Positive Neutrino Masses with DESI DR2 via Matter Conversion to Dark Energy

S. .P. Ahlen, A. Aviles, B. Cartwright, **K. S. Croker**, W. Elbers, D. Farrah, N. Fernandez, G. Niz, J. W. Rohlf, G. Tarlé, R. A. Windhorst, J. Aguilar, U. Andrade, D. Bianchi, D. Brooks, T. Claybaugh, A. de la Macorra, A. de Mattia, Biprateep Dey, P. Doel, J. E. Forero-Romero, E. Gaztañaga, S. Gontcho A Gontcho, G. Gutierrez, D. Huterer, M. Ishak, R. Kehoe, D. Kirkby, A. Kremin, O. Lahav, C. Lamman, M. Landriau, L. Le Guillou, M. E. Levi, M. Manera, R. Miquel, J. Moustakas, I. Pérez-Ràfols, F. Prada, G. Rossi, E. Sanchez, M. Schubnell, H. Seo, J. Silber, D. Sprayberry, M. Walther, B. A. Weaver, R. H. Wechsler, H. Zou (DESI Collaboration)

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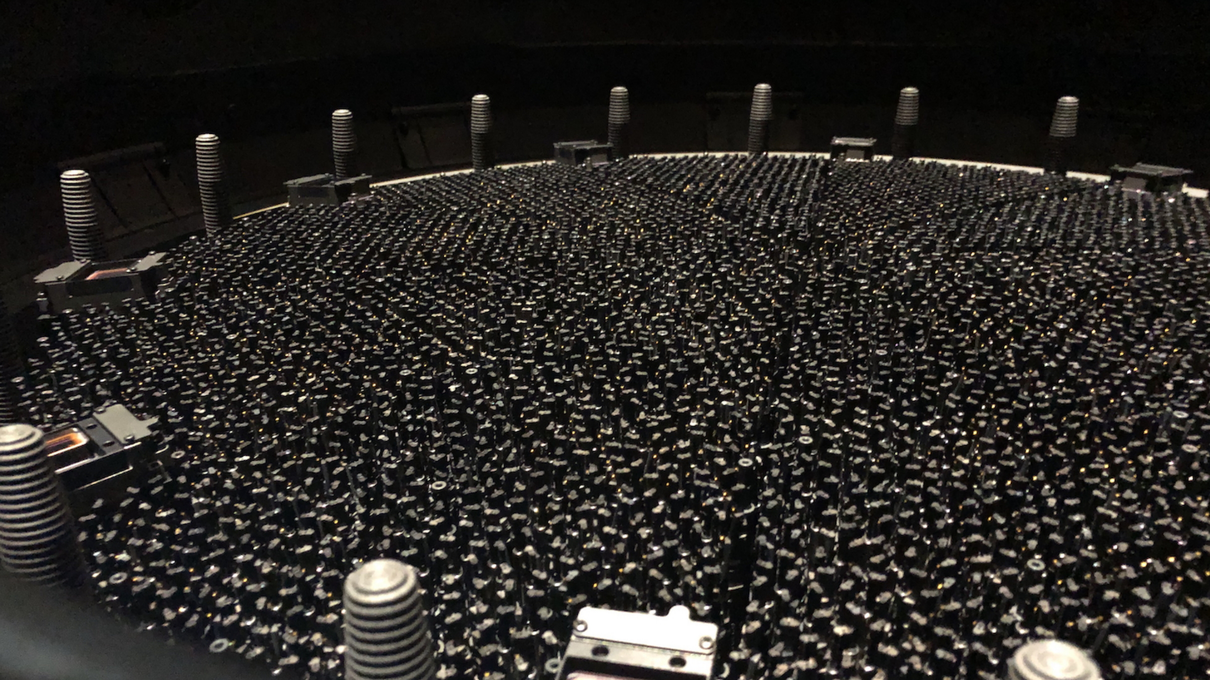
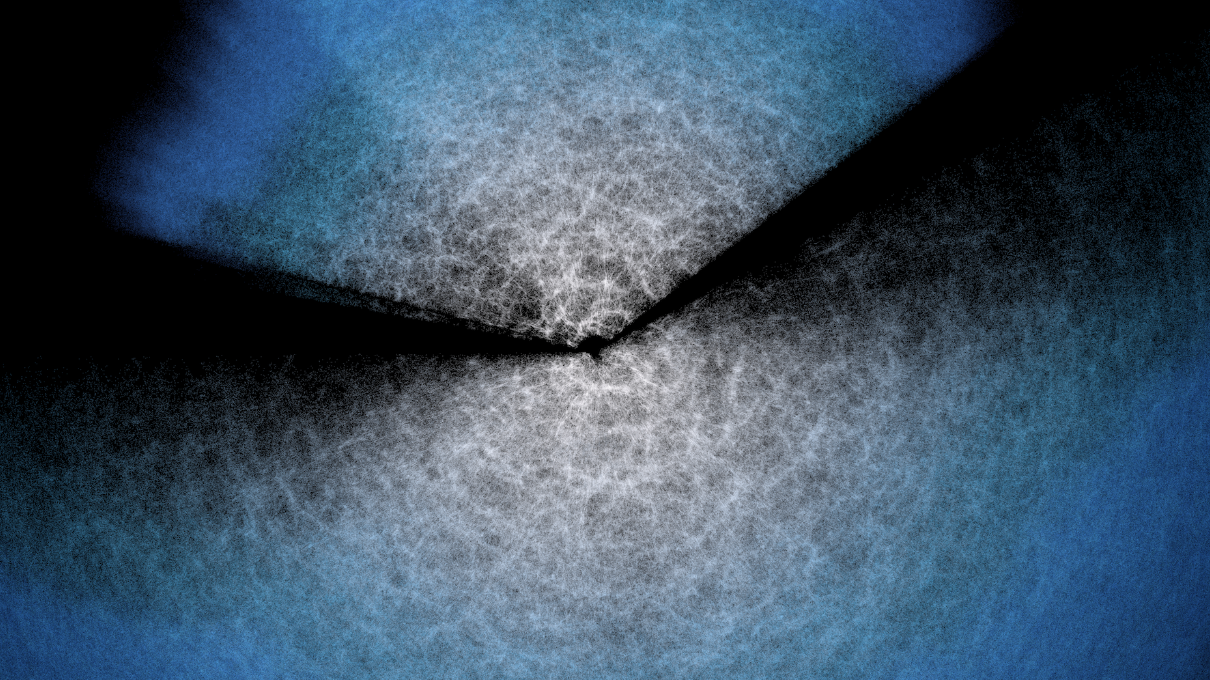
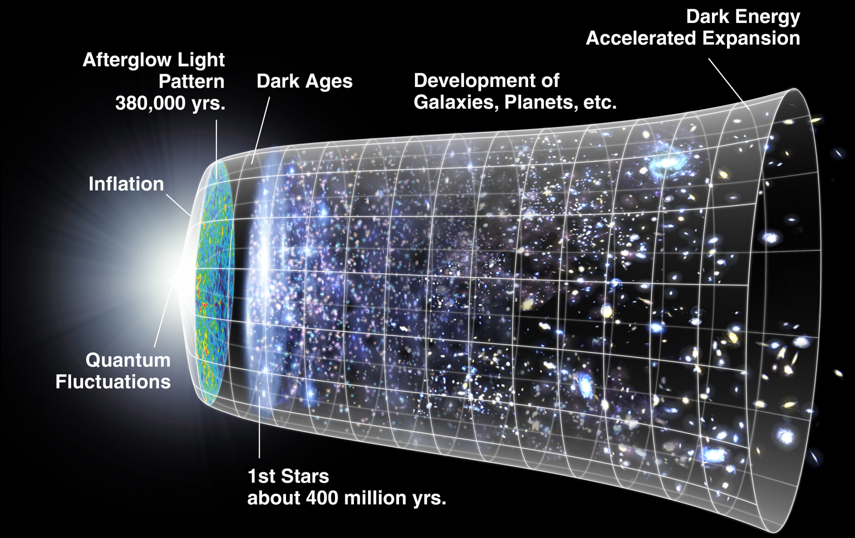




$a \to 0$
(Big Bang)



$a := 1$
(Today)

Image Credit: NASA / WMAP Science Team

Expansion Rate Determined by Universe Contents

DESI constrains the expansion rate, which is determined by Friedmann's equation,

$$H^2 = H_0^2 \left(\frac{8\pi G}{3H_0^2}\right) \left[\rho_{\rm DE}(a) + \rho_\nu(a) + \frac{\rho_b(1) + \rho_c(1)}{a^3} + \frac{\rho_\gamma(1)}{a^4}\right]$$

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- H_0, DE, and neutrinos remain flexible
- Massive neutrinos are cold, non-interacting, matter at late times

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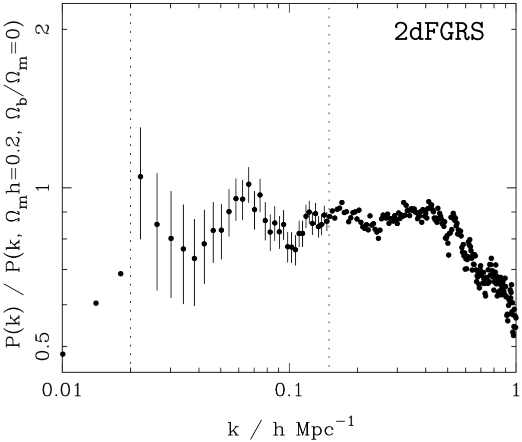
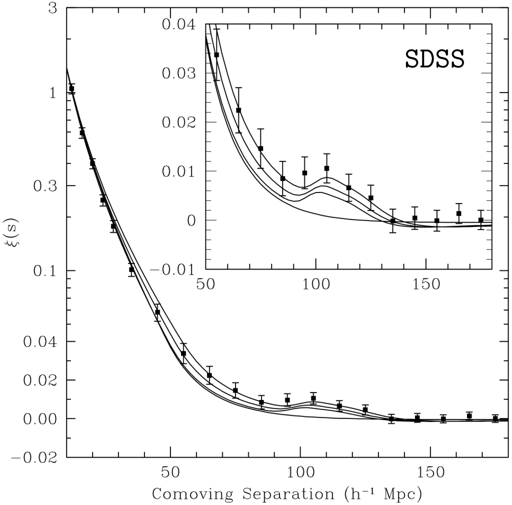
ΛCDM Model: $\rho_{\mathrm{DE}}(a) := \Lambda$ constant $\implies P_{\mathrm{DE}} = -\rho_{\mathrm{DE}}$

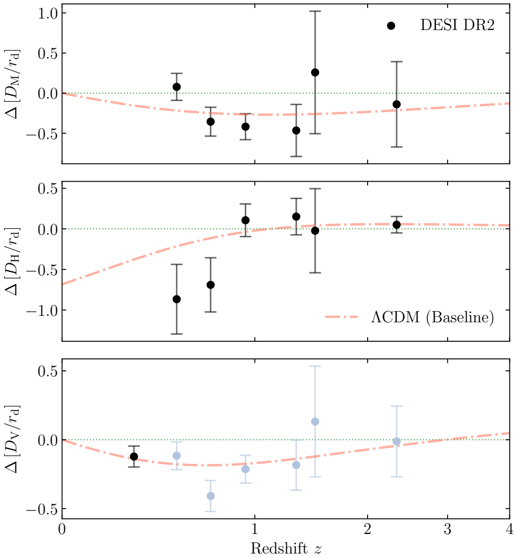




(Left) W. Percival, et al. *MNRAS* 327 1297 (2001); S. Cole, et al. *MNRAS* 362.2 505 (2005); (Right) D. Eisenstein, et al. *ApJ* 633 560 (2005)

DESI measures distances in r_d units

DESI measures the time-evolution of cosmological distances, relative to the BAO r_d



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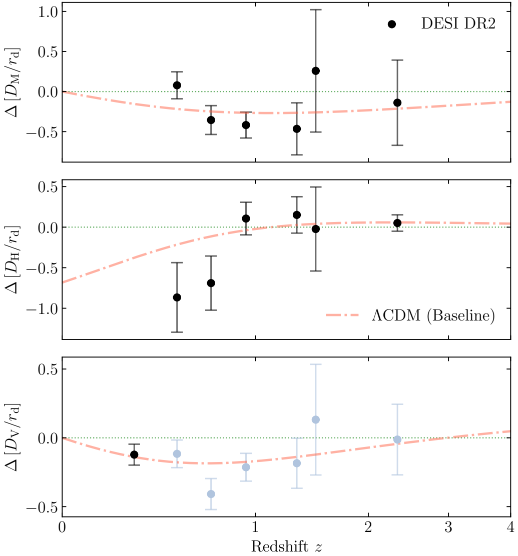

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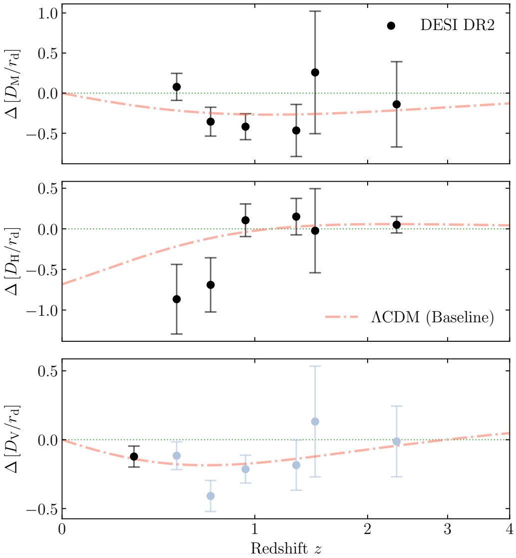

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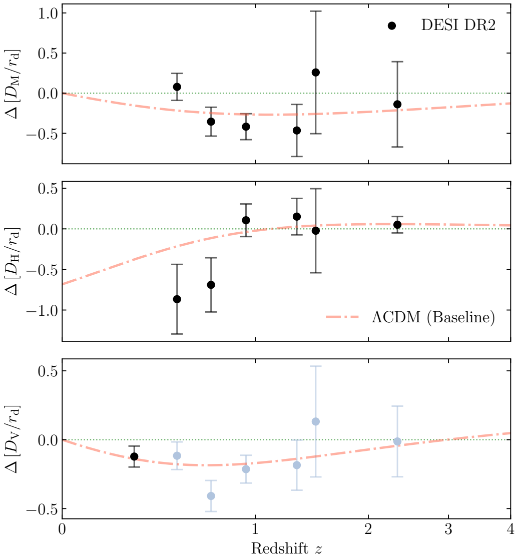

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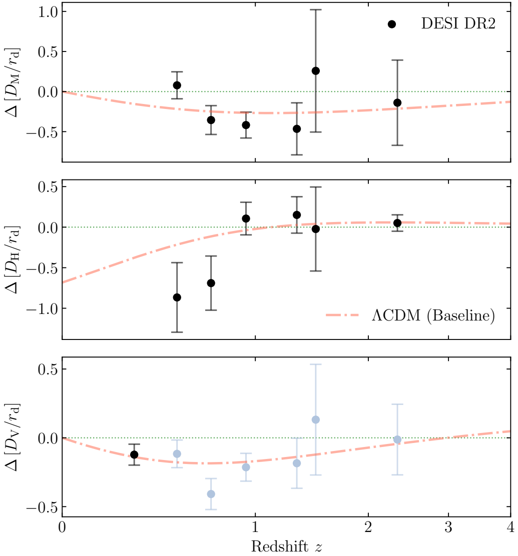
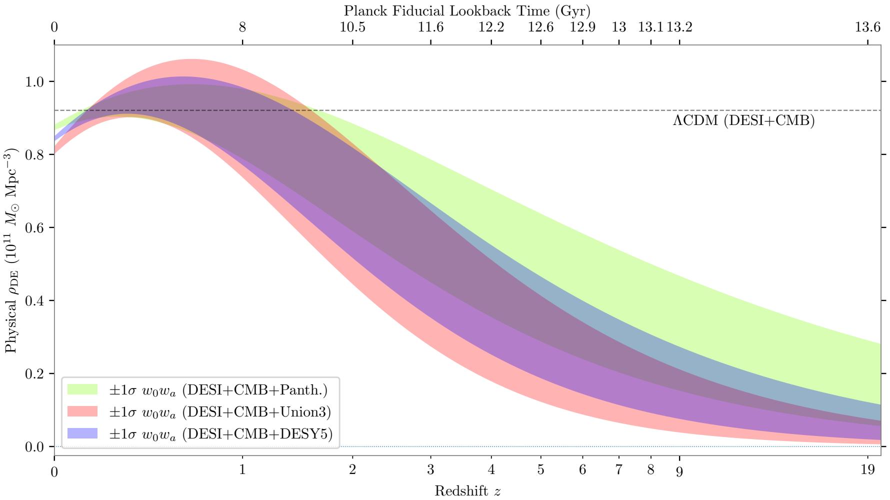
Time-evolution: ΛCDM at $z \sim 1089$ fit to CMB does not work at late-times



DESI characterizes time-dependence with $w_0 w_a$...



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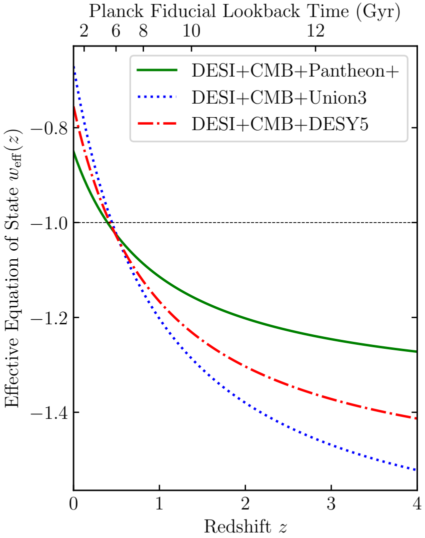
...but $w_0 w_a$ is a placeholder

Two parameters characterize the DE equation of state

$$P/\rho = w_{\text{eff}}(a) := w_0 + w_a(1-a)$$

Features

- Describes internal evolution between DE kinetic and potential DOF



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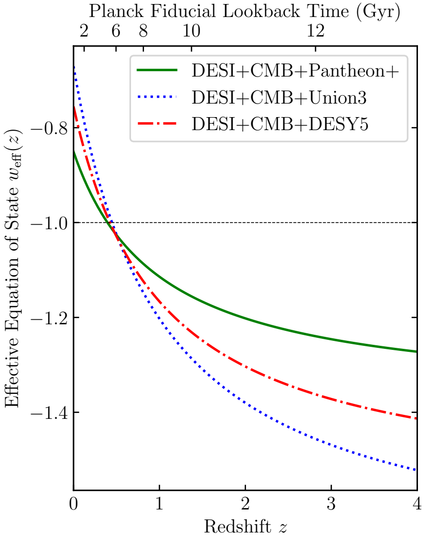

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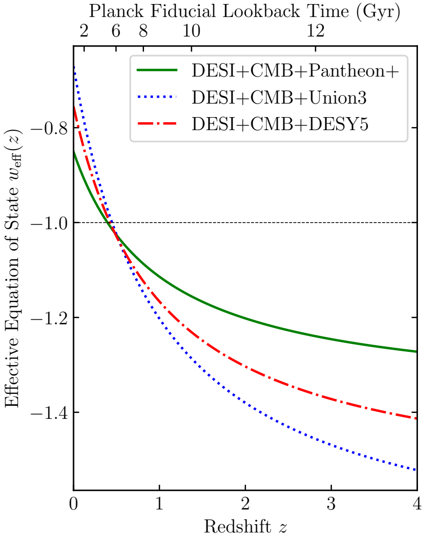
Causality violation: $w_{\text{eff}} < -1 \implies$ speed of sound greater than speed of light



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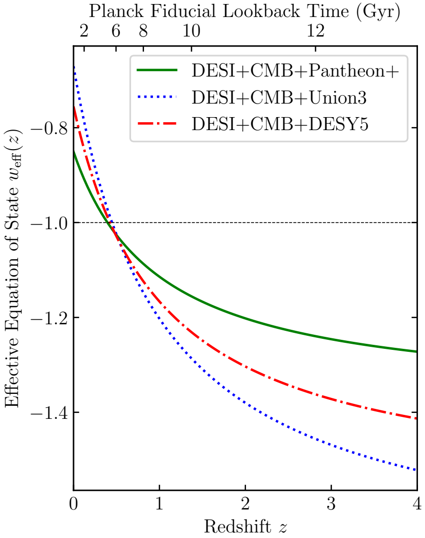
$$w_{\text{eff}} = -1 - \frac{a}{3\rho}\frac{\mathrm{d}\rho}{\mathrm{d}a}$$



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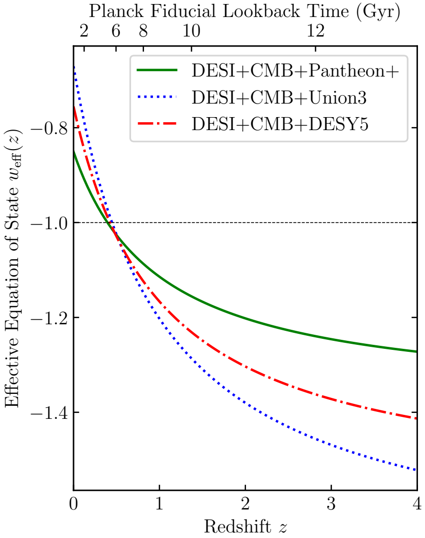

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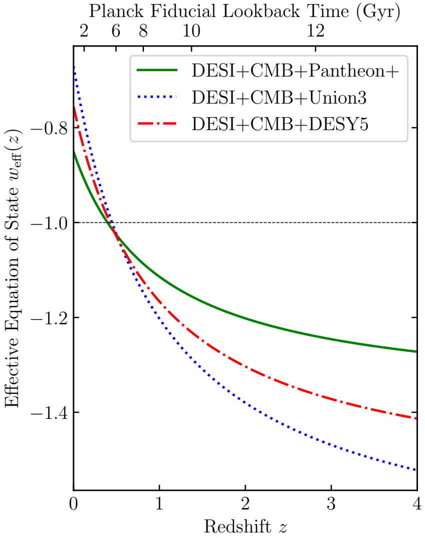
Interpretation: DESI data suggest energy injection from other species



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Constraints from oscillations

Massive neutrinos contribute as cold dark matter in the late universe.

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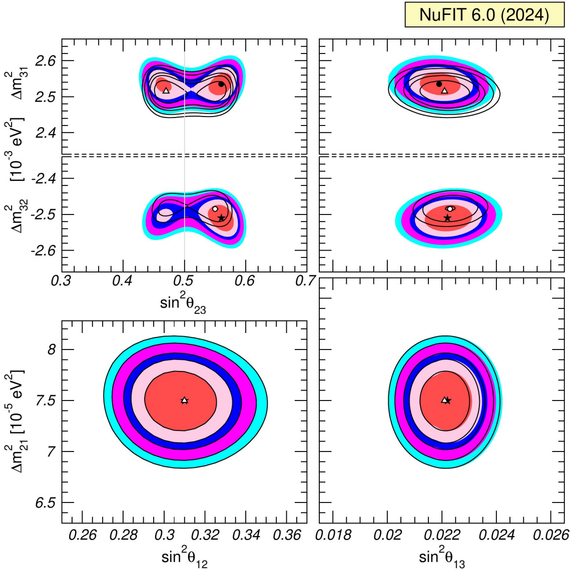

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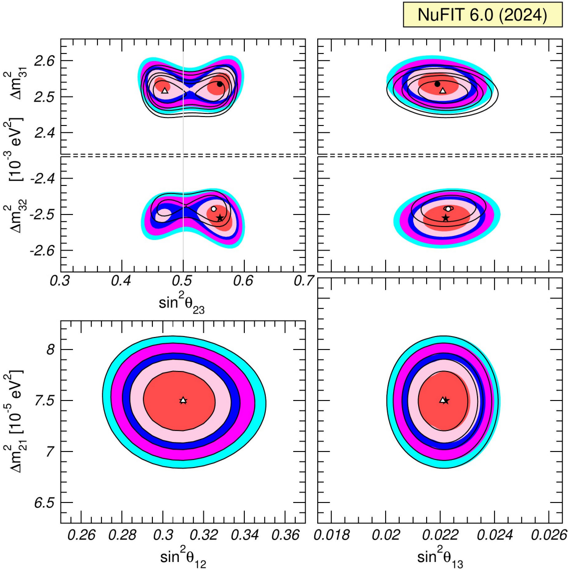

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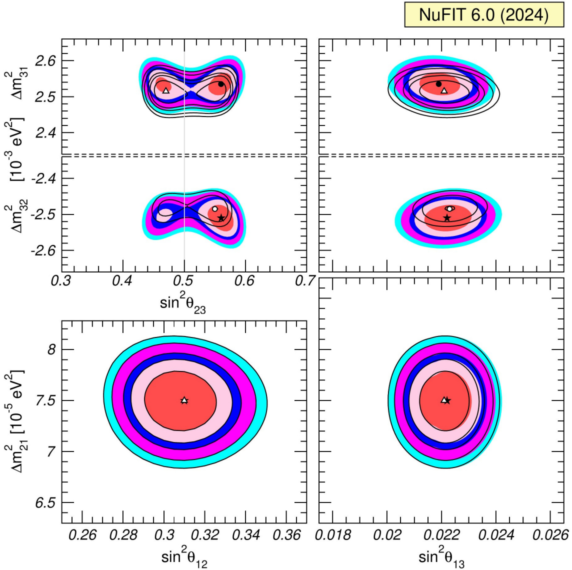

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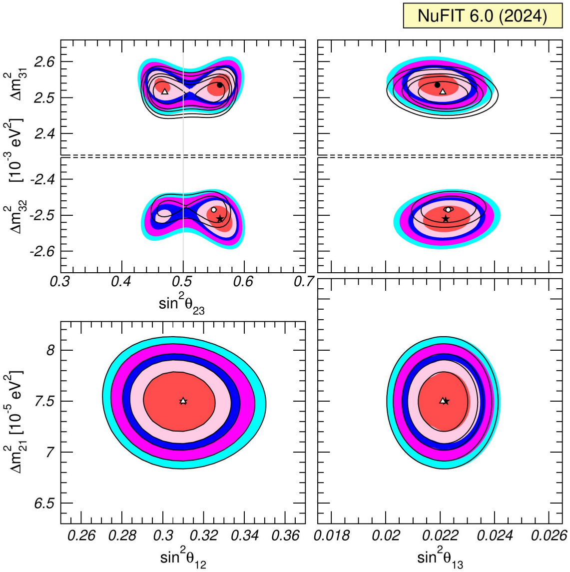

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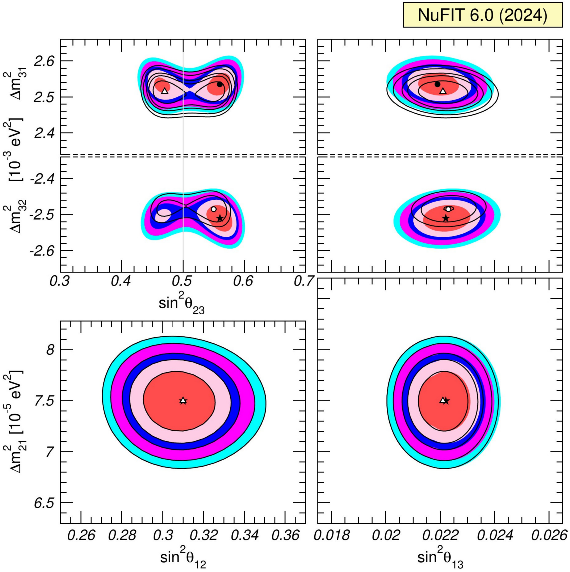
Providing a constraint on summed mass:

$$\sum m_\nu > \begin{cases} 0.05878 \pm 0.00023\,\mathrm{eV} & \text{(NO)}, \\ 0.09892 \pm 0.00041\,\mathrm{eV} & \text{(IO)}. \end{cases}$$



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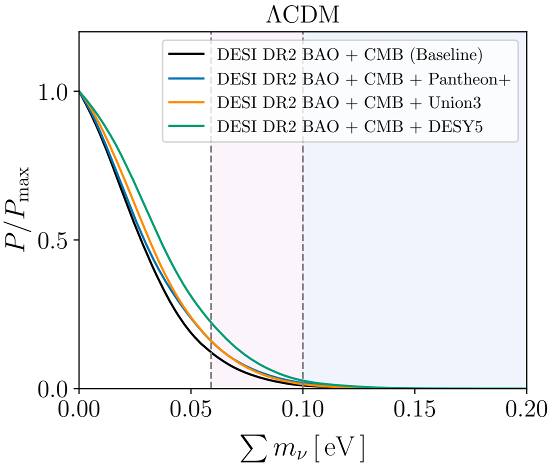

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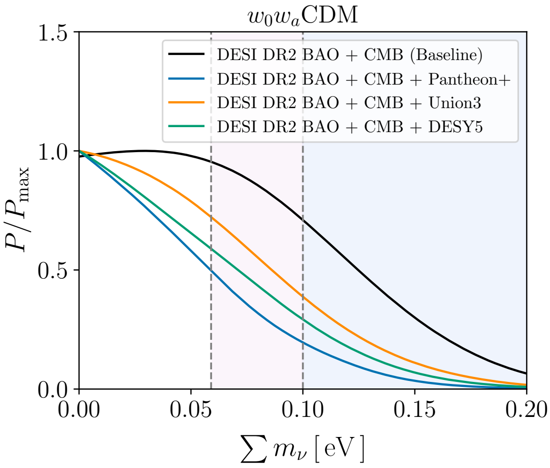

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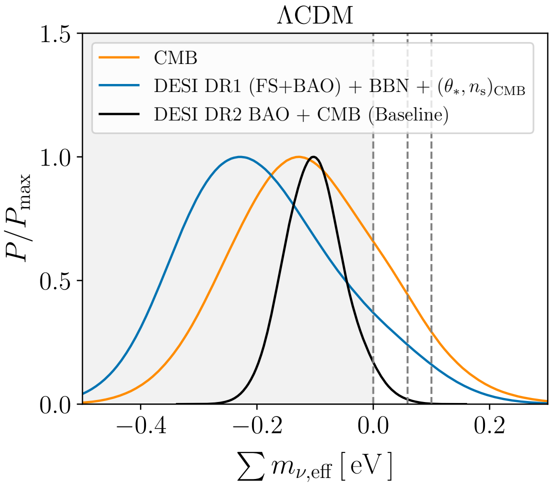

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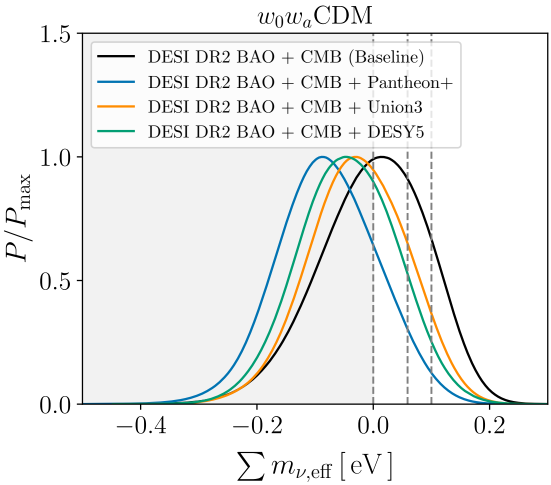
- $w_0 w_a$CDM cannot fix this: addition of SNe pulls the masses back to negative



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- And massive neutrinos are cold, non-interacting, matter at late times...

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Interpretation: DESI data prefer less matter than CMB implies

Conclusions from DESI fiducial analysis

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Conclusions from DESI fiducial analysis

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- $\sum m_{\nu,\mathrm{eff}} < 0 \implies$ less matter (baryons or CDM) today than at Big Bang
- "... negative values should be interpreted as a signature of unidentified systematic errors or possibly of **new physics which may be unrelated to neutrinos**..."

ALGEBRAIC PROPERTIES OF THE ENERGY-MOMENTUM TENSOR AND VACUUM-LIKE STATES OF MATTER

É. B. GLINER

A. F. Ioffe Physico-technical Institute, Academy of Sciences, U.S.S.R.

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J. Exptl. Theoret. Phys. (U.S.S.R.) **49**, 542-548 (August, 1965)

The physical interpretation of some algebraic structures of the energy-momentum tensor allows us to suppose that there is a possible form of matter, called the μ-vacuum, which macroscopically possesses the properties of vacuum. The assumption that an actually occurring vacuum is a μ-vacuum retains the Lorentz invariance of the Lagrangian (when gravitation is neglected) and preserves the theories based on the requirement of this invariance, and at the same time makes the Mach principle no longer logically convincing. The space time of a μ-vacuum is an Einstein space in the sense of Petrov's definition.[2] A uniform world of μ-vacuum has the de Sitter metric.

the condition μ = const, plays the role of the cosmological constant, which accordingly can be interpreted in the framework of the ordinary formalism of the general theory of relativity. If, on the other hand, we cannot neglect the matter other than the μ-vacuum, the analogy of the μ-vacuum density with the cosmological constant can be maintained only in so far as the interaction of this matter with the μ-vacuum is unimportant. Otherwise the condition μ = const does not hold, and the analogy with the cosmological constant is destroyed.

The differences between the structure of the energy-momentum tensor of μ-vacuum and that for ordinary matter, and the consequent differences between its equations of motion and its properties and the equations of motion and properties for ordinary matter show that if the μ-vacuum is real, then it is a specific form of matter. Since the equations of the general theory of relativity do not contain adequate information about the conditions of transition between different forms of matter, within the framework of this theory we cannot de-ing of particles of matter are annulled.

This situation is not utterly unrealistic. An attempt to describe phenomenologically the structure of an elementary charged particle would lead to the conclusion that inside the particle there must be a negative pressure which balances the electrostatic repulsion. This raises the thought that in an ultradense state of matter, with the baryons so compressed that the meson fields which provide the interaction between them (repulsion!) cannot be produced, a continuous medium is formed in which the conditions correspond to an attraction between material elements and are described phenomenologically by a negative pressure. For example, such a state might be reached in gravitational collapse.

It would seem that a negative pressure should lead to an internal instability, and that if there are no volume forces of the type of the electrostatic repulsion it would lead to a contraction without limit. This is not true, however. Let us assume that compression actually leads to a negative pres-

General Relativity and Gravitation, Vol. 24, No. 3, 1992

Vacuum Nonsingular Black Hole†

Irina Dymnikova[1]

The spherically symmetric vacuum stress-energy tensor with one assumption concerning its specific form generates the exact analytic solution of the Einstein equations which for large r coincides with the Schwarzschild solution, for small r behaves like the de Sitter solution and describes a spherically symmetric black hole singularity free everywhere.

PHYSICAL REVIEW D **76,** 063510 (2007)

Cosmological expansion and local physics

Valerio Faraoni* and Audrey Jacques†

Physics Department, Bishop's University, 2600 College Street, Sherbrooke, Québec, Canada J1M 0C8

(Received 7 June 2007; published 24 September 2007)

The interplay between cosmological expansion and local attraction in a gravitationally bound system is revisited in various regimes. First, weakly gravitating Newtonian systems are considered, followed by various exact solutions describing a relativistic central object embedded in a Friedmann universe. It is shown that the ''all or nothing'' behavior recently discovered (i.e., weakly coupled systems are comoving while strongly coupled ones resist the cosmic expansion) is limited to the de Sitter background. New exact solutions are presented which describe black holes perfectly comoving with a generic Friedmann universe. The possibility of violating cosmic censorship for a black hole approaching the big rip is also discussed.

DOI: 10.1103/PhysRevD.76.063510 PACS numbers: 98.80.−k, 04.50.+h

I. INTRODUCTION

e issue of whether a planet, a star, or a galaxy expands
ving the rest of the universe is a problem of principle
neral relativity that still awaits a definitive answer.

perturbed by a transient and does not expand [22]. This work breaks free of the standard assumption of previous literature that the coupling (of a gravitationally, instead of electrically, bound system) is weak. However, it has two fundamental limitations: first, the cosmological backion (40) was imposed by McVittie to he accretion of cosmic fluid onto the umption e) of Ref. [1]). It corresponds in turn implies that the stress-energy $T_0^1 = 0$ and there is no radial flow. In Eq. (40) corresponds to the constancy of vard mass, $\dot{m}_H = 0$. It is important to he physically relevant mass (eventually cal size of the horizon or of the central o avoid making coordinate-dependent mass and size (cf., e.g., Refs. [18,55]),] of the central object. $m(t)$ is just a n a particular coordinate system.

little doubt that the McVittie metric nd of strongly gravitating central object, retation is not completely clear and is [10,12,15,16]. This metric reduces to solution in isotropic coordinates when FLRW metric if $m \equiv 0$. However, in (39) can not be interpreted as describing dded in a FLRW universe because it is sphere $\bar{r} = m/2$ (which reduces to the

FLRW universe and its Newtonian limit

It is of interest to study the behavior of a relativistic star embedded in a FLRW background with respect to the problem of local physics versus cosmological expansion. The Nolan interior solution [33] describes a relativistic star of uniform density in such a background. The metric is

$$ds^2 = -\left[\frac{1 - \frac{m}{\bar{r}_0} + \frac{m\bar{r}^2}{\bar{r}_0^3}\left(1 - \frac{m}{4\bar{r}_0}\right)}{\left(1 + \frac{m}{2\bar{r}_0}\right)\left(1 + \frac{m\bar{r}^2}{2\bar{r}_0^3}\right)}\right]^2 dt^2 + a^2(t)\frac{\left(1 + \frac{m}{2\bar{r}_0}\right)^6}{\left(1 + \frac{m\bar{r}^2}{2\bar{r}_0^3}\right)^2}\left(d\bar{r}^2 + \bar{r}^2 d\Omega^2\right) \tag{43}$$

in isotropic coordinates, where $\bar{r}_0$ is the star radius, $\frac{\dot{m}}{m} = -\frac{\dot{a}}{a}$ (the condition forbidding accretion onto the star surface), and $0 \leq \bar{r} \leq \bar{r}_0$. The interior metric is regular at the center and is matched to the exterior McVittie metric at $\bar{r} = \bar{r}_0$ by imposing the Darmois-Israel junction conditions. The energy density is uniform and discontinuous at the surface $\bar{r} = \bar{r}_0$, while the pressure is continuous. These quantities are given by [33]

$$\mathcal{A}_{\Sigma_0}(t) = \iint_{\Sigma_0} d\theta d\varphi \sqrt{g_{\Sigma_0}} = 4\pi a^2(t)\bar{r}_0^2\left(1 + \frac{m(t)}{2\bar{r}_0}\right)^4, \tag{47}$$

where $g_{ab}|_{(\Sigma_0)}$ is the metric on Σ_0 at a fixed time t and g_{Σ_0} is its determinant. By using the Schwarzschild curvature coordinate $r \equiv \bar{r}(1 + \frac{m}{2\bar{r}})^2$, one has

$$\mathcal{A}_{\Sigma_0}(t) = 4\pi a^2(t) r_0^2. \tag{48}$$

The star surface is comoving with the cosmic substratum and the proper curvature radius of the star is $r_{\text{phys}}(t) = a(t)\bar{r}_0(1 + \frac{m}{2\bar{r}_0})^2$. Therefore, we have a local relativistic object with strong field which is perfectly comoving at all times: in this case the cosmic expansion wins over the local dynamics.

It is interesting to compute the generalized Tolman-

$$\frac{\partial P}{\partial r} + (P + \rho)\ldots$$

In the Newtonian ... equation reduces ...

where $\rho = m(\frac{4\pi}{3}\ldots$ potential. This e... obtained from E... curvature radius. ... of hydrostatic eq... uniform density s...

$$\frac{dP}{\ldots} + \frac{d\Phi_N}{\ldots}$$

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$$\frac{\mathrm{d}N_{BH}}{\mathrm{d}\mathcal{V}} \propto \frac{1}{a^3}.$$

KC & J. Weiner. *ApJ* 882.1 (2019): 19.; **KC**, J. Weiner, & D. Farrah. *PRD* 105.8 (2022): 084042.

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∴ the total energy density of BHs

$$\rho_{BH} = M_{BH}\frac{\mathrm{d}N_{BH}}{\mathrm{d}\mathcal{V}} = \text{constant}$$

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Cosmological conservation of stress-energy requires

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Conclusion: energized vacuum black holes, in aggregate, contribute as a DE species, just as you would expect from simply averaging over their stress-energy

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Dark Energy from Cosmologically Coupled BHs

Hypothesis: all DE comes from stellar-collapse BHs

$$\rho_b := \begin{cases} \dfrac{C\omega_b^{\mathrm{proj}}}{a^3} & a < a_i \\ \dfrac{C\omega_b^{\mathrm{proj}}}{a^3} - \dfrac{\Xi}{a^3}\displaystyle\int_{a_i}^{a} \psi \frac{\mathrm{d}a'}{Ha'} & a \geqslant a_i \end{cases}$$

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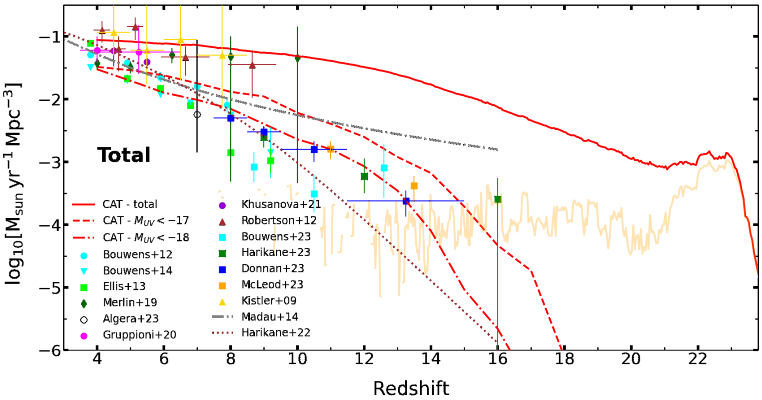
Evolution of DE density follows directly from conservation of stress-energy $\nabla_\mu T^\mu{}_\nu = 0$

$$\frac{\mathrm{d}\rho_{\mathrm{DE}}}{\mathrm{d}a} = \frac{\Xi}{Ha^4}\psi \qquad \rho_{\mathrm{DE}}(a_i) := 0$$

Cosmic star-formation rate density (SFRD) ψ

We use Trinca, et al. for $z > 4$ SFRD

- Accounts for faint sources



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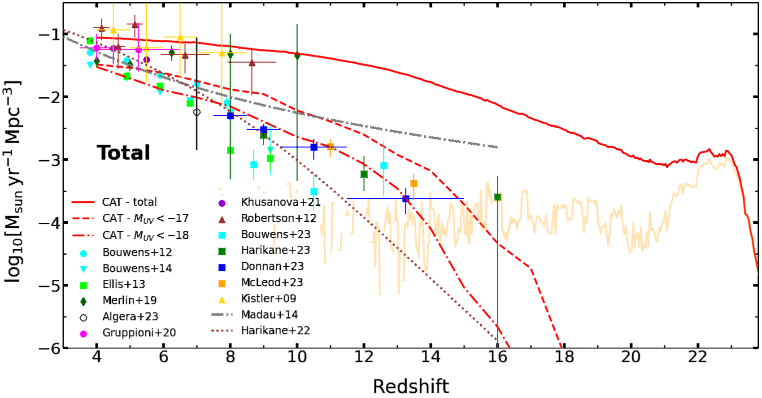

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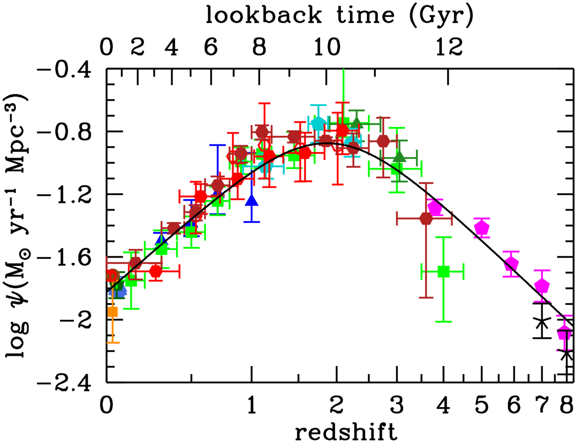

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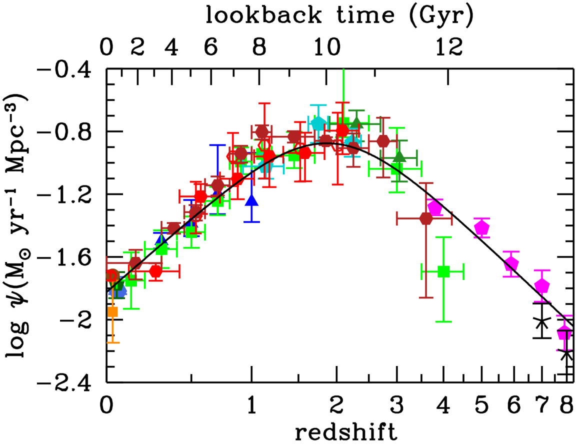

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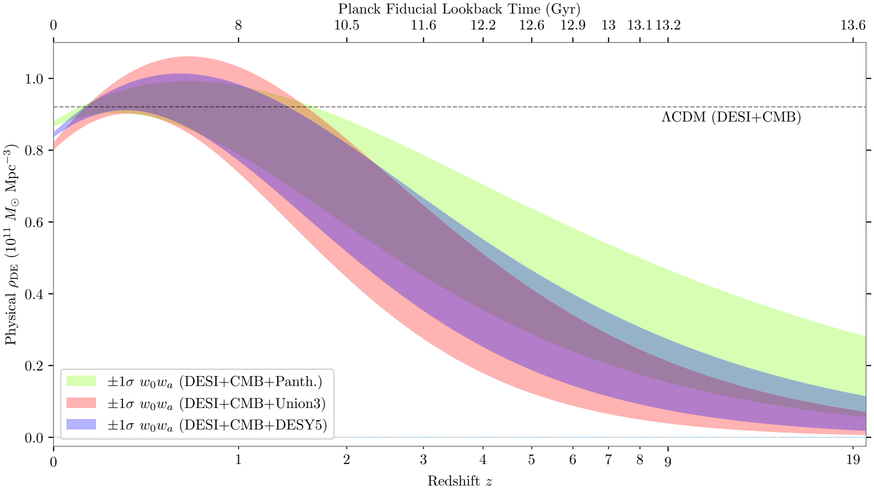
RESULT: Consistent time-evolution with 2 fewer parameters than $w_0 w_a$



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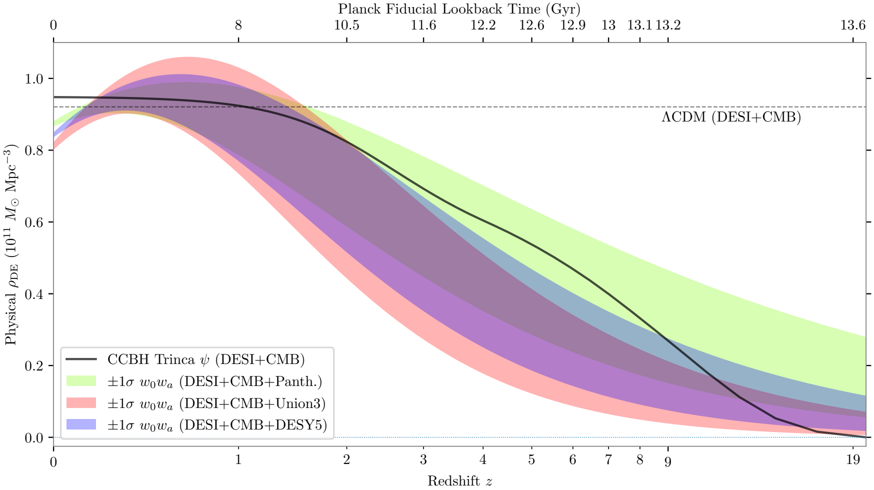

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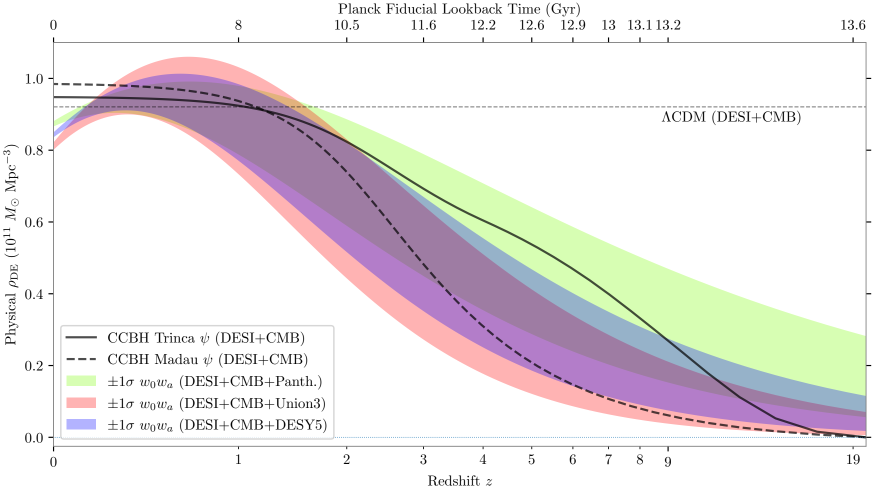

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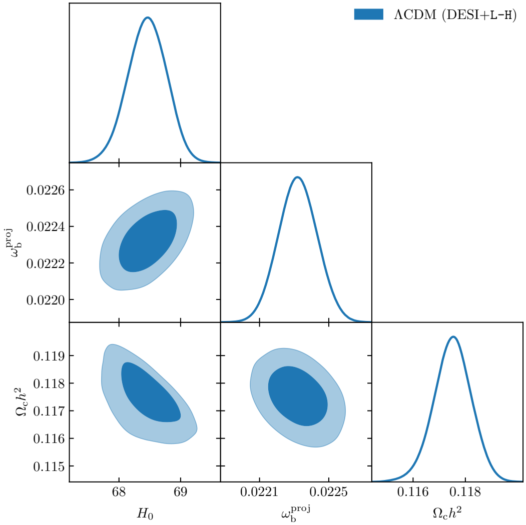

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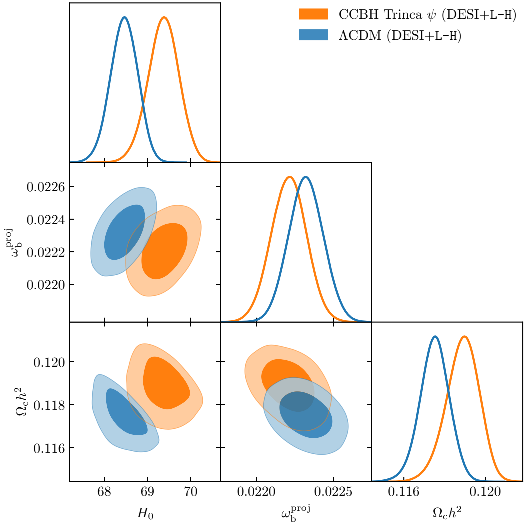

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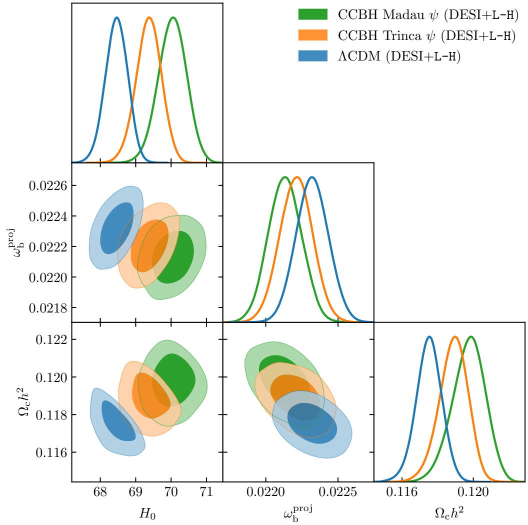

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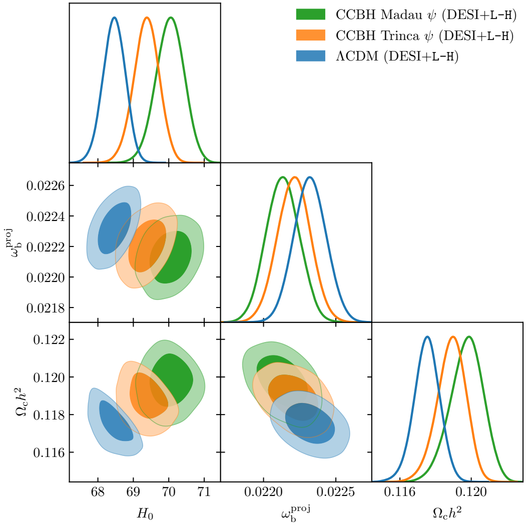

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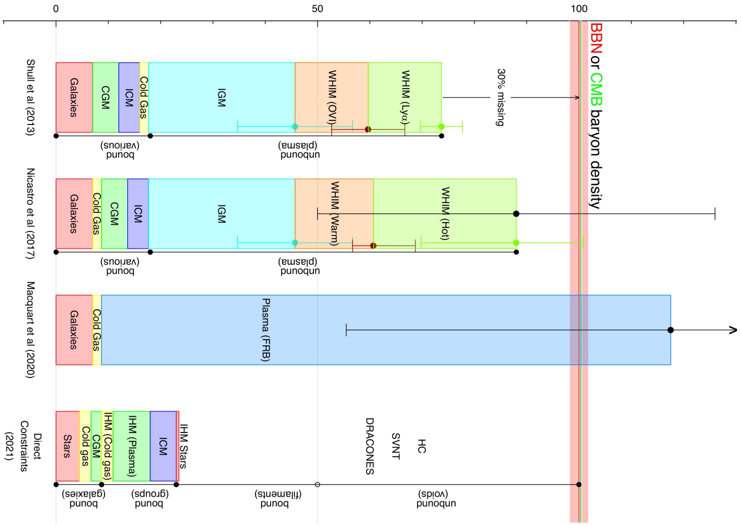
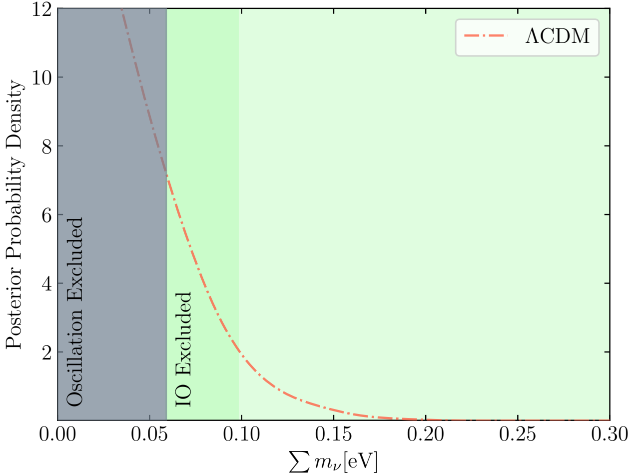


Figure: Driver, Simon. Nature Astronomy (2021) 5, 852–854

RESULT: Consistent summed neutrino masses

Conversion of baryon $\rho_{\rm b}$ into dark energy allows $\rho_\nu \propto \sum m_\nu$ to increase, at fixed total late-time matter density.



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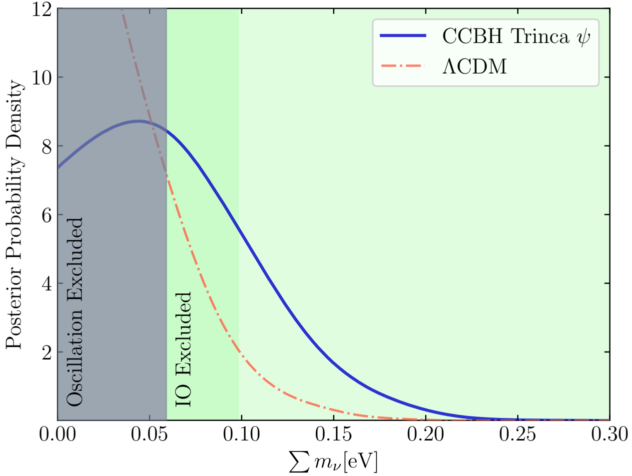

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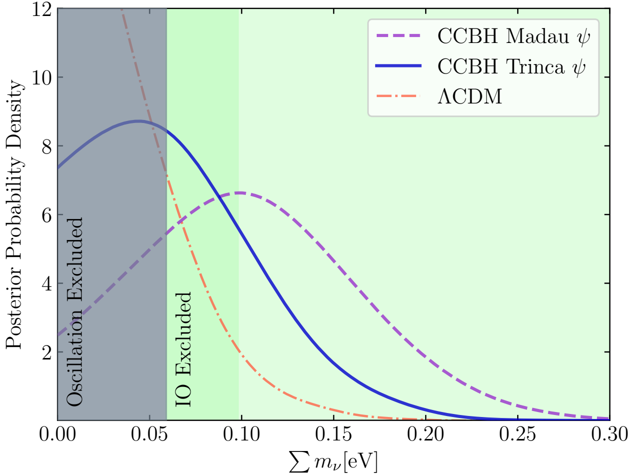

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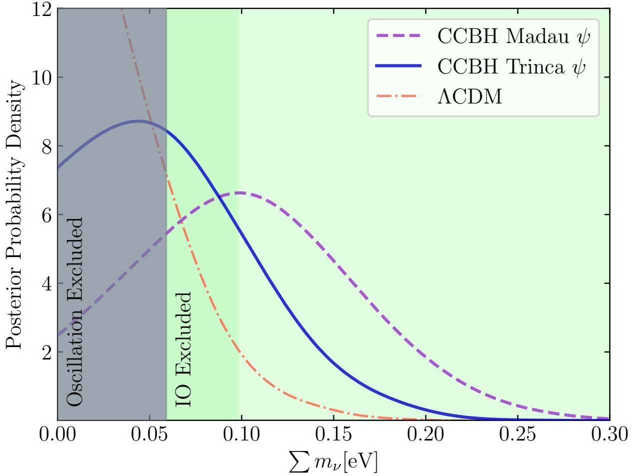
SFRD likely somewhere between



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- Acausal equation of state $w_{\mathrm{eff}} < -1$ signals energy injection from other species
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- Stellar-collapse to non-singular energized vacuum BHs converts baryons $\rightarrow$ DE and reproduces observed expansion history with two fewer parameters than $w_0 w_a$

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- Recovered $\sum m_\nu$ in good agreement with oscillation lower-bounds

Image: EHT Collaboration

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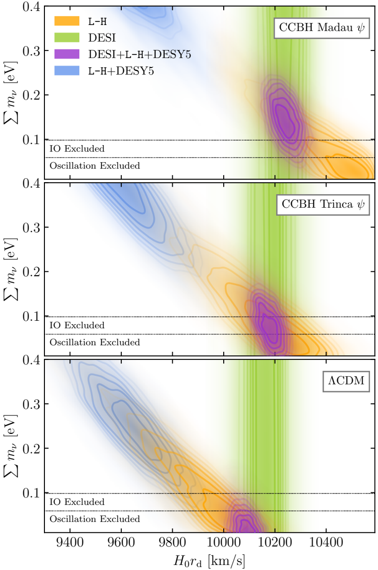
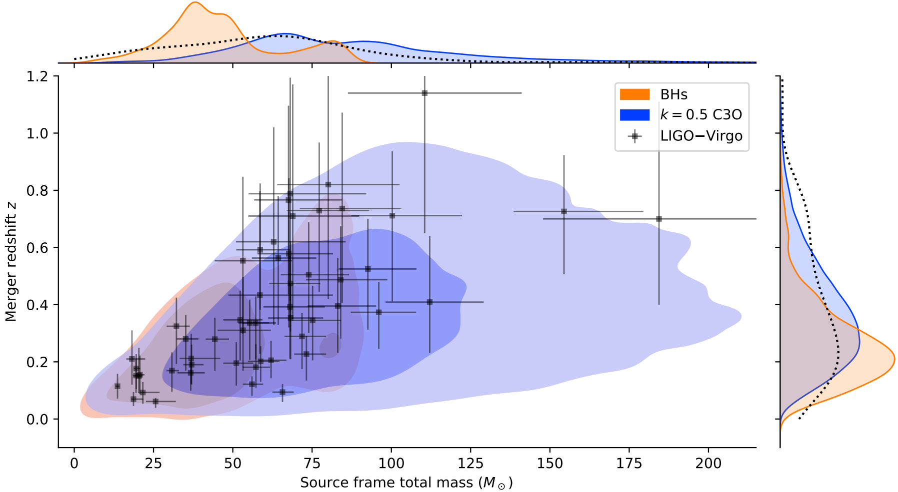
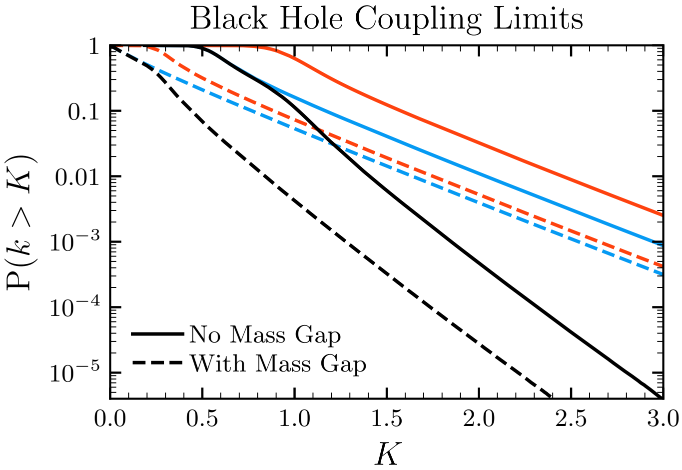


S. P. Ahlen, A. Aviles, B. Cartwright, **KC**, W. Elbers, D. Farrah, N. Fernandez, G. Niz, J.W. Rohlf, G. Tarle, R.A. Windhorst, et al. [arXiv:2504.20338]

Stellar mass tension: LIGO BBHs prefer $k \sim 0.5$

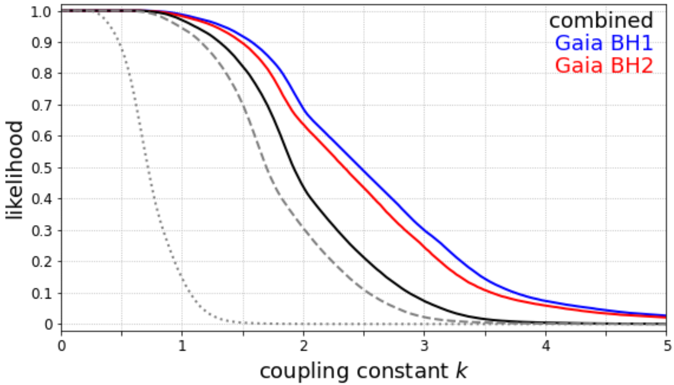


Stellar mass tension: Globular cluster BHs strongly prefer $k \leqslant 0.5$



Rodriguez, Carl L. *ApJL* 947 (2023): L12.

Stellar mass tension: *Gaia* DR3 BHs prefer $k \lesssim 0.75$



Andrae, Rene & El-Badry, Kareem. A&A Letters 674 (2023), L10

Cosmological equations describe effective fluids

Lemma

Let $A(\mathbf{k}, \eta)$ be the Fourier transform of some field $A(\mathbf{x}, \eta)$ that appears in $\mathcal{L}$. Let V denote the support of $A(\mathbf{k}, \eta)$. Then the Euler-Lagrange equations of motion for A are

$$\frac{\delta \mathcal{L}}{\delta A}(\mathbf{x}, \eta) * \mathcal{F}^{-1}\left[\mathbf{1}_V\right] = 0,$$

where $*$ denotes convolution, $\mathcal{F}^{-1}$ denotes the inverse Fourier transform, and $\mathbf{1}$ denotes the indicator function.

Proved: **KC**, J. Weiner, & D. Farrah. *PRD* 105.8 (2022): 084042.

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Recall that convolution is a "sliding average":

$$f * g := \int f\left(\mathbf{x}'\right) g\left(\mathbf{x} - \mathbf{x}'\right)\, \mathrm{d}\mathbf{x}'$$

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If $A(\mathbf{x}, \eta)$ is unconstrained in Fourier-space,

$$V := \operatorname{supp} A(\mathbf{k}, \eta) = \mathbb{R}^3$$

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Then the inverse Fourier transform is the Dirac delta

$$\mathcal{F}^{-1}\left[1\right] = \delta^3(\mathbf{x})$$

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And the familiar Euler-Lagrange equations are recovered

$$\int \frac{\delta\mathcal{L}}{\delta A}\left(\mathbf{x}'\right)\delta^3\left(\mathbf{x}-\mathbf{x}'\right)\ \mathrm{d}\mathbf{x}' = \frac{\delta\mathcal{L}}{\delta A}\left(\mathbf{x}\right) = 0.$$

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Spatial convolution integral comes directly from the action integral

$$\delta S = \int_\eta \int_\mathbf{x} \frac{\delta \mathcal{L}}{\delta A} \delta A \, \mathrm{d}\mathbf{x} \, \mathrm{d}\eta = \int_\eta \left\langle \frac{\delta \mathcal{L}}{\delta A}, \delta A \right\rangle_{L^2} \mathrm{d}\eta$$

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Cosmological metric is Fourier constrained

By definition of the model,

$$g_{\mu\nu} := a^2(\eta)\left[\eta_{\mu\nu} + \epsilon h^{(1)}_{\mu\nu}(\mathbf{x}, \eta) + \cdots\right].$$

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- In Fourier space

$$\operatorname{supp} a(\mathbf{k}) = \{(0,0,0)\}$$

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Apply the Lemma to the Einstein-Hilbert action

Gravitational DOF is unaltered by convolution:

$$\frac{\delta \mathcal{L}_{\mathrm{EH}}}{\delta a} * \mathcal{F}^{-1}\left[\mathbf{1}_{(0,0,0)}\right]$$

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but unapproximated stress-tensor DOFs are necessarily filtered by the EL convolution:

$$\frac{\delta \mathcal{L}_{M}}{\delta a} * \mathcal{F}^{-1}\left[\mathbf{1}_{(0,0,0)}\right] \propto a^3 \frac{4\pi G}{3} \int_{\mathcal{V}} T^{\mu}{}_{\mu}\left(\mathbf{x}', \eta\right) \mathrm{d}\mathbf{x}'$$

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Trace is frame invariant $\implies$ these are **microphysical** degrees of freedom:

$$a^3\frac{4\pi G}{3}\left\langle \underbrace{-\rho(\mathbf{x},\eta) + \sum_i \mathcal{P}_i(\mathbf{x},\eta)}_{\text{Microphysical eigenvalues!}} \right\rangle_{\mathcal{V}} = -\frac{\mathrm{d}^2 a}{\mathrm{d}\eta^2}$$

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