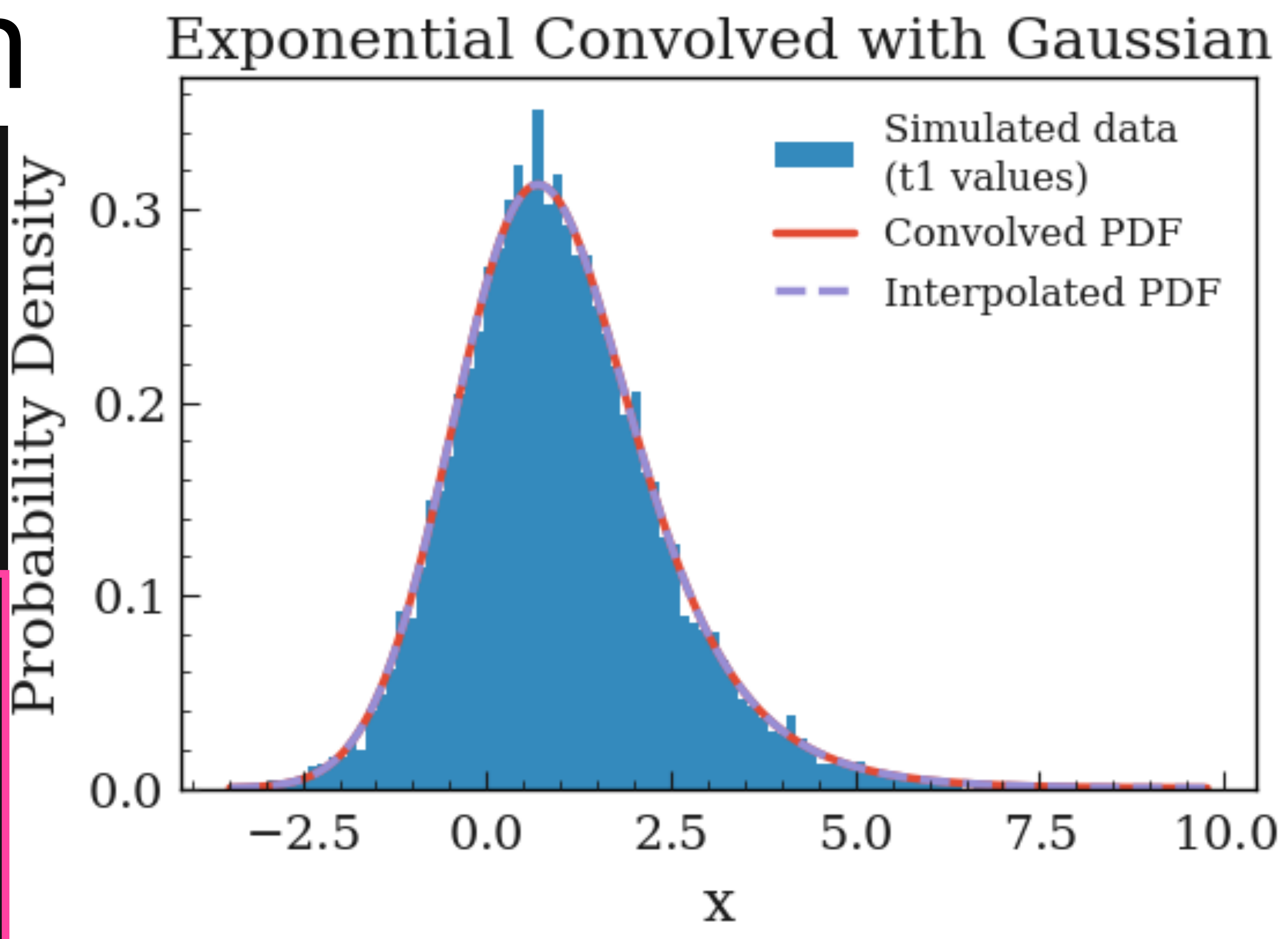
This gives us four integrals to solve for, which python takes over 24h for (analytically)
Need analytical form so I have a fit function in the end dependent on u and v (smeared times)

Is there a way to solve the integral numerical without setting values for u, v and λ obtaining a functional form to fit???

Okay let's take a step back and start with an easy convolution of a single exponential with a gaussian function!

Easy numerical convolution

```
# Parameters
t = 1.0    # decay parameter
m = 0.0    # mean of Gaussian
s = 1.0    # std dev of Gaussian

x = np.random.exponential(scale=t, size=10000)

x_data, x_dataError = AddGaussianNoise(
        x, 0.0, mu=m, sigma=s)

def true_pdf(x, t=1.0):
    return np.exp(-x / t) / t #if x >= 0 else 0.0

# Gaussian smearing function
def gaussian(x, mu=0.0, sigma=1.0):
    return norm.pdf(x, loc=mu, scale=sigma)
# Convolution: numerically integrate f(x') * g(x - x')
def convolved_pdf(x, t=1.0, sigma=1.0):
    integrand = lambda x_prime: true_pdf(x_prime, t) * gaussian(x - x_prime, sigma=sigma)
    result, _ = quad(integrand, 0, np.inf)  # since f(x') = 0 for x' < 0
    return result

x_vals = np.linspace(np.min(x_data), np.max(x_data), 10000)

y_vals = np.array([convolved_pdf(x, t=1.0, sigma=1.0) for x in x_vals])

f_func = interp1d(x_vals, y_vals, kind='cubic', bounds_error=False, fill_value=0.0)

def Interpolated_Func(x_vals, func):
    f_int_ForPlot = func(x_vals)
    return f_int_ForPlot
```



- Generate smeared data
- Calculate the convolution of an exponential and gaussian
- How to get from a numerical result back to a function with parameters that can be fitted?

100k events Gen $\lambda = 0.002\ \text{ps}^{-1}$		Wrong tagging bins (fraction of data)						
		w_{qr0} (15.5%)	w_{qr1} (15.8%)	w_{qr2} (16.5%)	w_{qr3} (13.4%)	w_{qr4} (11.6%)	w_{qr5} (11.0%)	w_{qr6} (16.2%)
Wrong tag ratio	Normal	0.0372 ± 0.0217	-0.0007 ± 0.0038	0.0069 ± 0.0018	0.0016 ± 0.0013	0.0007 ± 0.0011	0.0022 ± 0.0007	0.0021 ± 0.0003
	All 5%	0.0020 ± 0.0005	0.0021 ± 0.0005	0.0029 ± 0.0005	0.0020 ± 0.0005	0.0015 ± 0.0005	0.0018 ± 0.0005	0.0018 ± 0.0005
	All 25%	0.0028 ± 0.0013	0.0023 ± 0.0013	0.0052 ± 0.0013	0.0024 ± 0.0015	-0.0014 ± 0.0017	0.0019 ± 0.0015	0.0004 ± 0.0014
	All 30%	0.0018 ± 0.0016	0.0010 ± 0.0016	0.0062 ± 0.0017	0.0018 ± 0.0020	-0.0032 ± 0.0023	0.0002 ± 0.0020	-0.0002 ± 0.0019
	All 32%	0.0003 ± 0.0018	0.0004 ± 0.0018	0.0069 ± 0.0018	0.0015 ± 0.0022	-0.0044 ± 0.0026	0.0023 ± 0.0023	0.0002 ± 0.0021
	All 33%	0.0011 ± 0.0019	-0.0001 ± 0.0019	0.0075 ± 0.0019	0.0018 ± 0.0022	-0.0055 ± 0.0028	0.0017 ± 0.0025	0.0000 ± 0.0022
	All 40%	0.0030 ± 0.0034	-0.0001 ± 0.0035	0.0129 ± 0.0031	0.0016 ± 0.0038	-0.0116 ± 0.0049	0.0017 ± 0.0043	0.0012 ± 0.0036

- Small wrong tag ratios observe good fit precision
- Lager wrong tag ratios observe larger fit bias
- Larger wrong tag ratio, bias can switch from positive to negative bias → quantization effect in choosing how many events go into each wrong tag bin?

100k events Gen $\lambda = 0.002\ \text{ps}^{-1}$		Wrong tagging bins (fraction of data)						
		w_{qr0} ($\sim 14.3\%$)	w_{qr1} ($\sim 14.3\%$)	w_{qr2} ($\sim 14.3\%$)	w_{qr3} ($\sim 14.3\%$)	w_{qr4} ($\sim 14.3\%$)	w_{qr5} ($\sim 14.3\%$)	w_{qr6} ($\sim 14.3\%$)
Wrong tag ratio	Normal	0.0003 ± 0.0204	0.0072 ± 0.0043	0.0014 ± 0.0019	0.0011 ± 0.0014	0.0024 ± 0.0009	0.0017 ± 0.0007	0.0019 ± 0.0003
	All 5%	0.0014 ± 0.0005	0.0014 ± 0.0005	0.0020 ± 0.0005	0.0023 ± 0.0005	0.0021 ± 0.0004	0.0019 ± 0.0005	0.0023 ± 0.0005
	All 25%	0.0020 ± 0.0013	0.0033 ± 0.0013	0.0024 ± 0.0012	0.0012 ± 0.0015	0.0043 ± 0.0012	0.0012 ± 0.0014	0.0038 ± 0.0014
	All 30%	0.0030 ± 0.0016	0.0026 ± 0.0018	0.0019 ± 0.0017	0.0018 ± 0.0020	0.0041 ± 0.0017	0.0013 ± 0.0019	0.0044 ± 0.0017
	All 32%	0.0030 ± 0.0018	0.0039 ± 0.0020	0.0016 ± 0.0019	0.0010 ± 0.0023	0.0050 ± 0.0019	0.0012 ± 0.0020	0.0049 ± 0.0019
	All 33%	0.0029 ± 0.0019	0.0041 ± 0.0020	0.0021 ± 0.0020	0.0008 ± 0.0024	0.0050 ± 0.0020	0.0010 ± 0.0021	0.0052 ± 0.0020
	All 40%	0.0029 ± 0.0038	0.0058 ± 0.0039	0.0032 ± 0.0035	0.0019 ± 0.0046	0.0119 ± 0.0035	0.0013 ± 0.0039	0.0075 ± 0.0037

1M events Gen $\lambda = 0.002\ \mathrm{ps}^{-1}$		Wrong tagging bins (fraction of data)						
		w_{qr0} (15.5%)	w_{qr1} (15.8%)	w_{qr2} (16.5%)	w_{qr3} (13.4%)	w_{qr4} (11.6%)	w_{qr5} (11.0%)	w_{qr6} (16.2%)
Wrong tag ratio	Normal	0.0073 ± 0.0063	0.0021 ± 0.0014	0.0015 ± 0.0006	0.0021 ± 0.0004	0.0016 ± 0.0003	0.002 ± 0.0002	0.0019 ± 0.0001
	All 5%	0.0021 ± 0.0002	0.0021 ± 0.0001	0.0021 ± 0.0002	0.0019 ± 0.0002	0.0016 ± 0.0002	0.0018 ± 0.0002	0.0018 ± 0.0002
	All 25%	0.0025 ± 0.0004	0.0018 ± 0.0004	0.0017 ± 0.0004	0.0019 ± 0.0005	0.0014 ± 0.0005	0.0015 ± 0.0005	0.0017 ± 0.0004
	All 30%	0.0023 ± 0.0006	0.0022 ± 0.0006	0.0014 ± 0.0005	0.0021 ± 0.0006	0.0015 ± 0.0007	0.0013 ± 0.0007	0.0020 ± 0.0006
	All 32%	0.0022 ± 0.0006	0.0019 ± 0.0006	0.0014 ± 0.0006	0.0023 ± 0.0007	0.0014 ± 0.0008	0.0011 ± 0.0008	0.0023 ± 0.0006
	All 33%	0.0022 ± 0.0007	0.0019 ± 0.0007	0.0012 ± 0.0006	0.0023 ± 0.0007	0.0016 ± 0.0008	0.0013 ± 0.0008	0.0023 ± 0.0007
	All 40%	0.0030 ± 0.0012	0.0021 ± 0.0013	0.0014 ± 0.0010	0.0032 ± 0.0013	0.0031 ± 0.0015	0.0016 ± 0.0013	0.0032 ± 0.0012

1M events Gen $\lambda = 0.002\ \mathrm{ps}^{-1}$		Wrong tagging bins (fraction of data)						
		w_{qr0} ($\sim$ 14.3%)	w_{qr1} ($\sim$ 14.3%)	w_{qr2} ($\sim$ 14.3%)	w_{qr3} ($\sim$ 14.3%)	w_{qr4} ($\sim$ 14.3%)	w_{qr5} ($\sim$ 14.3%)	w_{qr6} ($\sim$ 14.3%)
Wrong tag ratio	Normal	0.0022 ± 0.0064	0.0035 ± 0.0014	0.0023 ± 0.0007	0.0021 ± 0.0004	0.0019 ± 0.0003	0.0020 ± 0.0002	0.0019 ± 0.0001
	All 5%	0.0020 ± 0.0002	0.0022 ± 0.0002	0.0022 ± 0.0002	0.0021 ± 0.0002	0.0018 ± 0.0002	0.0020 ± 0.0002	0.0019 ± 0.0002
	All 25%	0.0019 ± 0.0005	0.0020 ± 0.0004	0.0025 ± 0.0005	0.0020 ± 0.0005	0.0017 ± 0.0005	0.0016 ± 0.0004	0.0020 ± 0.0005
	All 30%	0.0017 ± 0.0007	0.0024 ± 0.0005	0.0023 ± 0.0006	0.0022 ± 0.0006	0.0021 ± 0.0006	0.0013 ± 0.0005	0.0022 ± 0.0006
	All 32%	0.0020 ± 0.0007	0.0026 ± 0.0006	0.0023 ± 0.0007	0.0018 ± 0.0006	0.0026 ± 0.0007	0.0015 ± 0.0006	0.0023 ± 0.0007
	All 33%	0.0021 ± 0.0008	0.0027 ± 0.0007	0.0024 ± 0.0007	0.0016 ± 0.0007	0.0027 ± 0.0008	0.0016 ± 0.0007	0.0022 ± 0.0008
	All 40%	0.0018 ± 0.0013	0.0033 ± 0.0012	0.0023 ± 0.0012	0.0013 ± 0.0012	0.0035 ± 0.0013	0.0009 ± 0.0011	0.0025 ± 0.0013

- With more data we observe fit values in same fraction of data agree with each other within its errors
- Bias seems to have statistic nature!