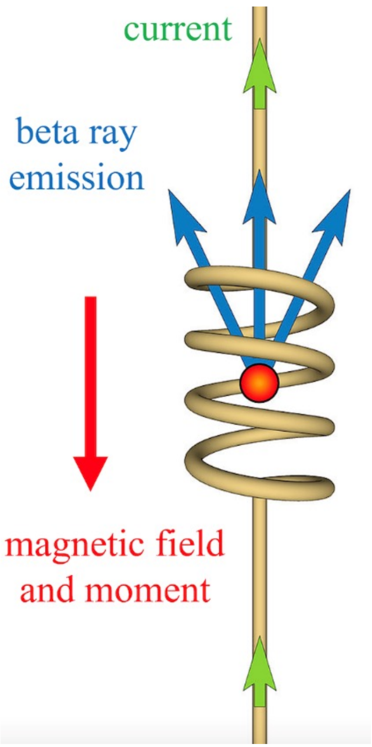
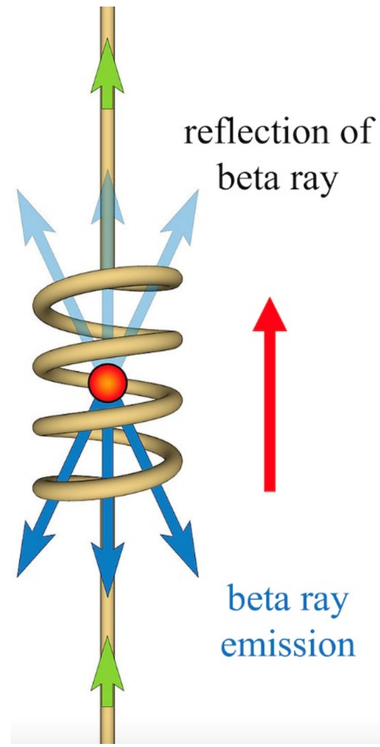
Symmetries and Particle Physics



UHM Physics & Astronomy Colloquium
Mar 04, 2025

Xiaochuan Lu

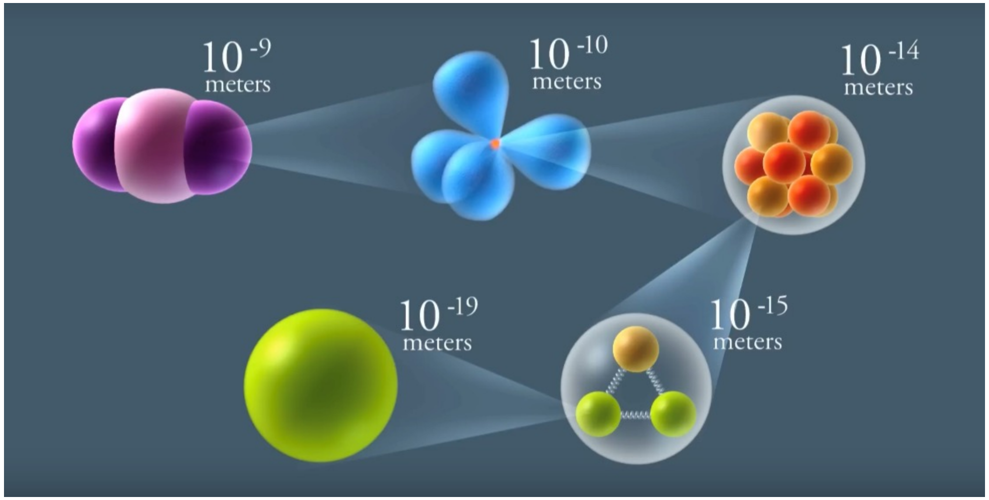
University of California, San Diego



The Approach of Particle Physics

- keep zooming in to shorter and shorter distances
- try to identify elementary building blocks (particles)
- study the interactions among them

molecules atoms nucleus



quarks protons, neutrons

The Approach of Particle Physics

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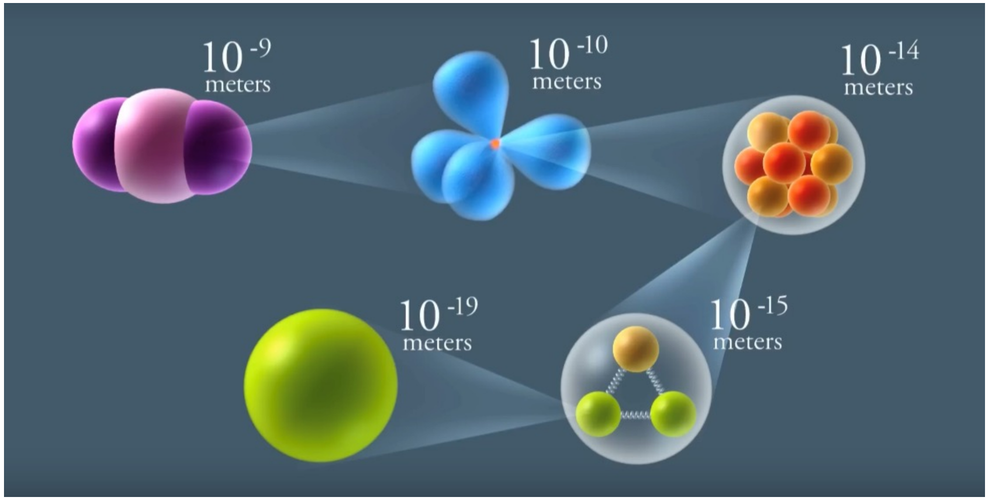
Compelling idea

Technical challenges

Conceptual limitations

molecules atoms nucleus



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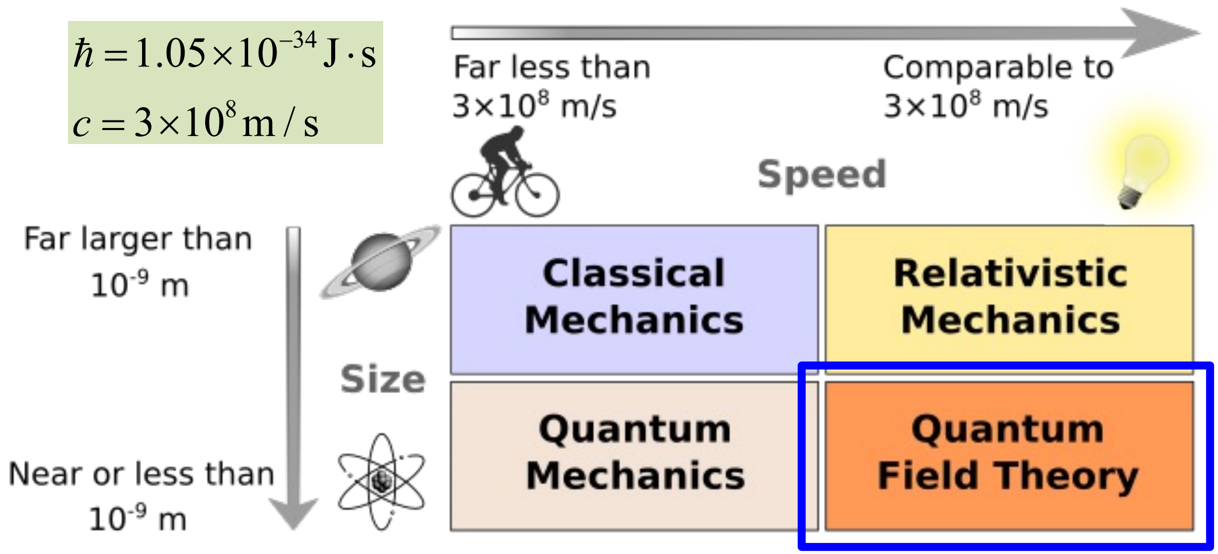
$\hbar = 1.05 \times 10^{-34}\ \mathrm{J \cdot s}$

$c = 3 \times 10^{8}\ \mathrm{m/s}$



$\hbar \to 0$

$\hbar = 1$

$c \to \infty$ $\qquad$ $c = 1$

The Approach of Particle Physics

- keep zooming in to shorter and shorter distances
- try to identify elementary building blocks (particles)
- study the interactions among them

Compelling idea

There are always physics at even shorter distances that are beyond our experimental resolution

Technical challenges

--- Identifying the *relevant* building blocks at a given scale

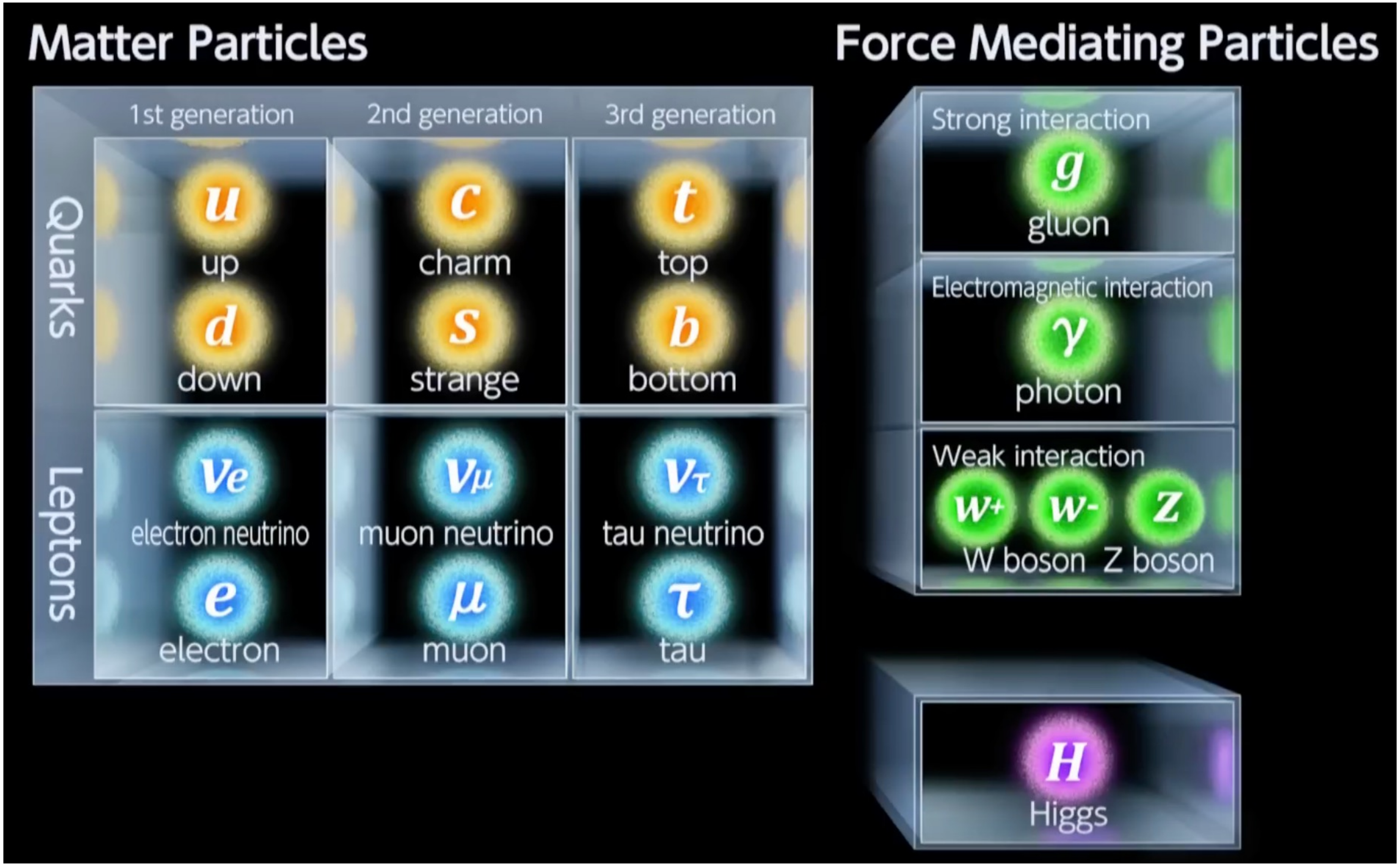
--- Studying the *effective* interactions among them

Conceptual limitations

We are always probing an "Effective Field Theory"

Technically known as the need for "Renormalization"

Currently understood: Standard Model of Particle Physics



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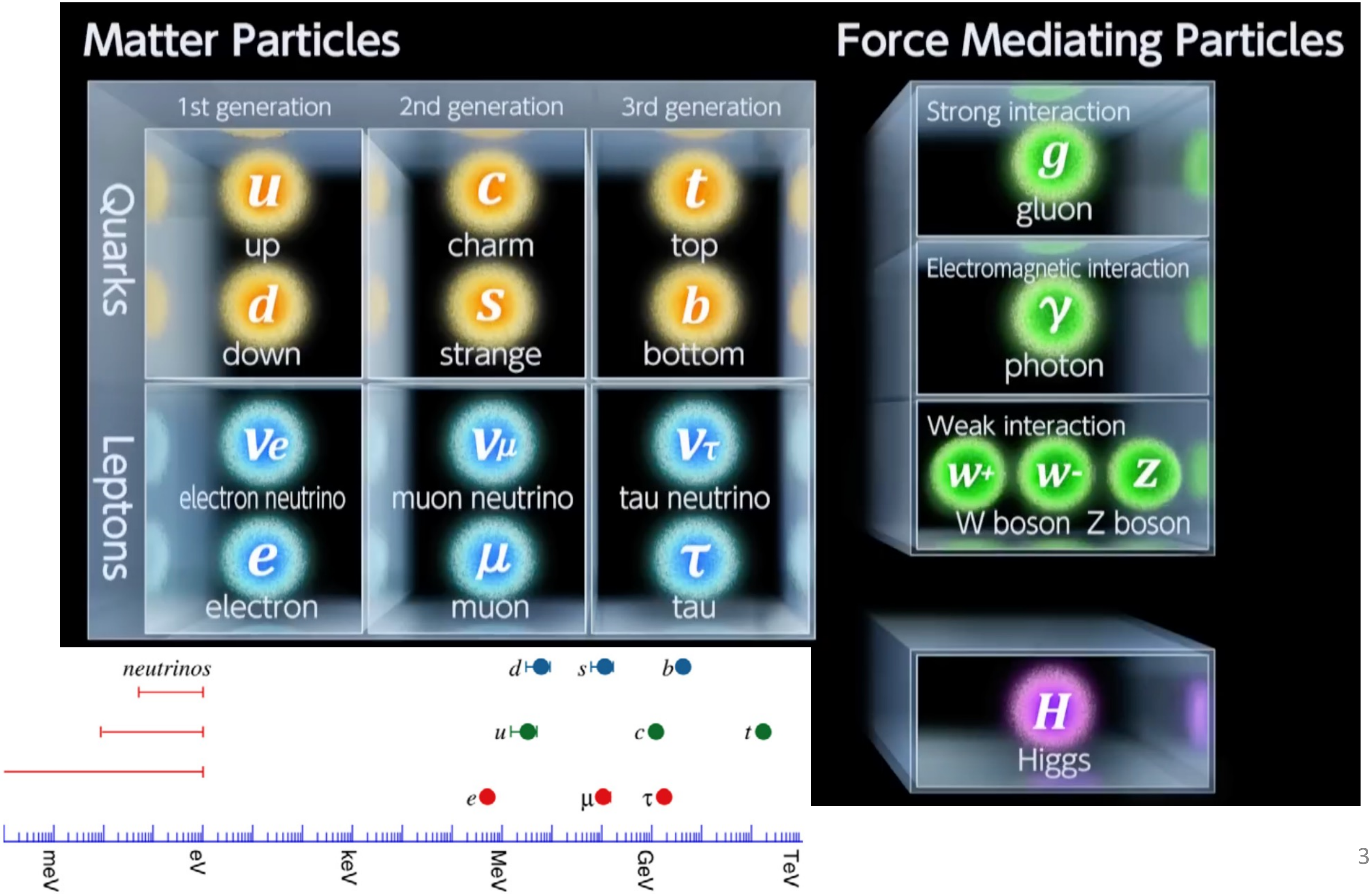
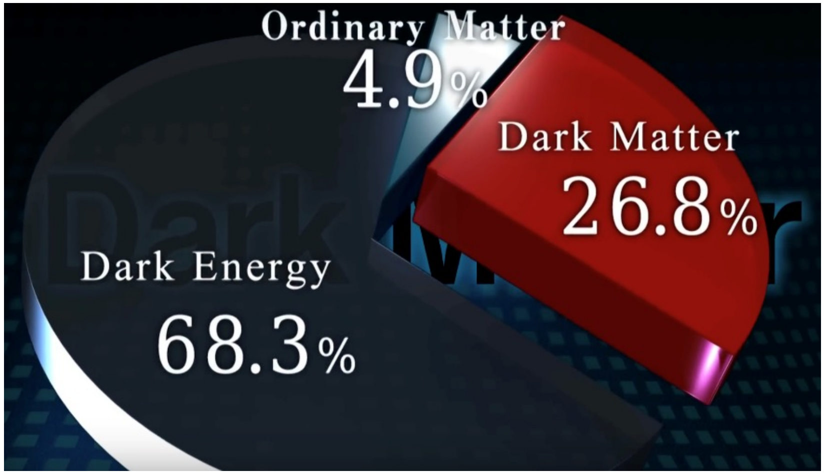
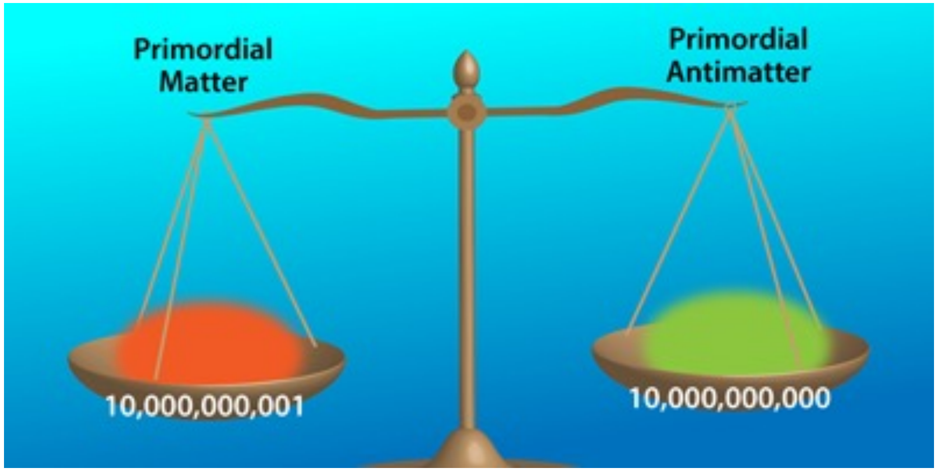
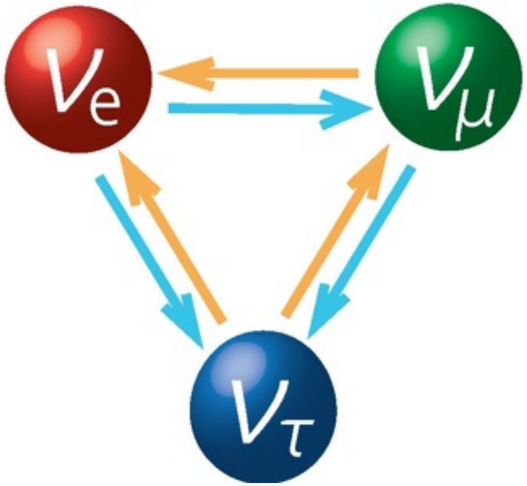


Yet to be understood: Physics beyond the Standard Model

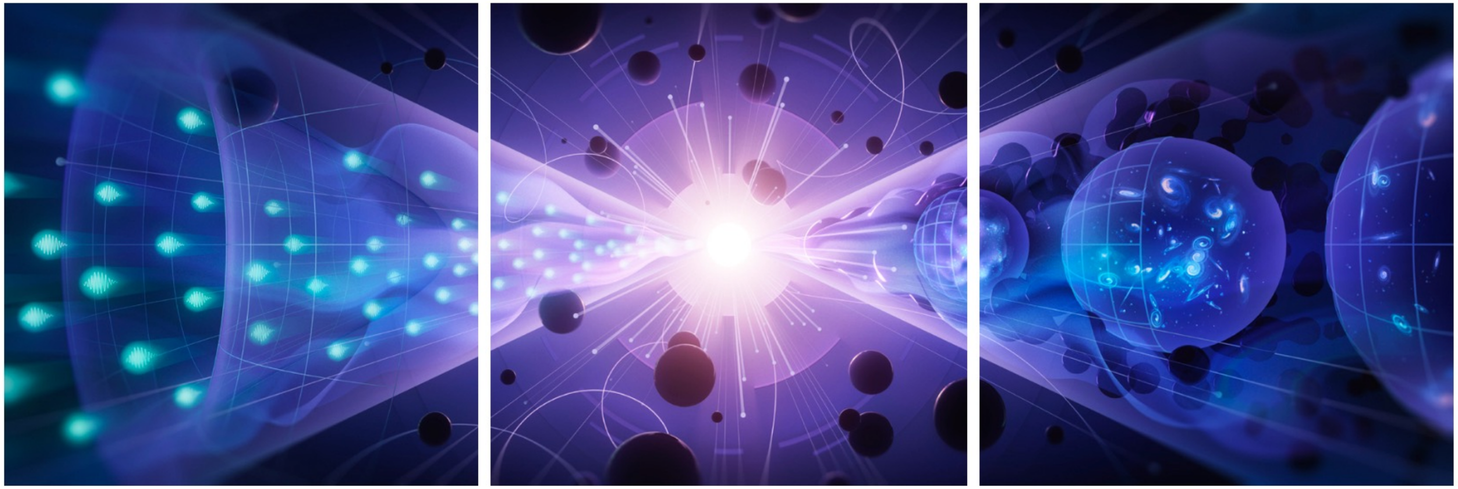






- nature of dark matter
- neutrino masses and oscillations
- matter-antimatter asymmetry
- ……

10-20 year Plan: three *Science Themes* and six *Science Drivers*



Decipher the Quantum Realm	Explore New Paradigms in Physics	Illuminate the Hidden Universe
Elucidate the Mysteries of Neutrinos	Search for Direct Evidence of New Particles	Determine the Nature of Dark Matter
Reveal the Secrets of the Higgs Boson	Pursue Quantum Imprints of New Phenomena	Understand What Drives Cosmic Evolution

2023 Particle Physics Project Prioritization Panel (P5) Report, Hitoshi Murayama, Karsten Heeger, et al.

How to tackle the problem as a theorist?

Today: Symmetries + Effective Field Theory (EFT)

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- Extremely illuminating in establishing a theory

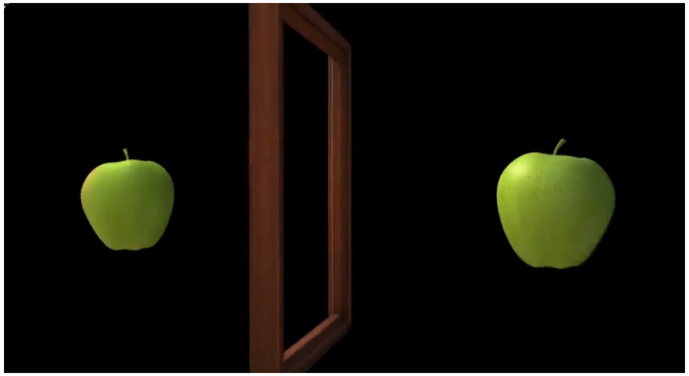
How to tackle the problem as a theorist?

Today: Symmetries + Effective Field Theory (EFT)

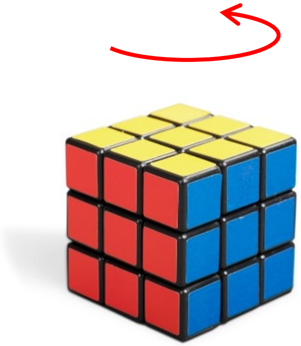
- Extremely illuminating in establishing a theory

  - Robust parameterization of new physics
  - Systematic way of implementing symmetries

Symmetry: Invariance under a transformation



reflection: parity



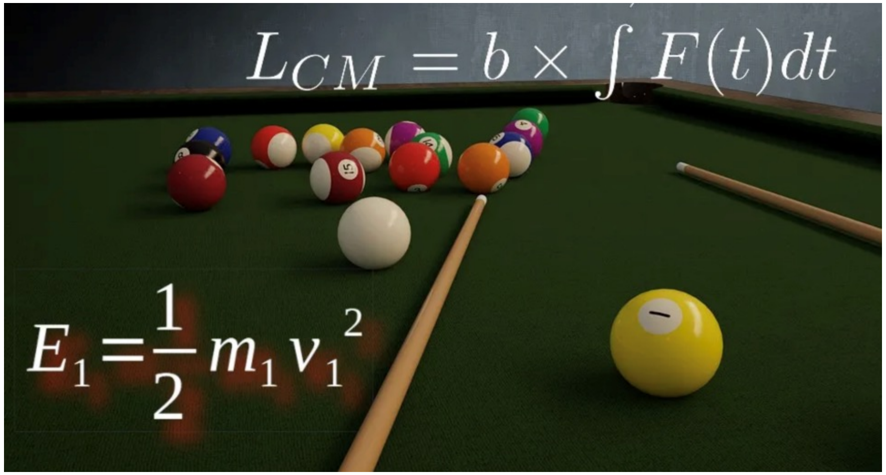
discrete rotations

continuous rotations

Profound Implications: Conservation Laws

Noether's theorem

- **Energy**
  time translation invariance
- **Momentum**
  space translation invariance
- **Angular Momentum**
  space rotation invariance



Conservation Laws (symmetries) in Particle Physics

- Electric charge conservation: -1 -1 0 0

muon decay : $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

Conservation Laws (symmetries) in Particle Physics

➢ Electron flavor number conservation:	0	1	-1	0
➢ Electric charge conservation:	-1	-1	0	0
muon decay :	μ^-	$\rightarrow e^-$	$+ \bar{\nu}_e$	$+ \nu_\mu$

$$\text{muon decay}: \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

Conservation Laws (symmetries) in Particle Physics

	μ^-	e^-	$\bar{\nu}_e$	ν_μ
➢ Muon flavor number conservation:	1	0	0	1
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Conservation Laws (symmetries) in Particle Physics

- Muon flavor number conservation
- Electron flavor number conservation
- Electric charge conservation

$$\text{muon decay}: \quad \mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$$

$$\text{pion decay}: \quad \pi^+ \rightarrow \mu^+ + \nu_\mu$$

$$\text{neutron beta decay}: \quad n \rightarrow p + e^- + \bar{\nu}_e$$

Conservation Laws (symmetries) in Particle Physics

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Baryon number: 1 1 0 0

Conservation Laws (symmetries) in Particle Physics

- Baryon Number Conservation
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Symmetries also lead to stable particles!

The lightest particle charged under a symmetry is stable

muon decay : $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

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Baryon number: 1 1 0 0

Conservation Laws (symmetries) in Particle Physics

- Baryon Number Conservation — proton is stable
- Muon flavor number conservation
- Electron flavor number conservation
- Electric charge conservation — electron is stable

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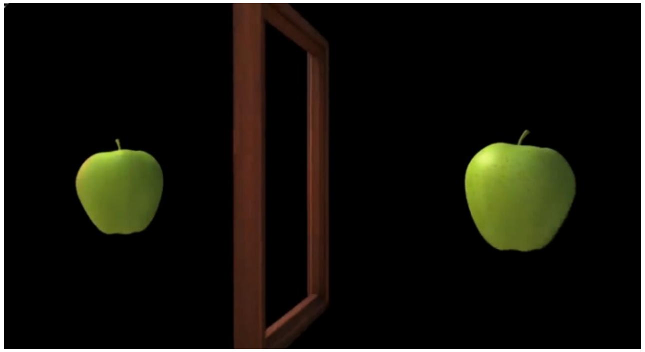
A breakthrough: Parity Violation

$\tau - \theta$ puzzle: Parity conservation had been taken for granted up to 1956

$$K^+(u\bar{s}) = \begin{cases} \theta^+ \rightarrow \pi^+ + \pi^0 & \text{parity even} \\ \tau^+ \rightarrow \pi^+ + \pi^+ + \pi^- & \text{parity odd} \end{cases}$$



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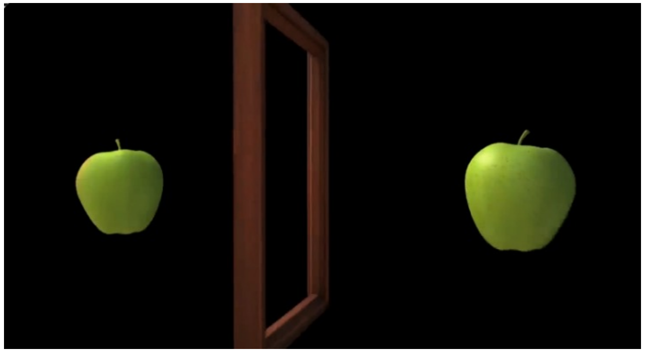

In 1956, T. D. Lee and C. N. Yang proposed parity violation as a solution to the puzzle

PHYSICAL REVIEW VOLUME 104, NUMBER 1 OCTOBER 1, 1956

Question of Parity Conservation in Weak Interactions*

T. D. LEE, *Columbia University, New York, New York*

AND

C. N. YANG,† *Brookhaven National Laboratory, Upton, New York*

(Received June 22, 1956)

The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

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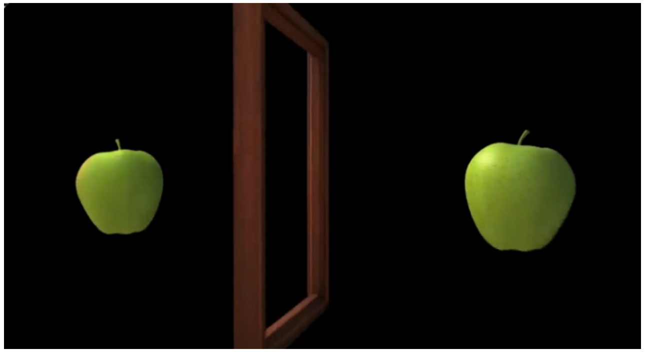

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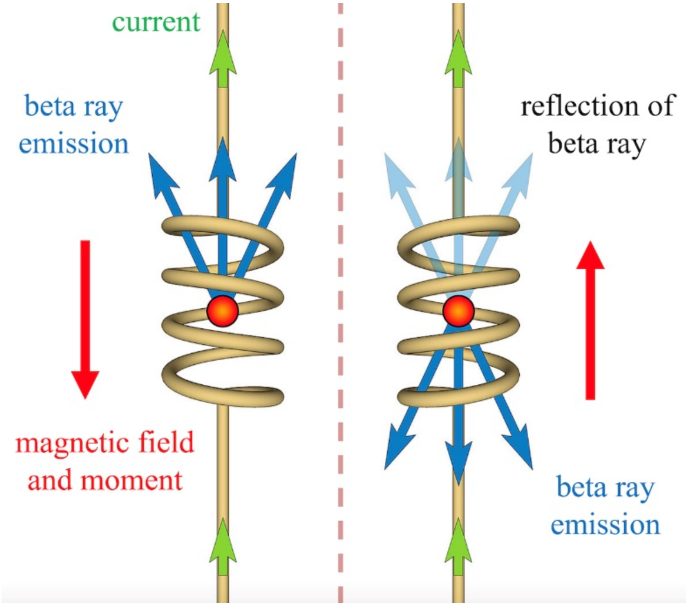

$$n \to p + e^{-} + \bar{\nu}_e$$

preferred direction of beta decay

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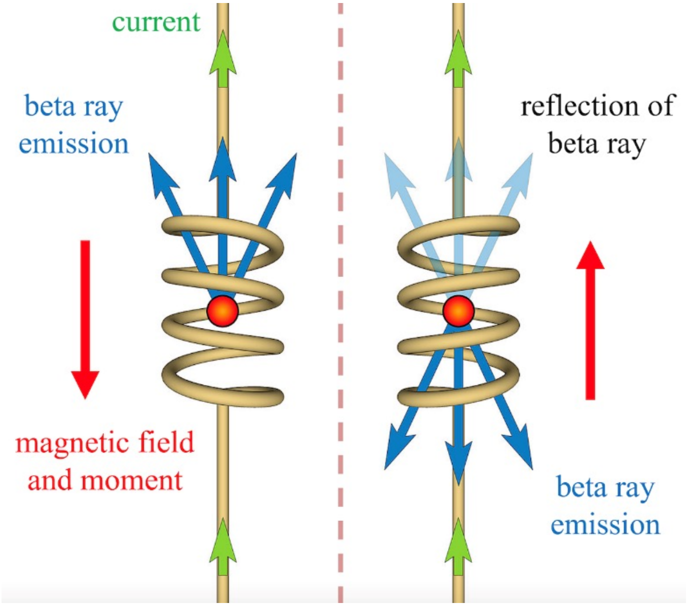
--- Weak interactions are chiral

--- Neutrinos are left-handed



$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

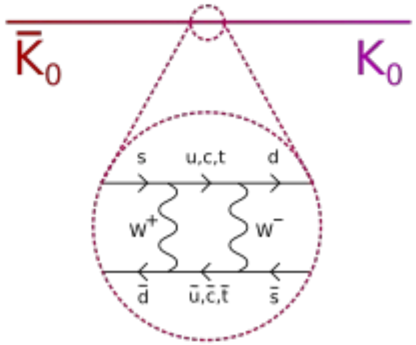
preferred direction of beta decay

Studying a symmetry/violation can be highly rewarding

Another breakthrough: CP Violation

- Maybe weak interactions preserve the combine of C and P --- CP symmetry?

P: Parity C: Charge Conjugation, switching particles with anti-particles



$K^0(d\bar{s}) - \bar{K}^0(\bar{d}s)$

mixing violates CP

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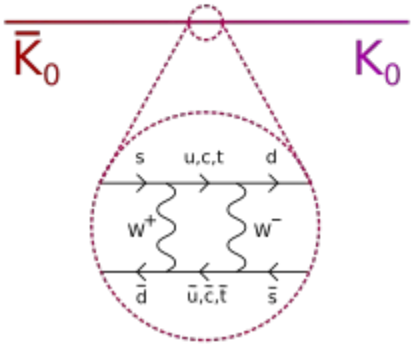

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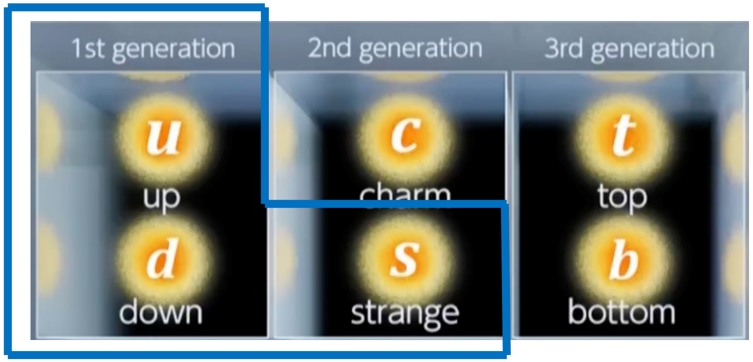

Led to the proposal of Cabibbo-Kobayashi-Maskawa (CKM) matrix with 6 quarks and 3 generations: only 3 quarks were observed at the time!

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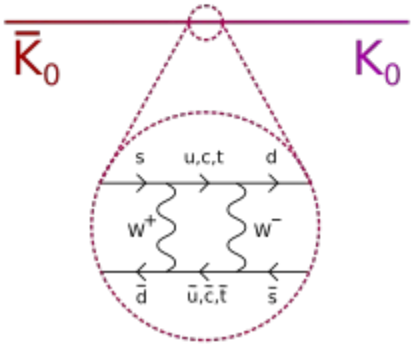

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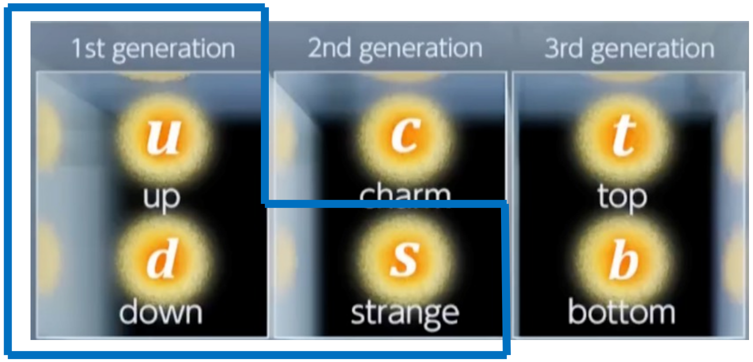

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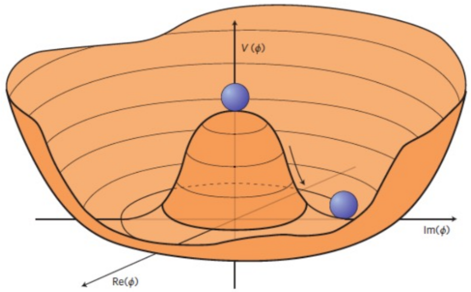
--- Uncovers the nature of weak interactions and flavor structure

--- Critical for the establishment of the Standard Model

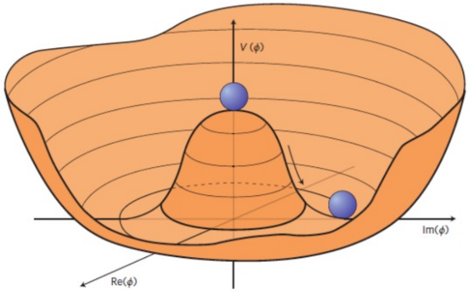
Symmetries



Symmetries in the Standard Model

- Spacetime translation and Lorentz invariance
- Charge symmetries $SU(3)_C \times SU(2)_L \times U(1)_Y$ --- strong, weak, E&M
- Spontaneous Symmetry Breaking from Higgs --- origin of masses

Symmetries



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Study symmetries to understand physics beyond the Standard Model

- Lepton number: neutrino masses and oscillations (Dirac or Majorana)
- Flavor symmetries and CP violations: matter-antimatter asymmetry
- Additional symmetries: new stable particle, dark matter candidate
- ……

How to tackle the problem as a theorist?

Today: Symmetries + Effective Field Theory (EFT)

- Extremely illuminating in establishing a theory

- Robust parameterization of new physics
- Systematic way of implementing symmetries

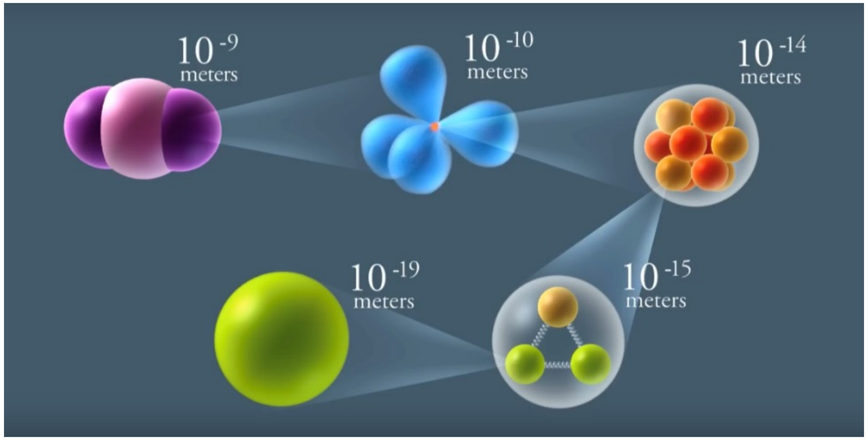
Suppose we observe a symmetry violation, or place a stringent bound on it, how do we make use of this data to pin down the possibilities of new physics?

Spirit of an Effective Field Theory (EFT)

- Build different theories at different scales
- At each scale, build the theory with relevant building blocks (particles/fields)
- Study *effective* interactions (operators) among them

$$\mathcal{L}_{\mathrm{EFT}}(\phi) = \sum_i c_i \mathcal{O}_i(\phi)$$

molecules atoms nucleus



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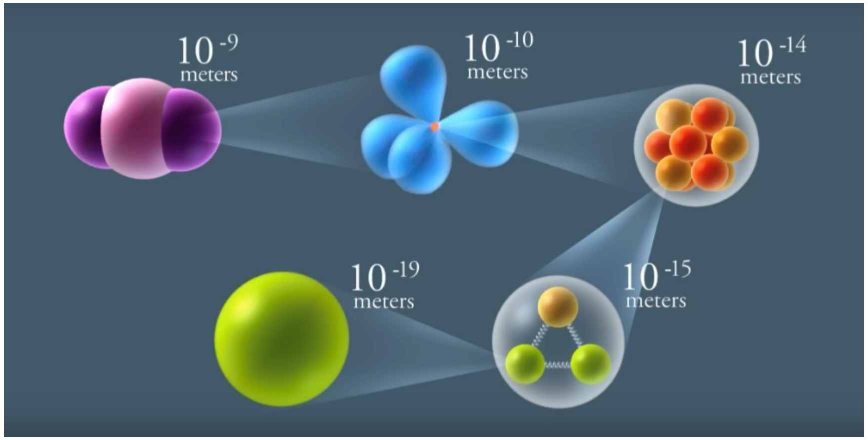

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There are always physics at shorter distances

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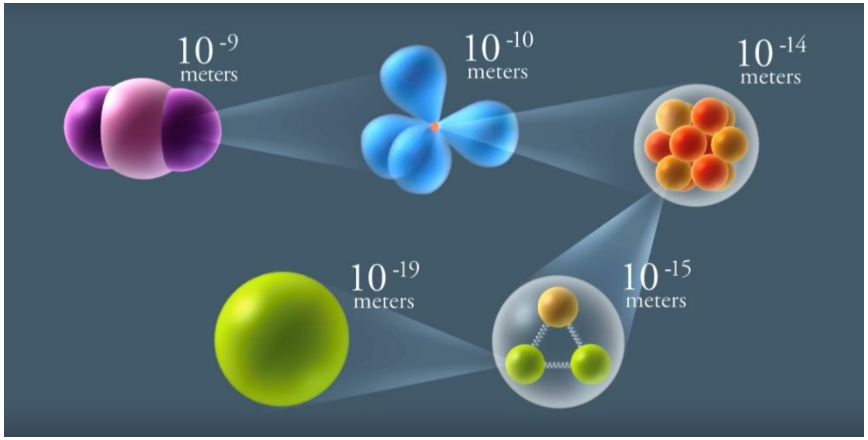


quarks protons, neutrons

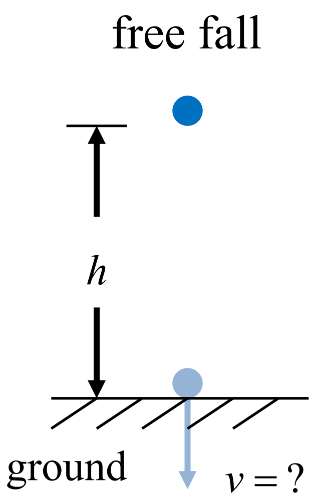
There are always physics at shorter distances

capture their effects

Include *enough* number of effective operators

A general spirit in theoretical physics

Example: Gravity near the surface of the Earth



$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh}$$

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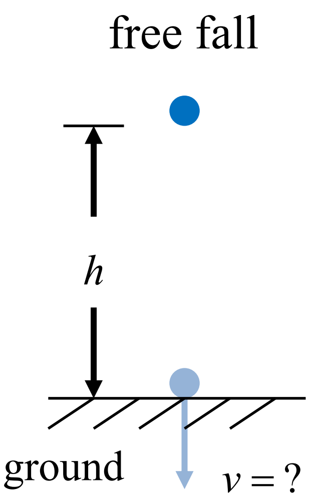

$$\frac{1}{2}mv^2 = mgh \quad \Rightarrow \quad v = \sqrt{2gh}$$

$$V(h) = V_0 + mgh \quad , \quad g = 9.8\,\mathrm{m/s^2}$$

An effective formula
works well in our labs

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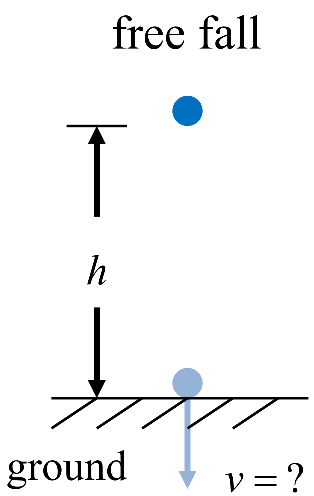

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- Newton's law of universal gravitation
- General Relativity
- Quantum Gravity

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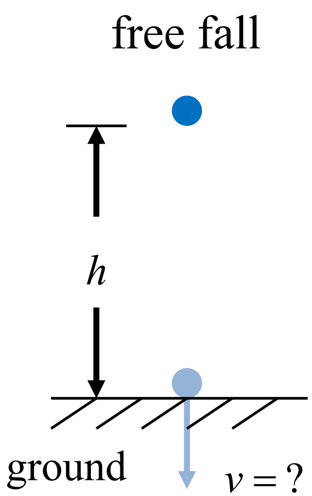

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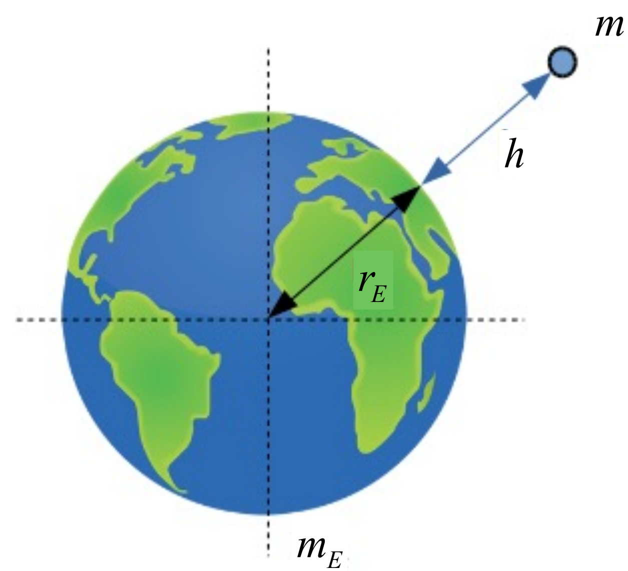

- Newton's law of universal gravitation
- General Relativity
- Quantum Gravity

$$V(h) = -\frac{G_N m_E m}{r_E + h} = -\frac{G_N m_E m}{r_E}\left(1 - \frac{h}{r_E} + \cdots\right)$$

$$h \ll r_E$$

$$g \equiv \frac{G_N m_E}{r_E^2} = 9.8\,\mathrm{m/s^2}$$

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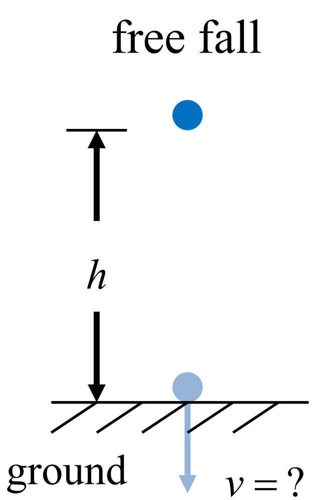

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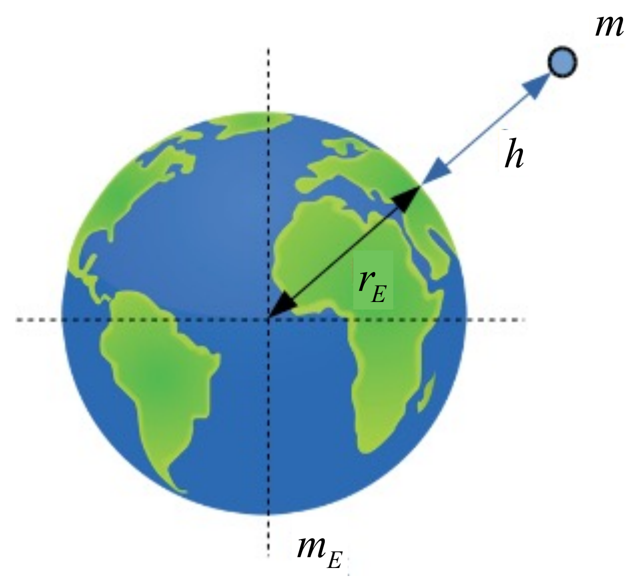
An effective formula works well in our labs



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- ➢ General Relativity

$$V(h) = V_0 + c_1 h + c_2 h^2 + c_3 h^3 + \cdots$$

- ➢ Quantum Gravity

$$g \equiv \frac{G_N m_E}{r_E^2} = 9.8\,\mathrm{m/s^2}$$

Discover more fundamental theories by measuring higher orders precisely

Symmetries are crucial in defining an EFT

$$V(h) = V_0 + c_1 h + c_2 h^2 + c_3 h^3 + \cdots$$

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$$V(h) = V(-h)$$

Symmetries are crucial in defining an EFT

$$V(h) = V_0 + c_1 h + c_2 h^2 + c_3 h^3 + \cdots \qquad V(h) = V(-h)$$

non-derivative terms

$$\mathcal{L}_{\rm EFT}\left(\phi_1, \phi_1^*, \partial_\mu\right) = c_{1a}\phi_1 + c_{1b}\phi_1^* + c_{2a}\phi_1^2 + c_{2b}\phi_1^{*2} + c_{2c}\phi_1\phi_1^* + \cdots$$

Symmetries are crucial in defining an EFT

$$V(h) = V_0 + \cancel{c_1 h} + c_2 h^2 + \cancel{c_3 h^3} + \cdots \qquad V(h) = V(-h)$$

non-derivative terms

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$$+1 \qquad -1 \qquad +2 \qquad -2 \qquad 0$$

	Charge
ϕ_1	$+1$
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Symmetries can be quite restrictive!

Effective Field Theory

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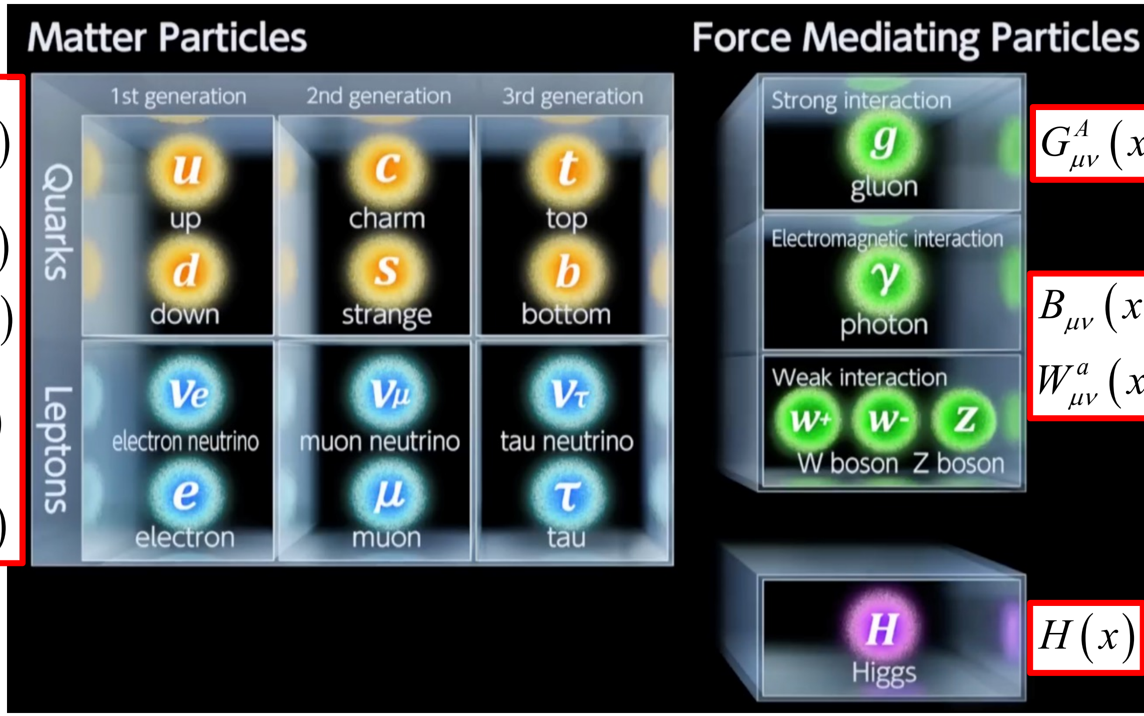
General Construction of an EFT: Fields + Expansion + Symmetries

Imposing different symmetries is defining different EFTs, which are probing different classes of new physics

Standard Model Effective Field Theory (SMEFT)

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \sim q(x)$$
$$u_R \sim u(x)$$
$$d_R \sim d(x)$$
$$\begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \sim l(x)$$
$$e_R \sim e(x)$$



$G^A_{\mu\nu}(x)$

$B_{\mu\nu}(x)$

$W^a_{\mu\nu}(x)$

$H(x)$

Fields ϕ_{SM}
$H(x)$
$q(x)$
$u(x)$
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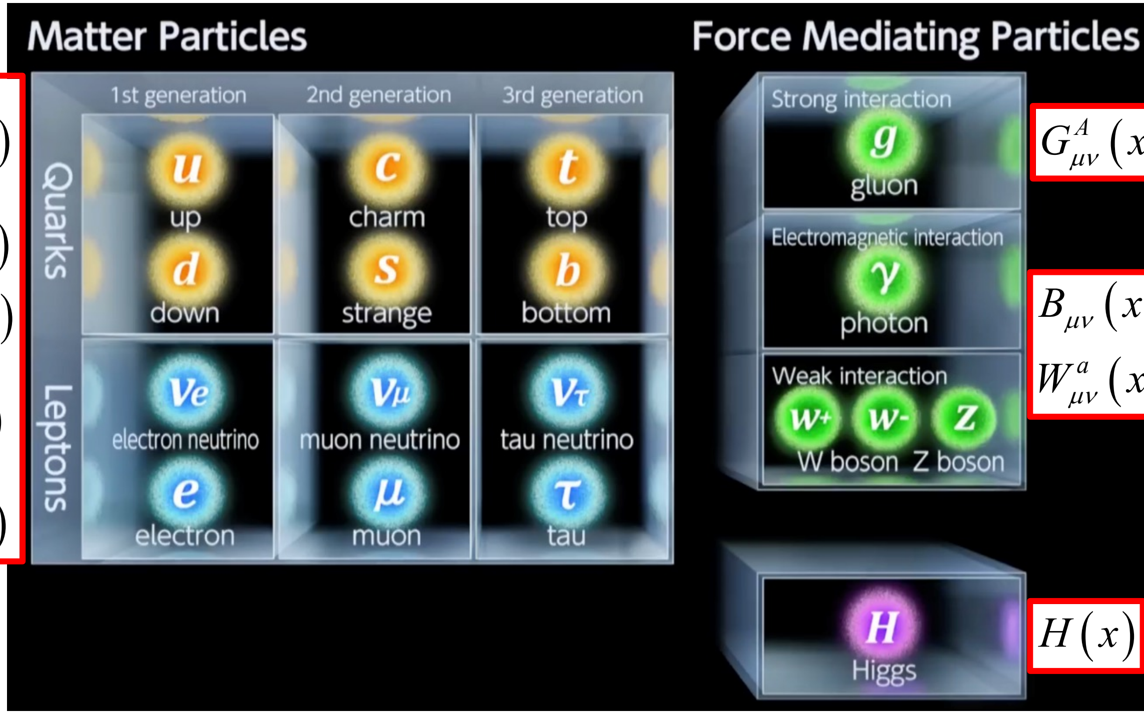

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$W^a_{\mu\nu}(x)$

$H(x)$

Expansion

Fields $\phi_{\rm SM}$	mass dim
$H(x)$	1
$q(x)$	3/2
$u(x)$	3/2
$d(x)$	3/2
$l(x)$	3/2
$e(x)$	3/2
$G^A_{\mu\nu}(x)$	2
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$$[\partial_\mu] = 1$$

Effective Field Theory

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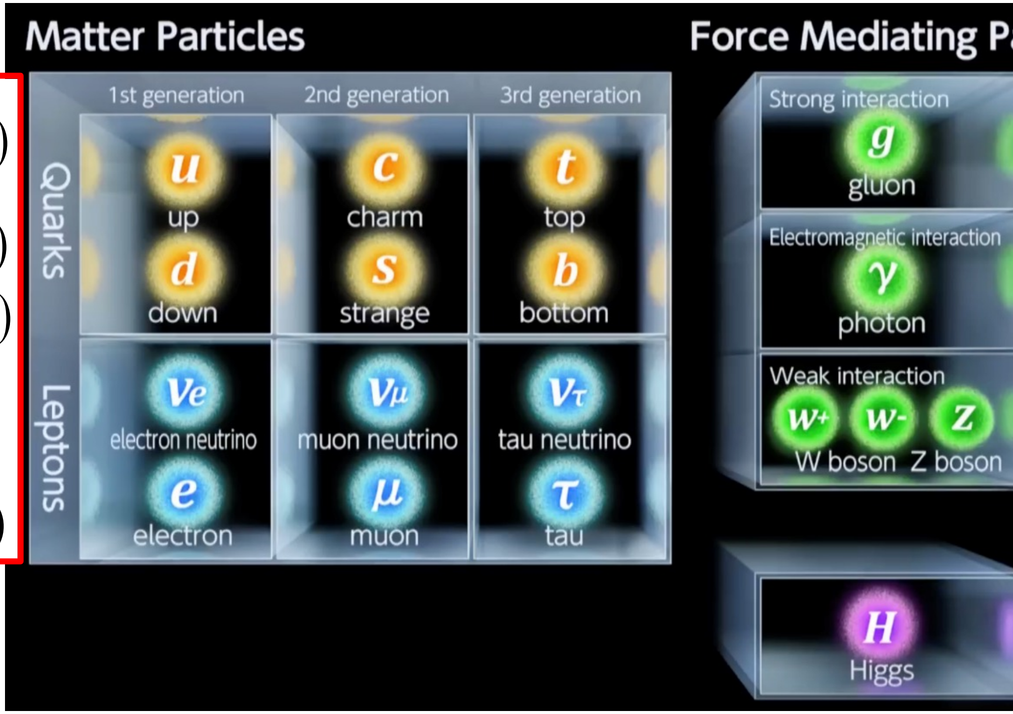

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Lorentz, $SU(3)_C \times SU(2)_L \times U(1)_Y$

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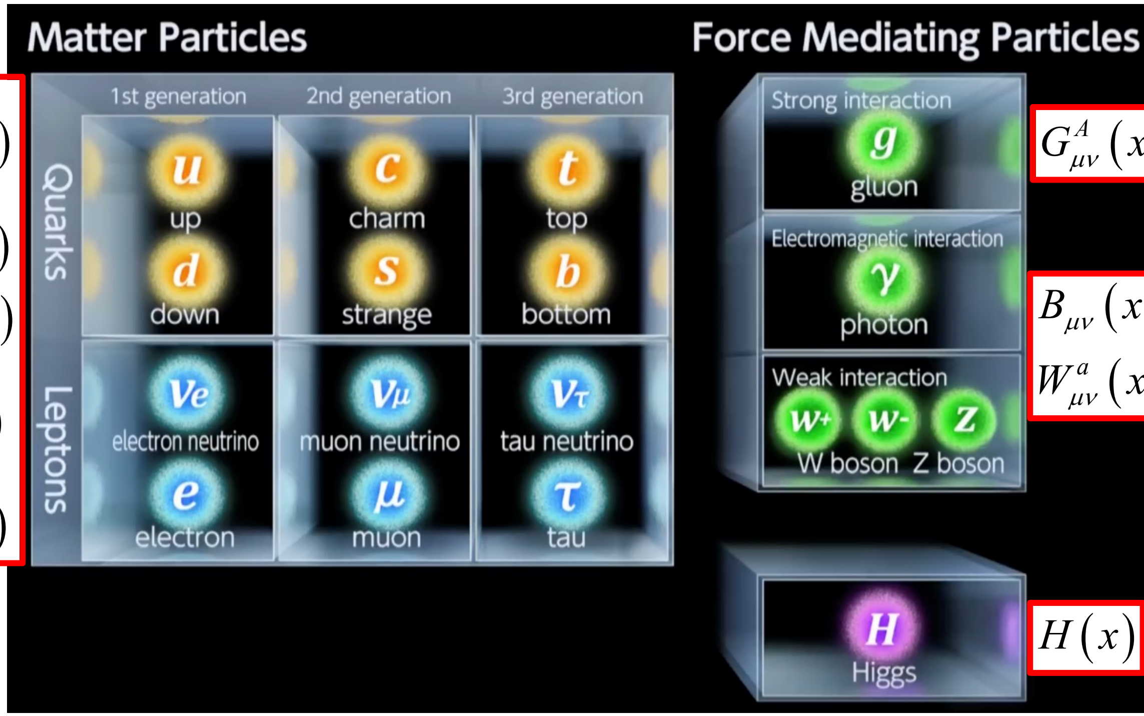

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$$\mathcal{L}_{\text{SM}} = \mathcal{L}^{(\dim\le 4)}_{\text{SMEFT}} = |DH|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \tfrac{1}{4} G^A_{\mu\nu} G^{A\mu\nu} - \tfrac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$- \lambda \left(|H|^2 - \tfrac{1}{2} v^2 \right)^2 - \left(\bar{q} Y_u u \tilde{H} + \bar{q} Y_d d H + \bar{l} Y_e e H + \text{h.c.} \right)$$

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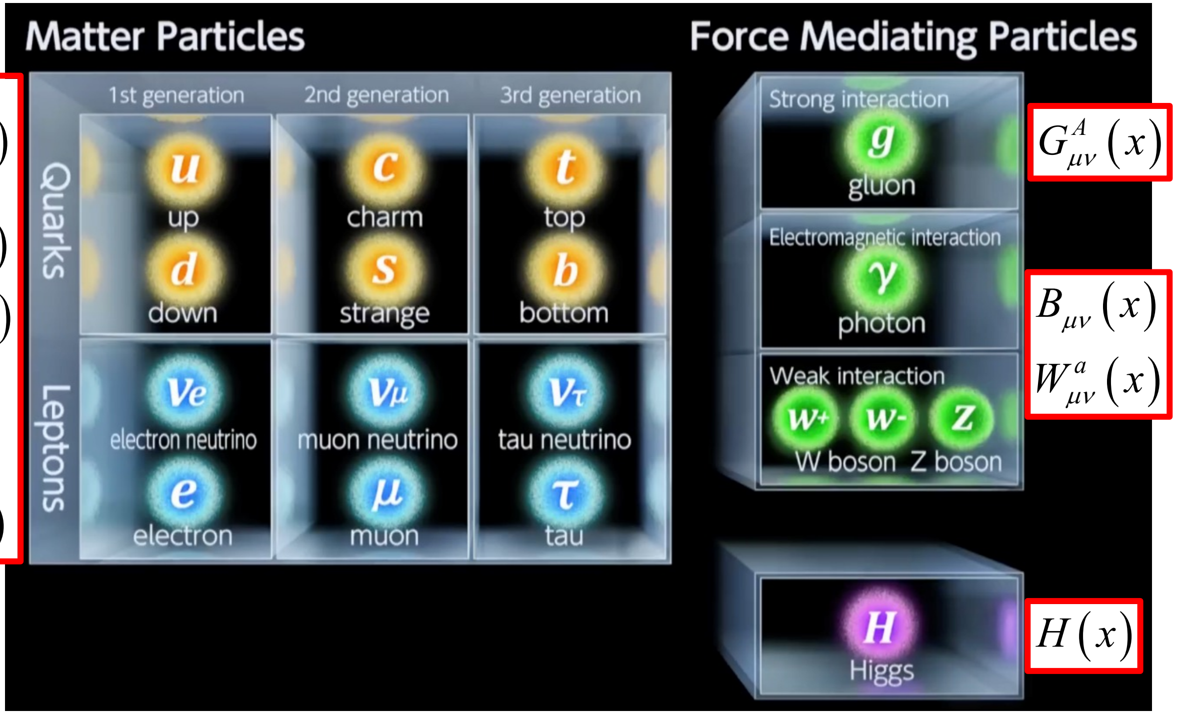

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$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \mathcal{L}^{(\text{dim-5})}_{\text{SMEFT}} + \mathcal{L}^{(\text{dim-6})}_{\text{SMEFT}} + \mathcal{L}^{(\text{dim-7})}_{\text{SMEFT}} + \cdots$$

$$\mathcal{L}^{(\text{dim-}k)}_{\text{SMEFT}} \sim \frac{1}{\Lambda^{k-4}} \mathcal{O}(\phi_{\text{SM}})$$

Example Operators in SMEFT

$\mathcal{L}_{\text{SMEFT}}^{(\text{dim-5})}$: $\frac{1}{\Lambda}\left(l_i^T i\gamma^0\gamma^2 l_k\right)\varepsilon^{ij}\varepsilon^{kl}H_j H_l + \text{h.c.}$ "Weinberg operator"

violates lepton number, generates neutrino masses

$\mathcal{L}_{\text{SMEFT}}^{(\text{dim-6})}$: $\frac{1}{\Lambda^2}\left(\bar{l}\gamma_\mu l\right)\left(\bar{q}\gamma^\mu q\right)$

impacts flavor physics, such as *B* meson decays

$\mathcal{L}_{\text{SMEFT}}^{(\text{dim-6})}$: $\frac{1}{\Lambda^2}|H|^6$

modifies Higgs potential, impacts Higgs trilinear coupling
electroweak phase transition, and baryogenesis

- Need to include them all (at low-orders) to give a robust parameterization

How many are there at mass dim-6? What about higher orders?

History of Enumerating SMEFT Operators

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  Henning, **XL**, Melia, and Murayama, arXiv: 1512.03433 — 993

Effective Field Theory

dim 6, $n_g = 1$

Grzadkowski, Iskrzynski, Misiak, and Rosiek, arXiv: 1008.4884

Fierz identity

$$\left(\bar{l}\gamma_\mu \tau^I l\right)\left(\bar{l}\gamma^\mu \tau^I l\right) = \left(\bar{l}\gamma_\mu l\right)\left(\bar{l}\gamma^\mu l\right)$$

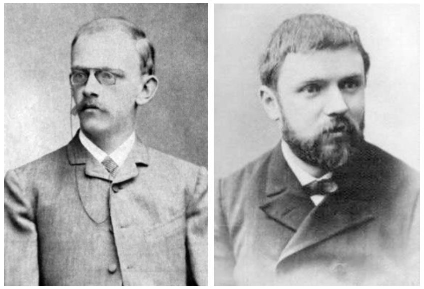
X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar{l}_p e_r \varphi)$
$Q_{\widetilde{G}}$	$f^{ABC} \widetilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar{q}_p u_r \widetilde{\varphi})$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi$		
$Q_{\widetilde{W}}$	$\varepsilon^{IJK} \widetilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p\sigma^{\mu\nu}$
$Q_{\varphi\widetilde{G}}$	$\varphi^\dagger\varphi\, \widetilde{G}^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p\sigma^{\mu}$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p\sigma^{\mu\nu}$
$Q_{\varphi\widetilde{W}}$	$\varphi^\dagger\varphi\, \widetilde{W}^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p\sigma^{\mu\nu}$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p\sigma^{\mu}$
$Q_{\varphi\widetilde{B}}$	$\varphi^\dagger\varphi\, \widetilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p\sigma^{\mu\nu}$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p\sigma^{\mu\nu}$
$Q_{\varphi\widetilde{W}B}$	$\varphi^\dagger\tau^I\varphi\, \widetilde{W}^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p\sigma^{\mu}$

Table 2: Dimension-six operators o

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating			
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$		
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$		
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$		
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k \sigma^{\mu\nu} u_t)$				

Table 3: Four-fermion operators.



Tool in Invariant Theory handling graded algebra

Hilbert-Poincaré Series $\mathcal{H}$

non-derivative terms

$$\mathcal{L}_{\text{EFT}}\left(\phi_1, \phi_1^*, \cancel{\partial}_\mu\right) = c_{1a}\,\phi_1 + c_{1b}\,\phi_1^* + c_{2a}\,\phi_1^2 + c_{2b}\,\phi_1^{*2} + c_{2c}\,\phi_1\phi_1^* + \cdots$$

	Charge
ϕ_1	$+1$
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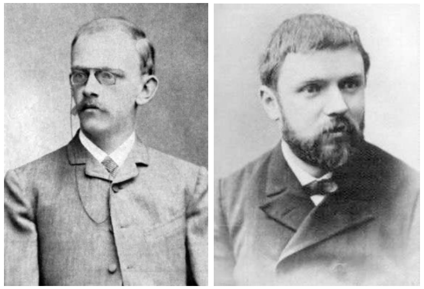

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ϕ_1	$+1$
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$$\mathcal{L}_{\text{EFT}}\left(\phi_1, \phi_1^*, \cancel{\partial}_\mu\right): \quad 1 \ , \quad \phi_1\phi_1^* \ , \quad \left(\phi_1\phi_1^*\right)^2 \ , \quad \cdots\cdots$$

Allowed operators by symmetry, one of each in above


Tool in Invariant Theory handling graded algebra

Hilbert-Poincaré Series $\mathcal{H}$

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ϕ_1	$+1$
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$$\mathcal{L}_{\text{EFT}}\left(\phi_1, \phi_1^*, \cancel{\partial}_\mu\right): \quad 1 \quad , \quad \phi_1\phi_1^* \quad , \quad \left(\phi_1\phi_1^*\right)^2 \quad , \quad \cdots\cdots$$

Allowed operators by symmetry, one of each in above

$$1 \;+\; \phi_1\phi_1^* \;+\; \left(\phi_1\phi_1^*\right)^2 \;+\; \cdots \;=\; \frac{1}{1-\phi_1\phi_1^*} = \mathcal{H}$$

Effective Field Theory

 

Tool in Invariant Theory handling graded algebra

Hilbert-Poincaré Series $\mathcal{H}$

non-derivative terms

	Charge
ϕ_1	$+1$
ϕ_1^*	-1

$$\mathcal{L}_{\text{EFT}}\left(\phi_1, \phi_1^*, \cancel{\partial}_\mu\right) = \cancel{c_{1a}\,\phi_1} + \cancel{c_{1b}\,\phi_1^*} + \cancel{c_{2a}\,\phi_1^2} + \cancel{c_{2b}\,\phi_1^{*2}} + c_{2c}\,\phi_1\phi_1^* + \cdots$$

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Allowed operators by symmetry, one of each in above

$$1 + \phi_1\phi_1^* + \left(\phi_1\phi_1^*\right)^2 + \cdots = \frac{1}{1-\phi_1\phi_1^*} = \mathcal{H}$$

	Charge
ϕ_1	$+1$
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$$\mathcal{L}_{\text{EFT}}\left(\phi_1, \phi_1^*, \phi_2, \phi_2^*, \cancel{\partial}_\mu\right):$$

$$1 \;+\; 4\left(\phi_i\phi_j^*\right) \text{ terms} \;+\; 9\left(\phi_i\phi_j\right)\left(\phi_k^*\phi_l^*\right) \text{ terms} \;+\; \cdots$$

$$\left(\phi_1+\phi_2\right)\left(\phi_1^*+\phi_2^*\right) \qquad \left(\phi_1^2+\phi_1\phi_2+\phi_2^2\right)\left(\phi_1^{*2}+\phi_1^*\phi_2^*+\phi_2^{*2}\right)$$

 

Tool in Invariant Theory handling graded algebra

Hilbert-Poincaré Series $\mathcal{H}$

non-derivative terms

$$\mathcal{L}_{\text{EFT}}(\phi_1, \phi_1^*, \cancel{\partial}_\mu) = \cancel{c_{1a}\phi_1} + \cancel{c_{1b}\phi_1^*} + \cancel{c_{2a}\phi_1^2} + \cancel{c_{2b}\phi_1^{*2}} + c_{2c}\phi_1\phi_1^* + \cdots$$

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Hilbert series can be efficiently computed with group representation theory

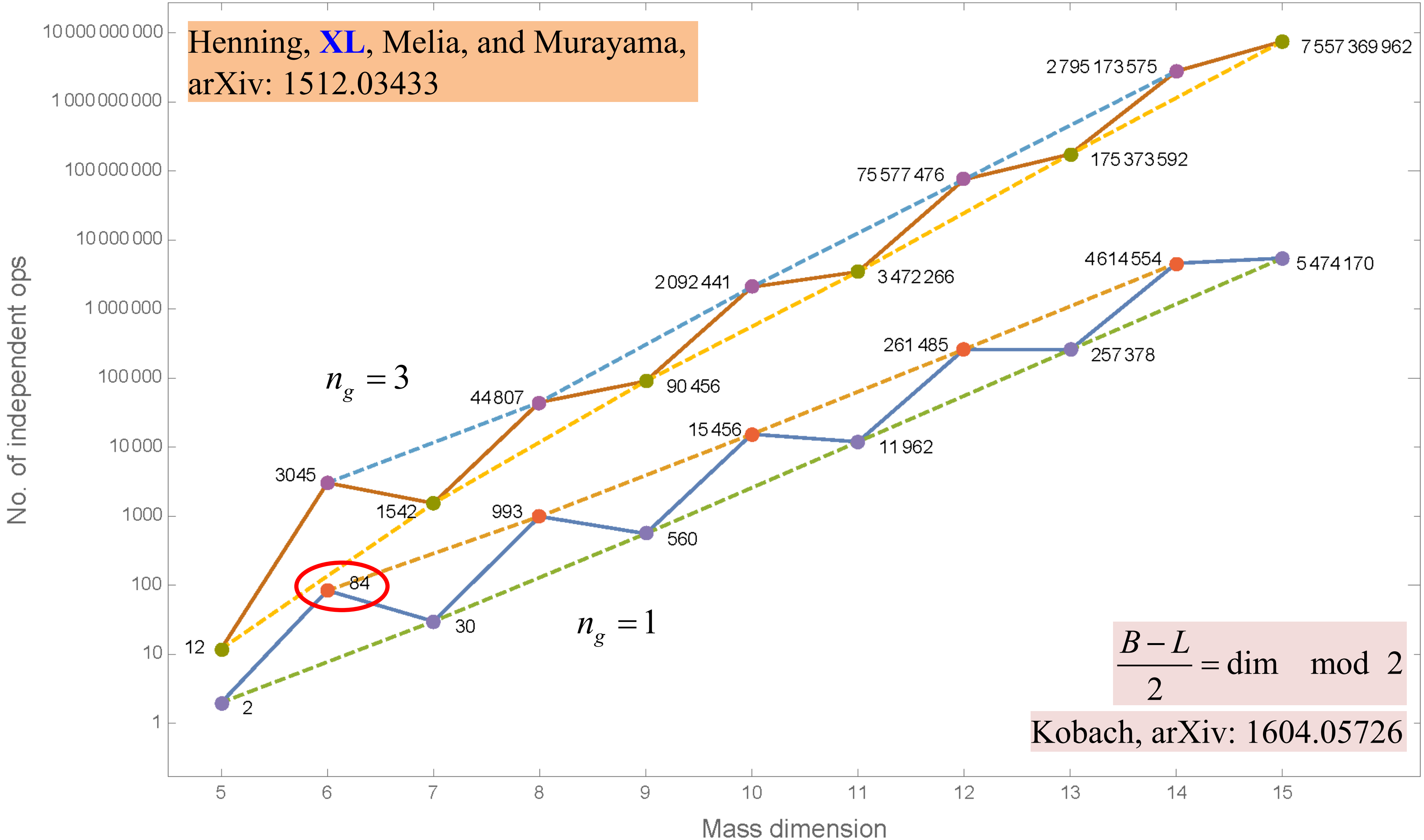
Hilbert Series for dim-6 SMEFT

Henning, **XL**, Melia, and Murayama, arXiv: 1512.03433

$$Q_{H\Box} = -\left(\partial_\mu |H|^2\right)^2 \quad , \quad Q_{HD} = \left|H^\dagger D_\mu H\right|^2$$

$$\mathcal{H}_{\text{SMEFT}}^{\text{dim-6}} = G_R^3 + G_L^3 + W_R^3 + W_L^3 + H^3 H^{\dagger 3} + \boxed{2H^2 H^{\dagger 2}\partial^2}$$
$$+ 2qq^\dagger HH^\dagger\partial + uu^\dagger HH^\dagger\partial + dd^\dagger HH^\dagger\partial + 2ll^\dagger HH^\dagger\partial + ee^\dagger HH^\dagger\partial + \left(du^\dagger H^2\partial + d^\dagger uH^{\dagger 2}\partial\right)$$
$$+ HH^\dagger G_R^2 + HH^\dagger G_L^2 + HH^\dagger W_R^2 + HH^\dagger W_L^2 + HH^\dagger B_R^2 + HH^\dagger B_L^2 + HH^\dagger B_R W_R + HH^\dagger B_L W_L$$
$$+ \left(uq^\dagger H^\dagger G_R + u^\dagger qHG_L\right) + \left(dq^\dagger HG_R + d^\dagger qH^\dagger G_L\right) + \left(uq^\dagger H^\dagger W_R + u^\dagger qHW_L\right) + \left(dq^\dagger HW_R + d^\dagger qH^\dagger W_L\right)$$
$$+ \left(uq^\dagger H^\dagger B_R + u^\dagger qHB_L\right) + \left(dq^\dagger HB_R + d^\dagger qH^\dagger B_L\right) + \left(el^\dagger HW_R + e^\dagger lH^\dagger W_L\right) + \left(el^\dagger HB_R + e^\dagger lH^\dagger B_L\right)$$
$$+ \left(uq^\dagger HH^{\dagger 2} + u^\dagger qH^2H^\dagger\right) + \left(dq^\dagger H^2H^\dagger + d^\dagger qHH^{\dagger 2}\right) + \left(el^\dagger H^2H^\dagger + e^\dagger lHH^{\dagger 2}\right)$$
$$+ 2q^2q^{\dagger 2} + 2qq^\dagger ll^\dagger + l^2l^{\dagger 2} + u^2u^{\dagger 2} + d^2d^{\dagger 2} + e^2e^{\dagger 2} + 2dd^\dagger uu^\dagger + dd^\dagger ee^\dagger + ee^\dagger uu^\dagger$$
$$+ 2uu^\dagger qq^\dagger + 2dd^\dagger qq^\dagger + ee^\dagger qq^\dagger + uu^\dagger ll^\dagger + dd^\dagger ll^\dagger + ee^\dagger ll^\dagger$$
$$+ \left(d^\dagger eql^\dagger + de^\dagger q^\dagger l\right) + \left(2duq^{\dagger 2} + 2d^\dagger u^\dagger q^2\right) + \left(2euq^\dagger l^\dagger + 2e^\dagger u^\dagger ql\right)$$
$$+ \left(duql + d^\dagger u^\dagger q^\dagger l^\dagger\right) + \left(euq^2 + e^\dagger u^\dagger q^{\dagger 2}\right) + \left(q^3 l + q^{\dagger 3} l^\dagger\right) + \left(deu^2 + d^\dagger e^\dagger u^{\dagger 2}\right) \quad \rightarrow \quad 84$$

Number of SMEFT operators from Hilbert series method



Applications of Hilbert Series after SMEFT Solved

- νSMEFT — Liao and Ma, arXiv: 1612.04527
- NRQED and HQET — Kobach and Pal, arXiv: 1704.00008, 1810.02356
- GR SMEFT — Ruhdorfer, Serra, and Weiler, arXiv: 1908.08050
- QCD Chiral Lagrangian — Graf, Henning, **XL**, Melia, and Murayama, arXiv: 2009.01239
- HEFT — Graf, Henning, **XL**, Melia, and Murayama, arXiv: 2211.06275
  Sun, Wang, and Yu, arXiv: 2211.11598
- Nonlinear O(N) Model — Bijnens, Gudnason, Yu, and Zhang, arXiv: 2212.07901
- Supersymmetric EFTs — Delgado, Martin, and Wang, arXiv: 2212.02551, 2305.01736
- EFTs for Axion-like Particles — Grojean, Kley, and Yao, arXiv: 2307.08563

Compare different options of symmetries

Total number of dim-6 SMEFT operators	3045
Imposing baryon or lepton number	2499
+ flavor universality	76
+ CP invariance	53

mostly about flavor structures

Alonso, Jenkins, Manohar, and Trott, arXiv: 1312.2014

Alonso, Chang, Jenkins, Manohar, and Shotwell, arXiv: 1405.0486

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Accidental Symmetry

$$\mathcal{H}_{\text{SMEFT}}^{\text{dim}-6} = \mathcal{H}_{\text{SMEFT}}^{\text{dim}-6,\, B-L}$$

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How to systematically find accidental symmetries in an EFT?

Grinstein, **XL**, Miro, and Quilez, arXiv: 2412.05359

Cohen, Craig, **XL**, and Sutherland
arXiv: 2008.08597, 2108.03240

Is SMEFT enough?

- There are extra light particles
  - --- right-handed neutrinos "νSMEFT"
  - --- axion-like particles "aSMEFT"
  - --- other light dark matter candidates

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$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{SMEFT}}^{(\dim\leq 4)} = \left|DH\right|^2 + \sum_{\psi=q,u,d,l,e} \bar{\psi} i \not{D} \psi - \tfrac{1}{4} G_{\mu\nu}^{A} G^{A\mu\nu} - \tfrac{1}{4} W_{\mu\nu}^{a} W^{a\mu\nu} - \tfrac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$- \lambda \left(\left|H\right|^2 - \tfrac{1}{2} v^2 \right)^2 - \left(\bar{q}\, Y_u\, u\, \tilde{H} + \bar{q}\, Y_d\, d\, H + \bar{l}\, Y_e\, e\, H + \text{h.c.} \right)$$

$\dfrac{1}{4\left|H\right|^2} \left(\partial_\mu \left|H\right|^2 \right)^2 \notin \mathcal{L}_{\text{SMEFT}}$, because it is singular at $H = 0$!

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Cohen, Craig, **XL**, and Sutherland
arXiv: 2008.08597, 2108.03240

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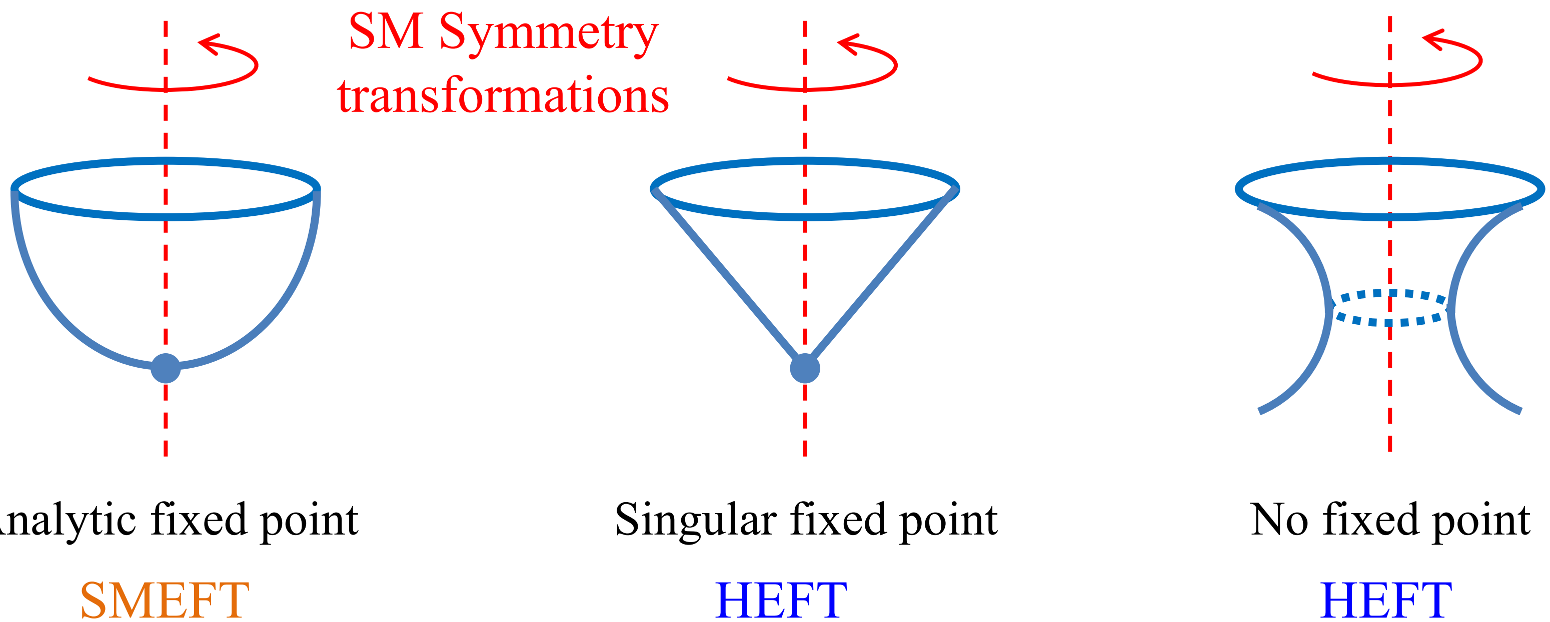
- Singularity can be faked by a bad choice of coordinate

Geometric Formulation of Symmetries

Higgs field space manifold



$$\mathcal{L}_{\text{EFT}} \supset \frac{1}{2} g_{ab}(\phi)\left(\partial_\mu \phi_a\right)\left(\partial^\mu \phi_b\right)$$

Alonso, Jenkins, and Manohar
arXiv: 1605.03602

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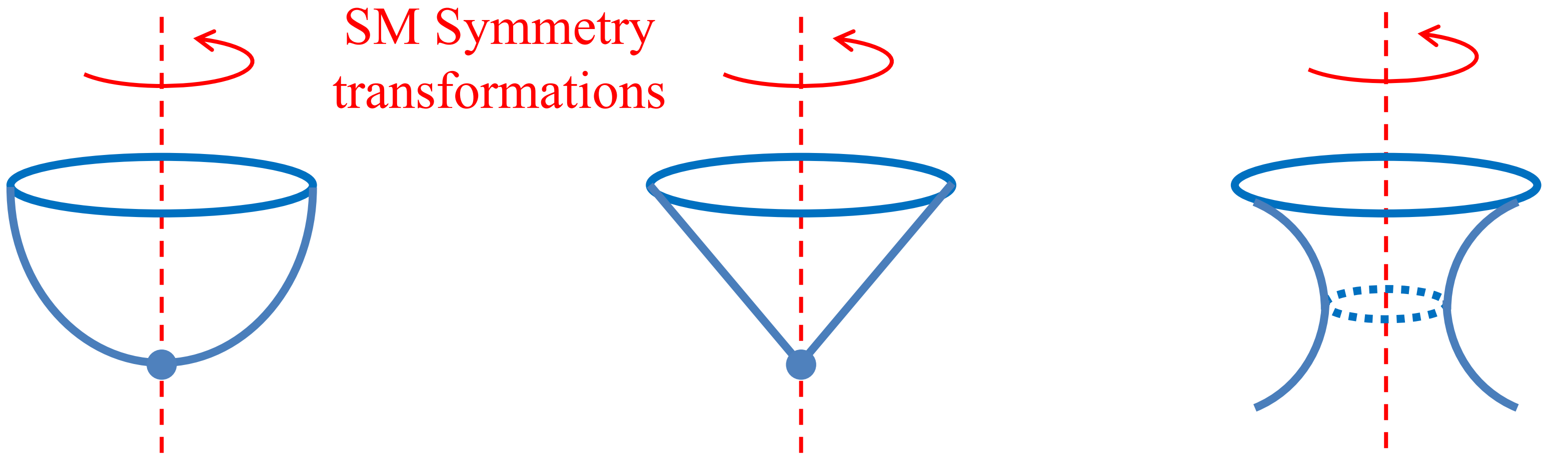

Analytic fixed point	Singular fixed point	No fixed point
SMEFT	HEFT	HEFT
$\mathcal{L}_{\text{EFT}} \supset \frac{1}{2} g_{ab}(\phi)(\partial_\mu \phi_a)(\partial^\mu \phi_b)$	New particles with mass dominantly from SM Higgs	New particles with SM symmetry breaking

Alonso, Jenkins, and Manohar
arXiv: 1605.03602

Cohen, Craig, **XL**, and Sutherland, arXiv: 2008.08597, 2108.03240

Banta, Cohen, Craig, **XL**, and Sutherland, arXiv: 2110.02967

Summary and Outlook

- Symmetry Principles are powerful and illuminating. They will guide us to understand physics beyond the Standard Model

- Effective Field Theory is a systematic parameterization of new physics. In particular, Standard Model Effective Field Theory gives a robust parameterization of physics beyond the Standard Model

- Hilbert series method enables us to systematically impose symmetries in an EFT, and to compare the consequences of different symmetries

  --- Facilitate the study of lepton number, flavor, CP violations, etc.

- Geometric formulation of symmetries identifies the true singularities

  --- Point us to new physics scenarios in which SMEFT is not enough