Charting the Uncharted

Hamiltonian Truncation and Real-Time Dynamics in Strongly Coupled Field Theories

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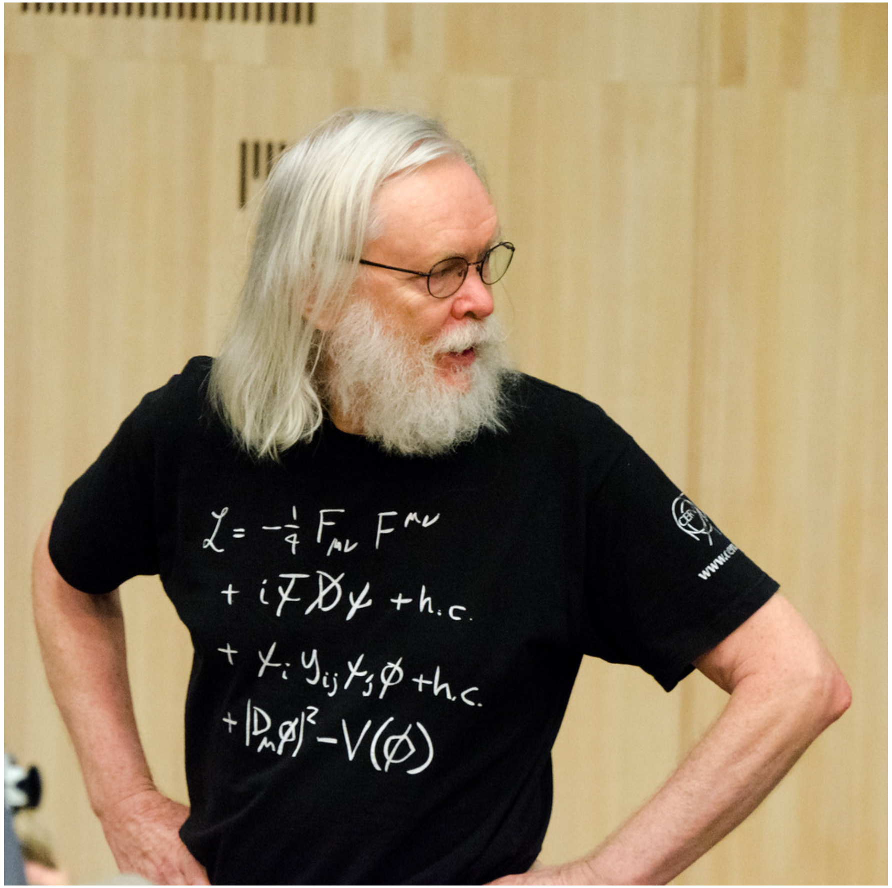
Based on work done in collaboration with
N. Craig, O. Delouche, K. Farnsworth, A. Fee, T. Melia, H. Murayama, R. Rattazzi, M. Riembau,
F. Riva, M. Stadlbauer, J. Thompson, M. Walters, et. al.

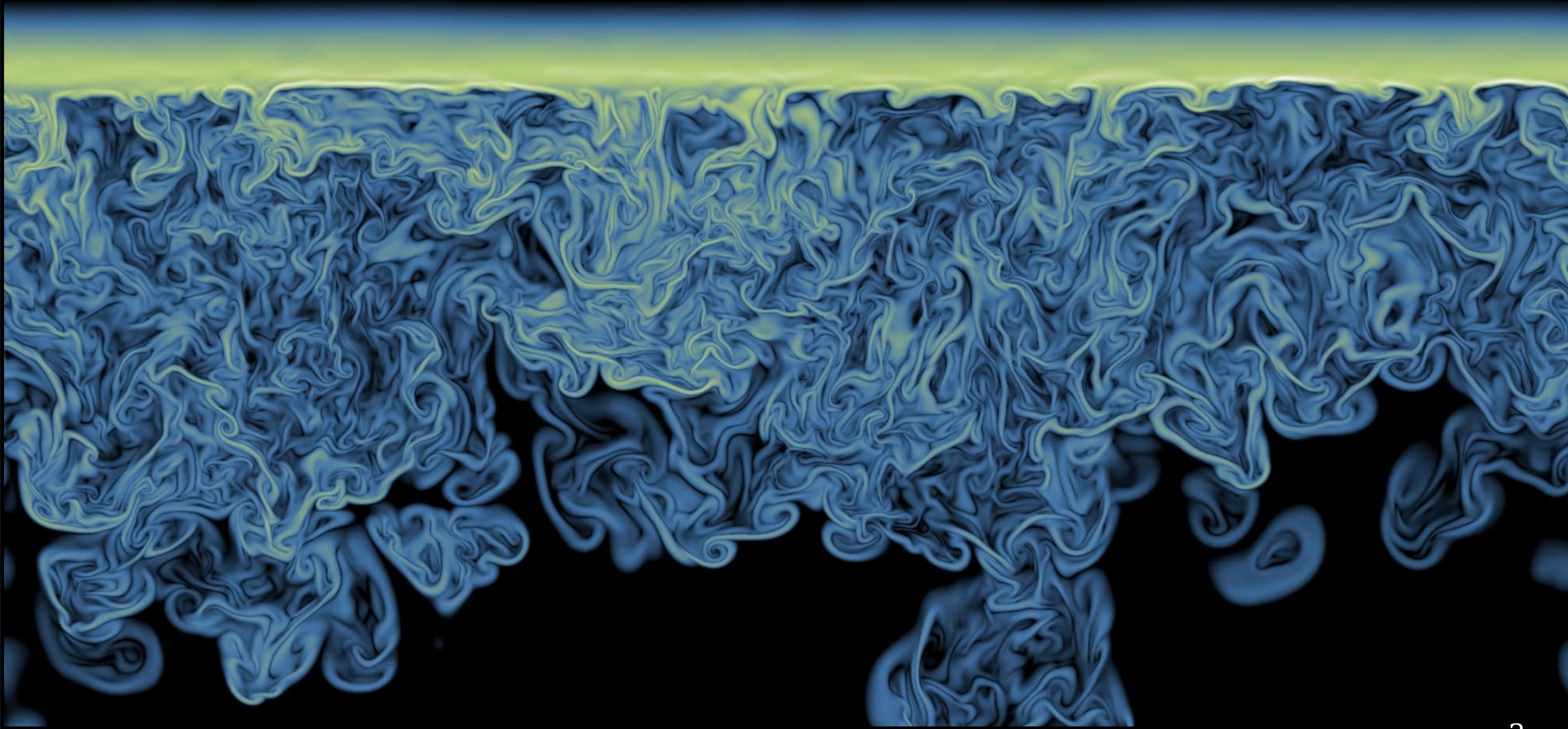
University of Hawaii Colloquium
06/Mar/2025

Our universe in 16 kB

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
&+ i\overline{\psi}\not{D}\psi + \text{h.c.} \\
&+ \psi_i y_{ij}\psi_j\phi + \text{h.c.} \\
&+ |D_\mu\phi|^2 - V(\phi)
\end{aligned}
$$

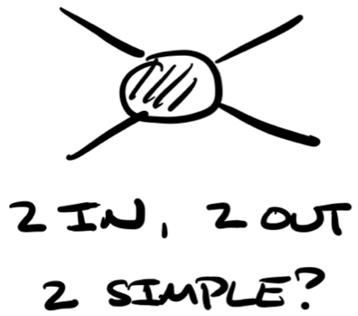


this from that



Scope of QFT

Particle physics deals with the **simplest** possible **systems**



⇒ The **scope** of questions **is** severely **narrow**

⇒ **"More is different" -P. Anderson**

⇒ **Abundance of "new physics"** lurking within theories we "know"

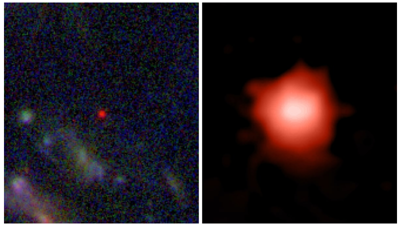
The universe at different scales

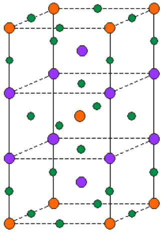
Standard Model

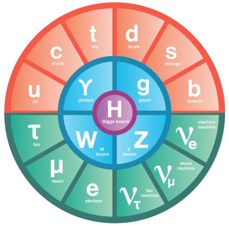
New Particles

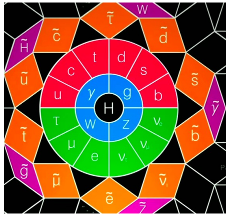
10^{27} 10^{20} 10^{10} 1m 10^{-10} 10^{-15} 10^{-35}

Very big → Very Small









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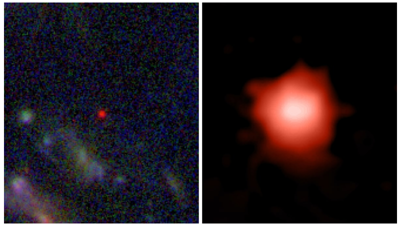

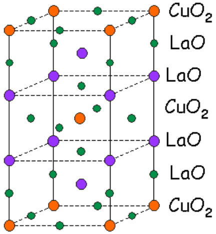

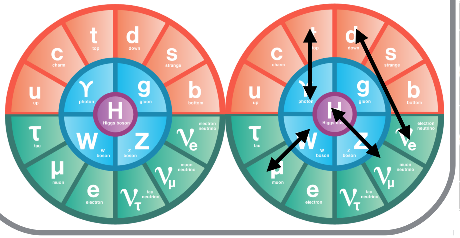

Effective **F**ield **T**heory:

New Interactions

- Model independent
- Exhaustive
- Guide for experiments

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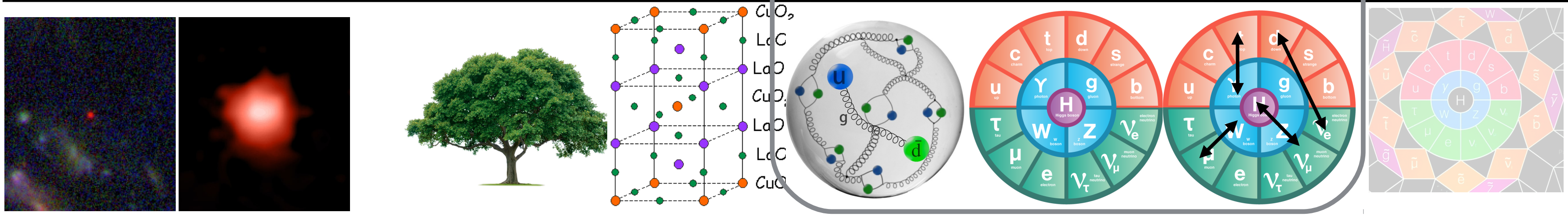

New Physics under our nose: **QCD**

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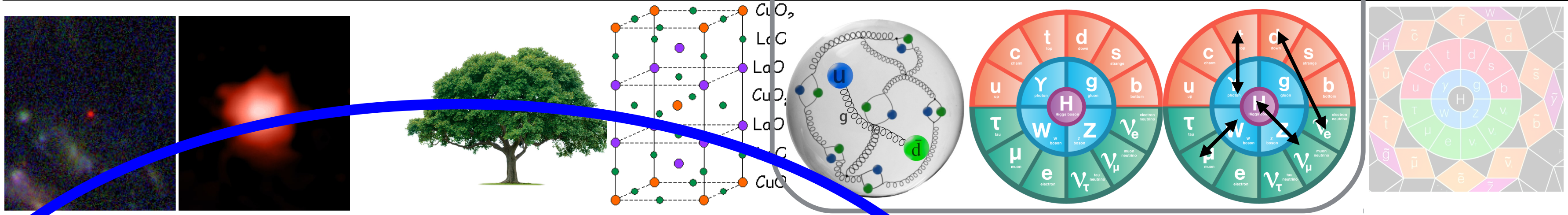
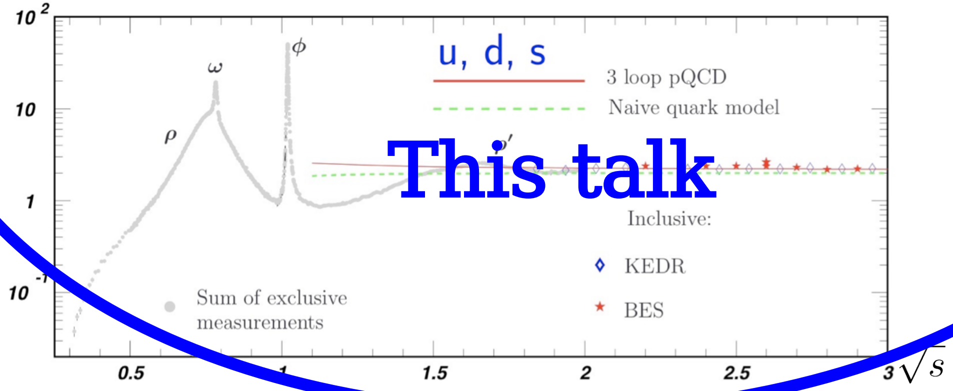


New Physics under our nose: **QCD**

New numerical methods for strong coupling:
Hamiltonian truncation

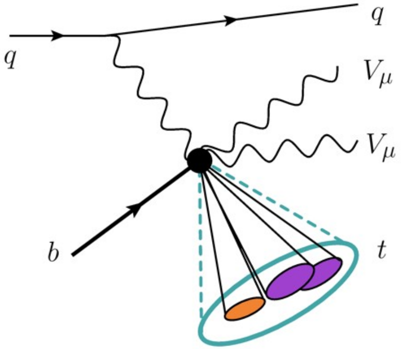


This talk

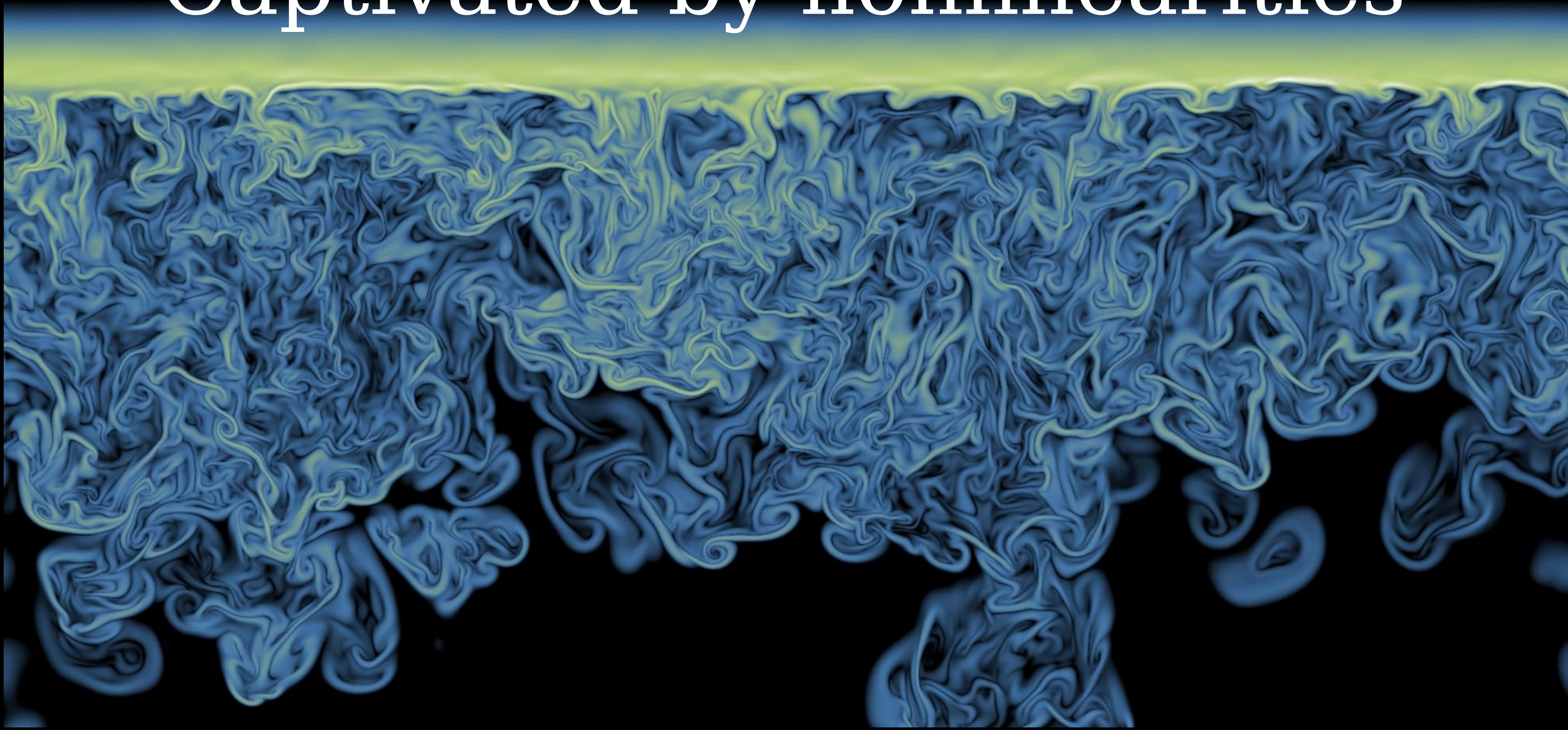
Effective **F**ield **T**heory:

New Interactions

- Model independent
- Guide for experiments



Captivated by nonlinearities



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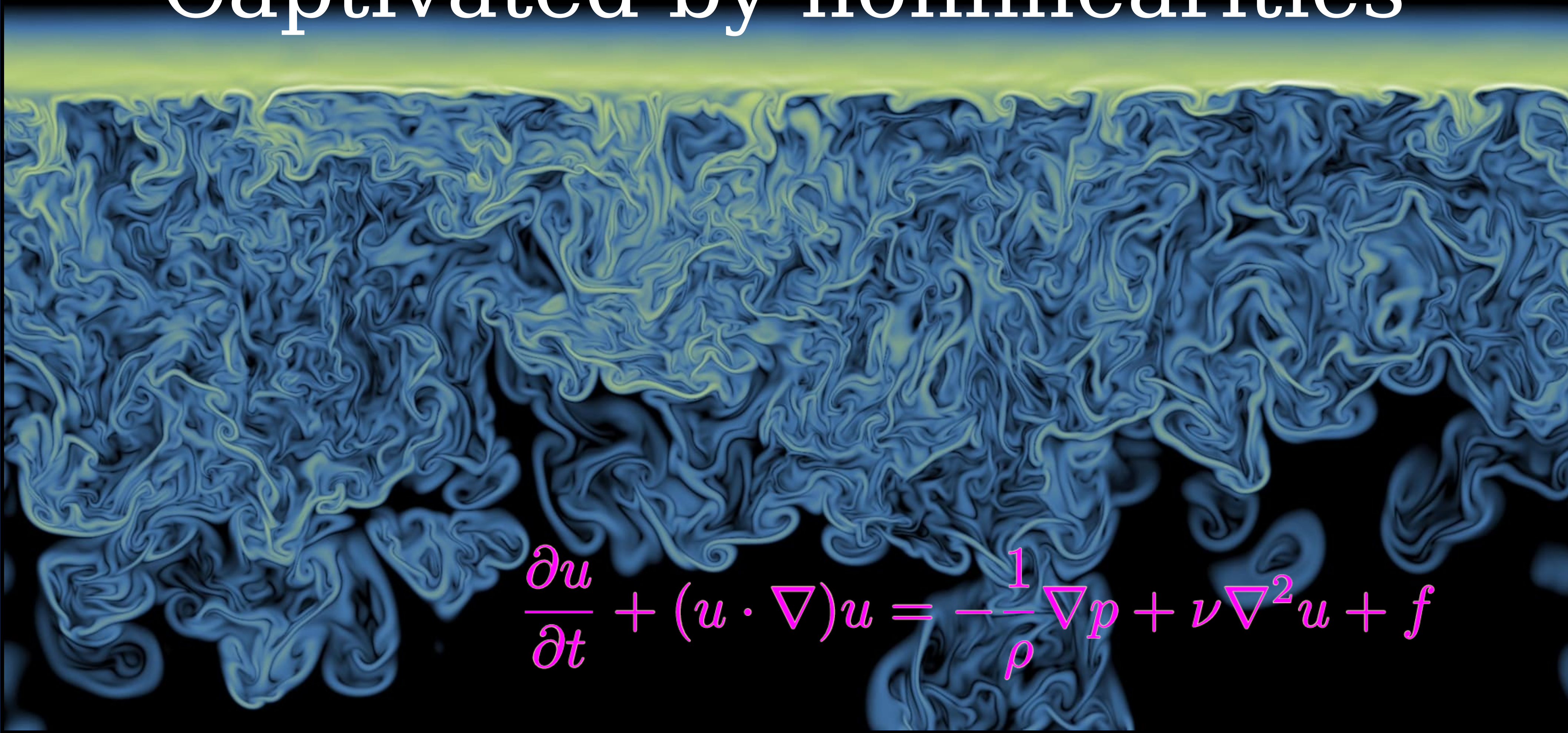
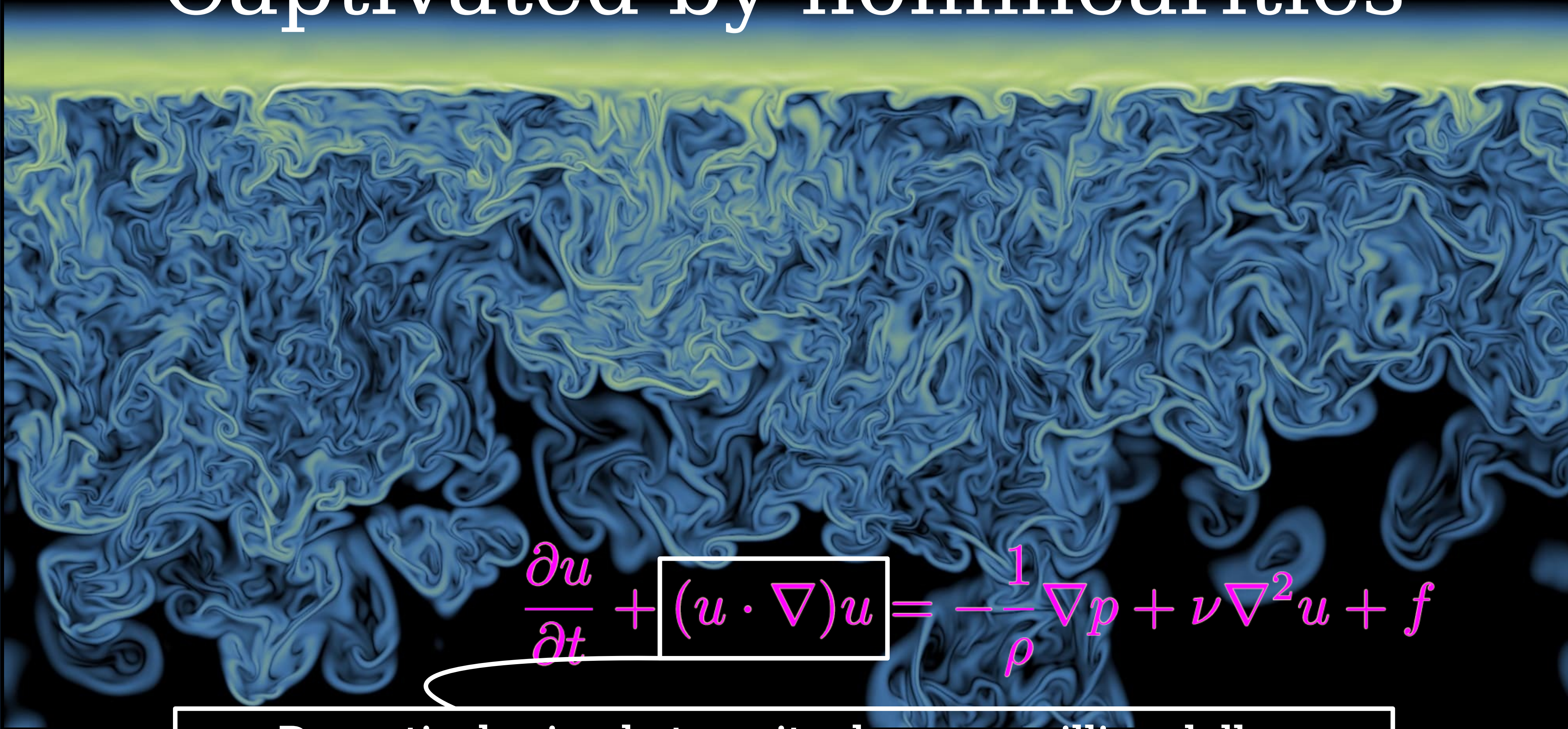


$$\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + f$$

Captivated by nonlinearities

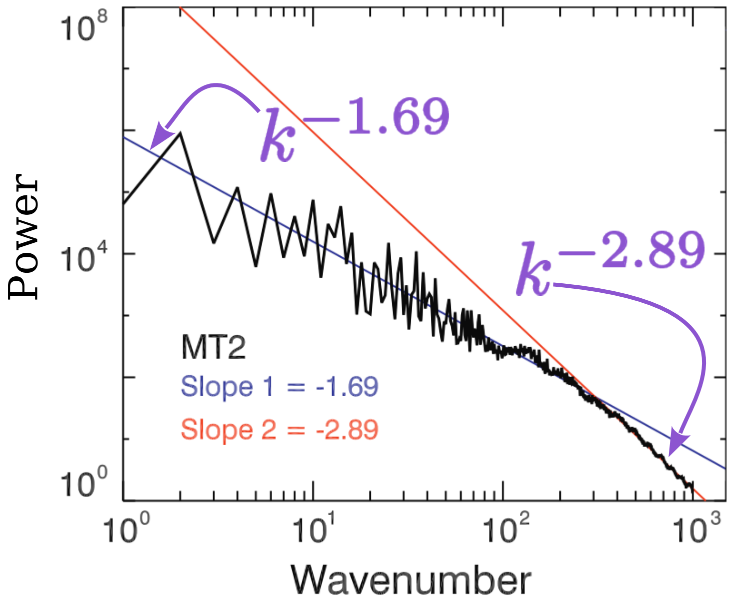


$$\frac{\partial u}{\partial t} + \boxed{(u \cdot \nabla) u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u + f$$

Deceptively simple to write down...a million dollar problem to solve

Captivated by nonlinearities

Choi & Snowman (2011)
DOI: 10.1016/j.icarus.2011.10.001



yet exhibits various **UNIVERSAL** behavior

Captivated by nonlinearities

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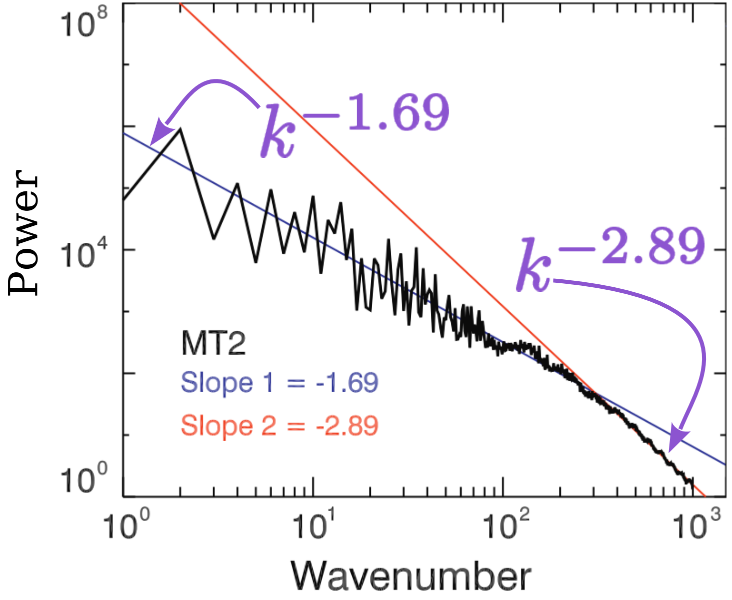

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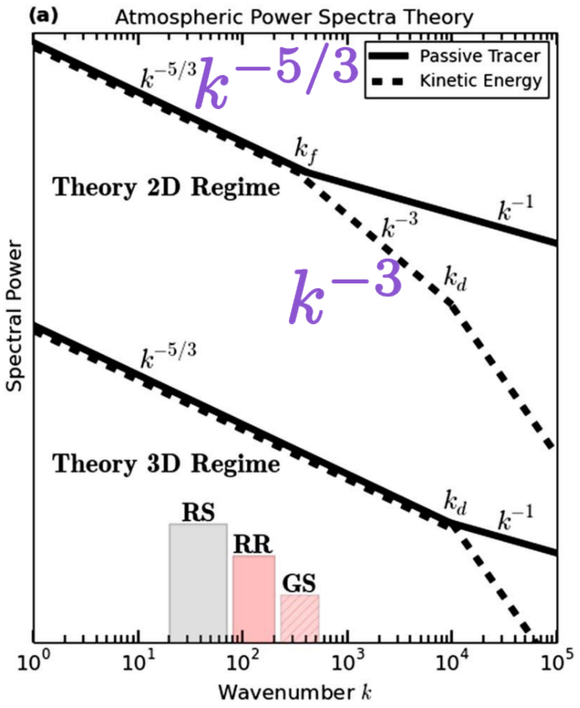
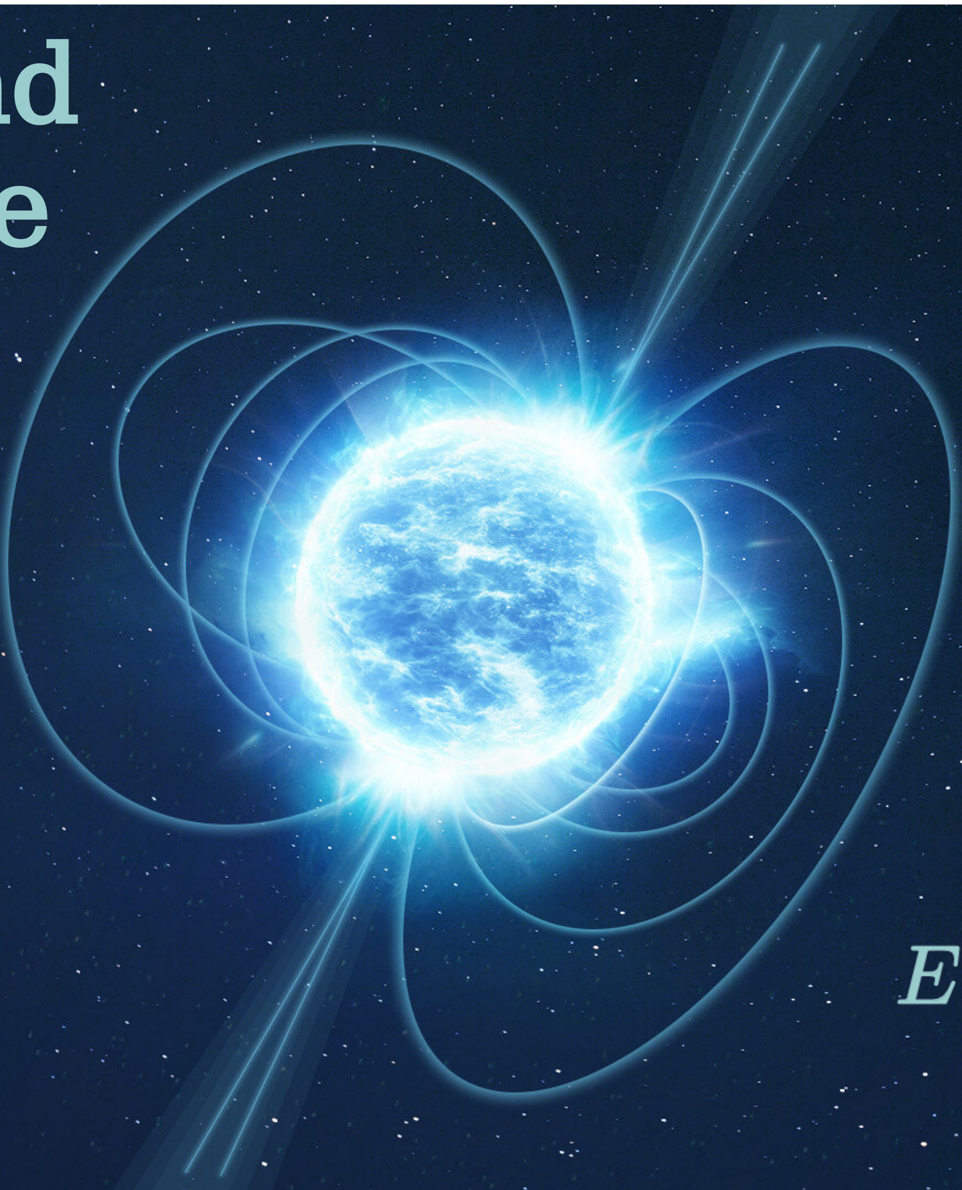


Casentino et. al. (2019)
DOI: 10.1029/2018JE005762

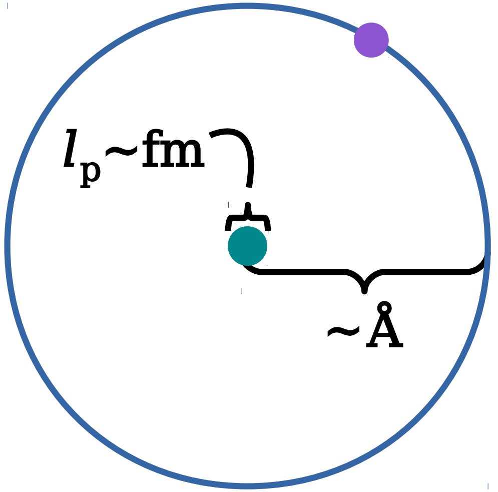
exotic and awesome



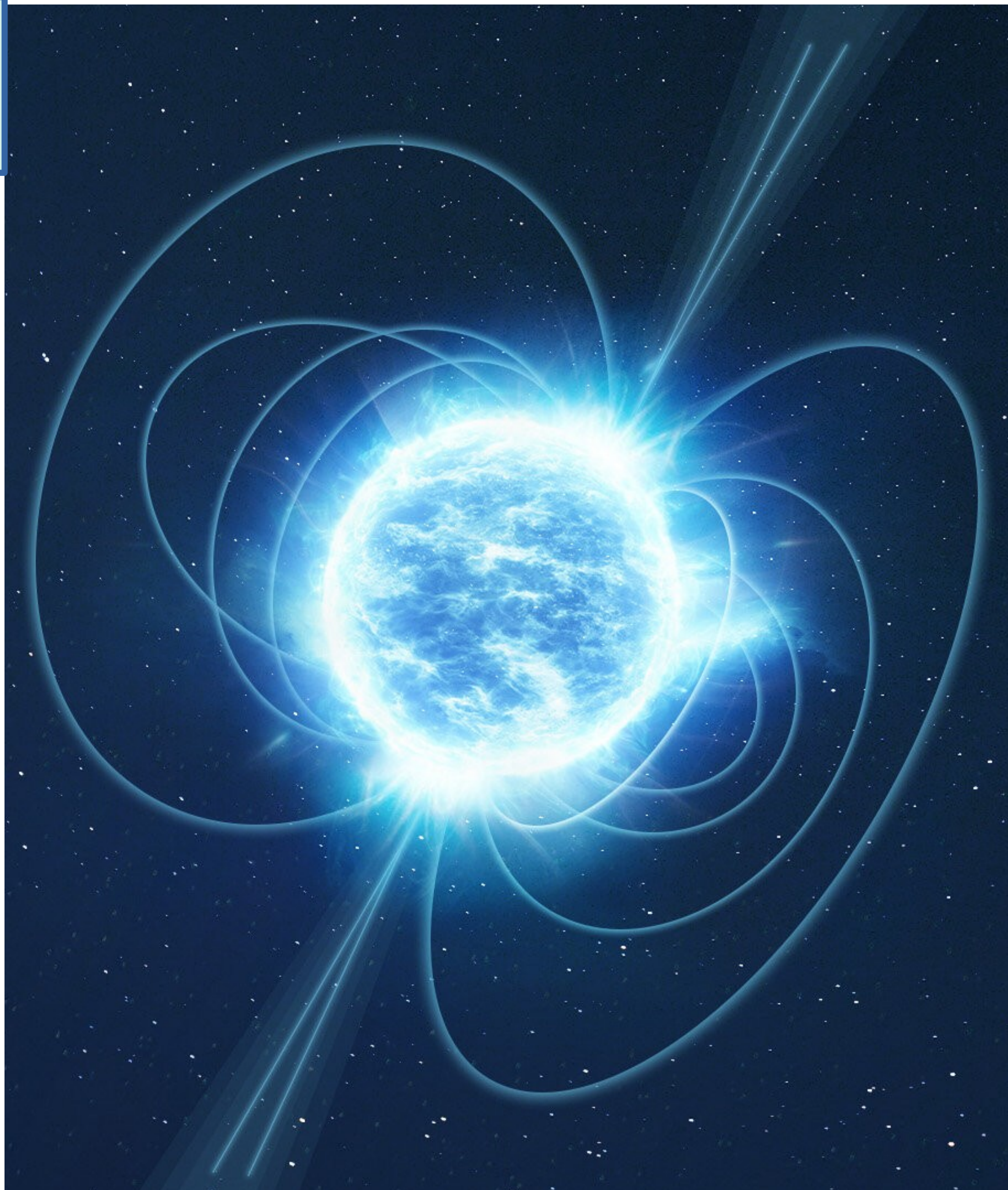
$$\rho \sim (\mathrm{GeV})^4$$

$$E \sim m_p c^2 \sim \mathrm{GeV}$$

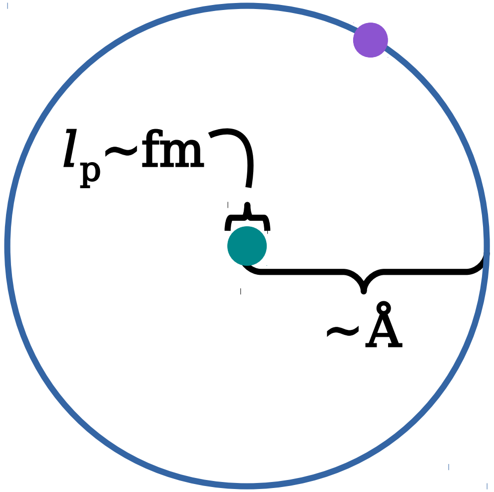
filling an atom



$$\frac{\hbar c}{l_p} \sim \mathrm{GeV}$$



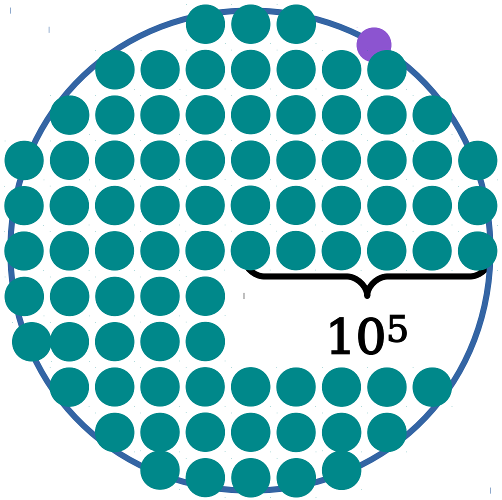
filling an atom

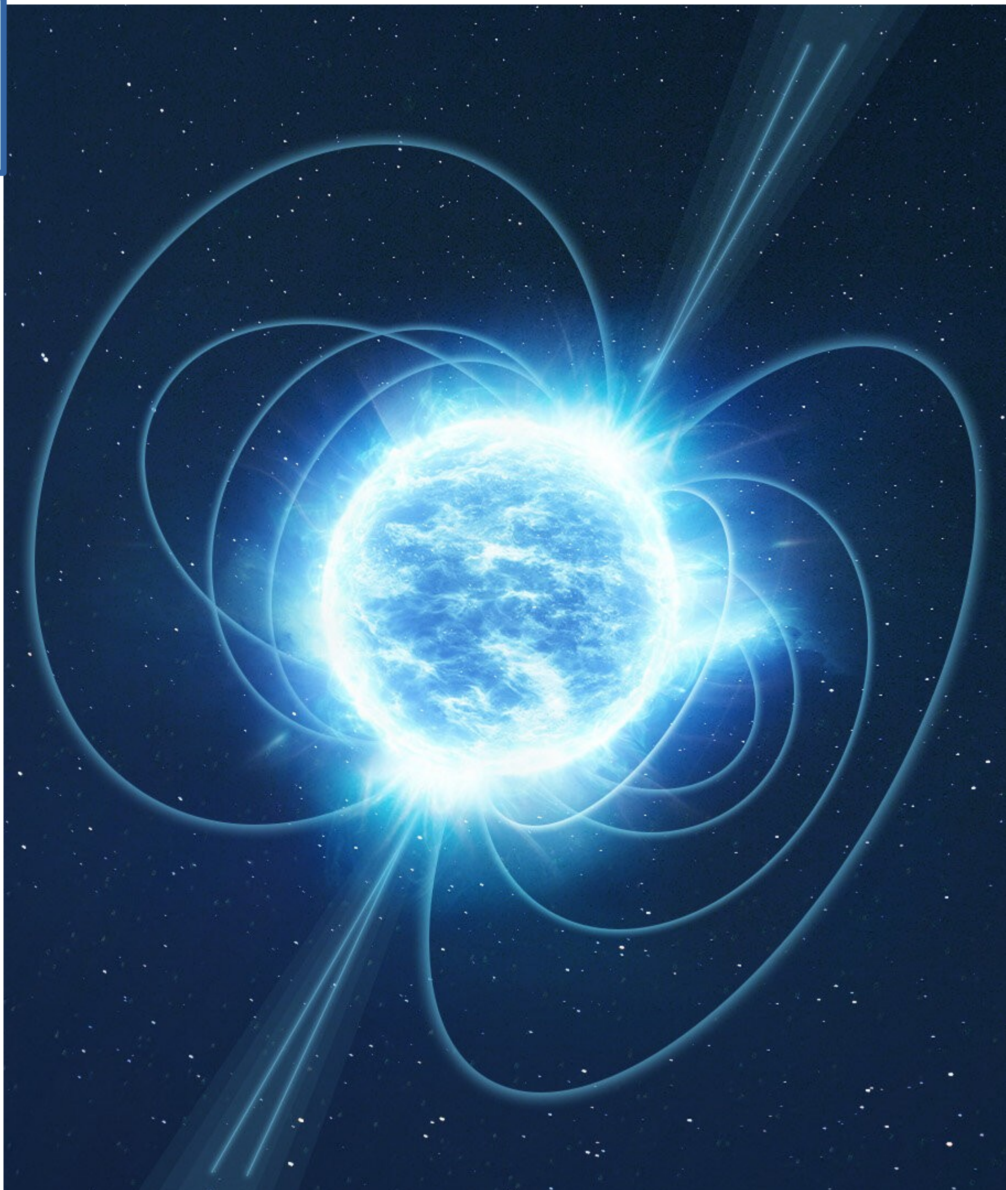


$$\frac{\hbar c}{l_p} \sim \text{GeV}$$

relative density

$$\sim (10^5)^3 = 10^{15}$$





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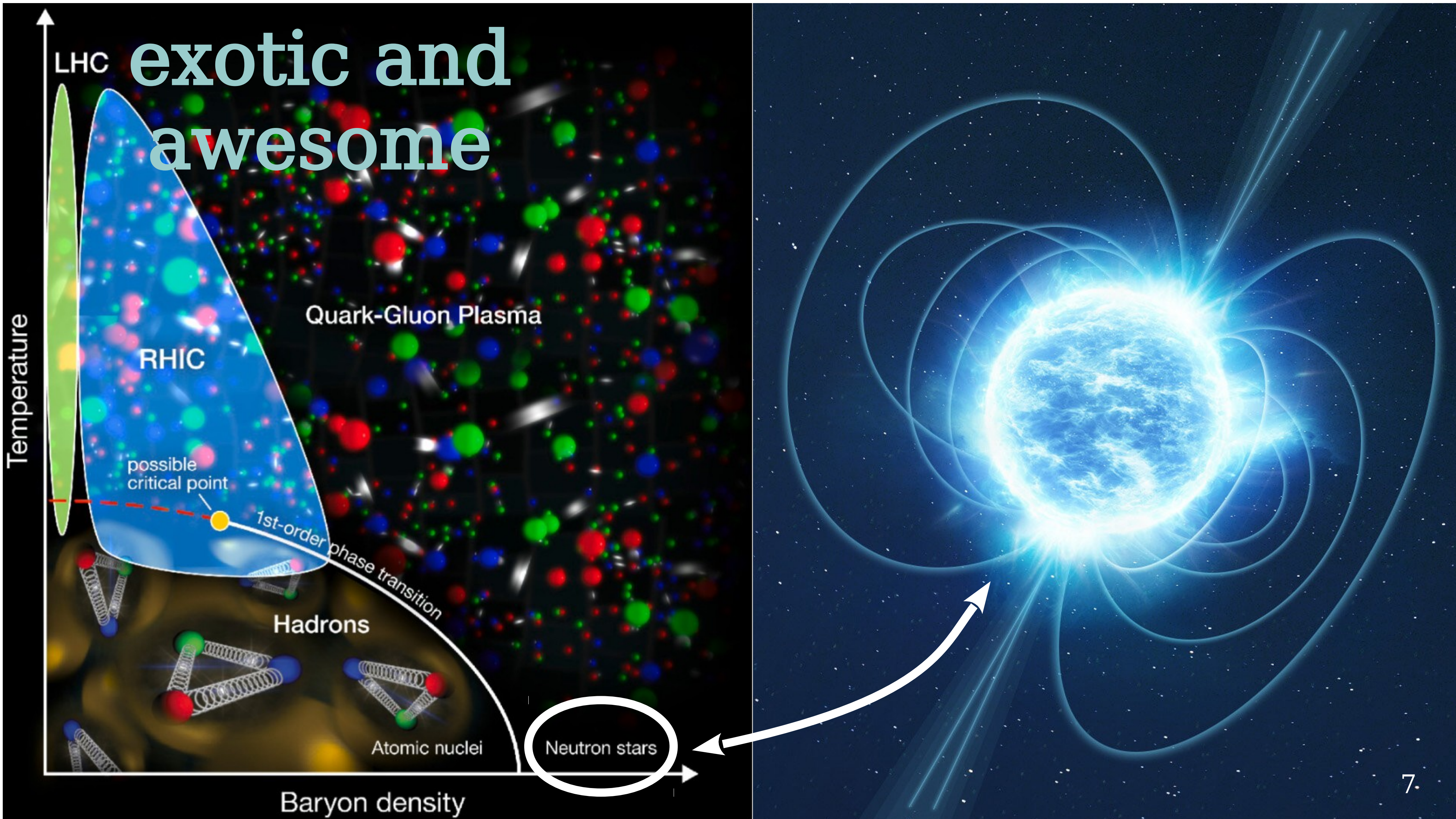
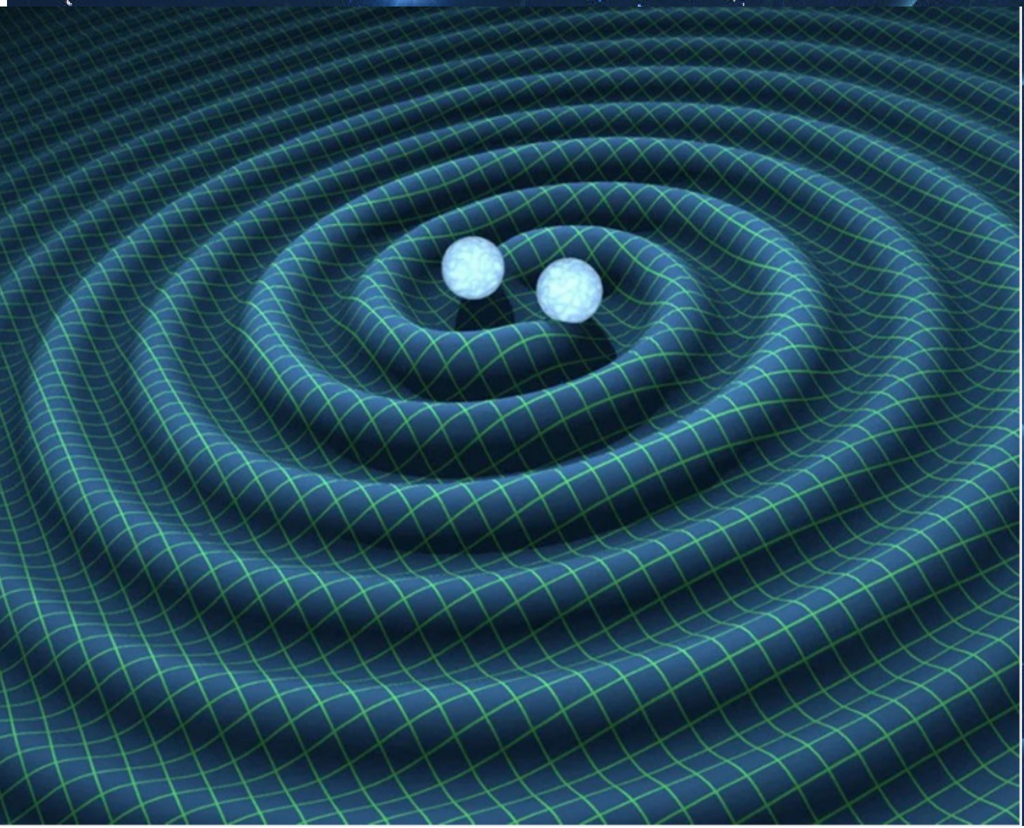
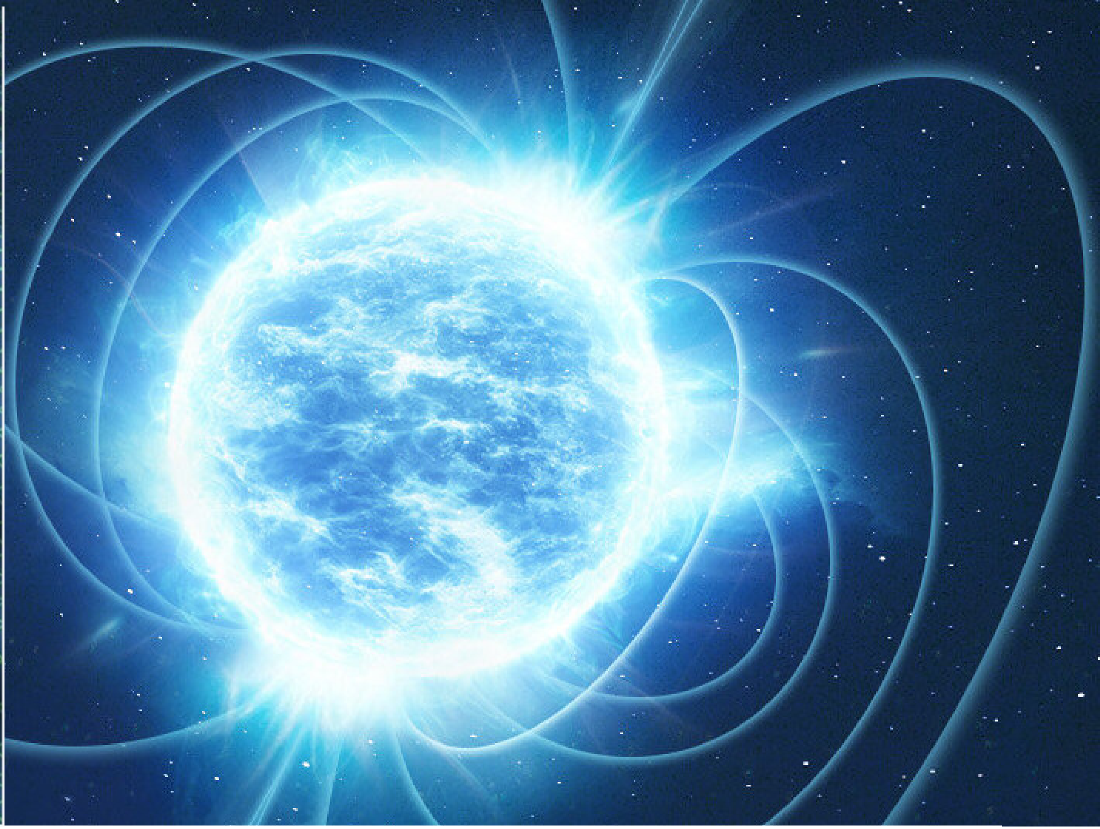
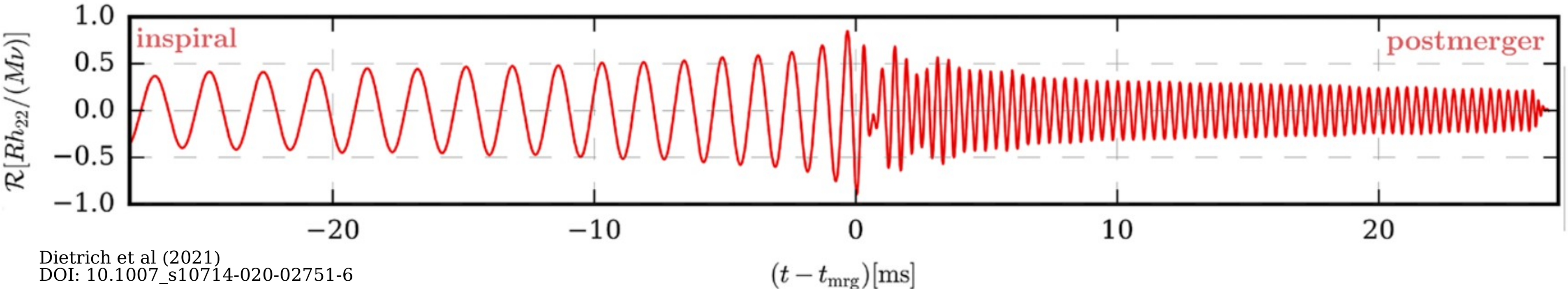
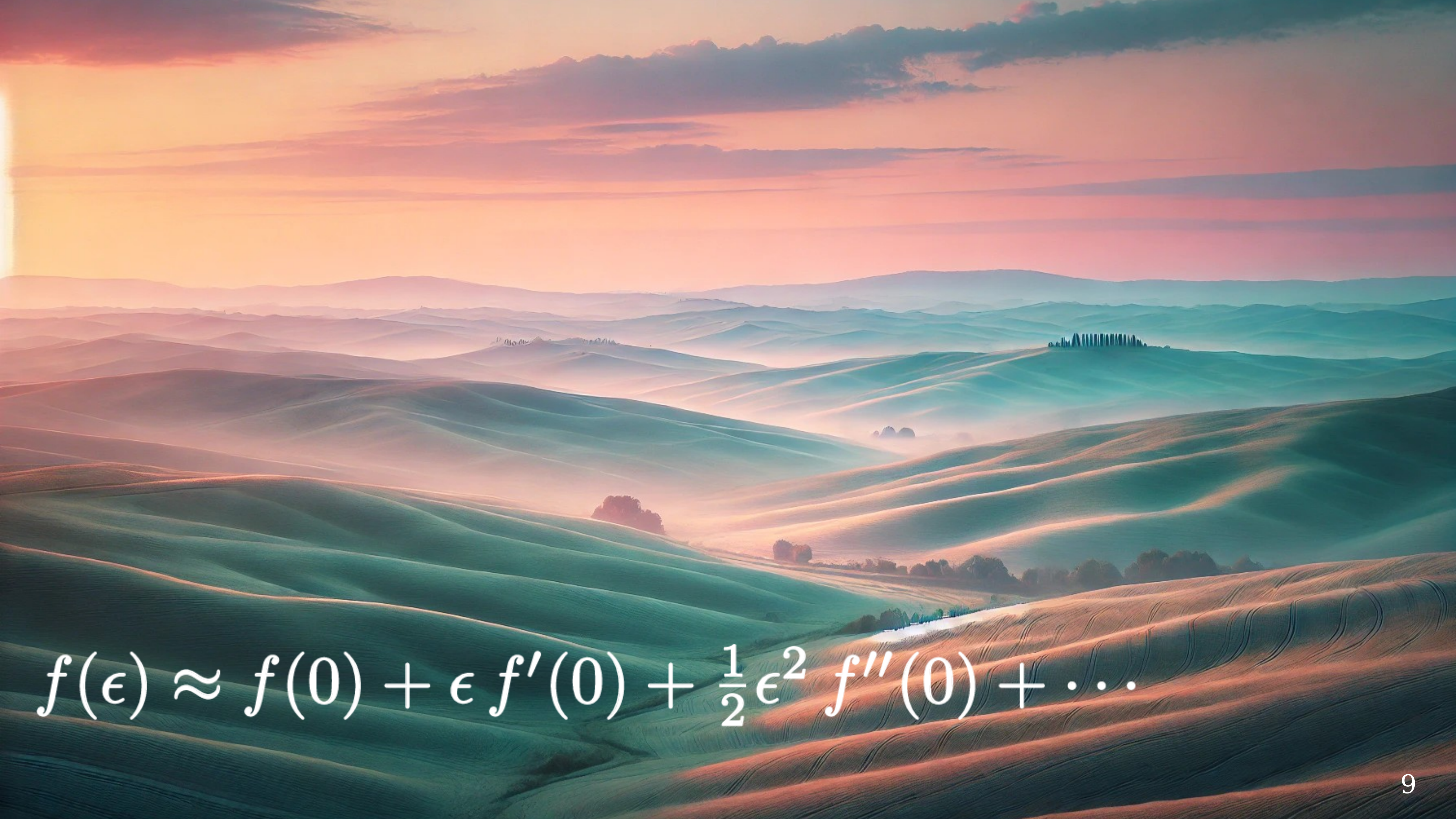








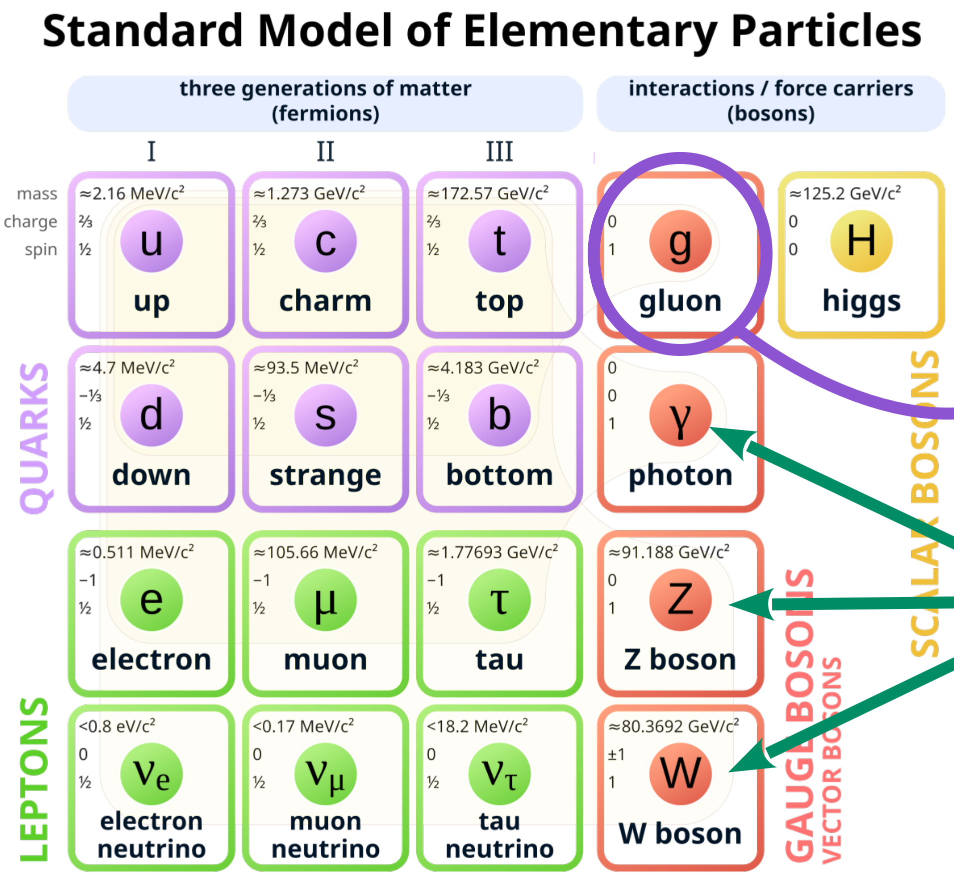
Dietrich et al (2021)
DOI: 10.1007_s10714-020-02751-6



$$f(\epsilon) \approx f(0) + \epsilon\, f'(0) + \frac{1}{2}\epsilon^2\, f''(0) + \cdots$$

$$f(g) = e^{-1/g^2}$$

The wild child of the Standard Model

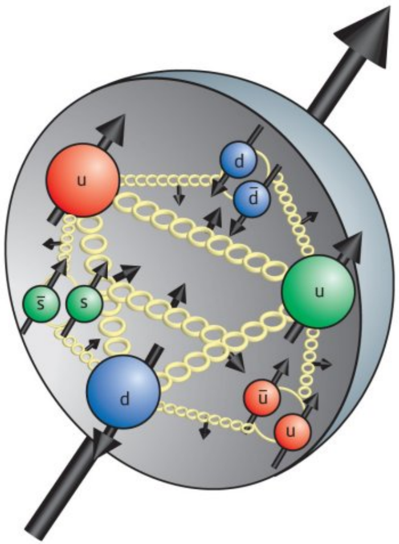


nonperturbative

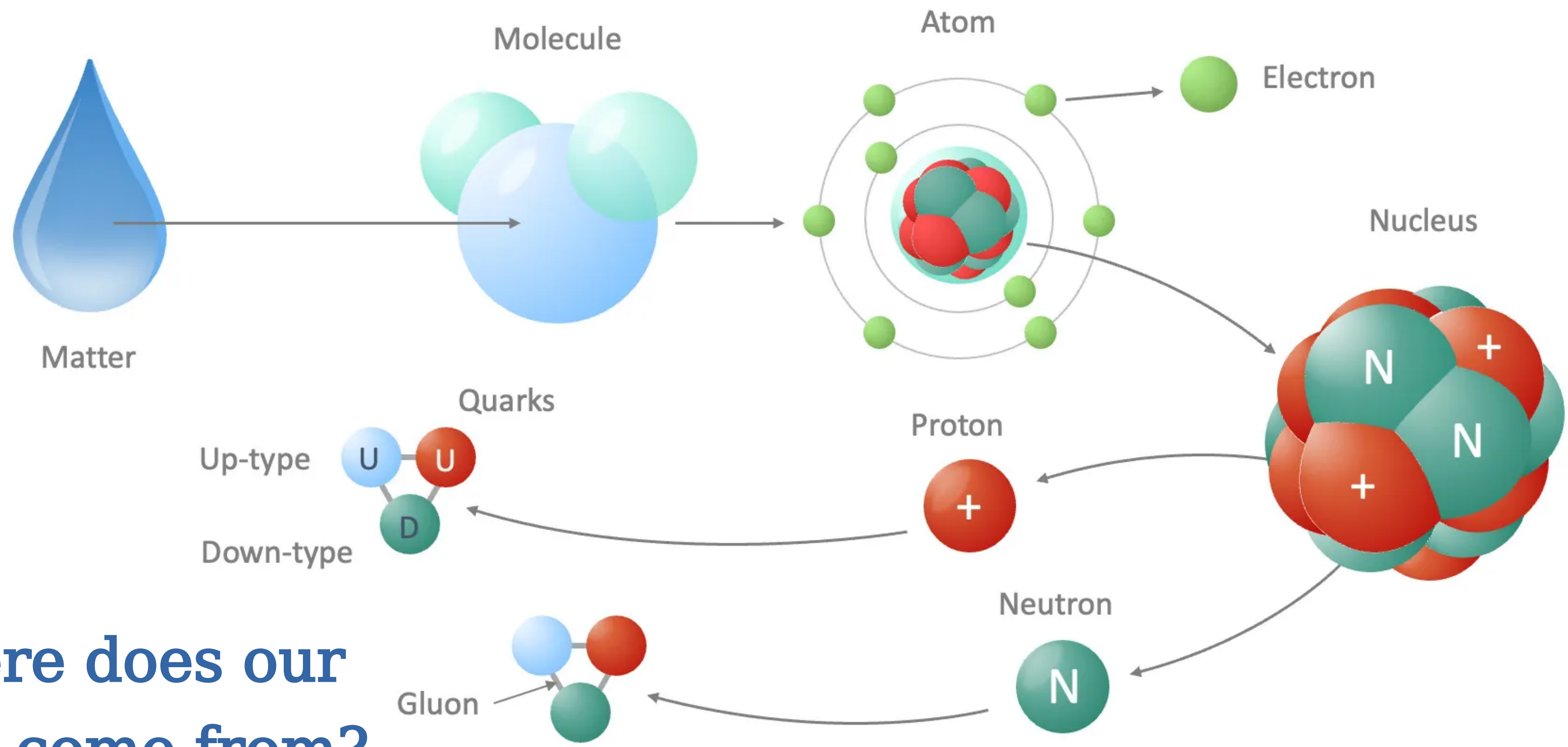
(at the scale of a proton)

$\Lambda_{\mathrm{QCD}} \sim m_p \sim \mathrm{GeV}$

perturbative



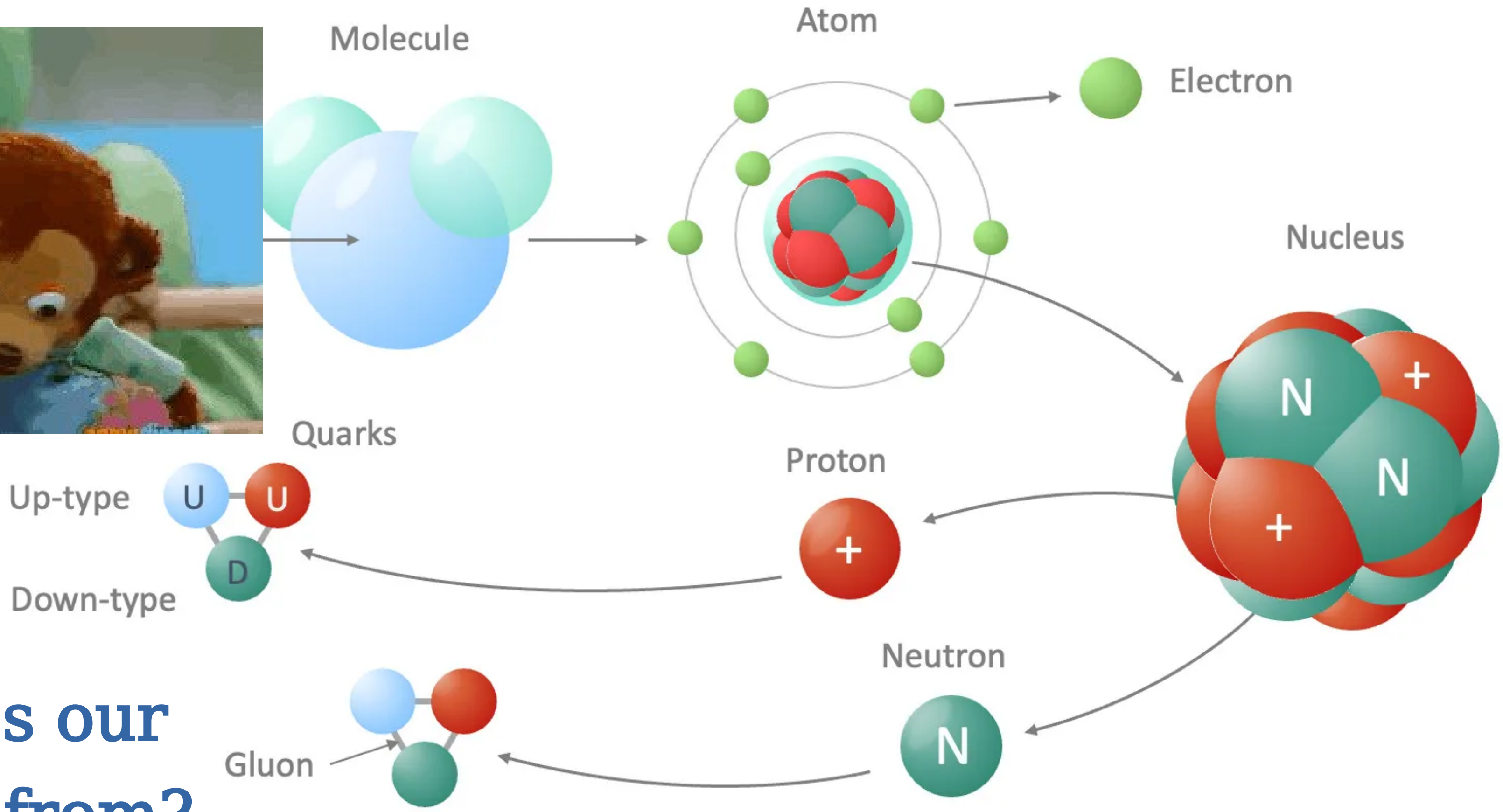
Matter from Molecule to Quark



Where does our mass come from?

Designed by PoweredTemplate

Matter from Molecule to Quark



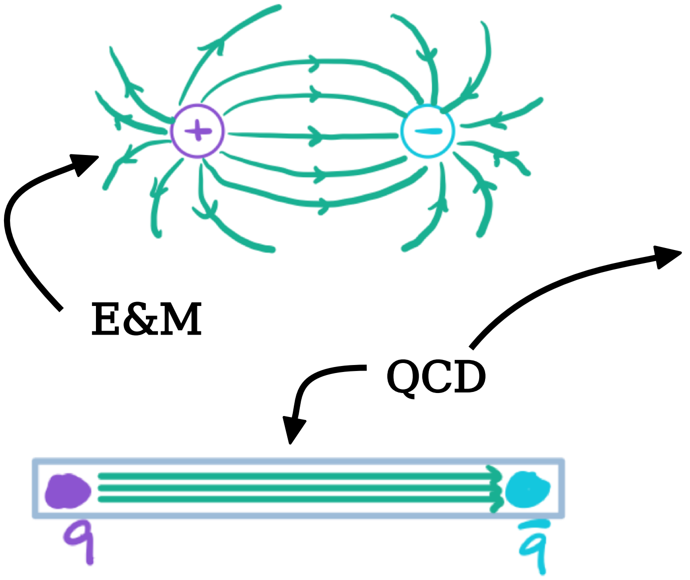
Where does our mass come from?

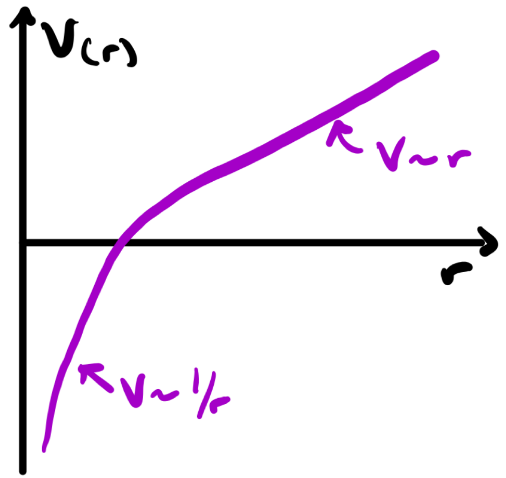
Most striking feature of QCD is confinement

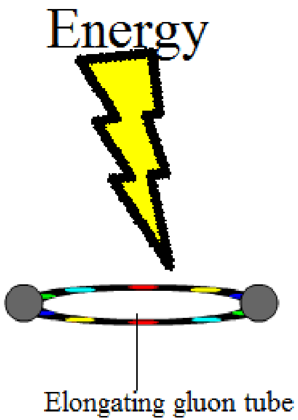
⇒ Inherently a strongly coupled (nonperturbative) phenomenon

⇒ A 50+ year old problem

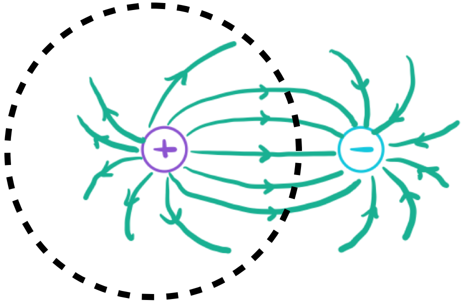
The wild child of the Standard Model

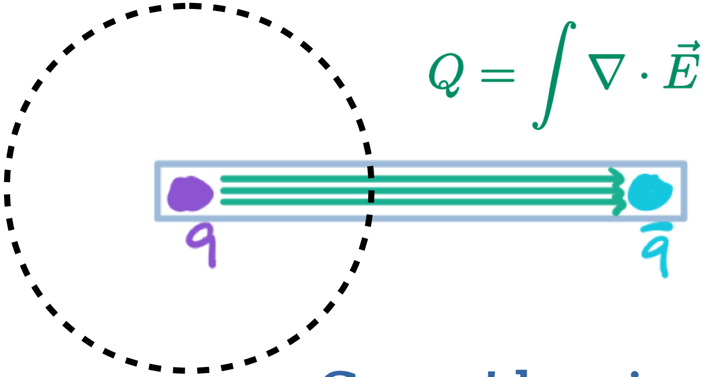




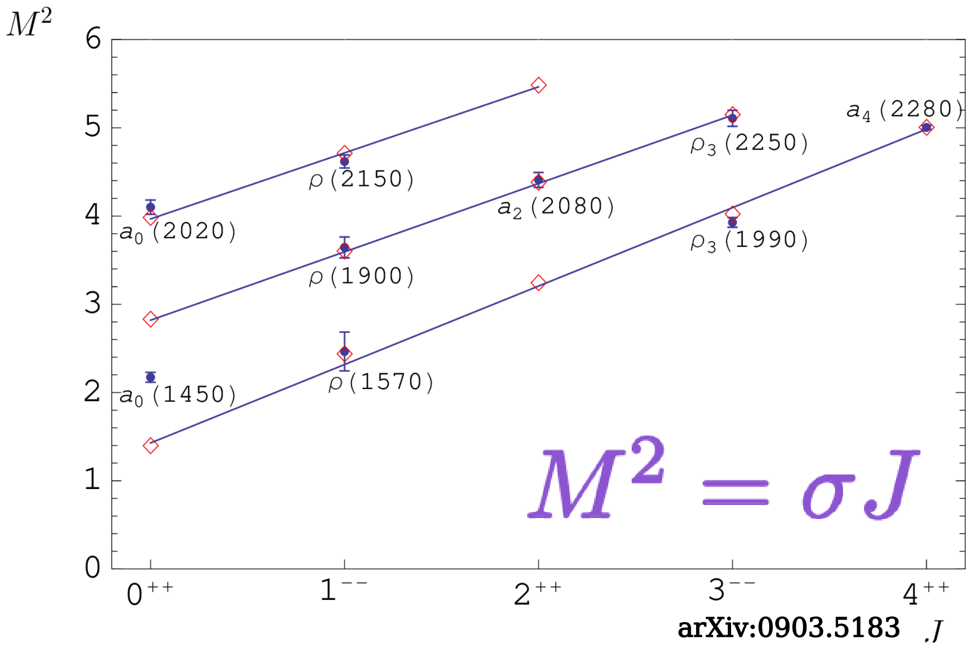


Linearly confining potential





$$Q = \int \nabla \cdot \vec{E} = \int \vec{E} \cdot d\vec{A}$$



arXiv:0903.5183

Gauss' law implies constant E-field

$$V(r) = \sigma r$$

$$\sigma \sim \Lambda_{\mathrm{QCD}}^2 \sim \mathrm{GeV}^2$$

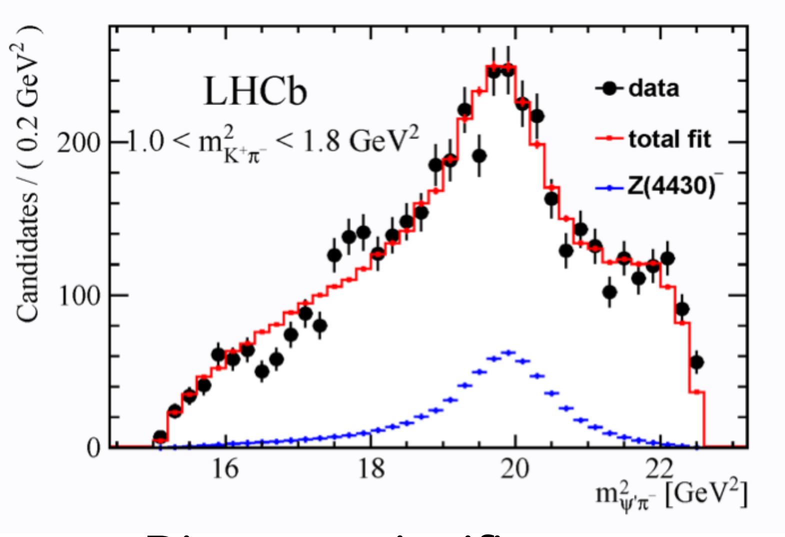
The mysteries are more than just a proliferation of $\sim$'s

The mysteries are more than just a proliferation of $\sim$'s

How many $\boxed{\text{new}}$ particles have been discovered at the LHC?

qualitative mysteries

Tetraquark Z(4430)



<u>Discovery significance</u>

Belle 2008: 6.5σ

LHCb 2014: 13.9σ

The 23 exotic hadrons discovered at the LHC

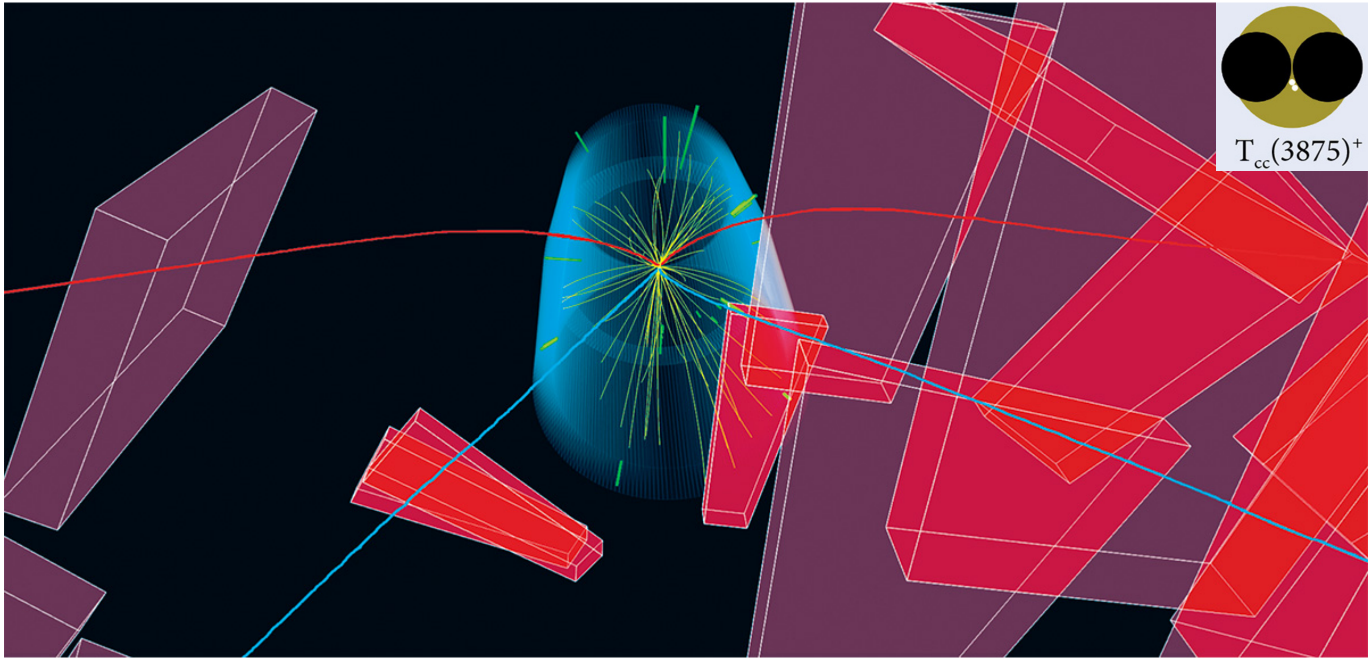
Ordered by mass

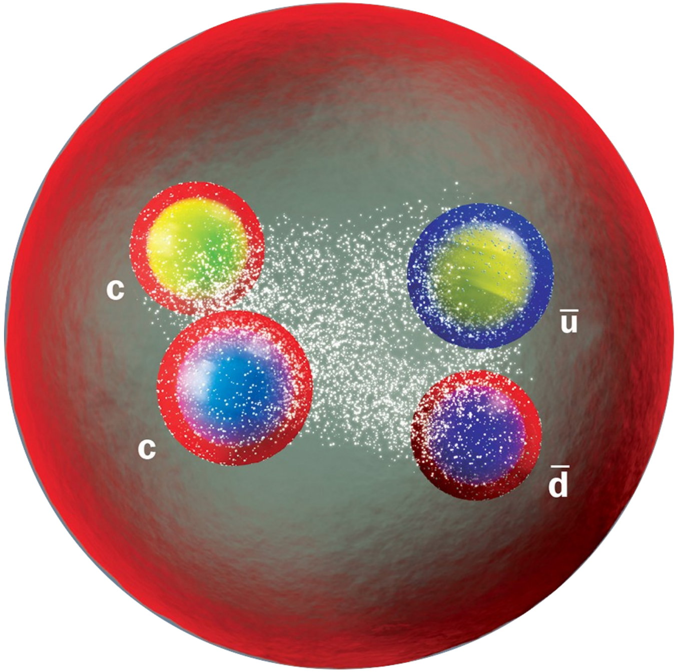
Exotic state		J^P	Mass [MeV]	Width [MeV]	Discovery reference
$T^*_{cs0}(2870)^0$	$c\bar{d}s\bar{u}$	0^+	2866 ± 7	57 ± 13	LHCb 2020 *Phys. Rev. D* **102** 112003
$T^*_{c\bar{s}0}(2900)^0$	$c\bar{s}\bar{u}d$	0^+	2892 ± 21	119 ± 29	LHCb 2023 *Phys. Rev. Lett.* **131** 041902
$T^*_{cs1}(2900)^0$	$c\bar{d}s\bar{u}$	1^-	2904 ± 5	110 ± 12	LHCb 2020 *Phys. Rev. D* **102** 112003
$T^*_{c\bar{s}0}(2900)^{++}$	$c\bar{s}u\bar{d}$	0^+	2921 ± 26	137 ± 36	LHCb 2023 *Phys. Rev. Lett.* **131** 041902
$T_{cc}(3875)^+$	$cc\bar{u}\bar{d}$		3874.83 ± 0.11	0.41 ± 0.17	LHCb 2022 *Nature Phys.* **18** 751
$\chi_{c0}(3960)$	$c\bar{c}(s\bar{s})$	0^+	3956 ± 11	43 ± 15	LHCb 2023 *Phys. Rev. Lett.* **131** 071901
$T_{c\bar{c}\bar{s}1}(4000)^0$	$c\bar{c}d\bar{s}$	1^+	3991^{+15}_{-20}	105 ± 34	LHCb 2023 *Phys. Rev. Lett.* **131** 131901
$T_{c\bar{c}\bar{s}1}(4000)^+$	$c\bar{c}u\bar{s}$	1^+	4003^{+7}_{-15}	131 ± 30	LHCb 2021 *Phys. Rev. Lett.* **127** 082001
$\chi_{c1}(4010)$	$c\bar{c}(q\bar{q})$	1^+	$4012.5^{+5.5}_{-5.4}$	63 ± 9	LHCb 2024 *Phys. Rev. Lett.* **133** 131902
$\chi_{c1}(4140)$	$c\bar{c}(s\bar{s})$	1^+	4148 ± 7	28^{+24}_{-22}	CMS 2014 *Phys. Lett. B* **734** 261
$T_{c\bar{c}\bar{s}1}(4220)^+$	$c\bar{c}u\bar{s}$	1^+	4220^{+50}_{-40}	233^{+110}_{-90}	LHCb 2021 *Phys. Rev. Lett.* **127** 082001
$\chi_{c1}(4274)$	$c\bar{c}(s\bar{s})$	1^+	4273^{+19}_{-9}	56^{+14}_{-16}	LHCb 2017 *Phys. Rev. Lett.* **118** 022003
$P_{c\bar{c}}(4312)^+$	$c\bar{c}uud$		4312^{+7}_{-1}	$9.8^{+4.6}_{-5.2}$	LHCb 2019 *Phys. Rev. Lett.* **122** 222001
$P_{c\bar{c}s}(4338)^0$	$c\bar{c}sud$	$1/2^-$	4338.2 ± 0.8	7 ± 1.8	LHCb 2023 *Phys. Rev. Lett.* **131** 031901
$P_{c\bar{c}}(4380)^+$	$c\bar{c}uud$		4380 ± 30	205 ± 88	LHCb 2015 *Phys. Rev. Lett.* **115** 072001
$P_{c\bar{c}}(4440)^+$	$c\bar{c}uud$		4440^{+4}_{-5}	21^{+10}_{-11}	LHCb 2019 *Phys. Rev. Lett.* **122** 222001
$P_{c\bar{c}}(4457)^+$	$c\bar{c}uud$		4457^{+4}_{-2}	$6.4^{+6}_{-2.8}$	LHCb 2019 *Phys. Rev. Lett.* **122** 222001
$\chi_{c0}(4500)$	$c\bar{c}(s\bar{s})$	0^+	4506^{+16}_{-19}	92 ± 30	LHCb 2017 *Phys. Rev. Lett.* **118** 022003
$X(4630)$	$c\bar{c}(s\bar{s})$		4630^{+20}_{-110}	174^{+137}_{-78}	LHCb 2021 *Phys. Rev. Lett.* **127** 082001
$\chi_{c1}(4685)$	$c\bar{c}(s\bar{s})$	1^+	4684^{+15}_{-17}	130 ± 40	LHCb 2021 *Phys. Rev. Lett.* **127** 082001
$\chi_{c0}(4700)$	$c\bar{c}(s\bar{s})$	0^+	4704^{+17}_{-26}	120^{+52}_{-45}	LHCb 2017 *Phys. Rev. Lett.* **118** 022003
$T_{c\bar{c}c\bar{c}}(6600)$	$c\bar{c}c\bar{c}$		6552 ± 16	124^{+46}_{-42}	CMS 2024 *Phys. Rev. Lett.* **132** 111901
$T_{c\bar{c}c\bar{c}}(6900)$	$c\bar{c}c\bar{c}$		6886 ± 16	168 ± 76	LHCb 2020 *Sci. Bull.* **65** 1983

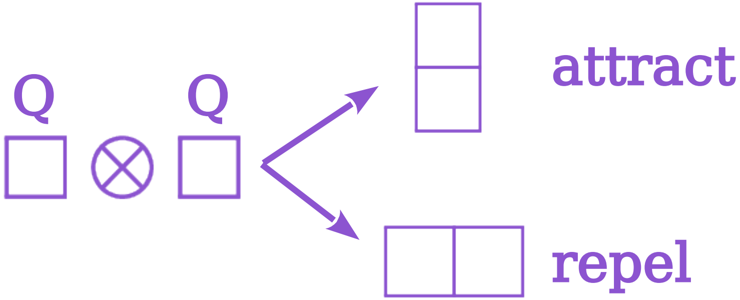
double hidden-charm tetraquark; double open-charm tetraquark; hidden-charm pentaquark; hidden-charm tetraquark; open-charm tetraquark

Data correspond to the discovery paper with selected additions

In QCD, two like charges...

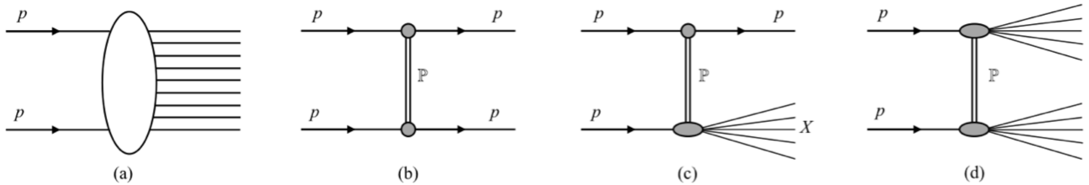


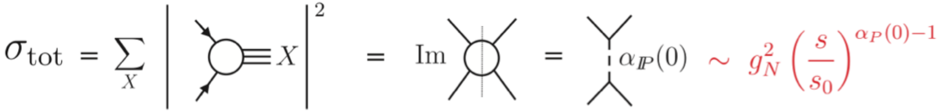




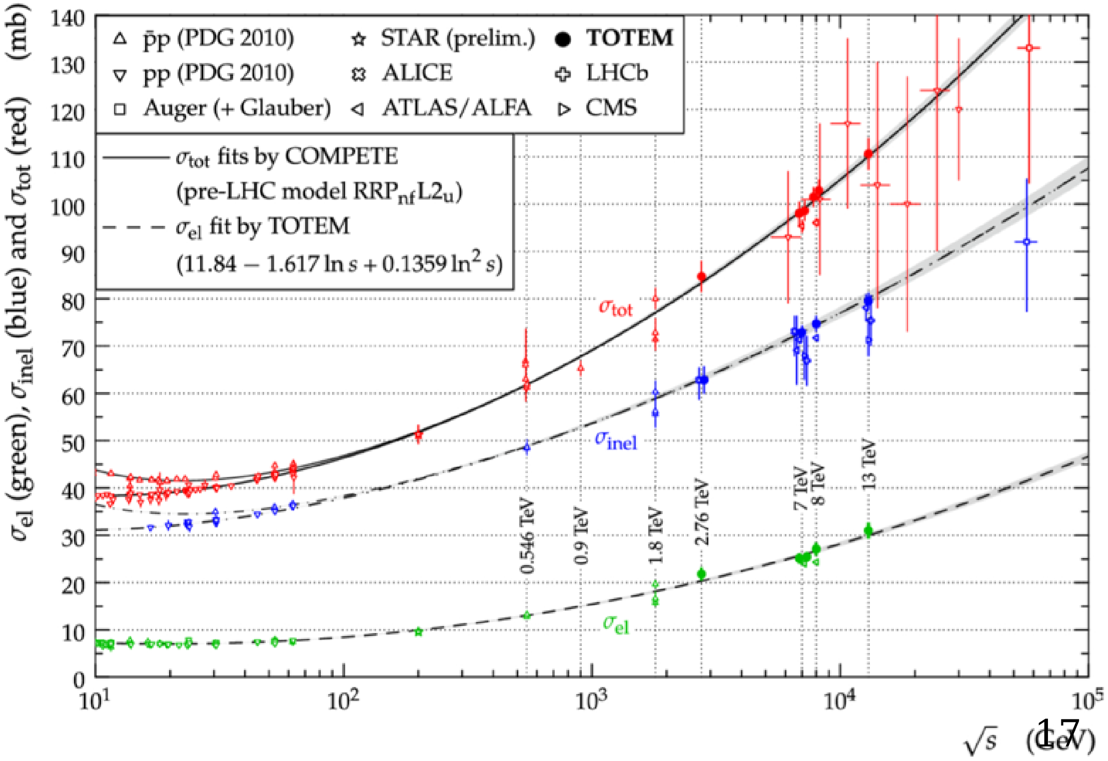
exotic bound states not predicted by the quark model

Dominantly soft (~GeV) exchange

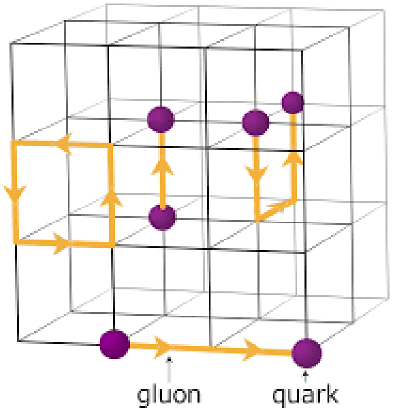




Pomeron modeled phenomenologically @ LHC



Current state-of-the-art: Lattice MC

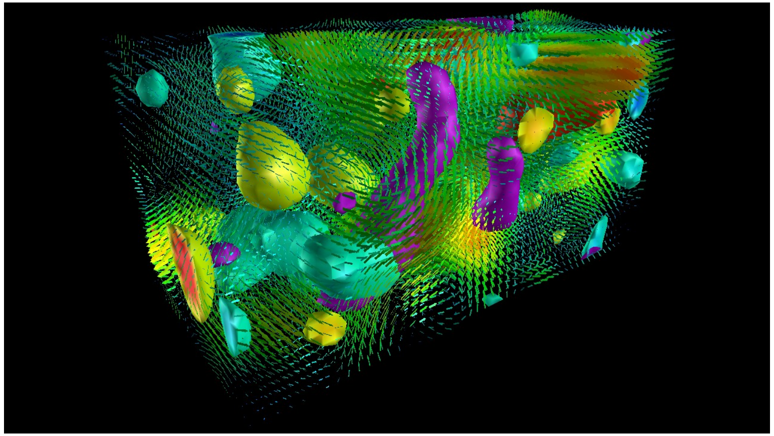


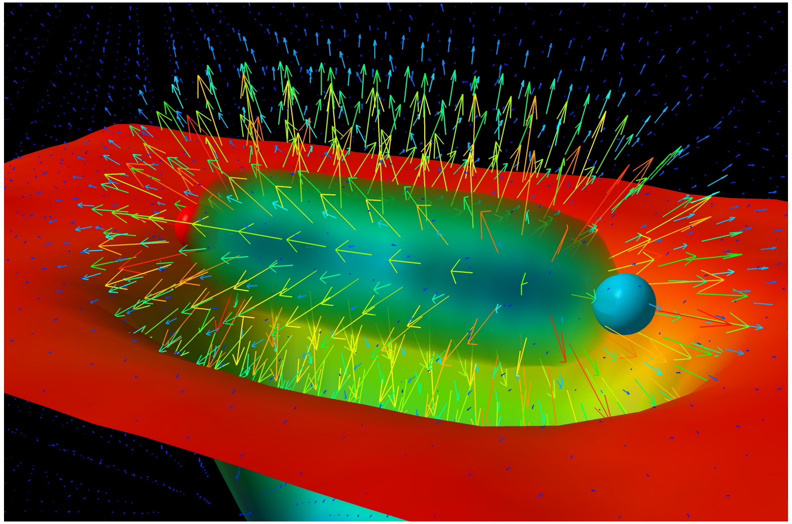
$$\int D\phi\, e^{iS[\phi]} \xrightarrow{\text{imaginary time}} \Rightarrow \int D\phi\, e^{-S_E[\phi]}$$

✓ **General nonperturbative method**

✓ **Tremendously successful**

- ⇒ e.g. hadron spectroscopy
- ⇒ crucial for experimental analyses





Derek Leinweber http://www.physics.adelaide.edu.au

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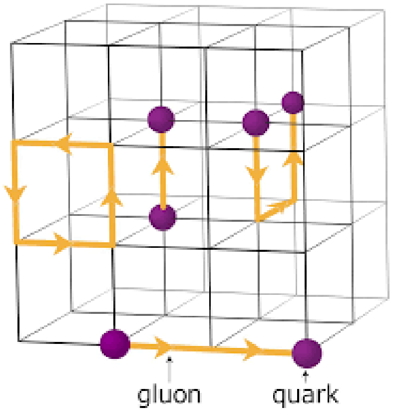

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imaginary time

$$\Rightarrow \int D\phi\, e^{-S_E[\phi]}$$

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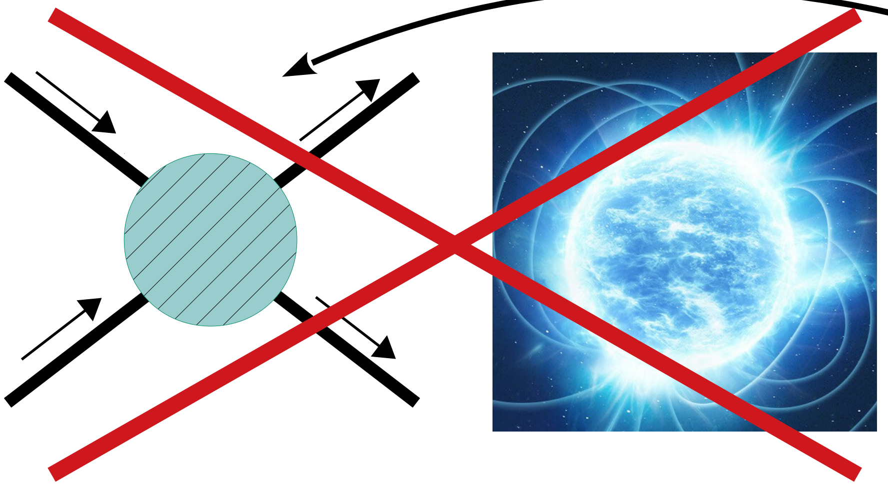
✗ **Inherently Euclidean**

- ⇒ No real time dynamics, e.g. scattering
- ⇒ “sign problem” → no chemical potential

✗ **No chiral fermions**

- ⇒ Can’t put the SM on the lattice!



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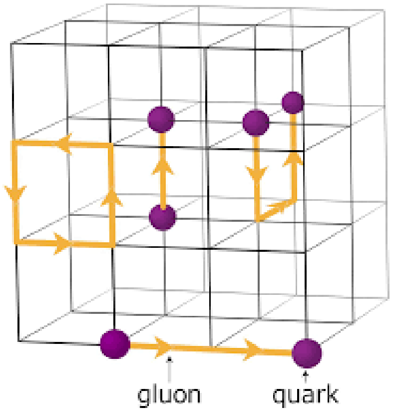

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NEED OTHER APPROACHES TO COMPLEMENT THE LATTICE!

$$P^\mu = \begin{pmatrix} H \\ \vec{P} \end{pmatrix}$$

Real time!

$$H = i\partial_t$$

Will present another approach:

Hamiltonian truncation

$$P^\mu = \begin{pmatrix} H \\ \vec{P} \end{pmatrix} \xrightarrow{\text{Real time!}} H = i\partial_t$$

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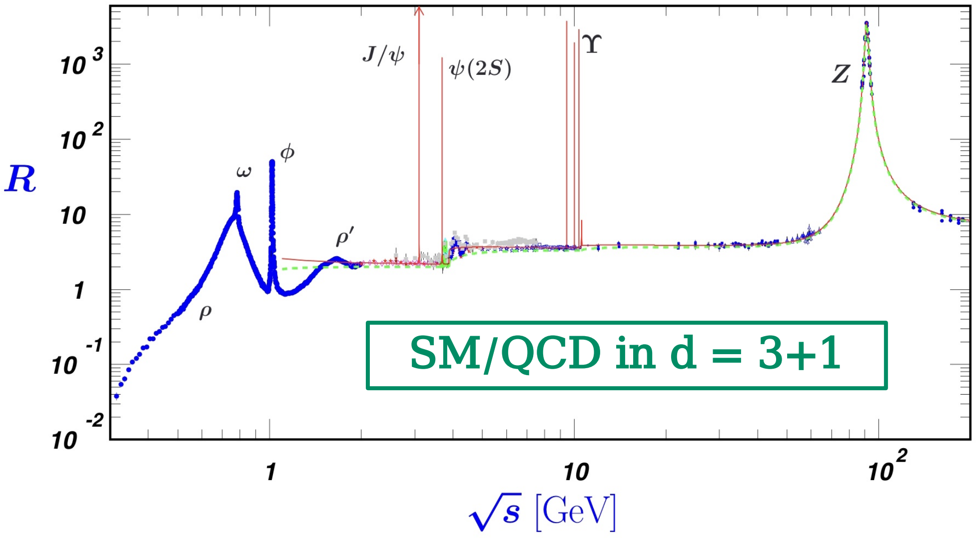
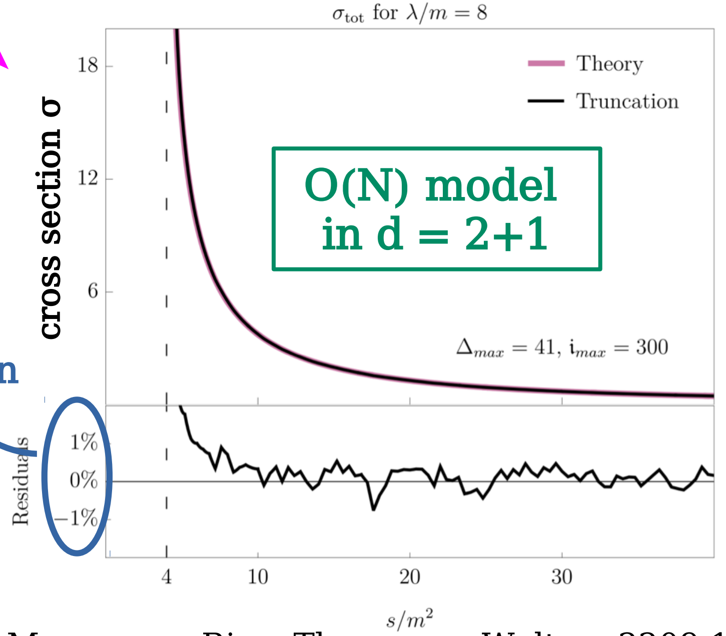
Where we want to go



Where we at

large coupling!

O(1%) precision



BH, Murayama, Riva, Thompson, Walters 2209.14306

$$P^\mu = \begin{pmatrix} H \\ \vec{P} \end{pmatrix} \xrightarrow{\text{Real time!}} H = i\partial_t$$

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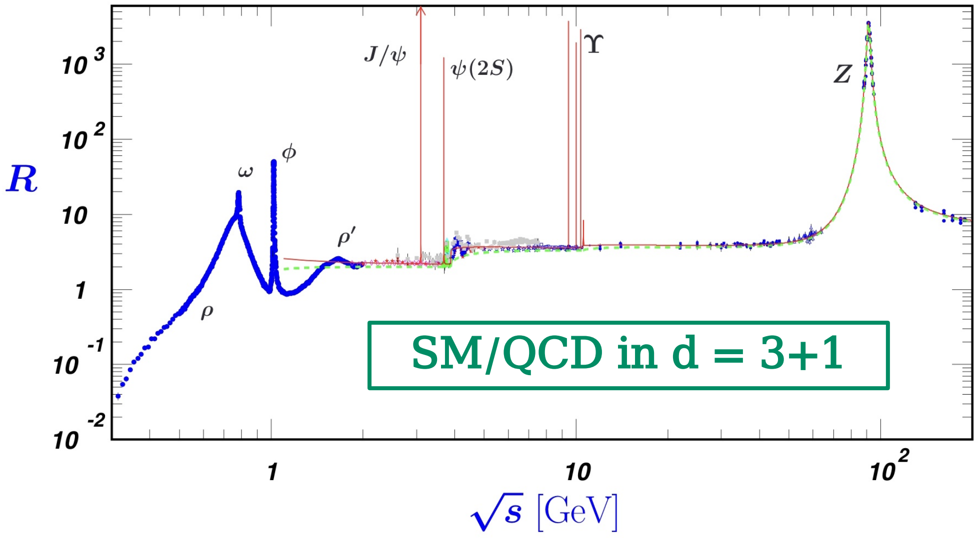


Is a theory collider absurd?

Putting the quantum in QFT

QFT = QM on an infinite # of d.o.f.

⇒ States live in a Hilbert space $\longrightarrow$ $|\psi\rangle \in \mathcal{H}$

⇒ They obey Schrödinger eqn $\longrightarrow$ $H\,|\psi_\alpha\rangle = E_\alpha\,|\psi_\alpha\rangle$

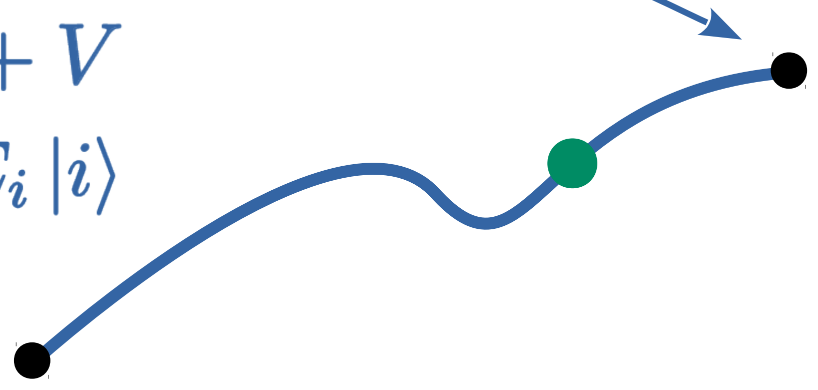
⇒ Operators act on states $\longrightarrow$ $\mathcal{O}(\hat{\phi}, \hat{\pi}),\ [\hat{\phi}, \hat{\pi}] \sim i$

the dumbest idea which might actually work

start from known system

$$H = H_0 + V$$

$$H_0 \, |i\rangle = E_i \, |i\rangle$$



deform with some relevant operator

$$V = \int d^{d-1}x \, \mathcal{O}_r(x)$$

$$\langle i | H_0 + V | j \rangle$$

compute matrix elements

diagonalize

result approximates true spectrum

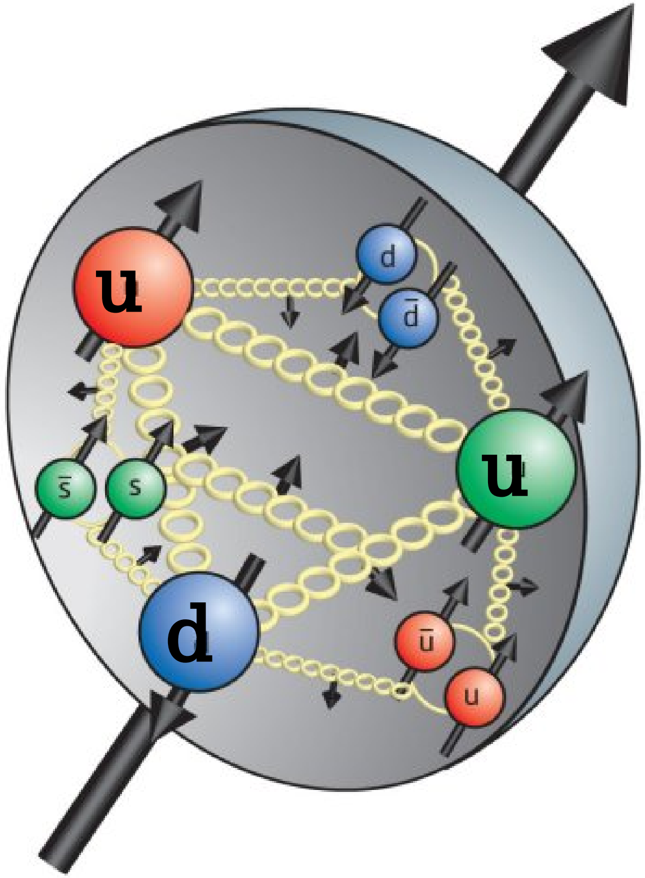
“Hamiltonian truncation”

HT output

$$|\psi\rangle = \sum_{\chi \in \mathcal{H}_{\mathrm{UV}}} c_\chi \, |\chi\rangle$$

e.g.

$$|p\rangle = c_{uud}\,|uud\rangle + \cdots$$



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discrete basis

putting theory
“in a box”

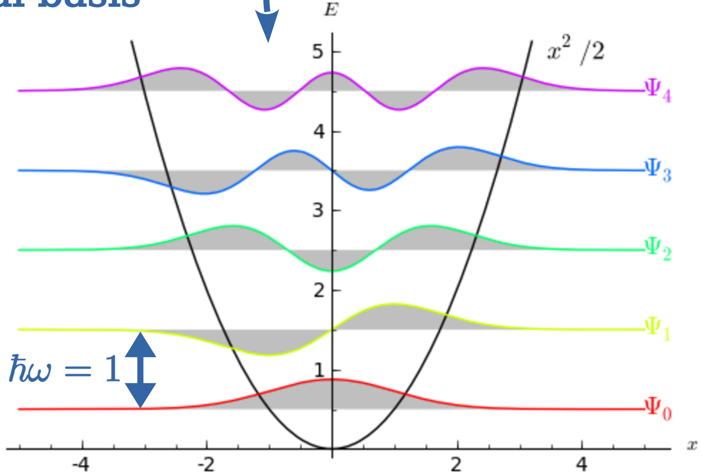
Example: anharmonic oscillator

$$H_0 = \frac{1}{2}p^2 + \frac{1}{2}x^2 \qquad V = \lambda x^4$$

$\Rightarrow$ perturbation theory doesn't converge (asymptotic series)

Example: anharmonic oscillator

initial basis



$\Delta E = \hbar\omega = 1$

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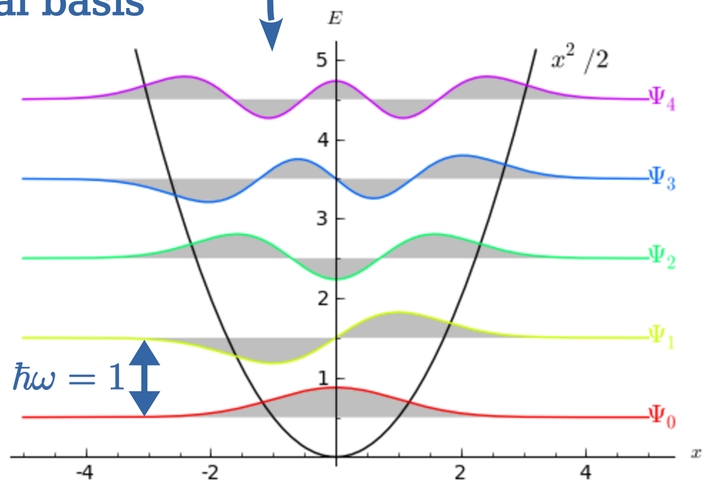
$$|0\rangle\,,\ |1\rangle\,,\ |2\rangle\,,\ldots \qquad H_0\,|n\rangle = \left(n + \frac{1}{2}\right)|n\rangle$$

Example: anharmonic oscillator

initial basis



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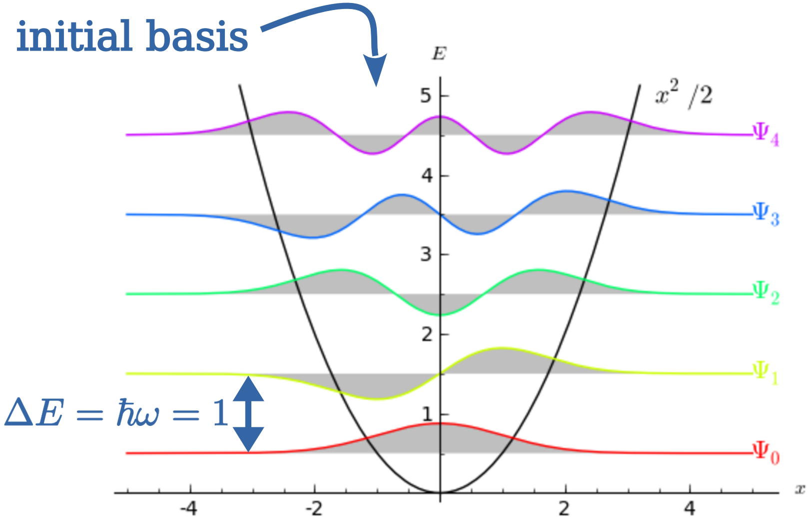

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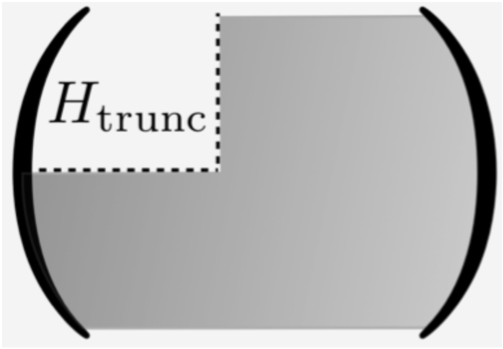
$$\langle m|V|n\rangle = \frac{\lambda}{4}\langle m|\left(a + a^\dagger\right)^4|n\rangle$$



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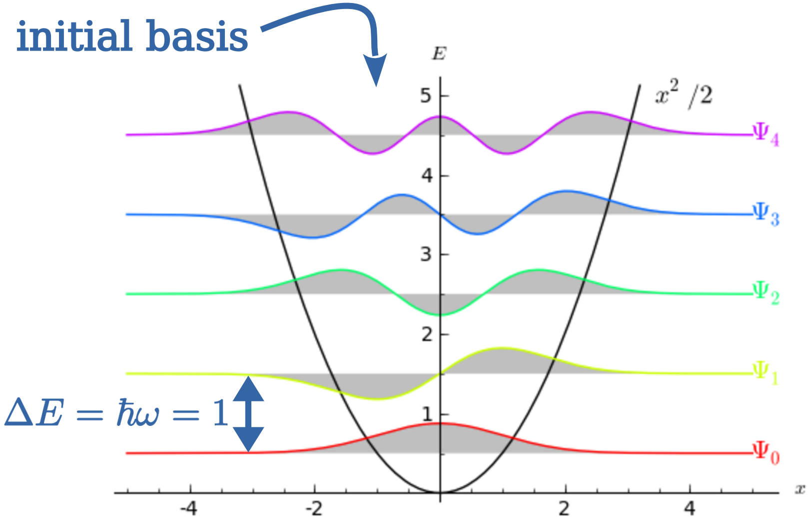

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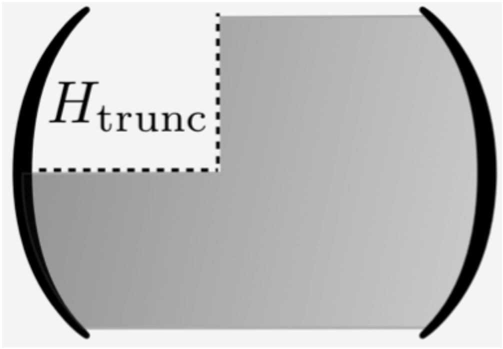
$\Rightarrow$ diagonalize



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$$H_0 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{5}{2} \end{pmatrix} \quad V = \lambda \begin{pmatrix} \frac{3}{4} & \frac{3}{\sqrt{2}} \\ \frac{3}{\sqrt{2}} & \frac{39}{4} \end{pmatrix}$$

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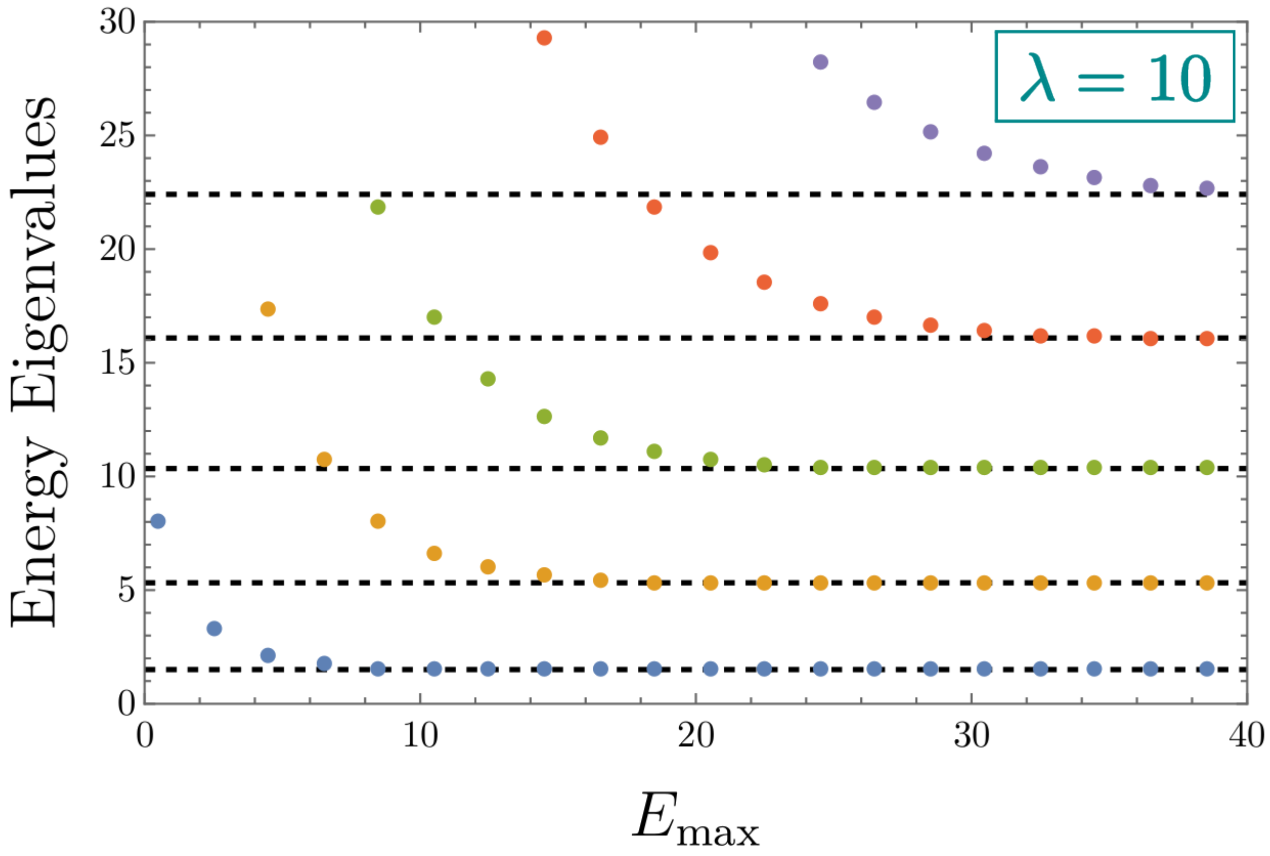
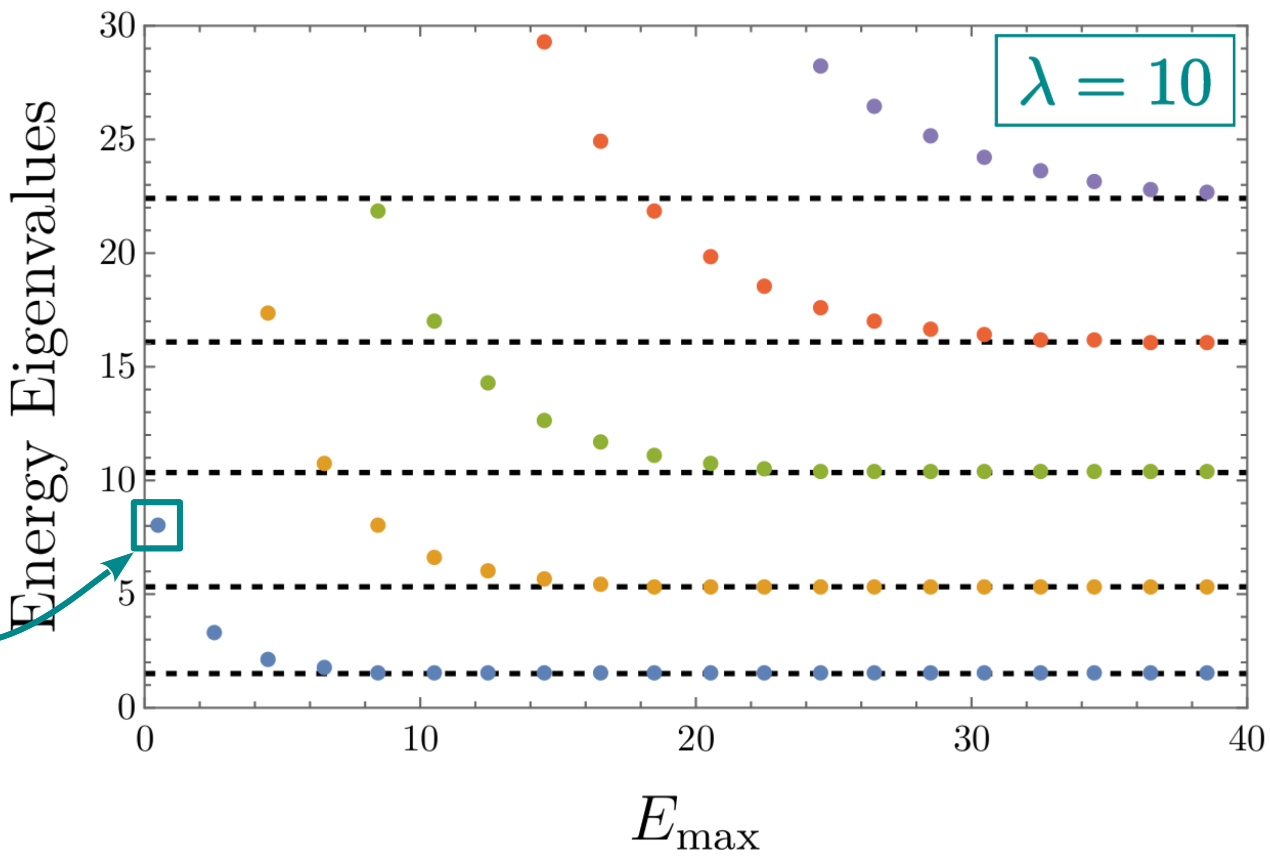
$\Rightarrow$ diagonalize $\Rightarrow$ eigenvalues: $\dfrac{3}{2} + \dfrac{21}{4}\lambda \pm \sqrt{1 + \dfrac{9}{4}\lambda(11\lambda + 4)}$

Example: anharmonic oscillator



Example: anharmonic oscillator

$$\frac{1}{2} + \frac{3}{4}\lambda = 8$$



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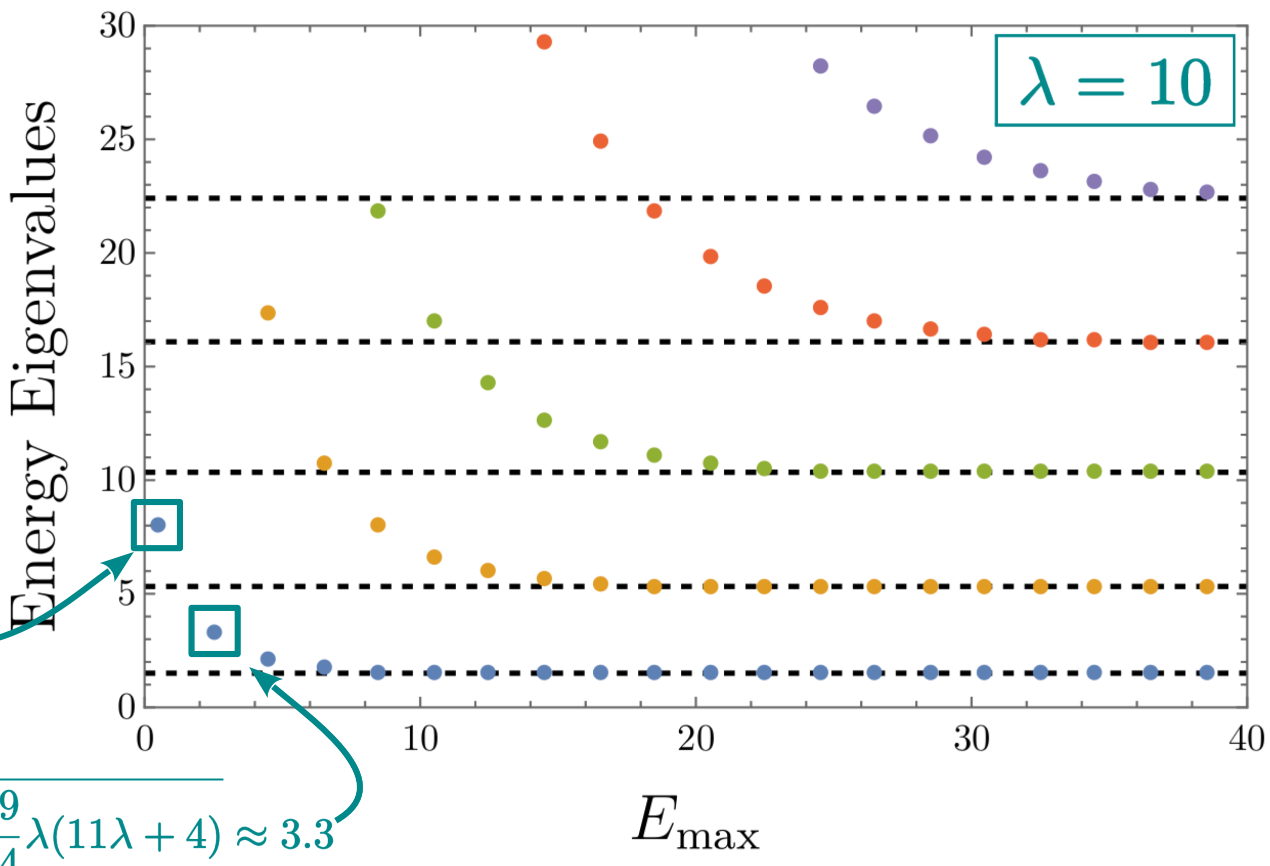

$$\frac{1}{2} + \frac{3}{4}\lambda = 8$$

$$\frac{3}{2} + \frac{21}{4}\lambda - \sqrt{1 + \frac{9}{4}\lambda(11\lambda + 4)} \approx 3.3$$

Example: anharmonic oscillator

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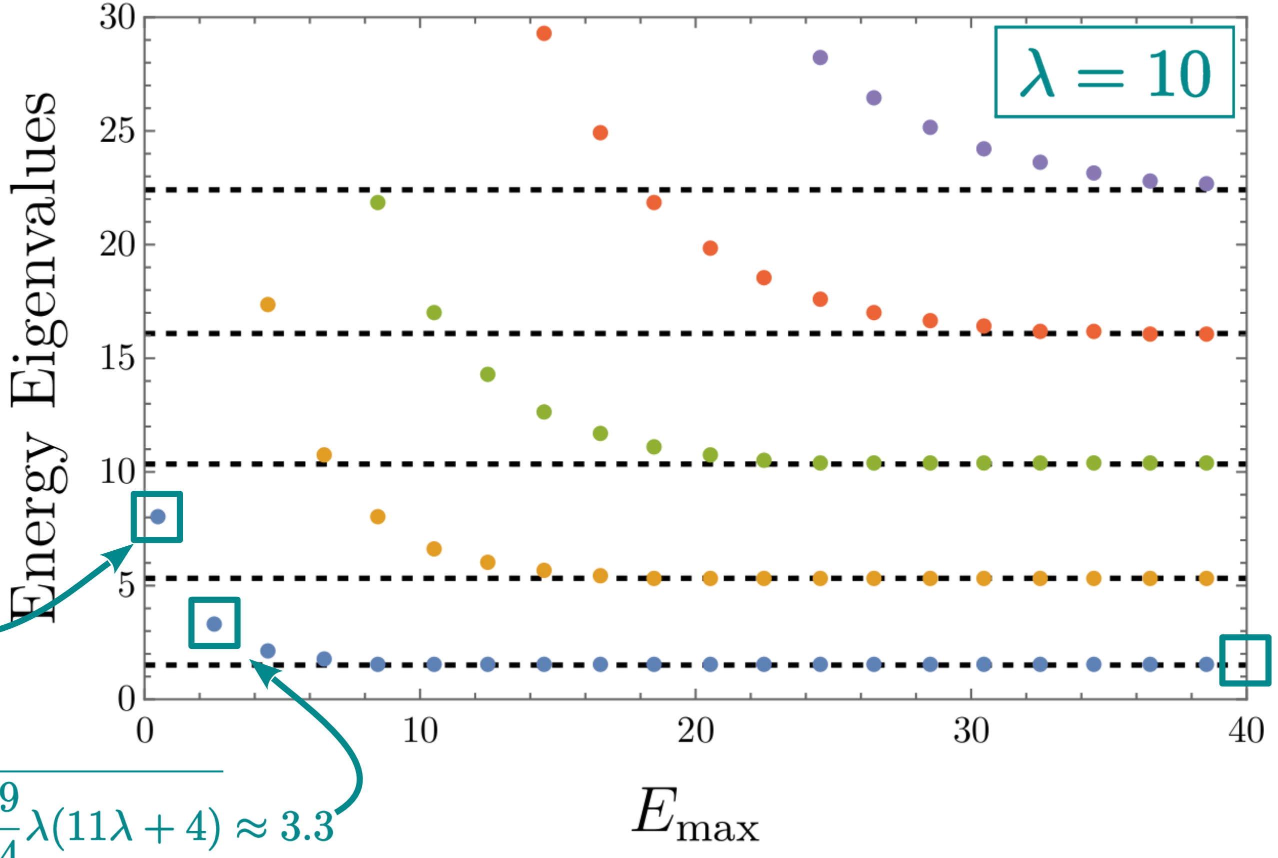
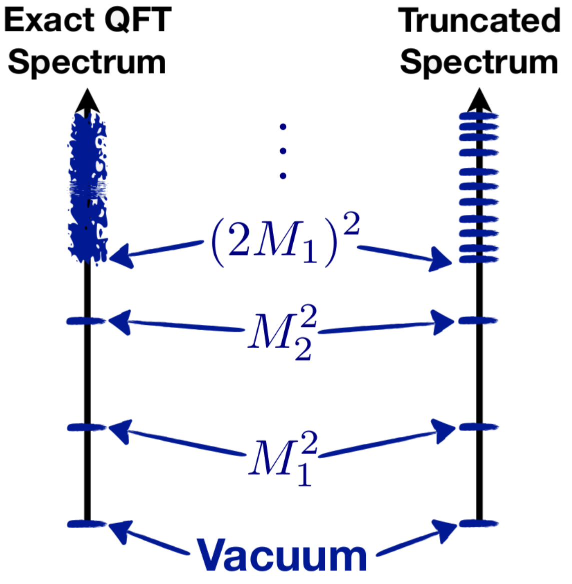


$$E_{\text{exact}} \approx 1.5$$

$$\frac{3}{2}+\frac{21}{4}\lambda-\sqrt{1+\frac{9}{4}\lambda(11\lambda+4)} \approx 3.3$$

HT output



$$|\psi\rangle = \sum_{\chi \in \mathcal{H}_{\mathrm{UV}}} c_\chi \, |\chi\rangle$$

basis choice?

truncated sum

→ convergence?

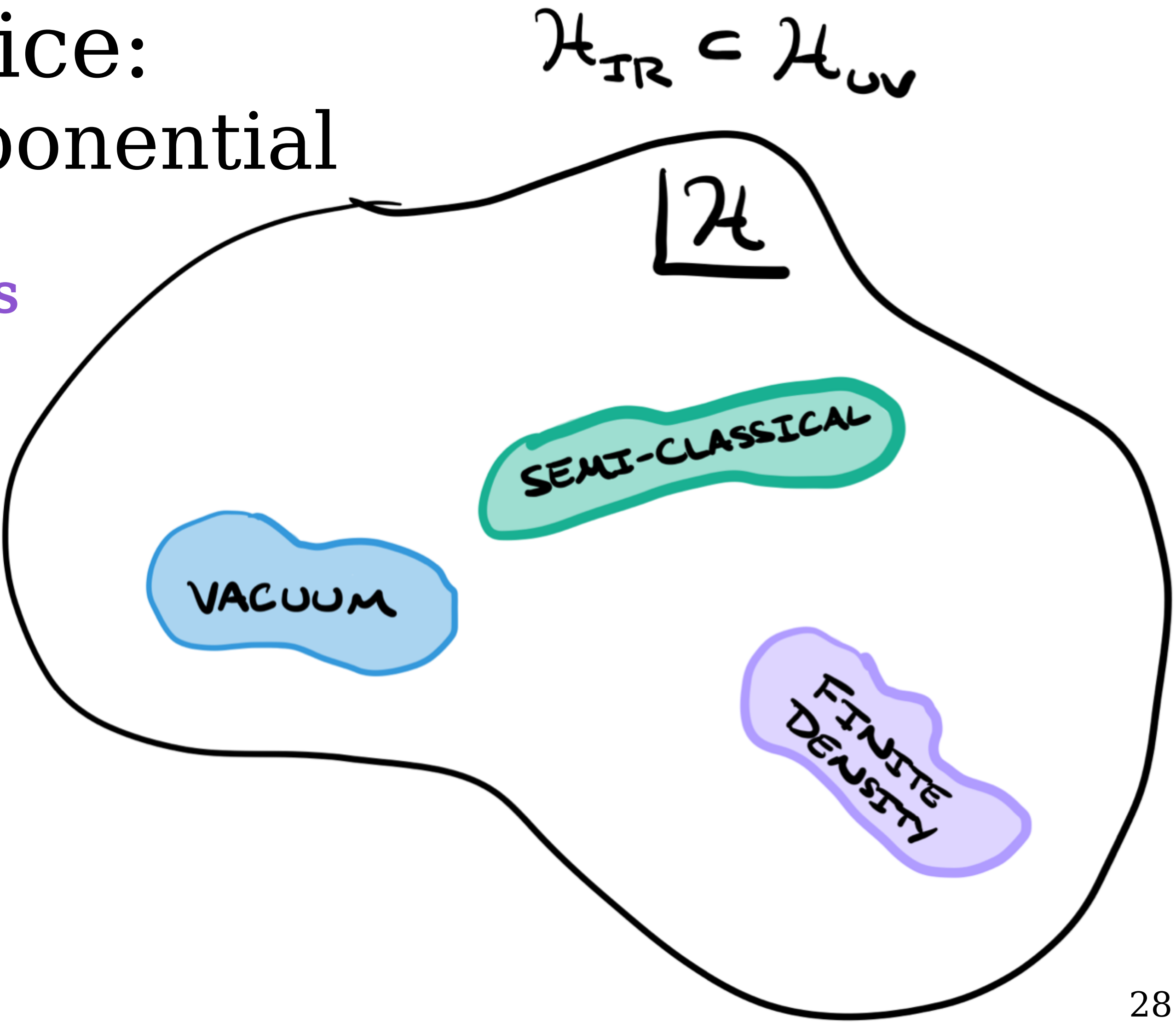
not independent questions

Basis choice:
fighting the exponential

Quantum Hilbert spaces grow exponentially

$$\rho(E) \propto \exp\left(\beta_d E^{\frac{d-1}{d}}\right)$$

$\Rightarrow$ How to isolate the relevant sector for desired physics?



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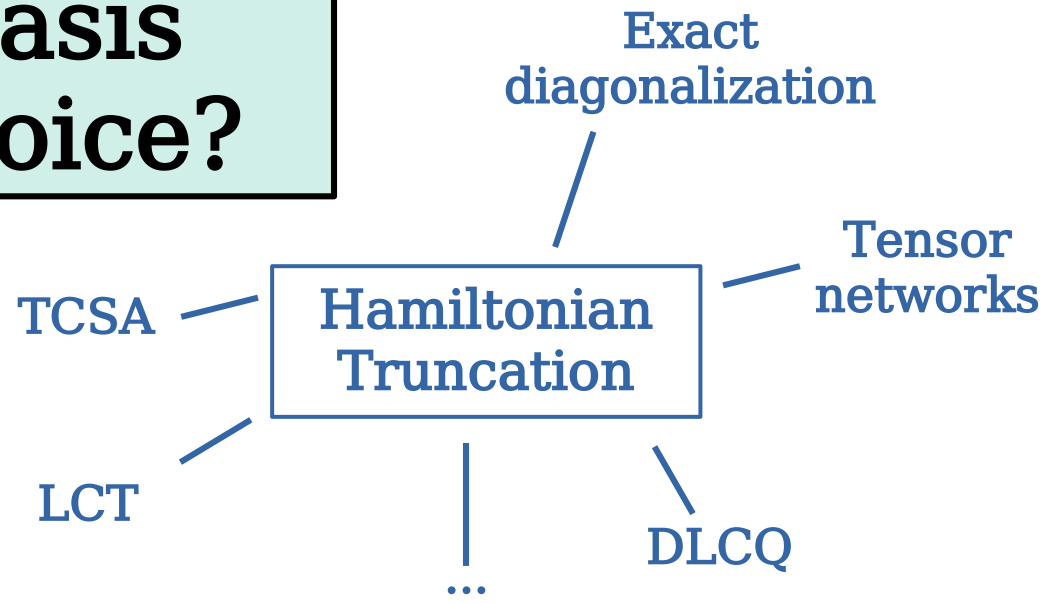
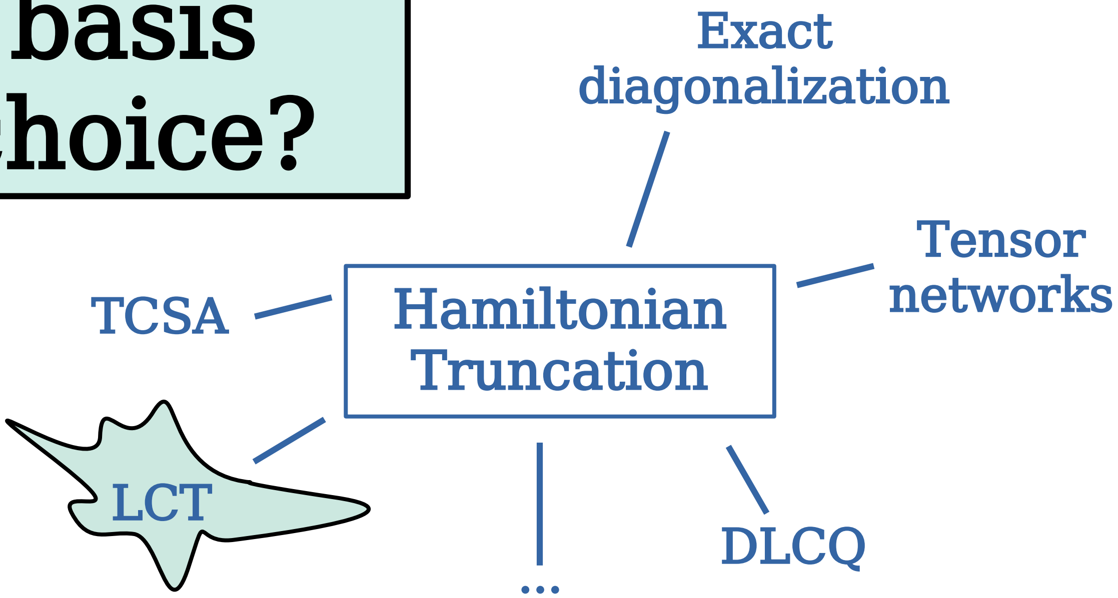


LCT: Lightcone Conformal Truncation
TCSA: Truncated Conformal Space Approach
DLCQ: Discretized Light-Cone Quantization
MPS: Matrix Product State
PEPS: Projected Entangled Pair States

basis choice?



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basis choice?

Exact diagonalization

Tensor networks

MPS/PEPS

⇒ organizing principle: information content

TCSA

Hamiltonian Truncation

LCT

...

DLCQ

plane waves

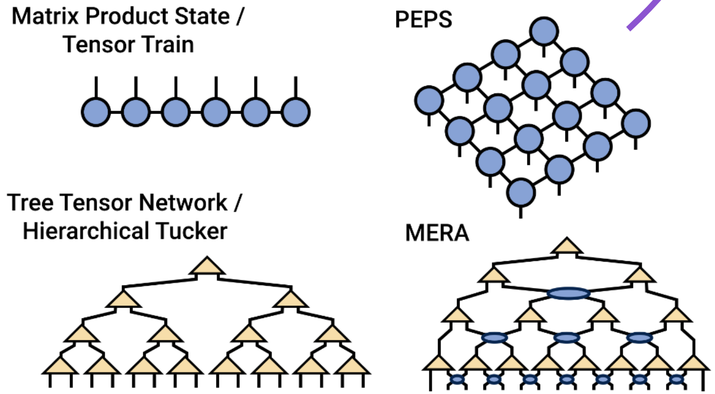
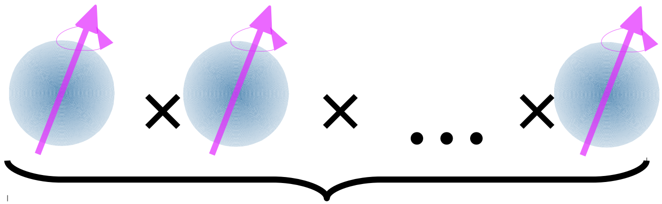


Fig: tensornetwork.org

Partial waves (conformal basis)

⇒ organizing principle: spacetime symmetry



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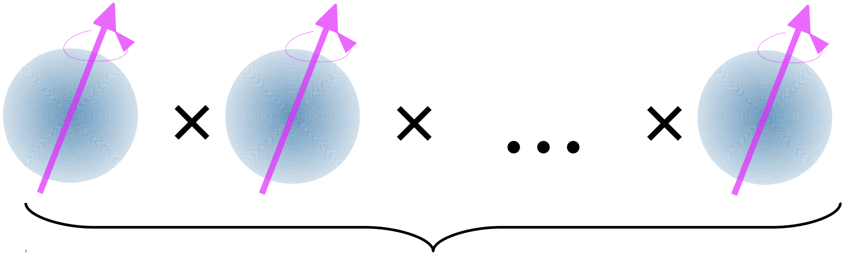
Partial waves/phase space harmonics



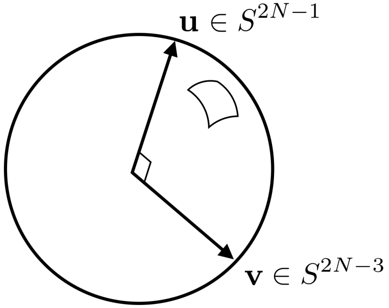
Don't treat independently—couple together and **ask about the collection of particles**

phase space measure

$$|\psi\rangle = \int d\Pi_{\text{LIPS}}\, |\vec{p}_1 \cdots \vec{p}_n\rangle \underbrace{\langle \vec{p}_1 \cdots \vec{p}_n | \psi\rangle}_{\psi(p_i) = \text{“wavefxn”}}$$

Free Hilbert space = wavefunctions on phase space

EFT amplitude bases
(see yesterday)



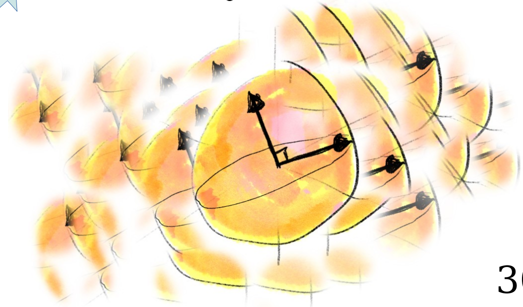
smart basis

$\Downarrow$

“spherical harmonics” on phase space

conformal basis

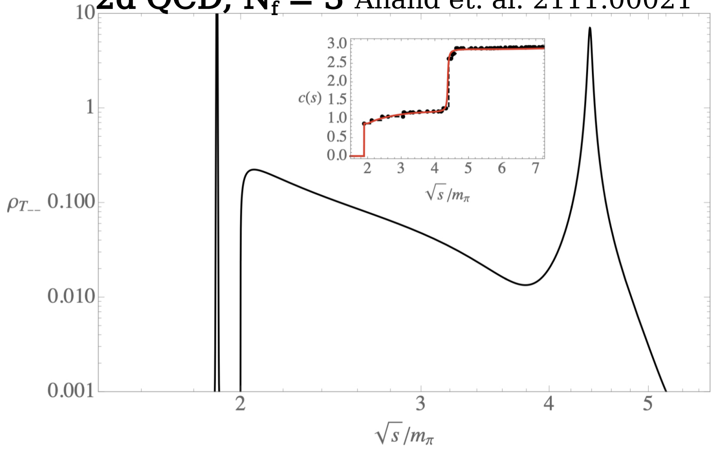
$$|\mathcal{O}(P)\rangle = \int dx\, e^{iP\cdot x} \mathcal{O}(x)\, |0\rangle$$

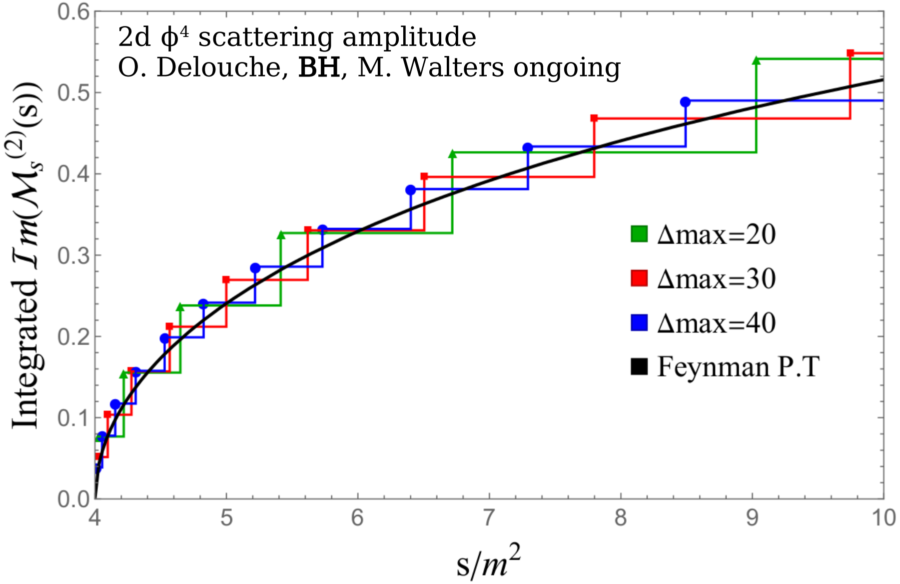


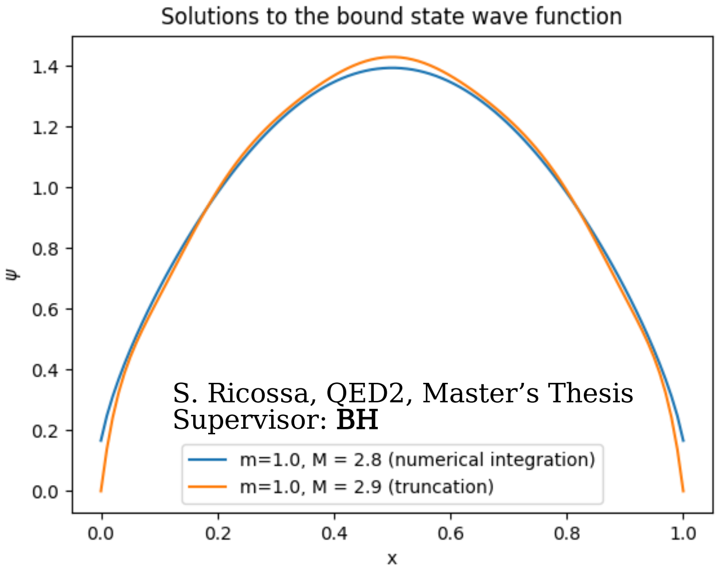
HT works splendidly in d = 1+1

- **Exponential improvement over naïve Fock basis**
  - # states = $p(\Delta_{max})$ = # partitions of the integer Δ_{max}
- **Laptop + Mathematica**

2d QCD, $N_f = 3$ Anand et. al. 2111.00021







$d>2$: Harder…but worth it

→ Requires "bigger" basis

→ 2 truncation parameters

→ Lots of relevant couplings in d = 2+1

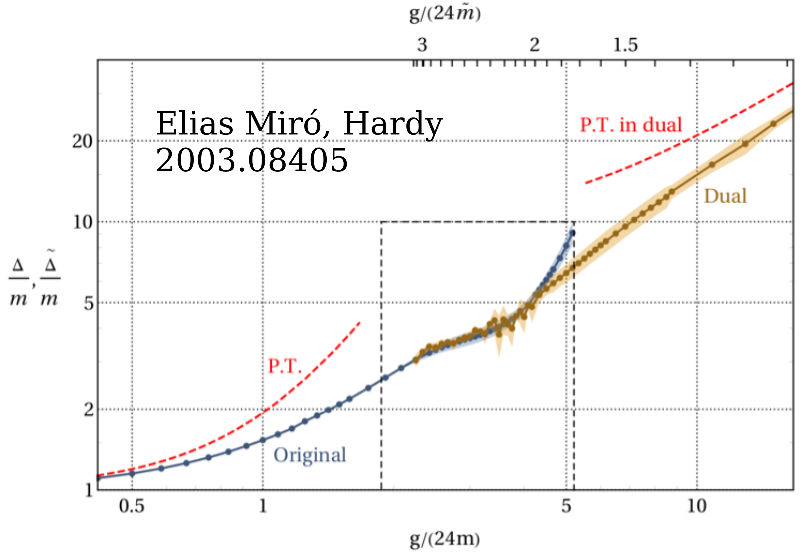
$$\lambda\phi^4 \;\; ; \;\; y\phi\bar{\psi}\psi \;\; ; \;\; \frac{1}{g^2}F^2 \, , \, gA_\mu J^\mu$$

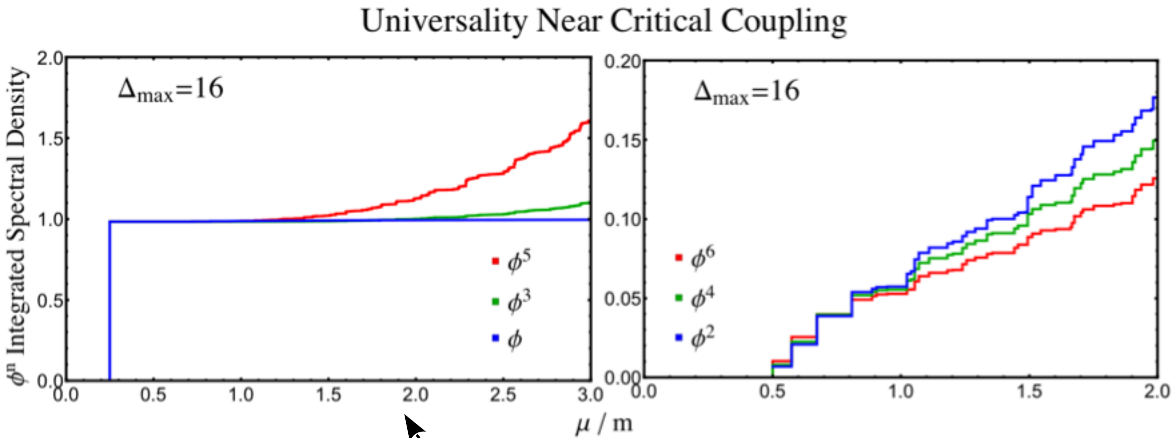
$$[\lambda] = 1 \;\; ; \;\; [y] = {}^1\!/_2 \;\; ; \;\; [g] = {}^1\!/_2$$

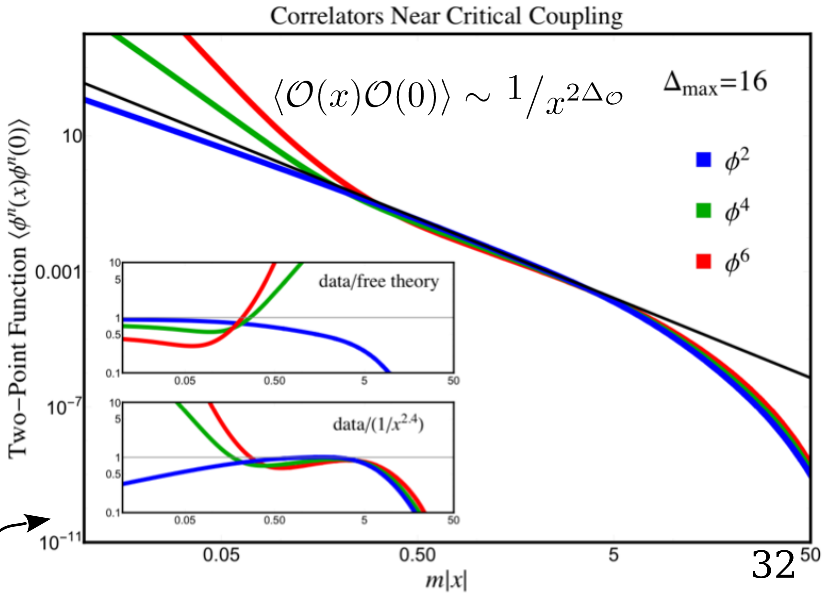
⇒ **lots of strong coupling!**

→ Fewer exact results

⇒ **uncharted territory!**







Anand, Katz, Khandker, Walters 2010.09730

Criticality in low dimensions

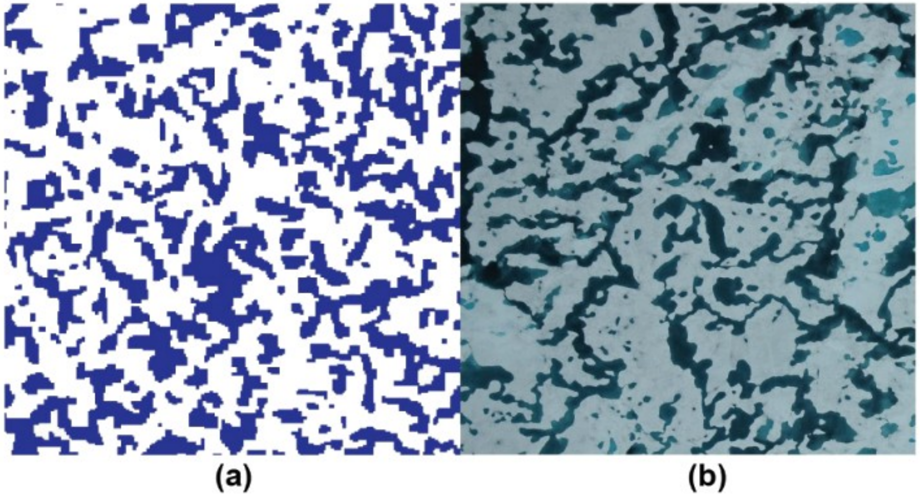


Figure 3. Comparison of real Arctic melt ponds with metastable equilibria in our melt pond Ising model. **3a.** Ising model simulation. **3b.** Real melt pond photo. Figure 3a courtesy of Yiping Ma, 3b courtesy of Donald Perovich.

Ising Model for Melt Ponds on Arctic Sea Ice

Yi-Pint Ma, Ivan Sudakov, Courtenay Strong and Kenneth M. Golden

Criticality in low dimensions

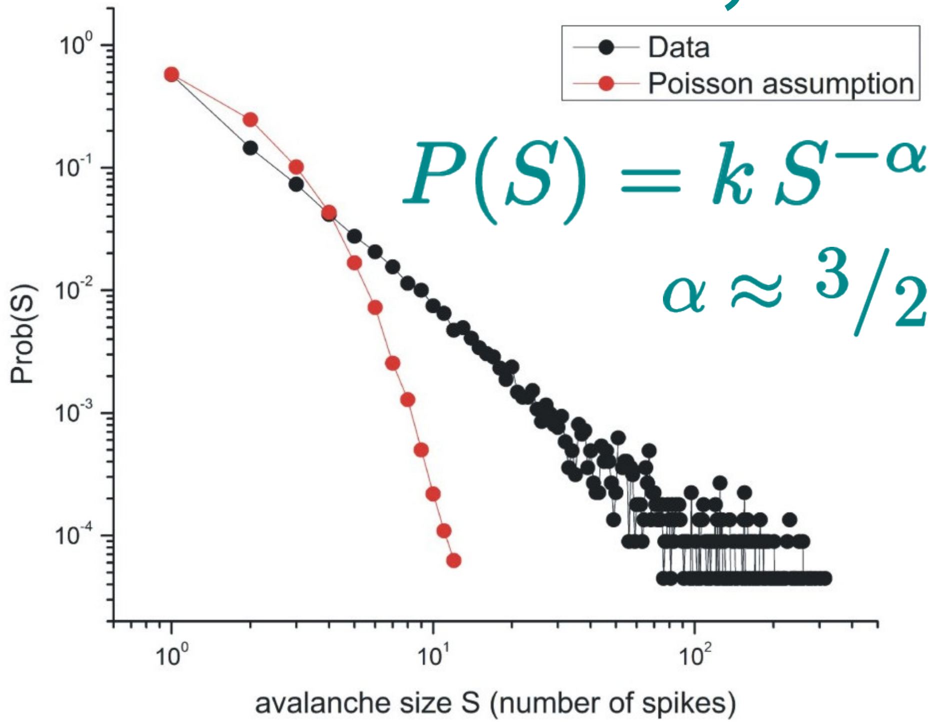
arctic sea ice

lizard skin patterns

described by the same critical theory (Ising model) → **UNIVERSALITY**

Critical Thinking

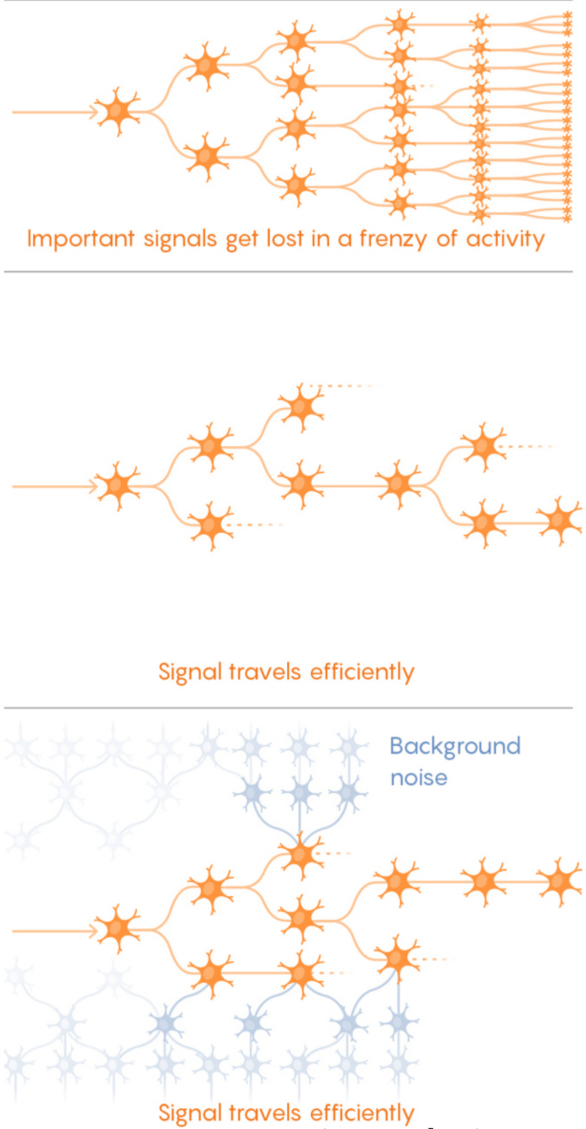
neuronal avalanches



$$P(S) = k\,S^{-\alpha}$$

$$\alpha \approx {}^{3}/_{2}$$

http://www.scholarpedia.org/article/Neuronal_avalanche



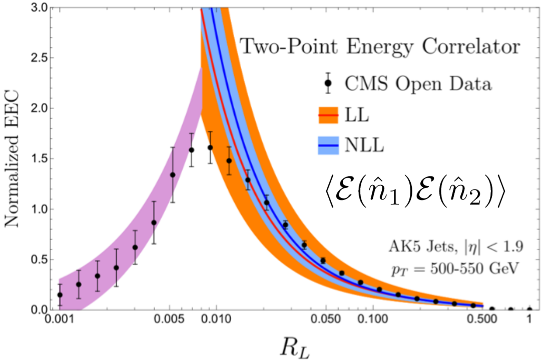
https://www.quantamagazine.org/brains-may-teeter-near-their-tipping-point-20180614/

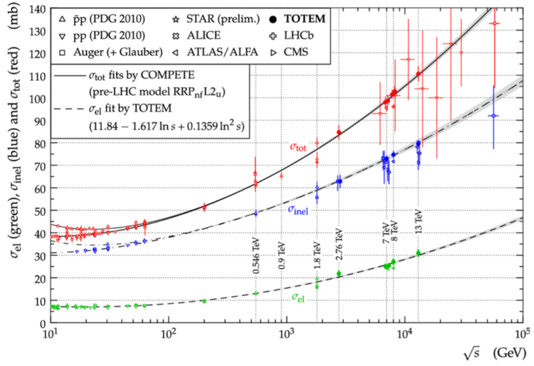
Truncation philosophy

1) Pick an observable

$$1 = \sum_i |\psi_i\rangle \langle\psi_i|$$

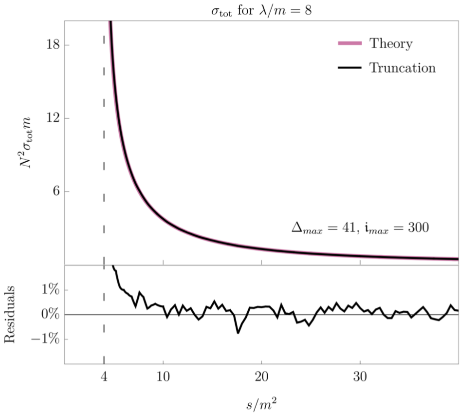
$$\langle T\{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_k(x_k)\}\rangle$$

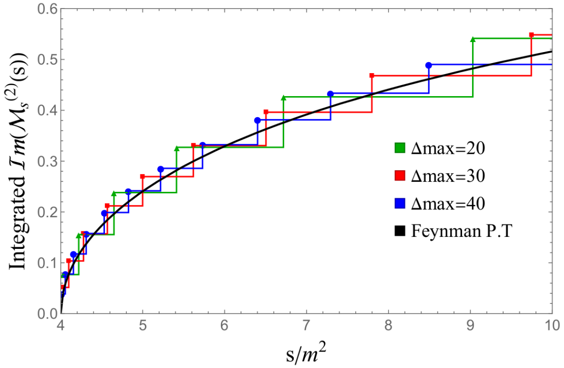




2) Learn to compute with Hamiltonian

3) Apply truncation





* $\langle \mathcal{O}(x)\mathcal{O}(0)\rangle = \sum_n \langle 0|\mathcal{O}(x)|n\rangle \langle n|\mathcal{O}(y)|0\rangle$

$H|n\rangle = E_n|n\rangle\,,\ \mathcal{O}(x) = e^{iPx}\mathcal{O}(0)e^{-iPx}$

things like

SPECTRAL INFO

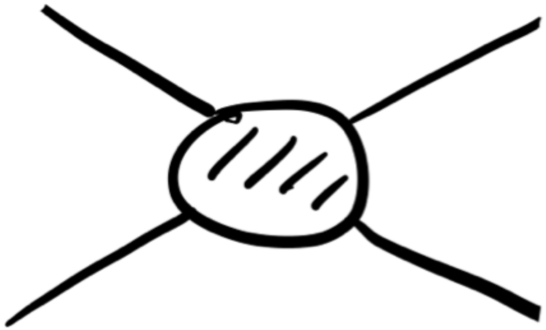
2-POINT FUNCTIONS*

super cool!

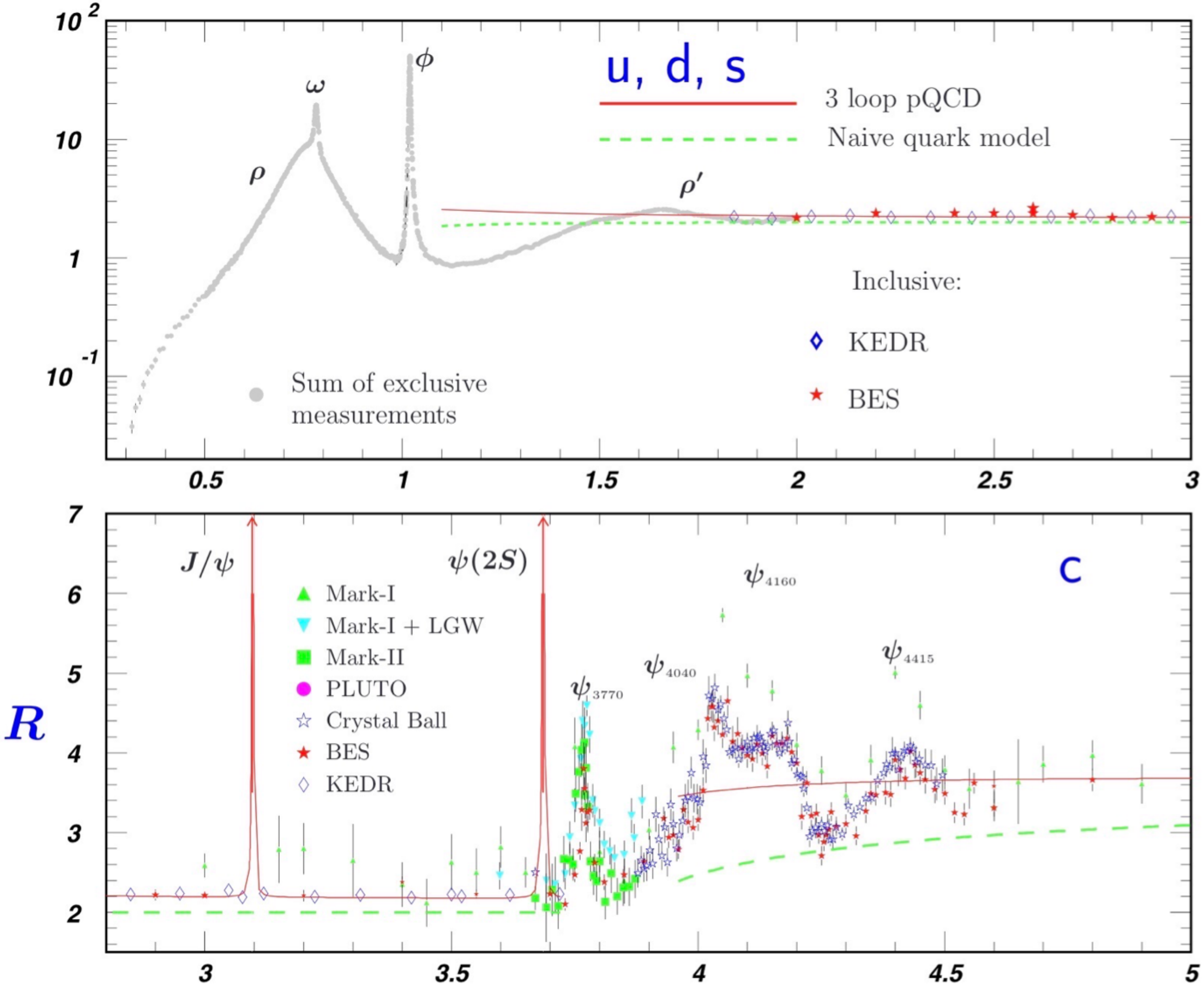
TIME TO GO AFTER THE

FUNDAMENTAL OBSERVABLE

IN RELATIVISTIC FIELD THEORY



The dream



u, d, s
3 loop pQCD
Naive quark model
Inclusive:
KEDR
BES
Sum of exclusive measurements
ρ
ω
ϕ
ρ′
c
J/ψ
ψ(2S)
Mark-I
Mark-I + LGW
Mark-II
PLUTO
Crystal Ball
BES
KEDR
R

Truncation output:

(approximate) spectrum $\Leftrightarrow \{E_i, |\psi_i\rangle\}, \; \hat{H}\,|\psi_i\rangle = E_i\,|\psi_i\rangle$

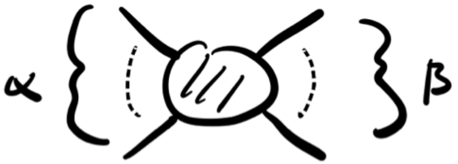
$\Rightarrow$ gives (approximate) **resolution of identity:** $1 \approx \sum_{i=1}^{N} |\psi_i\rangle\,\langle\psi_i|$

Fundamental question:

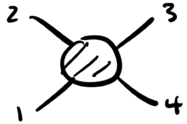
GIVEN THE ENERGY EIGENSTATES,

HOW DO YOU COMPUTE THE S-MATRIX?

How to compute $\mathcal{M}$ from $|\psi\rangle$?



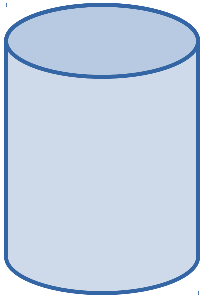
$S_{\beta\alpha} = \langle \psi_\beta^- | \psi_\alpha^+ \rangle$



$= \langle p_3 p_4; \text{out} | p_1 p_2; \text{in} \rangle$

PROBLEM: How are truncation states related to in/out-states?

Think finite volume e.g. $\mathbb{R} \times S^{d-1}$



DISCRETIZING continuum $\Rightarrow$ IR cutoff = finite "box"

Prevents formal identification of asymptotic states

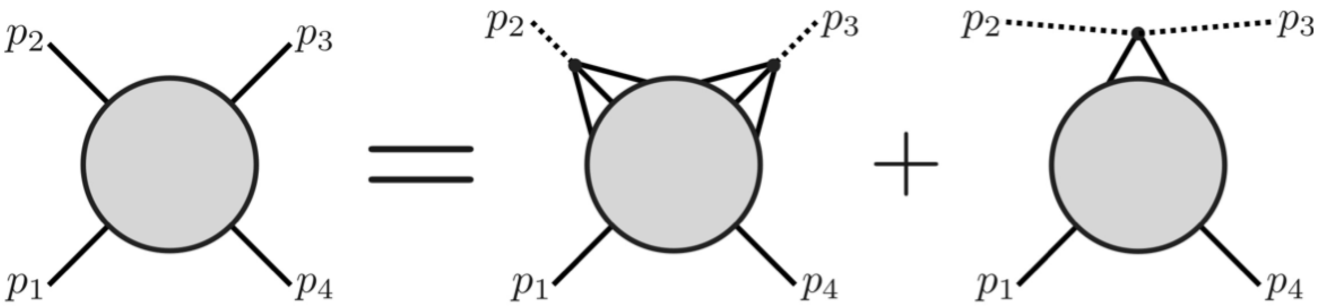
BH, Murayama, Riva, Thompson, Walters
arXiv:2209.14306

$\mathcal{M}$ from $|\psi\rangle$

scattering amplitude $\xrightarrow{\text{LSZ}}$ **correlation function**

$$\mathcal{M}(p_i) \sim (\Box_1 + m^2) \cdots (\Box_4 + m^2) \langle T\, \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \rangle$$

evaluate by inserting the identity $1 \approx \sum_{i=1}^{N} |\psi_i\rangle \langle \psi_i|$

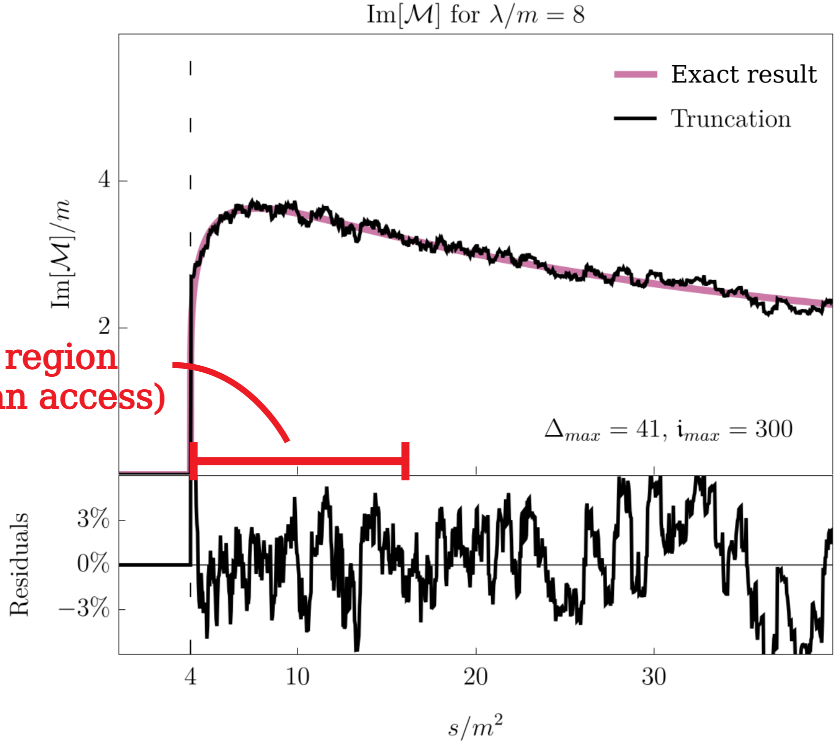


BH, Murayama, Riva, Thompson, Walters
JHEP 05 (2023) 197

results

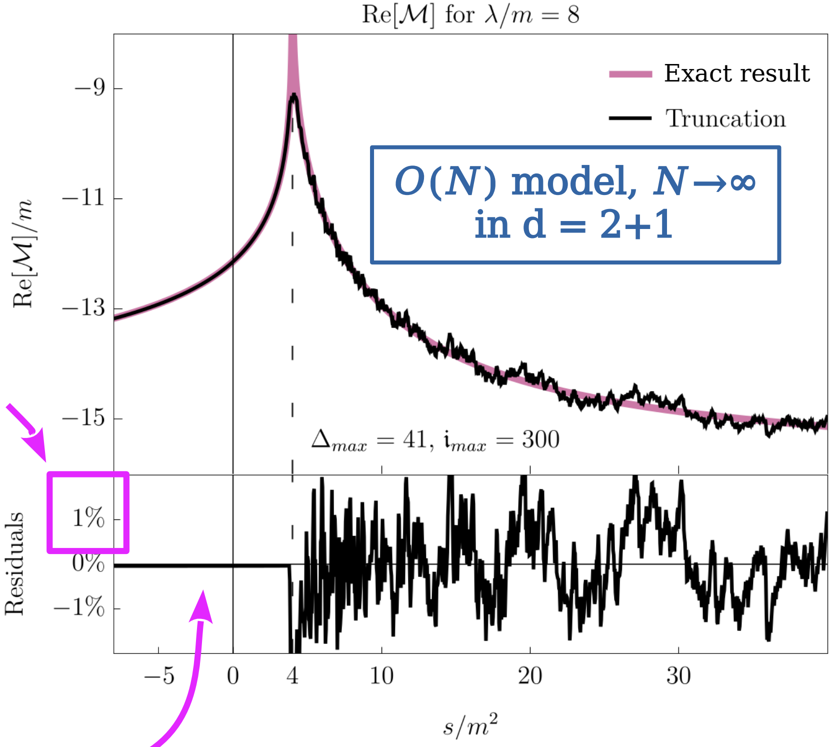
$O(N)$ model: repulsive interaction
$\Rightarrow$ no bound states

large coupling!



elastic region
(lattice can access)

!!



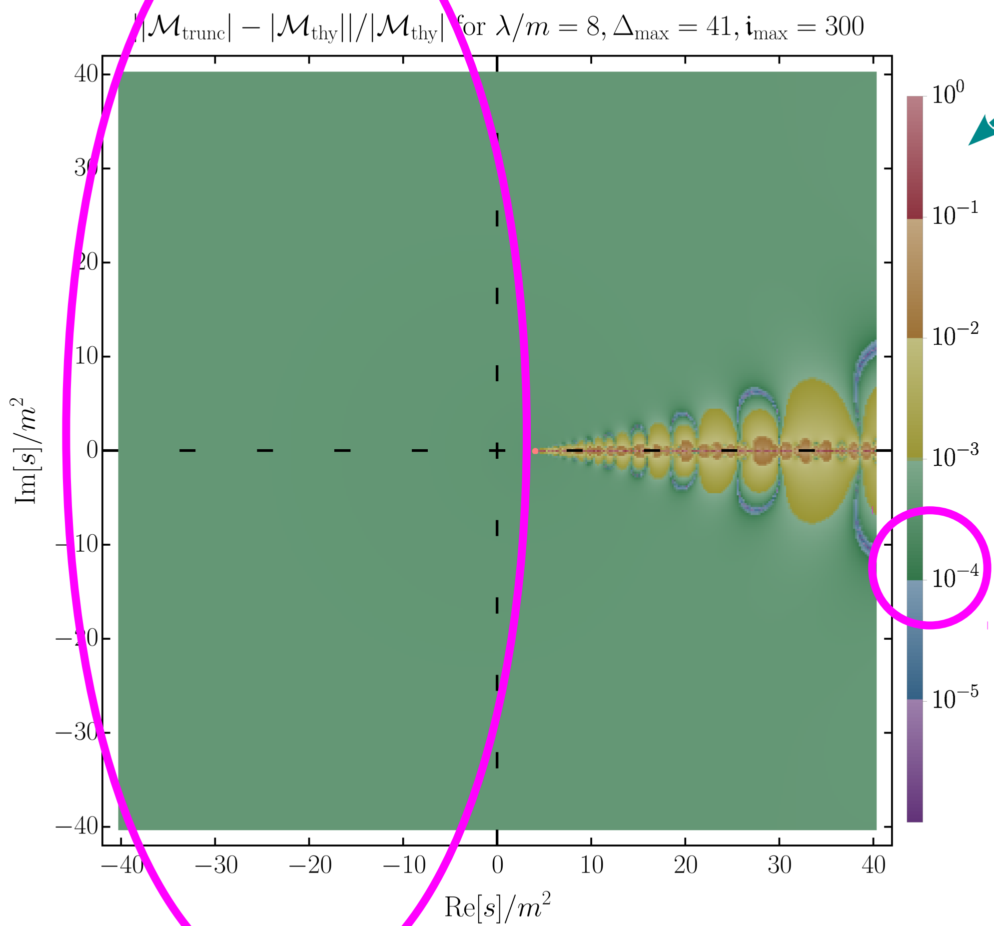
$O(N)$ model, $N \to \infty$
in d = 2+1

Clear appearance of threshold @ $s = 4m^2$

Best convergence outside physical regime

At high-E, perturbative regime: $\mathrm{Re}(\mathcal{M}) \sim \lambda = \mathrm{const}$, $\mathrm{Im}(\mathcal{M}) \sim \lambda^2/s$

results



complex s-plane

- Can explore analytic behavior
- Rapid convergence throughout complex plane

BH, Murayama, Riva, Thompson, Walters
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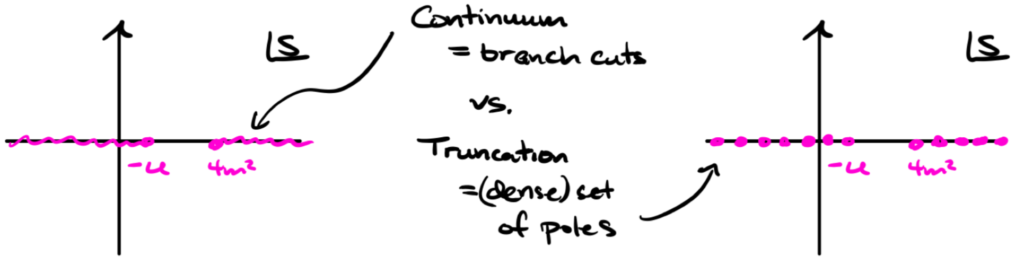
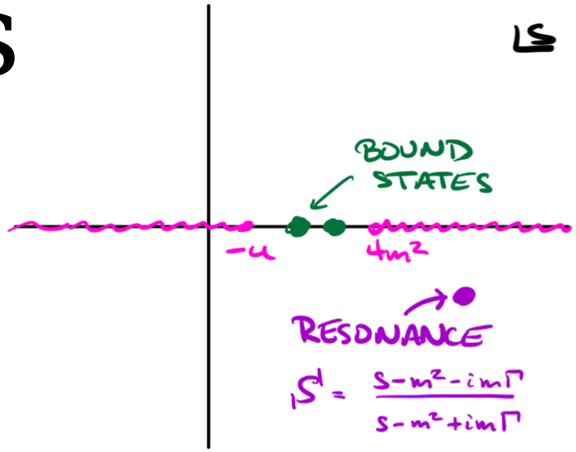
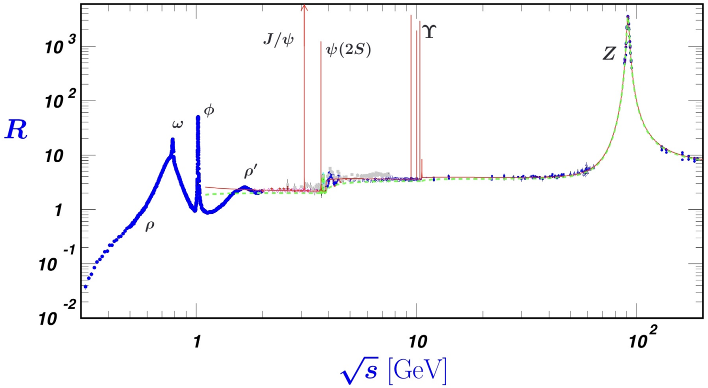
Scattering goals



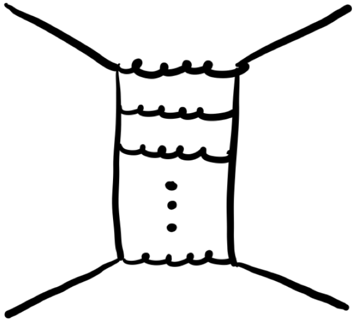
Probe analytic structure

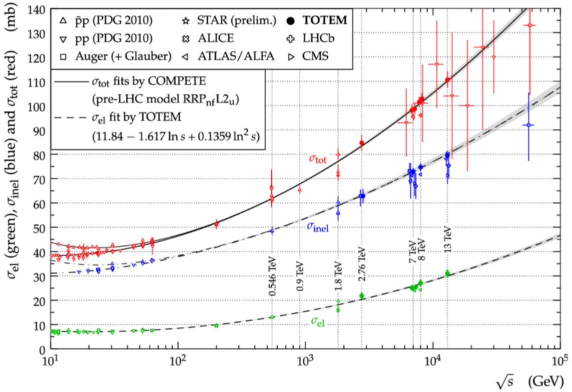




Bound state scattering from first principles

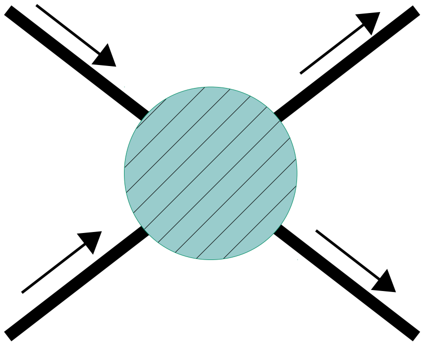
Forward scattering/ Regge physics



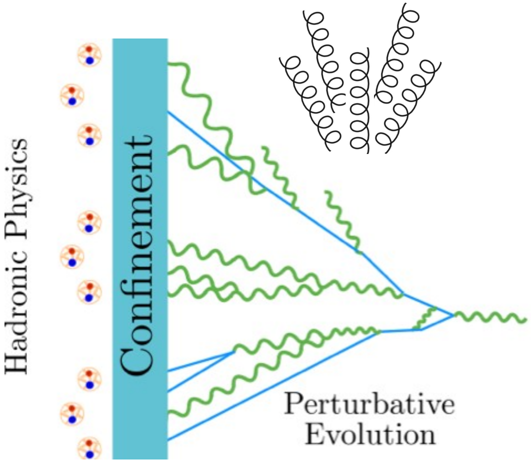


HT’s status in my brain

(simplified)



✓



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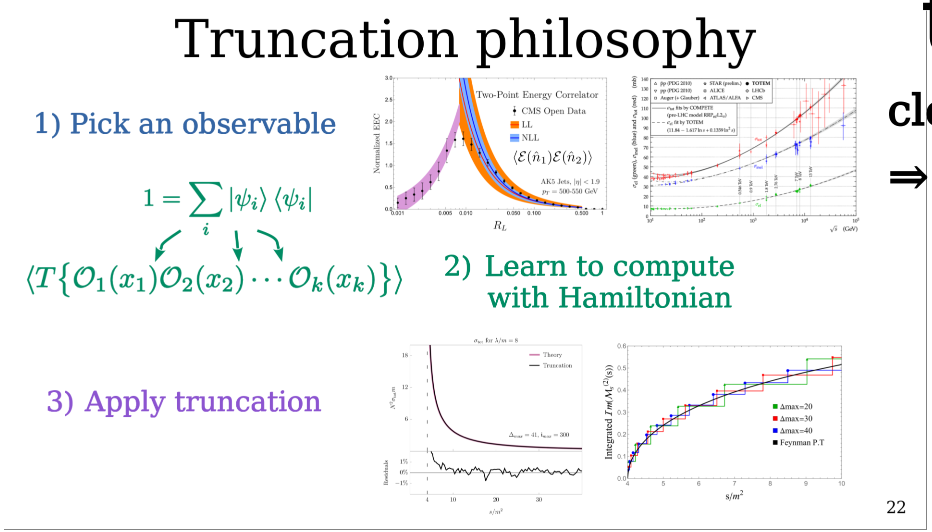
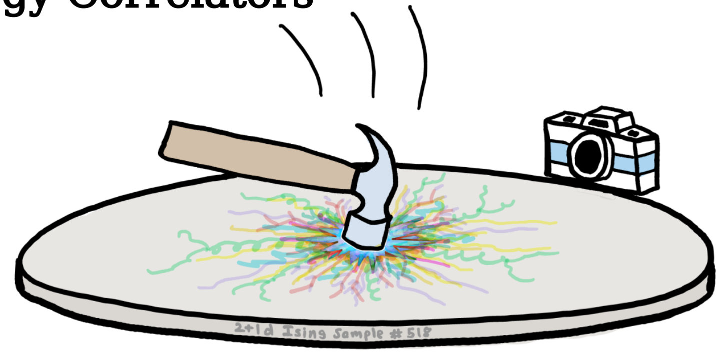


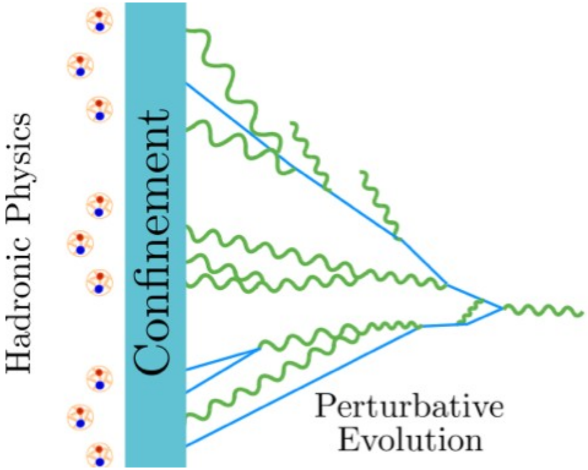
targeting hadronization

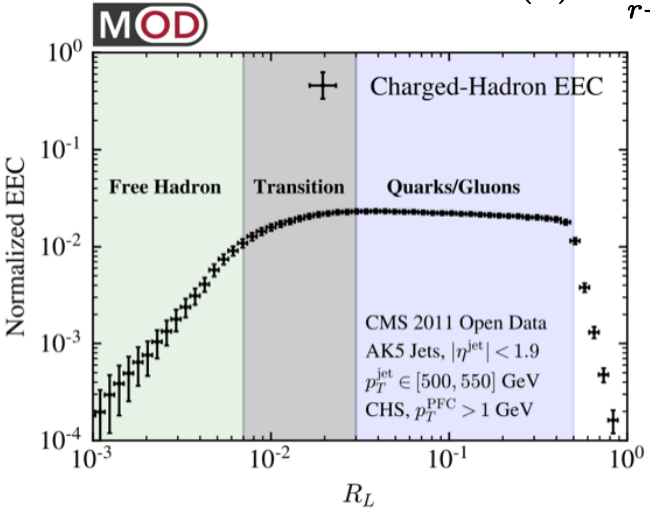
clean observables experimentally AND theoretically

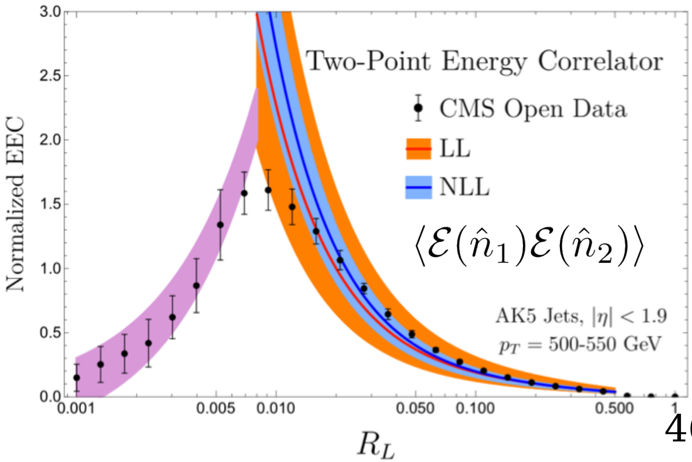
⇒ Energy Correlators



$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^{d-2} \int_0^\infty dt\, n^i T_{0i}(t, r\hat{n})$$



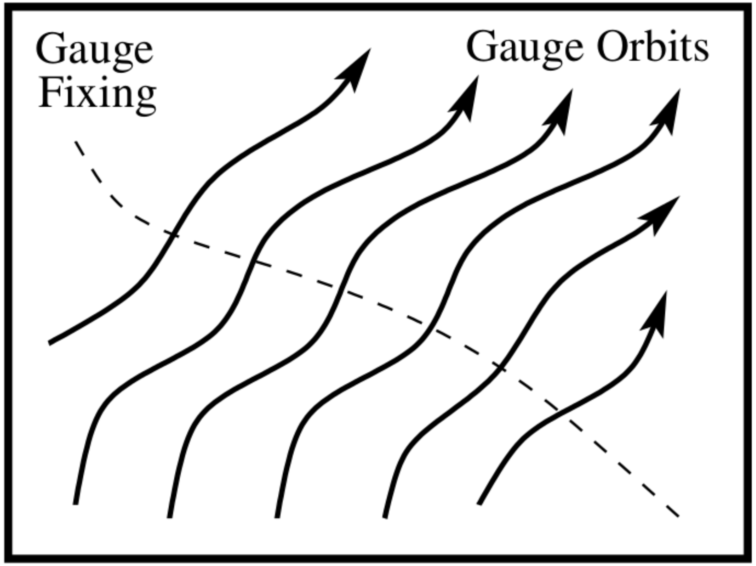


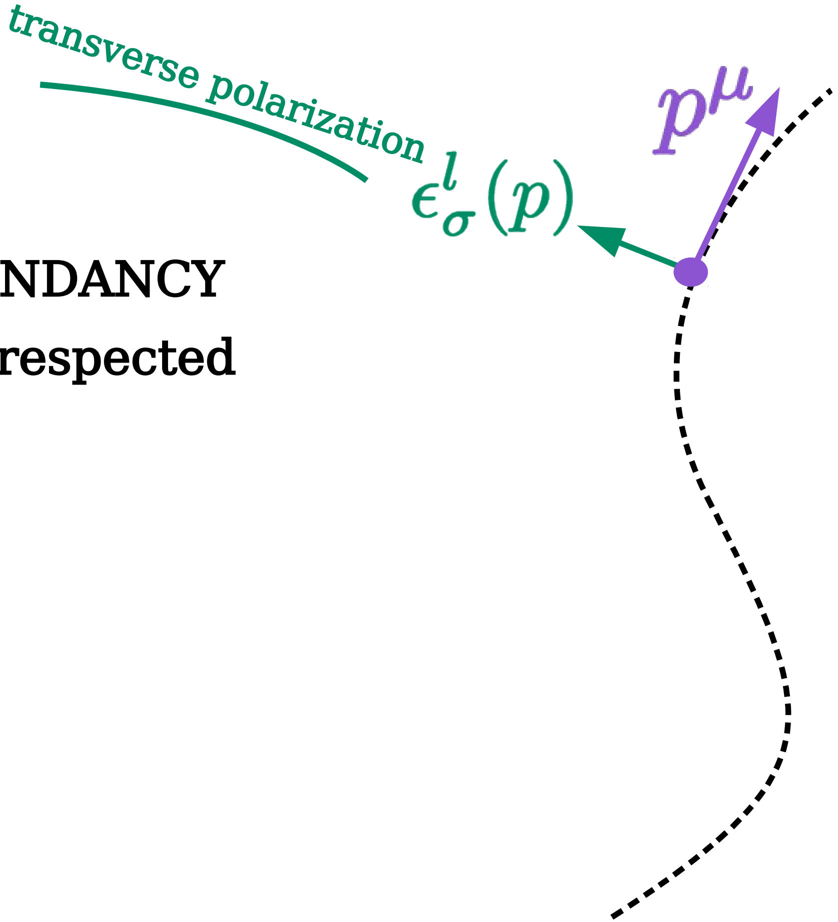


The challenge of gauge theory

- Gauge fields describe spin-1 bosons
  - Dynamical only for d ≥ 2+1

- Gauge "symmetry" is really a REDUNDANCY
  - Must ensure this redundancy is respected





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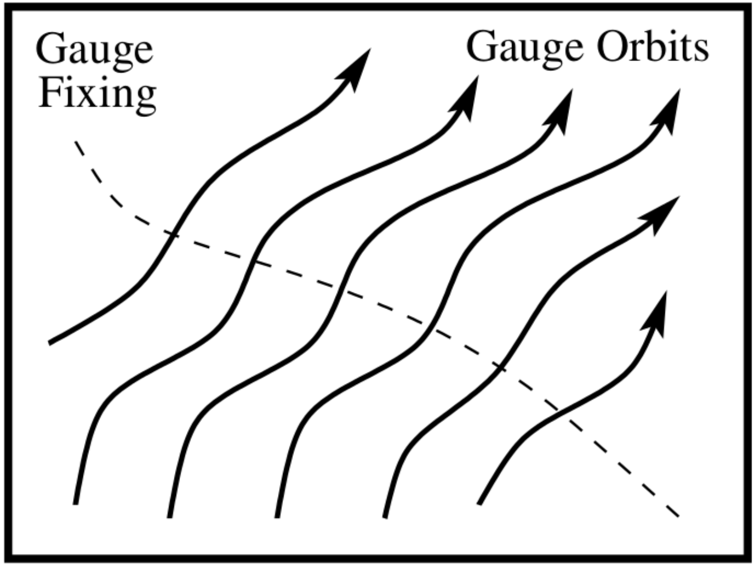
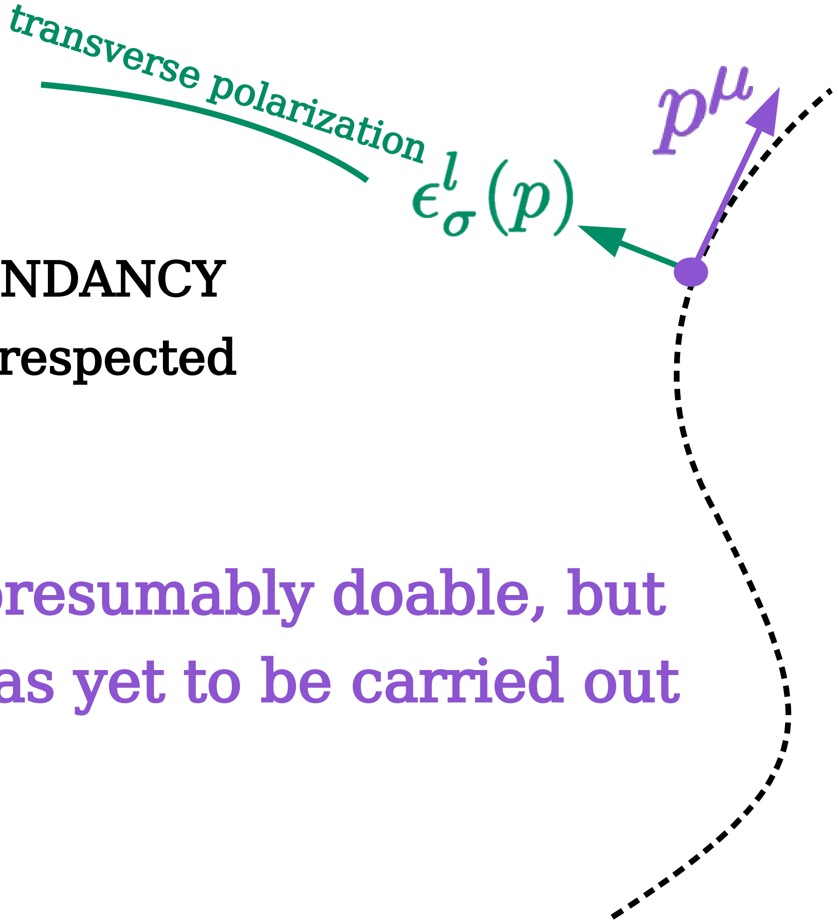
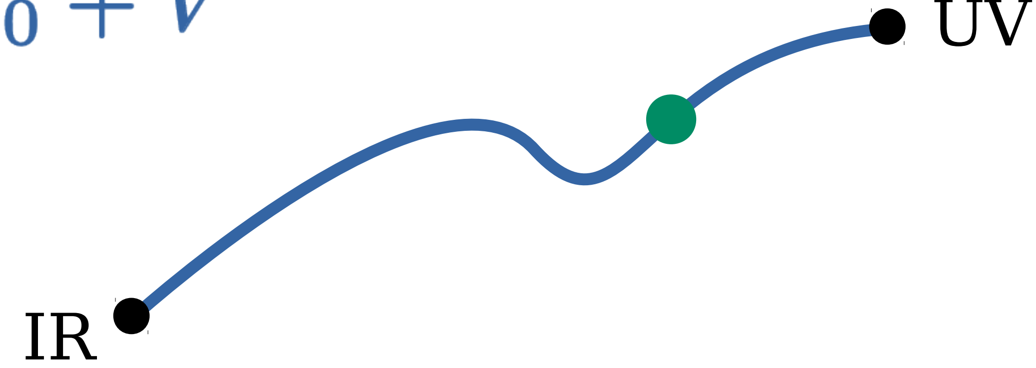




presumably doable, but has yet to be carried out

Go with the flow

$$H = H_0 + V$$



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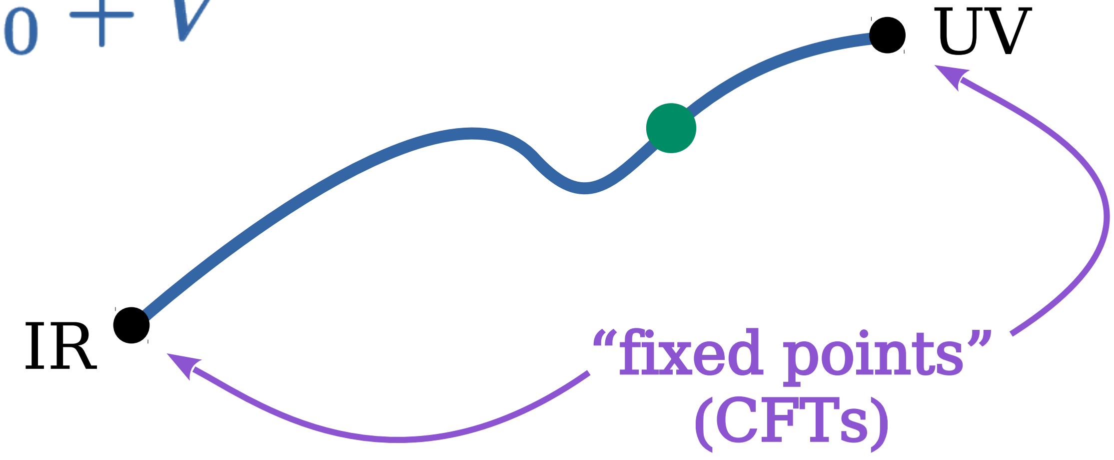
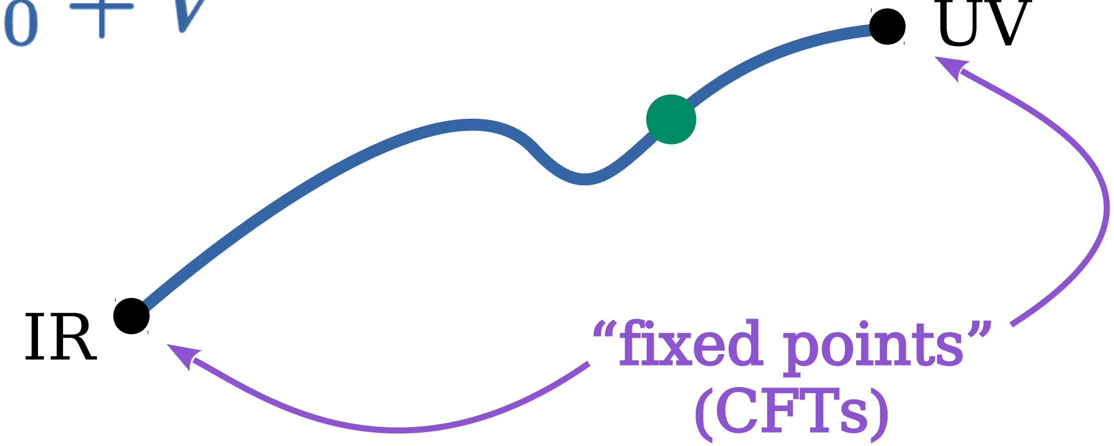


QFT = the “flow” between fixed points

Go with the flow

$$H = H_0 + V$$



QFT = the “flow” between fixed points

multiple theories can flow to SAME IR fixed point

$\Rightarrow$ UNIVERSALITY

UNIVERSALITY

⇒ what theories flow to QCD?

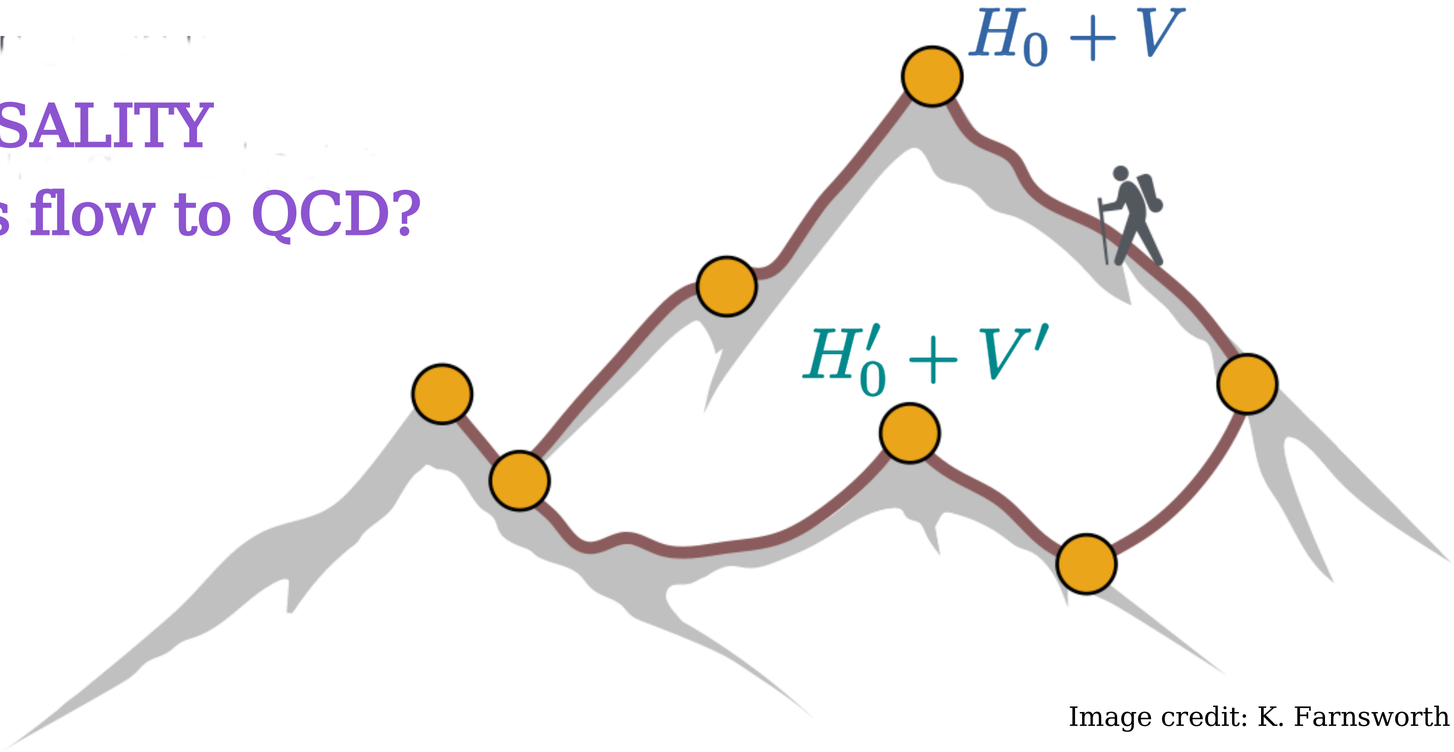


Image credit: K. Farnsworth

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$\Rightarrow$ what theories flow to ~~QCD~~? QED3

sensible (and fascinating!) starting point

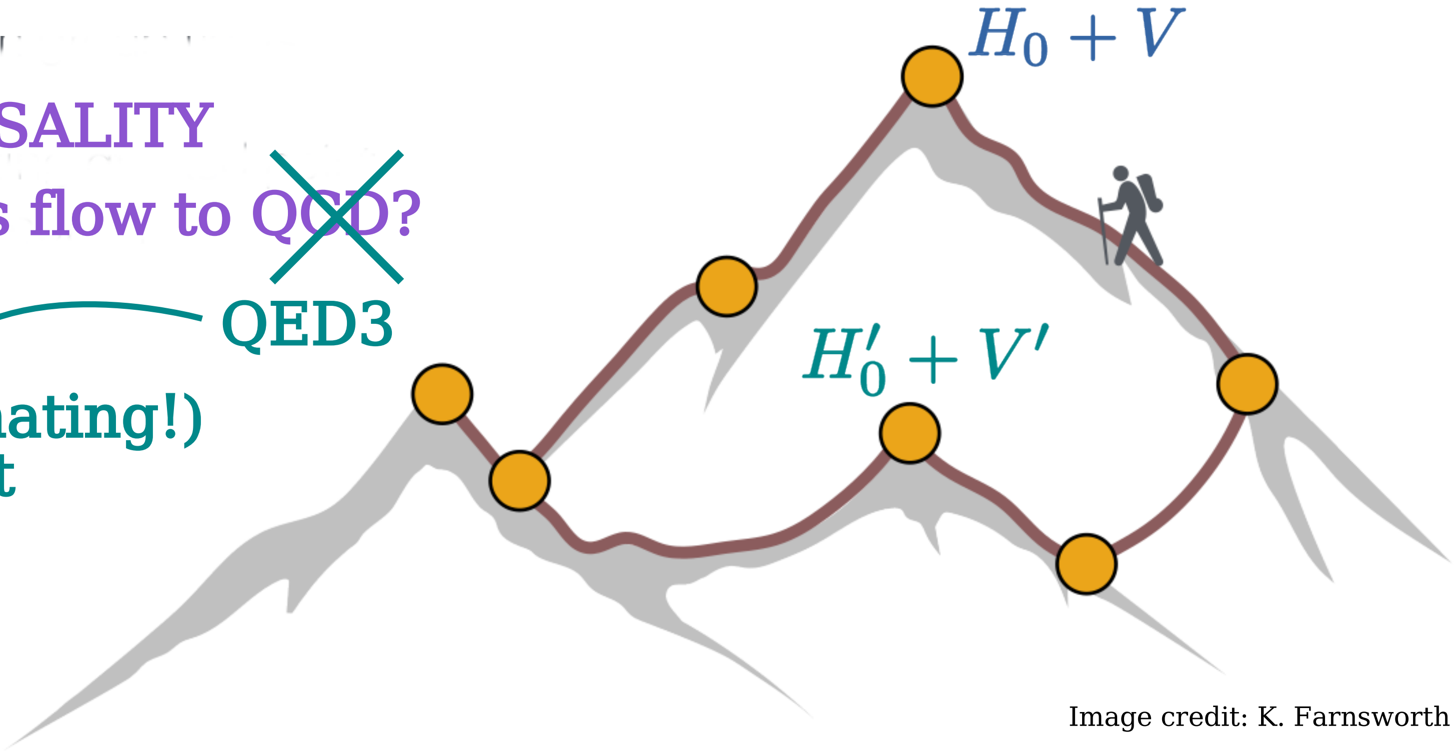

Image credit: K. Farnsworth

QED3

⇒ quantum induced linear confinement ✓ $V(r) \approx k\, r$

⇒ conformal window ✓

⇒ real-world relevance ✓

QED3

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H_0 = free electrons and photon

$$V \sim e\, A_\mu J^\mu$$

+: relevant (convergence under control)

+: gauge invariant (on-shell)

+: abelian

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H_0 = "solvable" $N_f \gg 1$ theory

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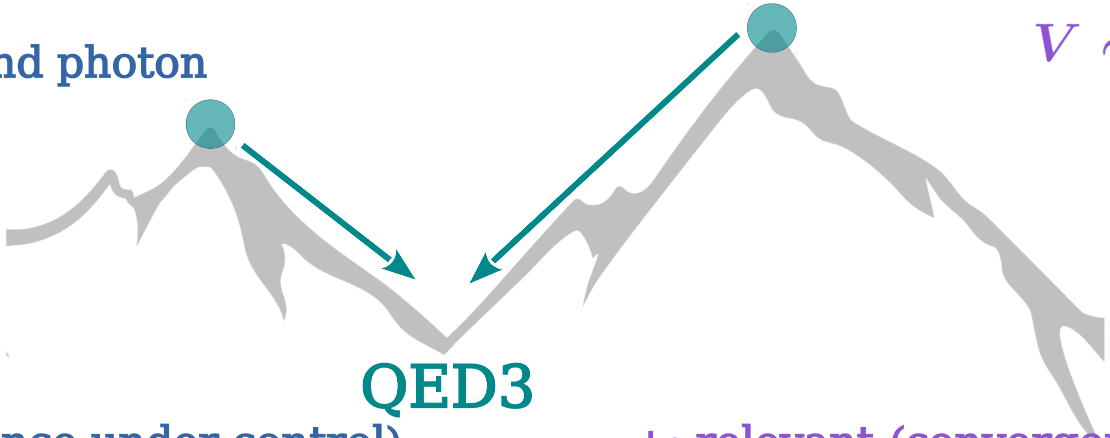

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−: need to "solve" H_0

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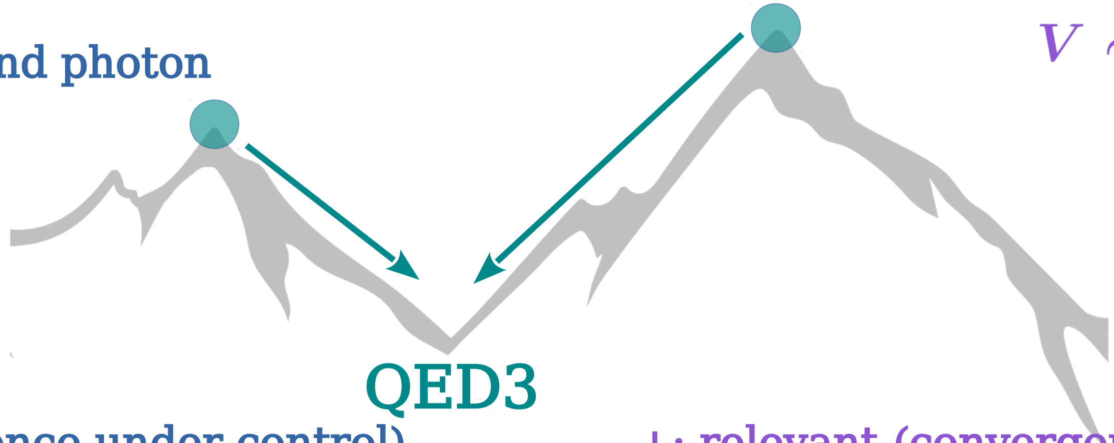

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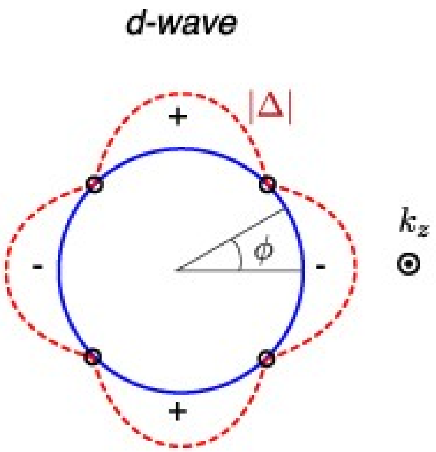
+: gauge invariant

−: need to "solve" H_0

Many analogs to QCD4, can test both approaches!

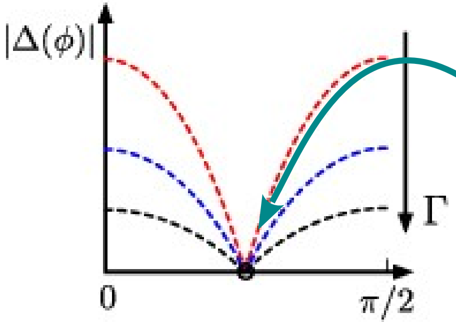
QED3 in the lab



relevant for

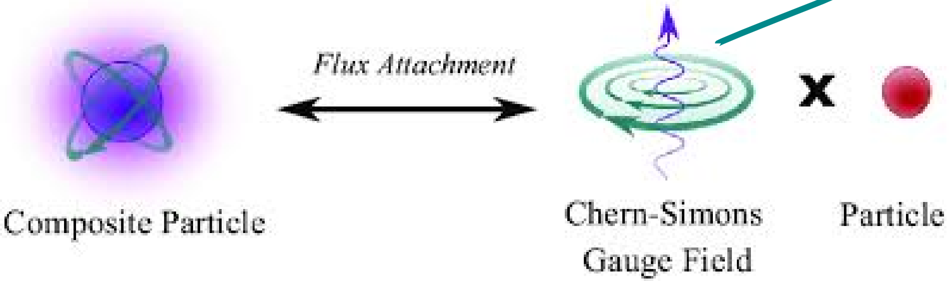
⇒ high-T_c superconductivity

⇒ half-filled Landau level



emergent Dirac fermions

emergent gauge field



Approaching QCD

H_0 = free quarks and gluons

$V \sim g\, A_\mu J^\mu + g\, A^2 \partial A$

H_0 = "solvable" interacting UV fixed point

e.g. Banks-Zaks, SU(3) $N_f = 16$

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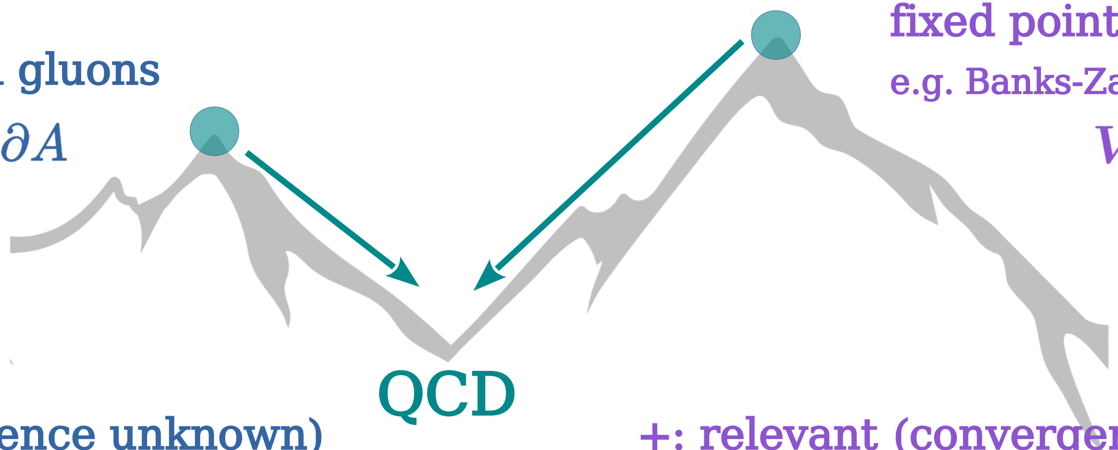

−: marginal (convergence unknown)

−: not gauge invariant

+: familiar

+: relevant (convergence under control)

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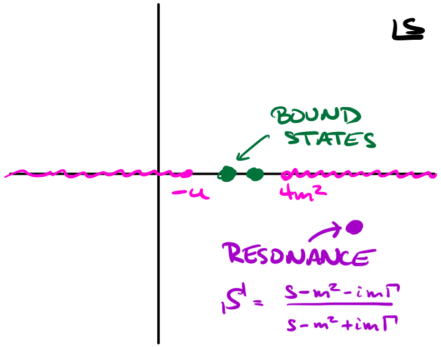
−: need to "solve" H_0

QED3 paves the route to QCD4

Future directions: HT



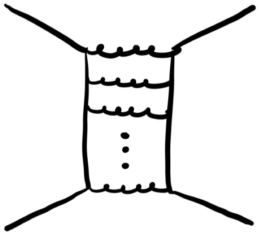
resonances

d = 1+1

- check against integrable systems
- complement S-matrix bootstrap
- ongoing with O. Delouche, M. Walters

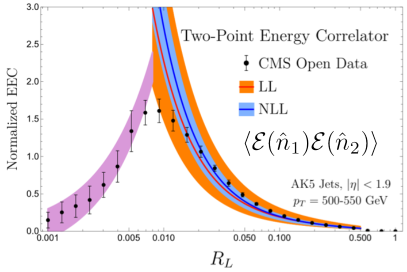
scattering

forward/Regge physics



- e.g. ϕ^3 or Yukawa in d=2+1

energy correlators

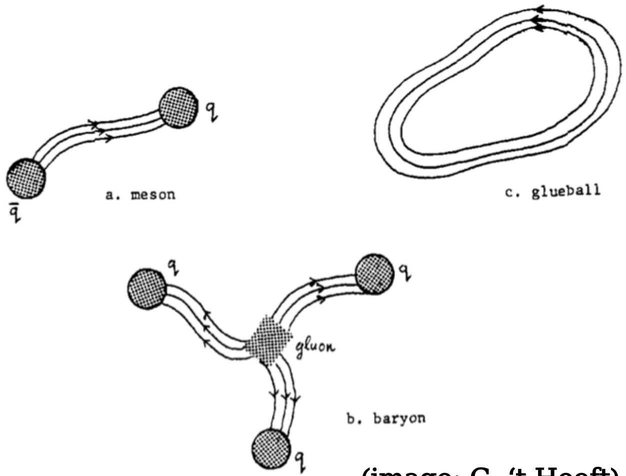


$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^{d-2} \int_0^\infty dt\, n^i T_{0i}(t, r\hat{n})$$

Future directions: HT

Banks-Zaks ⇒ QCD

⇒ Banks-Zaks data (ongoing)
with Karateev, Kosmopoulos, Ricossa, Riembau, Riva, Walters



(image: G. ‘t Hooft)

gauge theories

QED3

concrete mysteries; tension between methods; relevance to cond-mat

Two approaches:

1) Start from free theory
2) Start from interacting fixed point

– ongoing with J. Thompson, M. Walters, ...

gauge theories in d = 1+1

QED2

→screening vs confinement

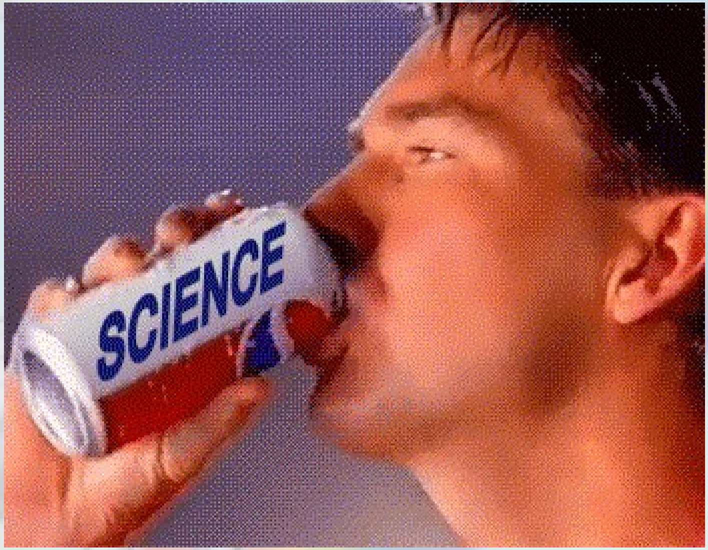
– ongoing with K. Farnsworth , S. Ricossa

PLENTY of projects, ranging from pheno, to formal, to numerical

⇒ something for everyone!

THANK YOU!

THANK YOU!



backup

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⇒ what theories flow to QCD?

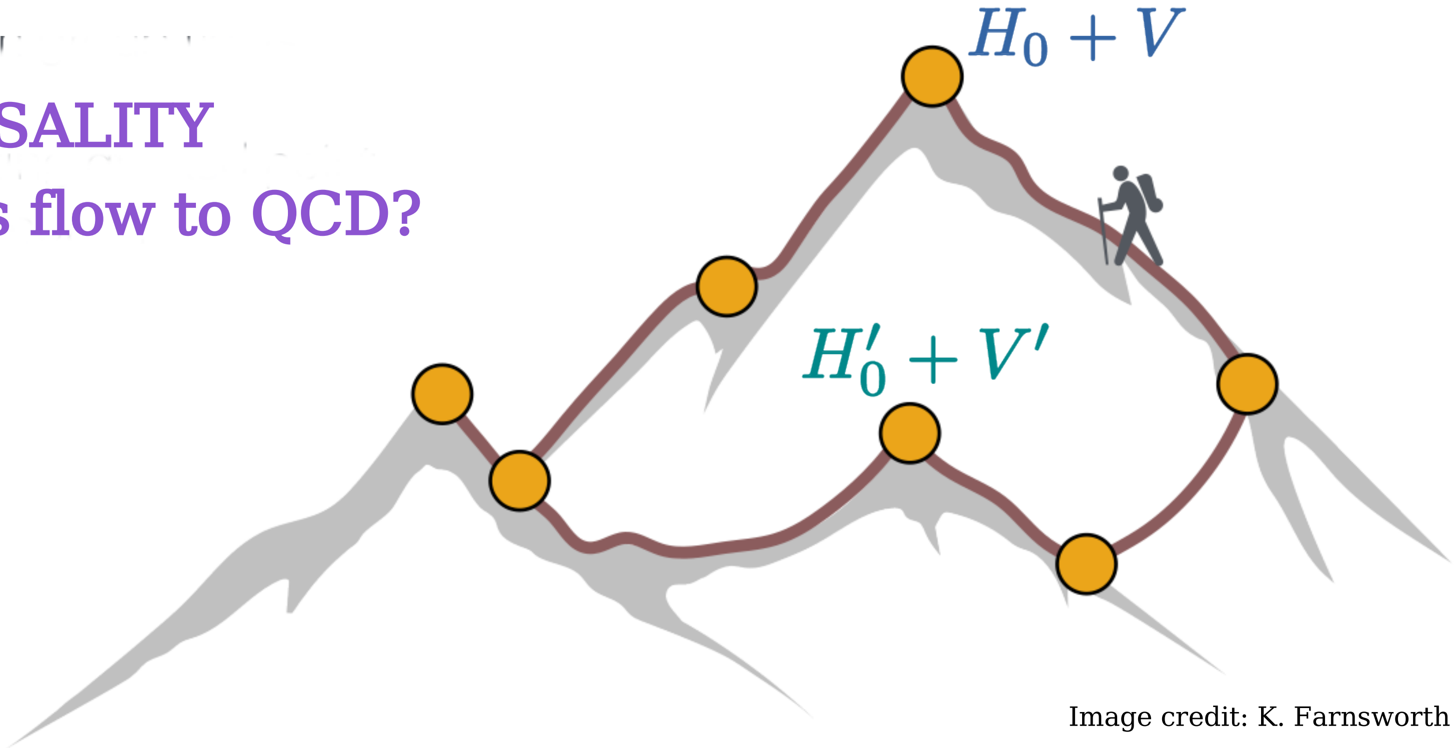

Image credit: K. Farnsworth

Approaching QCD

Approaching QCD

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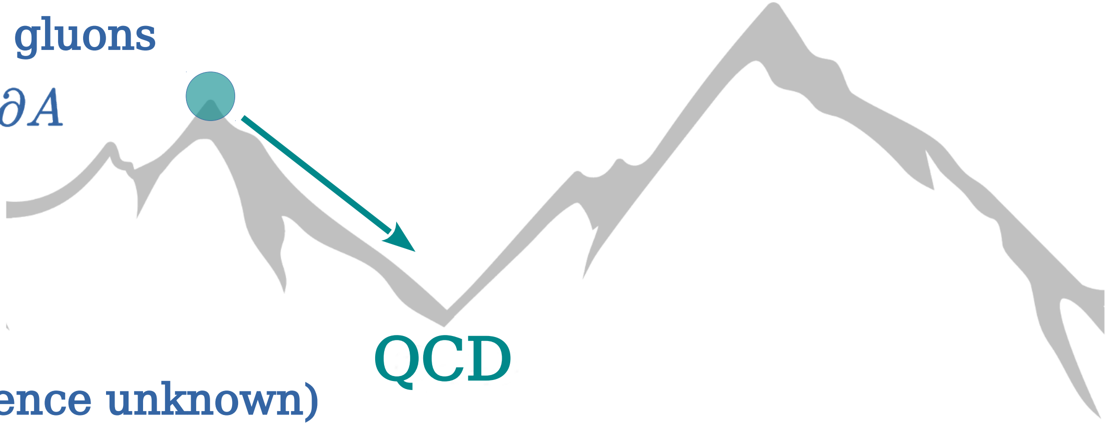

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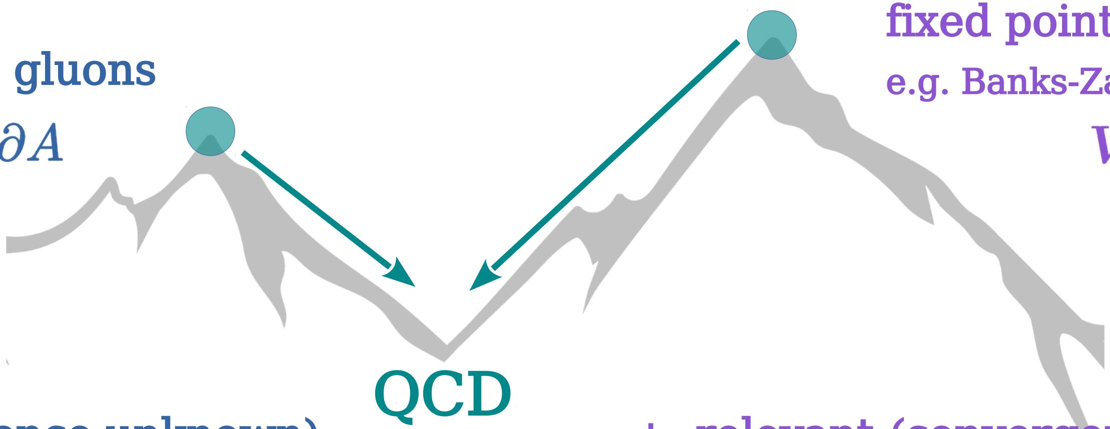

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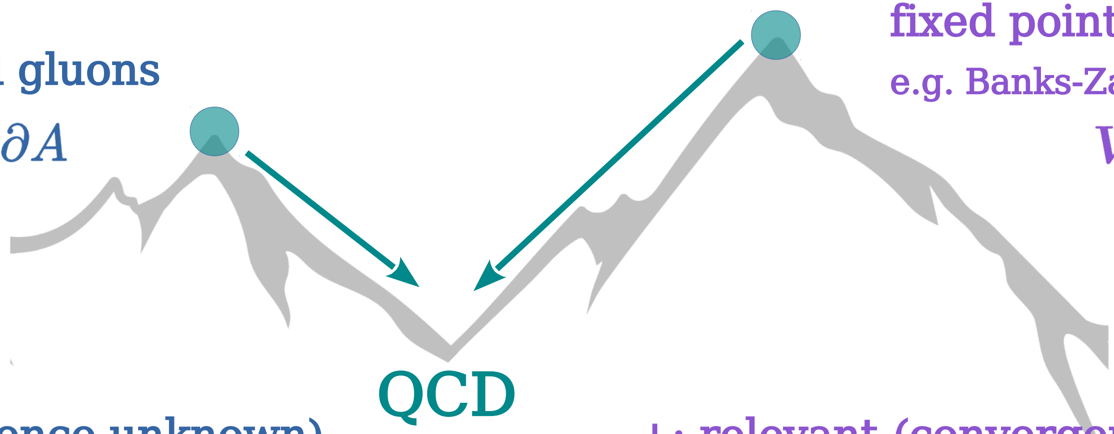

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big bite! ... something more digestible?

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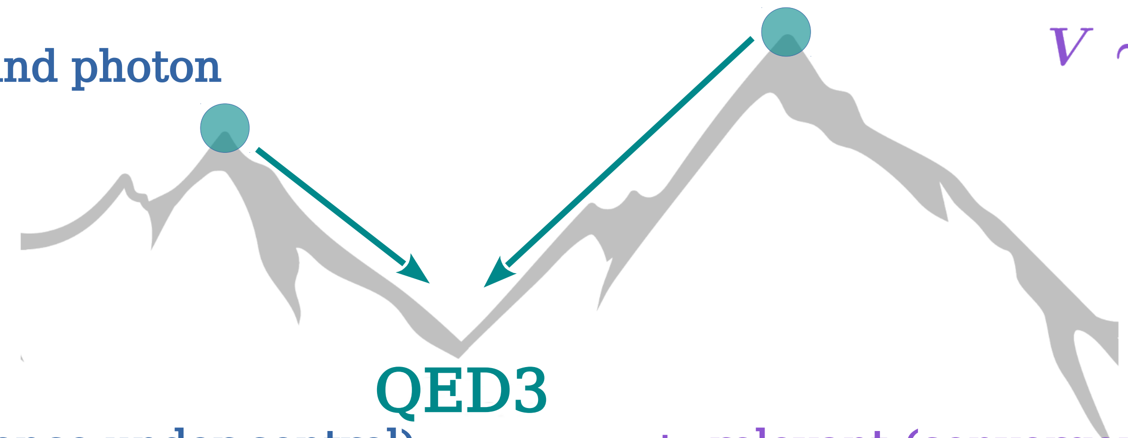

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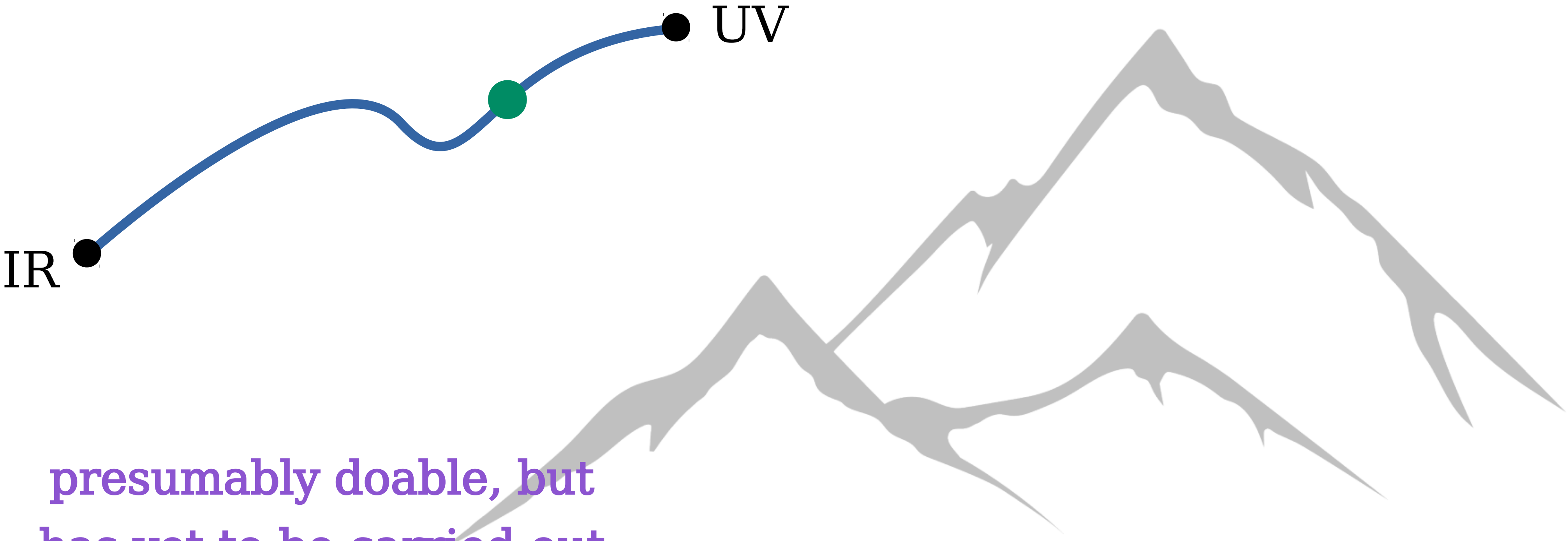
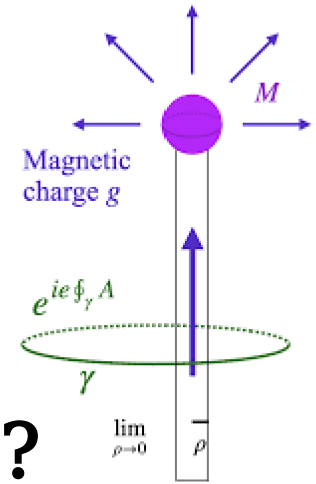
Computational dream slides



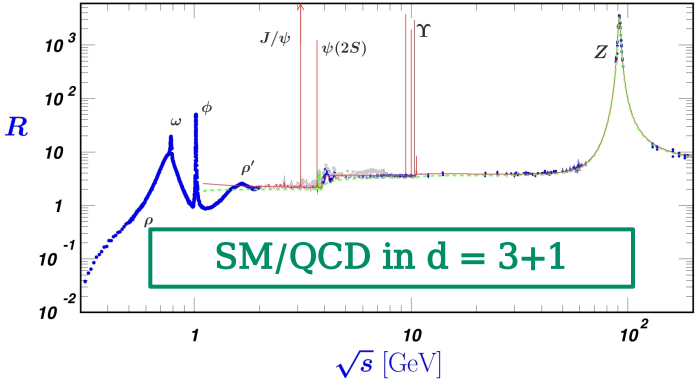
presumably doable, but
has yet to be carried out

Electromagnetism in the plane

big bite! ... something more digestible?

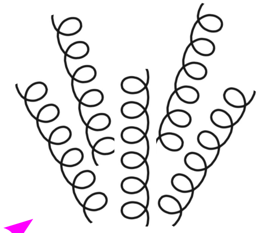


Gauge Theories & Truncation



If we want this

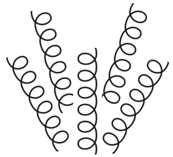
then we need these



two possible approaches

H_0 = free quarks and gluons

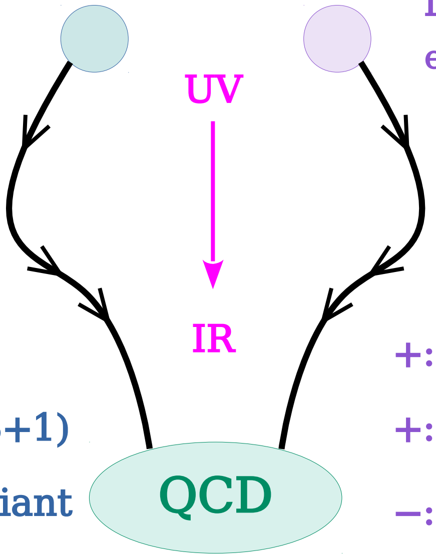
$V \sim g A_\mu J^\mu$
$+ g A^2 \partial A$



+: familiar

−: marginal (d = 3+1)

−: not gauge invariant



H_0 = "solvable" interacting UV fixed point

e.g. Banks-Zaks, SU(3) $N_f = 16$

$V \sim m\bar{q}q$

+: relevant

+: gauge invariant

−: need to "solve" H_0

Both options present for QED3 !

⇒ strongly coupled and confining

⇒ perturbative fixed point $N_f \gg 1$

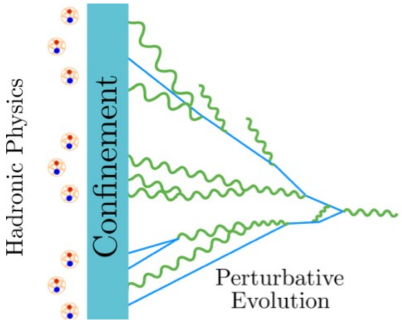
⇒ real world relevance (QHE, high T_c)

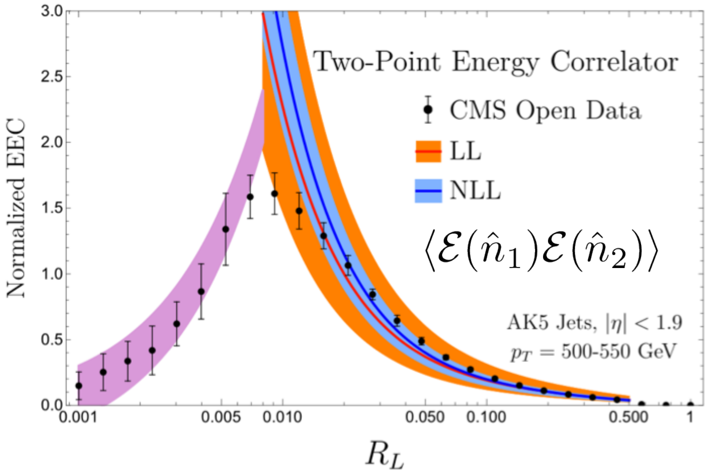
ambitious project already underway,
but it needs a permanent home!

Gauge theories

QCD in d = 3+1

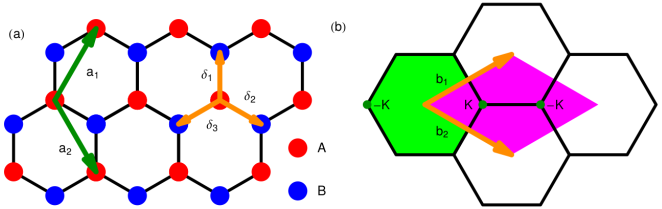
✓ Confining (for small N_f)





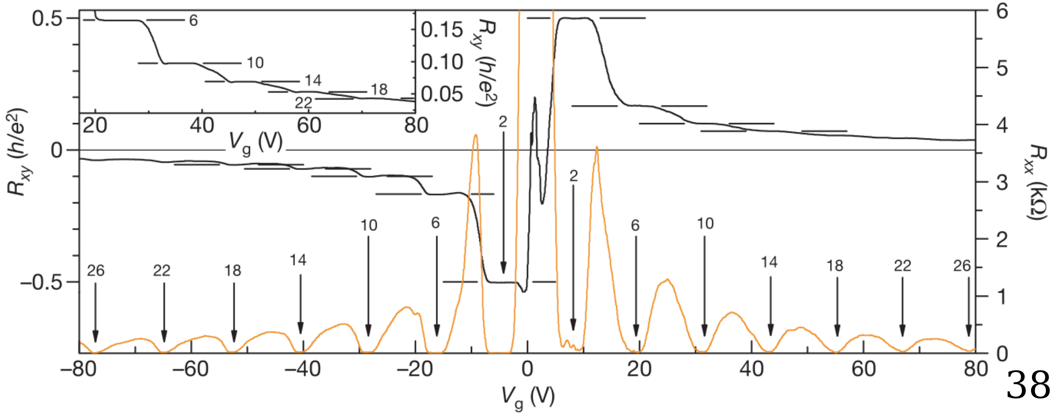
QED in d = 2+1

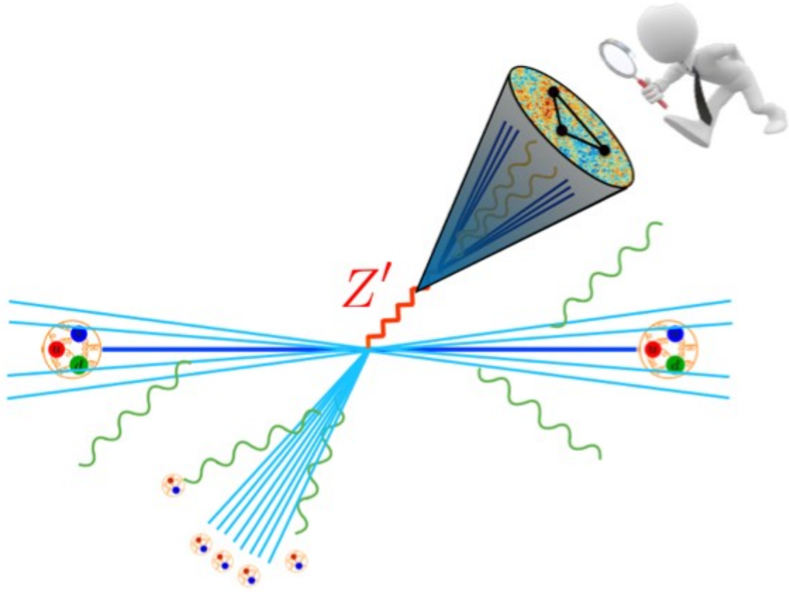
✓ Confining (for small N_f)



Graphene honeycomb lattice
Unconventional QHE
from: arXiv:0706.3016

QHE in graphene
Zhang et. al., Nature 438, 201-205 (2005)

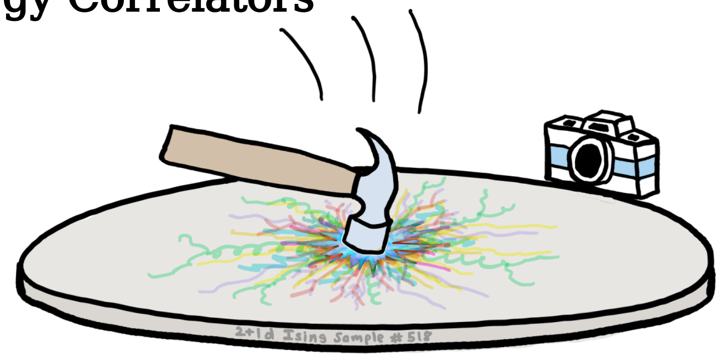




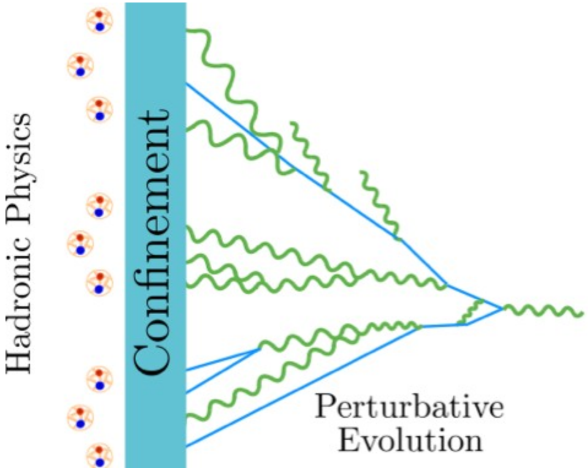
targeting hadronization

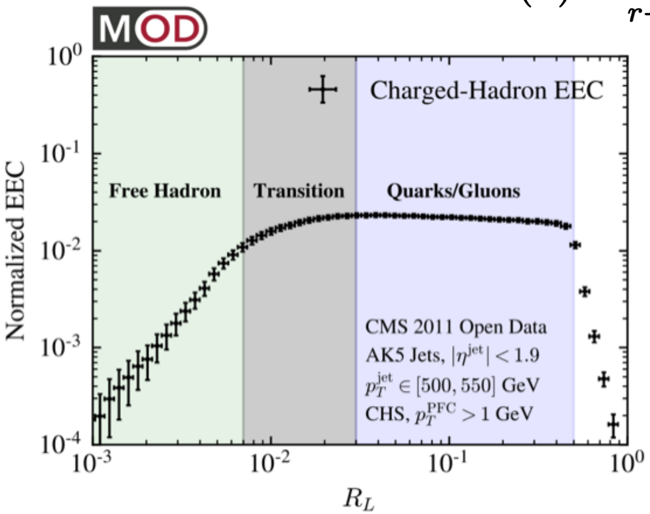
clean observables experimentally AND theoretically

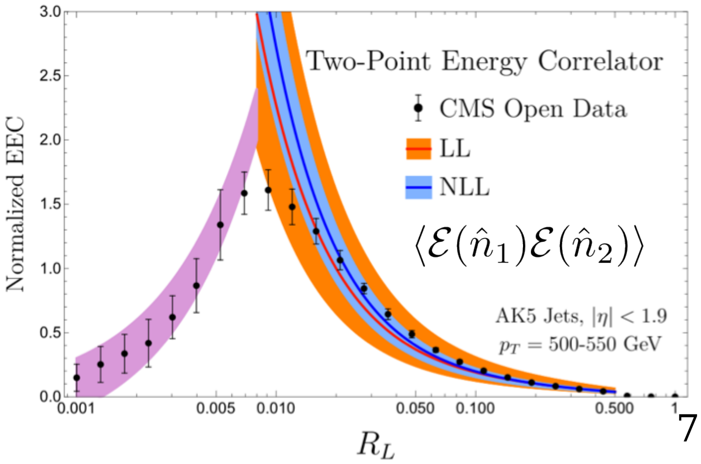
⇒ Energy Correlators



$$\mathcal{E}(\hat{n}) = \lim_{r\to\infty} r^{d-2} \int_0^\infty dt\, n^i T_{0i}(t, r\hat{n})$$





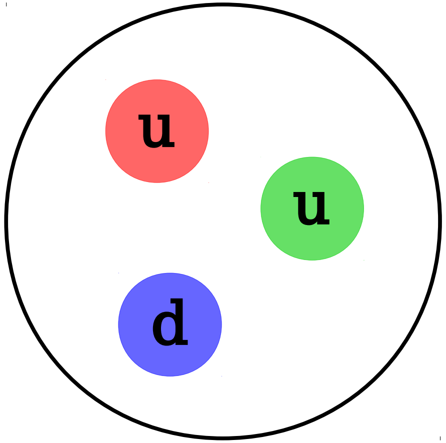


- Transition slide for gauge theories. Can I somehow use energy correlators (if I can, reference back to the “truncation philosophy”?
- The main hurdles: gauge redundancy (how to regularize) and marginal coupling
- Starting from H_0 an interacting fixed point
- Two paths slide
- QED3 (one or two slides)
-
- Can I slip in the dancing gif somewhere? Maybe with LSZ?

HT output

$$|\psi\rangle = \sum_{\chi \in \mathcal{H}_{\mathrm{UV}}} c_\chi \, |\chi\rangle$$

e.g.



$$|p\rangle = c_{uud}\, |uud\rangle + \cdots$$

Confinement as dual Meissner?

(Nambu, ‘t-Hooft, Mandelstam; Goddard, Nyuts, Olive; Witten, Osborne; Polyakov)

EM duality: $E \leftrightarrow B \qquad j_e \leftrightarrow j_m$ **response of test charges exchanged**

electric flux	magnetic flux
screened	confined
confined	screened

Is confinement in QCD driven by magnetic states condensing?

Confinement as dual Meissner?

(Nambu, 't-Hooft, Mandelstam; Goddard, Nyuts, Olive; Witten, Osborne; Polyakov)

EM duality: $\begin{pmatrix} E \\ B \end{pmatrix}$ $\begin{pmatrix} j_e \\ j_m \end{pmatrix}$

electric flux	magnetic flux
screened	confined
confined	screened

$$\begin{aligned} G &\leftrightarrow \widehat{G} \\ C(G) &= \pi_1(\widehat{G}) \\ \pi_1(G) &= C(\widehat{G}) \end{aligned}$$

e.g.

$$SU(N) \leftrightarrow SU(N)\big/Z_N$$

$$SO(2N+1) \leftrightarrow Sp(2N)$$

Such a mechanism is operative in a number of theories:

- **QED3**
- **4d susy gauge theories**

Often exhibit particle ↔ soliton duality

Question

Can we make it our responsibility to make a theory collider at the same time as building the next collider(s)?

[in the spirit of brainstorming how to get the future we want, I recommend taking a hard look at messaging]

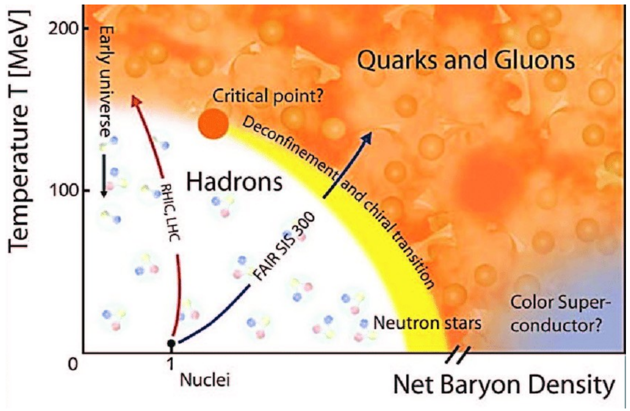
Observation/question

It appears (to me) that there is plenty of "new physics"

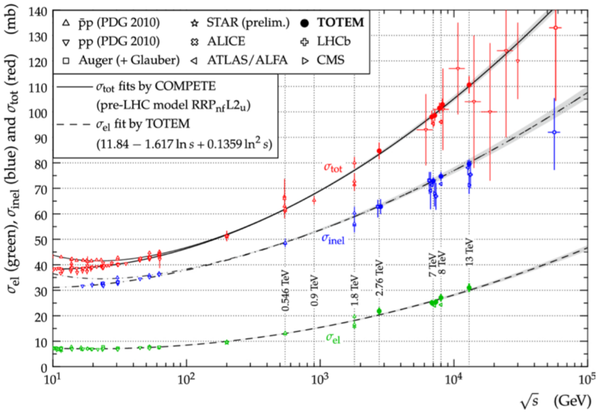
($\stackrel{\text{def}}{=}$ physics we don't know how to describe)

being discovered at colliders

Why doesn't this "count"?



[e.g.
Exotics,
diffractive scattering (Pomeron),
QGP,
hadronization (energy correlators),
...]



Reminder: two input ingredients

$$\text{STATES} \quad \Rightarrow \quad |\psi\rangle\,, \;\; \langle\psi|\psi\rangle < \infty$$

$$\underbrace{\langle\psi|H|\psi'\rangle}_{\text{Born level}} \quad \Leftarrow \quad \text{MATRIX ELEMENTS}$$

⇒ **Ingredients recyclable for many different theories**

Massless phase space

BH, T. Melia
1902.06747
1902.06754

⇒ **momentum conservation**

⇒ **on-shell**

⇒ **Lorentz invariance**

constraints define a manifold in phase space

$$\delta(p_1^2)\cdots\delta(p_n^2)\times\delta^4\big(P^\mu-(p_1^\mu+\cdots+p_n^\mu)\big)$$

use spinors

$$\delta^4\Big(P_{\alpha\dot\alpha}-(\lambda^1\widetilde{\lambda}^1+\cdots+\lambda^n\widetilde{\lambda}^n)_{\alpha\dot\alpha}\Big)$$

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Want a set of class functions on the manifold

→ generalized spherical harmonics

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use spinors

$$\delta^4\Big(P_{\alpha\dot\alpha}-(\lambda^1\widetilde{\lambda}^1+\cdots+\lambda^n\widetilde{\lambda}^n)_{\alpha\dot\alpha}\Big)$$

$$\lambda=\{\lambda_\alpha{}^i\}=\begin{pmatrix}\lambda_1{}^1 & \cdots & \lambda_1{}^N\\ \lambda_2{}^1 & \cdots & \lambda_2{}^N\end{pmatrix}$$

Want a set of class functions on the manifold

$\rightarrow$ **generalized spherical harmonics**

$$\lambda\to g\lambda U^T\ \left(\lambda_\alpha{}^i\to g_\alpha{}^\beta U^i{}_j\lambda_\beta{}^j\right)\quad\Rightarrow\quad P=\lambda\lambda^\dagger$$

$$g\in SL(2,\mathbb{C})\,,\ U\in U(N)\supset U(1)^N$$

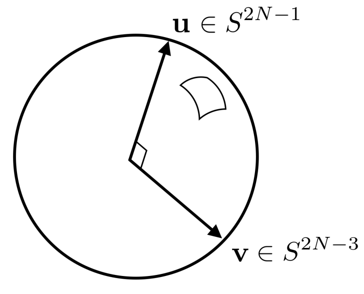
U(N) invariant!

$$\int d\Pi_n^P\ \Rightarrow\ \int d\lambda d\lambda^\dagger\delta(P-\lambda\lambda^\dagger)$$

BH, T. Melia
1902.06747
1902.06754

geometry of phase space

$$\delta^4(P - \lambda\lambda^\dagger) \xrightarrow{\text{c.o.m.}} P_{\alpha\dot\alpha} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} \left|\vec\lambda_1\right|^2 & \vec\lambda_1\cdot\vec\lambda_2^* \\ \vec\lambda_2\cdot\vec\lambda_1^* & \left|\vec\lambda_2\right|^2 \end{pmatrix}$$



$\Longleftrightarrow$

$$\vec v^2 = r^2$$
$$\vec u^2 = r^2$$
$$\vec v\cdot\vec u = 0$$

$\Longleftrightarrow$

geometry basically complex version of two orthogonal spheres

BH, T. Melia
1902.06747
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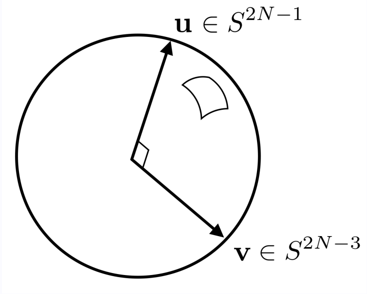
geometry of phase space

$$\delta^4(P - \lambda\lambda^\dagger) \xrightarrow{\text{c.o.m.}} P_{\alpha\dot{\alpha}} = \begin{pmatrix} M & 0 \\ 0 & M \end{pmatrix} = \begin{pmatrix} \left|\vec{\lambda}_1\right|^2 & \vec{\lambda}_1 \cdot \vec{\lambda}_2^* \\ \vec{\lambda}_2 \cdot \vec{\lambda}_1^* & \left|\vec{\lambda}_2\right|^2 \end{pmatrix}$$



$$\Longleftrightarrow \quad \vec{v}^2 = r^2, \quad \vec{u}^2 = r^2, \quad \vec{v} \cdot \vec{u} = 0 \quad \Longleftrightarrow$$

geometry basically complex version of two orthogonal spheres

$$G/H = U(N)/U(N-2)$$ "Stiefel manifold" $V_2(\mathbb{C}^N)$

Grassmannian ⊂ Stiefel $\quad G_2(\mathbb{C}^N) = U(N)/U(N-2) \times U(2)$

$\boxed{\textbf{states} \Leftrightarrow \textbf{harmonics on phase space}}$

"conformal – helicity duality"

$4d: \; SU(2,2) \times U(N)$

$3d: \; Sp(4,\mathbb{R}) \times O(N)$

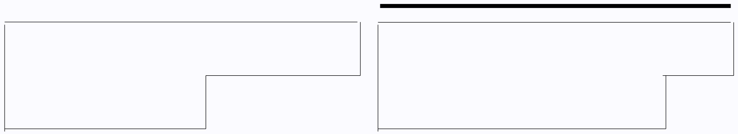
$2d: \; SL(2,\mathbb{R}) \times O(N)$

(math world: reductive dual pairs/Howe duality/oscillator representation)

upshot on Stiefel harmonics

harmonics labeled by Young diagrams

(with at most two rows)



these dictate specific polynomials in the spinors

comments:

1) each shape corresponds to operators

2) multiple operators belong to same shape

a) these involve particles with different spin

3) these operators are conformal primaries

Construct states algebraically

e.g.

$$|l, \mu = (\mu_1, \ldots, \mu_3)\rangle \simeq F^3$$

now apply U(N) lowering op:

$$L_- |l, \mu\rangle \sim |l, \mu'\rangle \simeq \widetilde{\psi} F \psi$$

Observation

we have significant representation and environment issues (to put it mildly, IMO)

physics, and theoretical physics in particular, do not have a good reputation

what does this mean for our future?