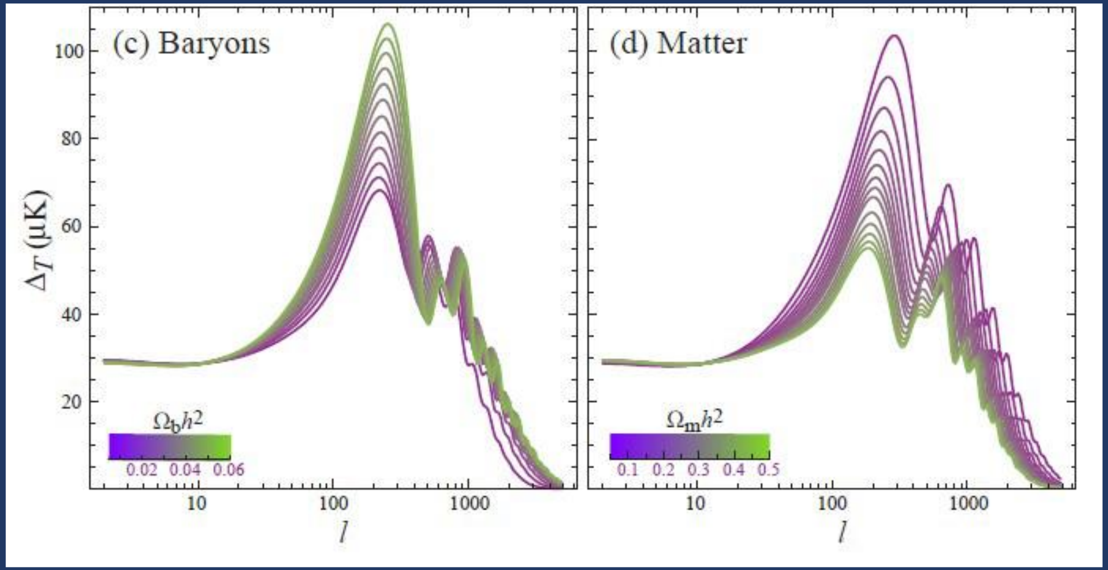
Ultralight Dark Matter and g-2

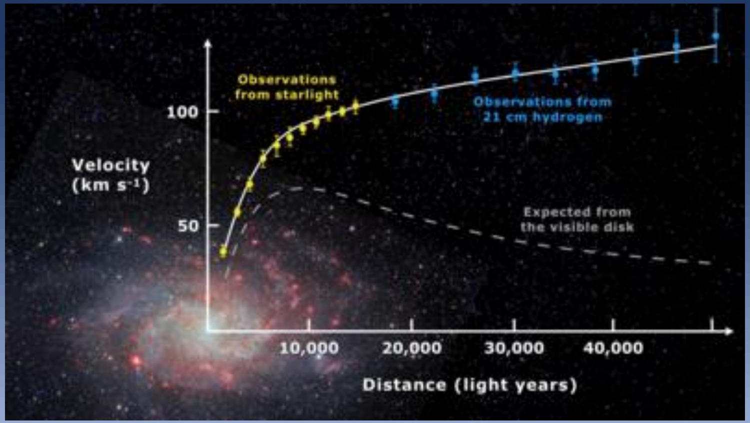
Jason L Evans

Tsung-Dao Lee Institute

Dark Matter: Evidence

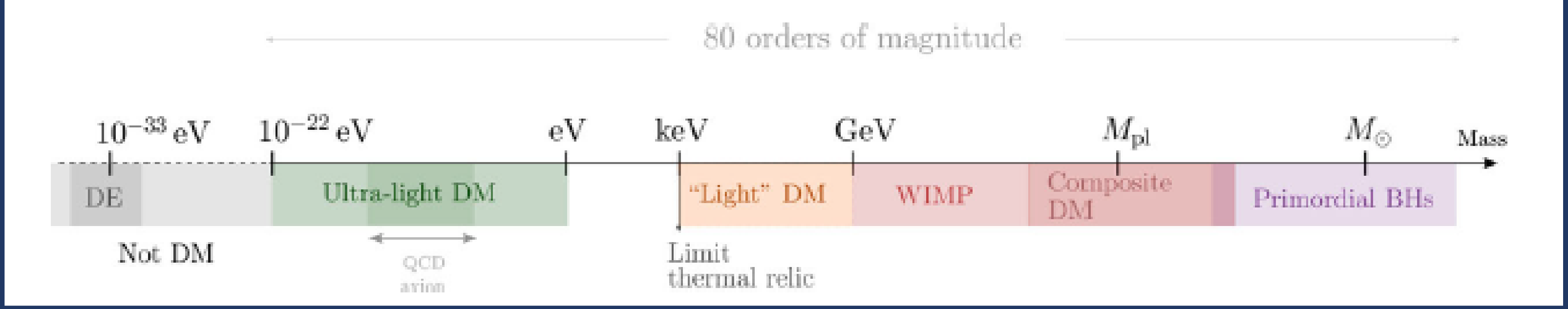
- Dark Matter
  - Gravitational Evidence





Dark Matter: What We Know

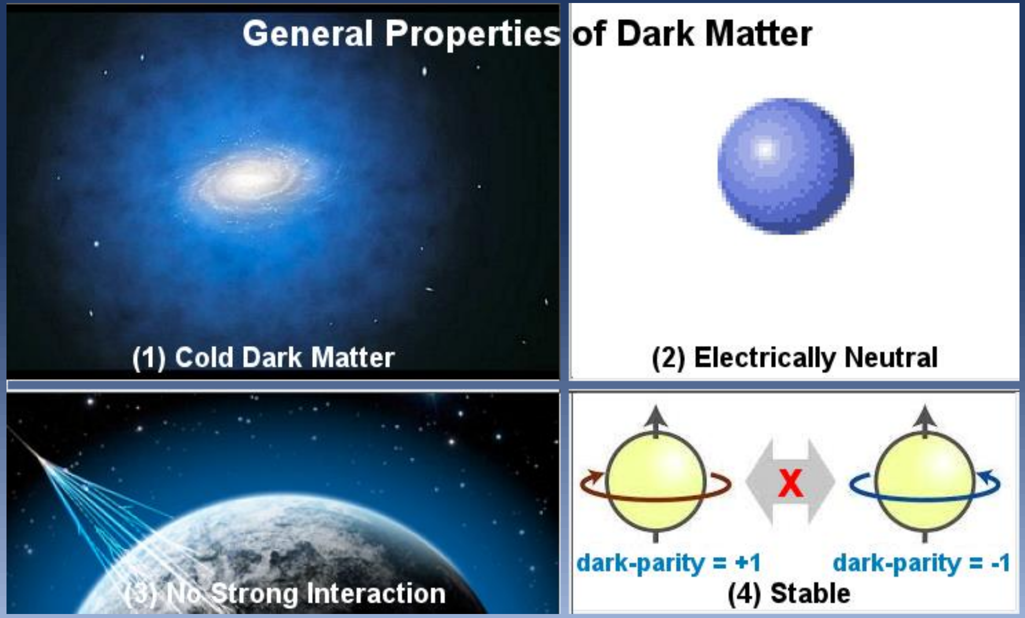
- Dark matter mass range poorly constrained



- DM should be cold

- $SU(3)XU(1)_{EM}$ Neutral

- Stable $\tau_{DM} \gg \frac{1}{H}$



Light Fermionic Dark Matter: Tremaine Gunn Bound

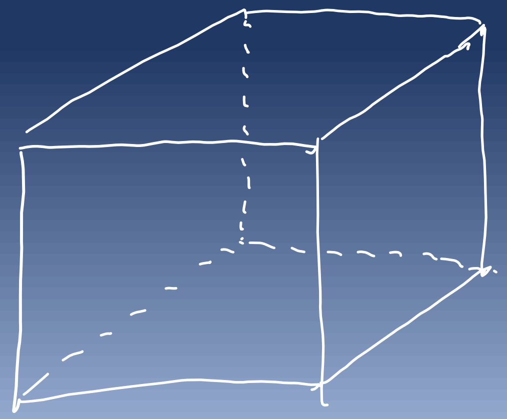
- Pauli-Exclusion Principle
  - No two fermions can occupy same energy state

- How many fermions can occupy a volume V?
  - Pauli-Exclusion limits two per energy state (spin)
  - 3D infinite square well, count states

$$-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$$

$$k_i = \frac{\pi}{L}n_i$$

Number of states

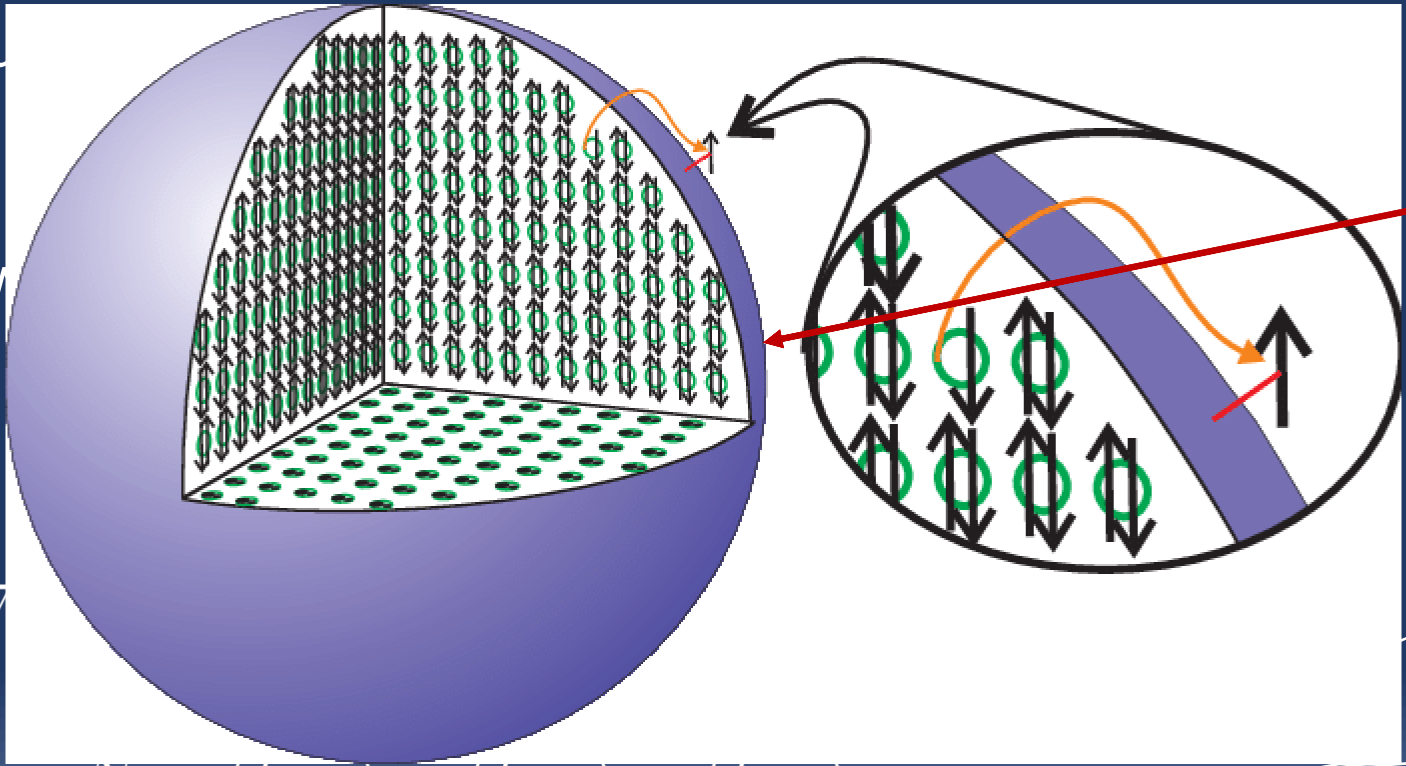


$$\psi(x,y,z) = N\sin(k_x x)\sin(k_y y)\sin(k_z z)$$

$$dN = dn_x dn_y dn_z = \frac{V}{\hbar^3}d^3p$$

Light Fermionic Dark Matter: Tremaine Gunn Bound

- Pau
- How



Galaxy is a sphere full of fermions

$$-\frac{\hbar^2}{2m}\nabla$$

$$\psi(x,y,z) = N\sin(k_x x)\sin(k_y y)\sin(k_z z)$$

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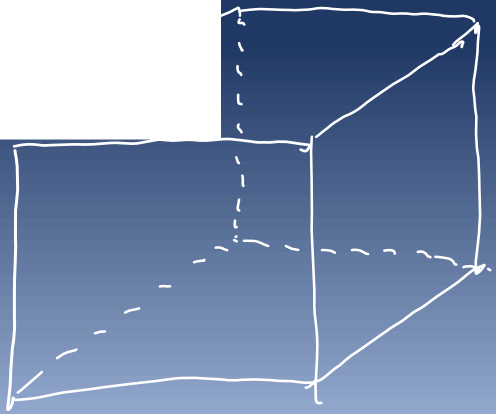


Light Fermionic Dark Matter: Tremaine Gunn Bound

- Pauli-Exclusion Principle
  - No two fermions can occupy same energy state

- How many fermions can occupy a volume V?
  - What does this say for a galaxy

Galaxy Size

Variance of velocity

$$M_V \approx \frac{\sigma^2 r}{G}$$

Virial Theorem Mass

$$N \approx \frac{Vp^3}{\hbar} \approx \frac{r^3 m^3 \sigma^3}{\hbar^3}$$

Number of particles for a given momentum

$$M_{MAX} \approx mN \approx \frac{r^3 m^4 \sigma^3}{\hbar^3}$$

Maximum mass in galaxy

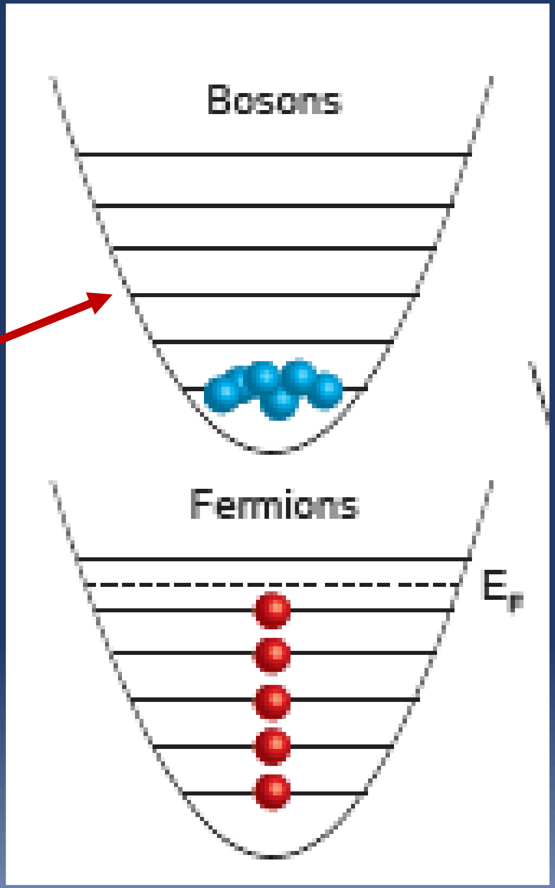
- Minimum dark matter mass $M_V > M_{MAX}$

$$m > \left(\frac{\hbar}{Gr^2\sigma}\right)^{1/4} = 20\ eV \left(\frac{r}{20\ \mathrm{kpc}}\right)^{-1/2} \left(\frac{\sigma}{200\ \mathrm{km/s}}\right)^{-1/4}$$

Light Bosonic Dark Matter

- Count the number of states for a boson
  - Basically, the same but can have ∞ particles per state

No lower bound on mass



Light Bosonic Dark Matter

- Count the number of states for a boson
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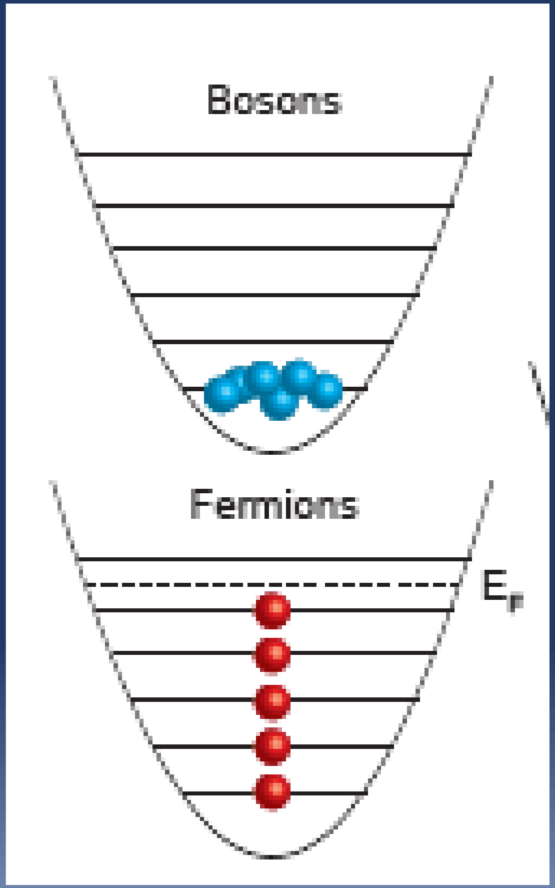
- Can there be a lower bound on the mass?



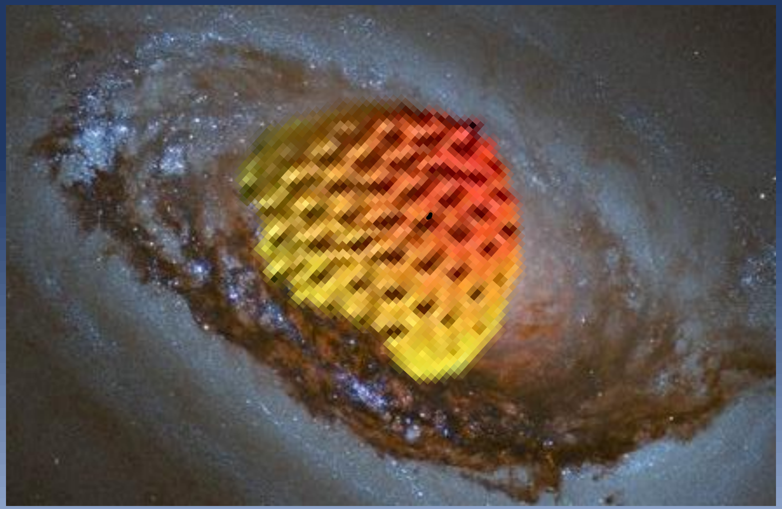
Ultralight Bosonic Dark Matter

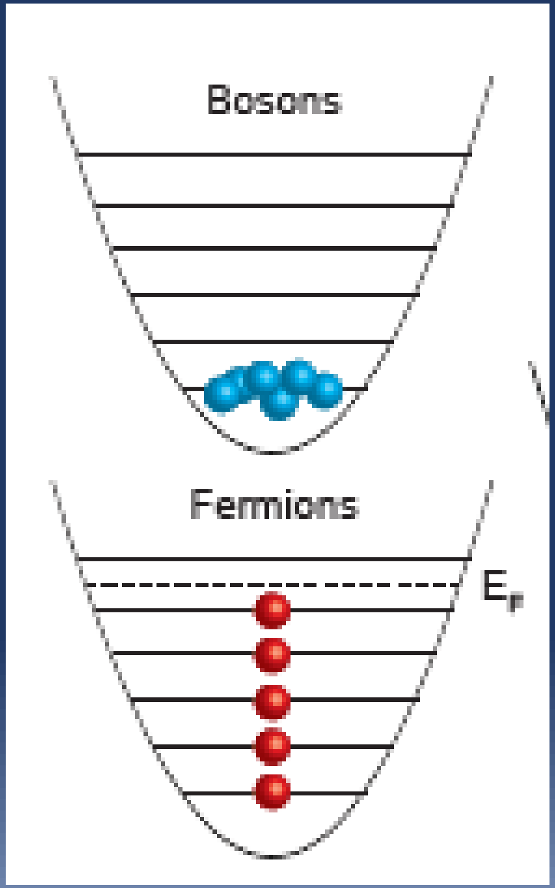
- Count the number of states for a boson
  - Basically the same, but can have ∞ particle per state
- Can there be a lower bound on the mass?
  - ULBD must be a condensate on “small” scales
  - Condensate must be smaller than galaxy

$\sim 100\ kpc$

$\sim \frac{1}{10^{-28}\ \mathrm{eV}}$

Real estimate $10^{-24}\ eV$ or even larger





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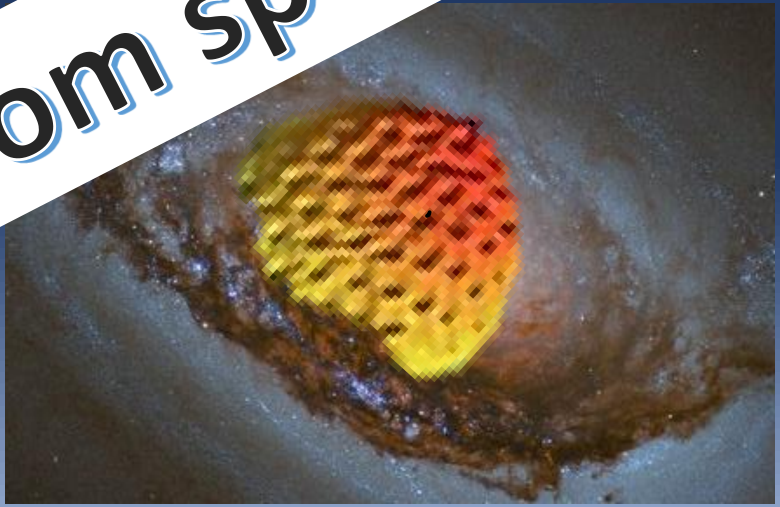
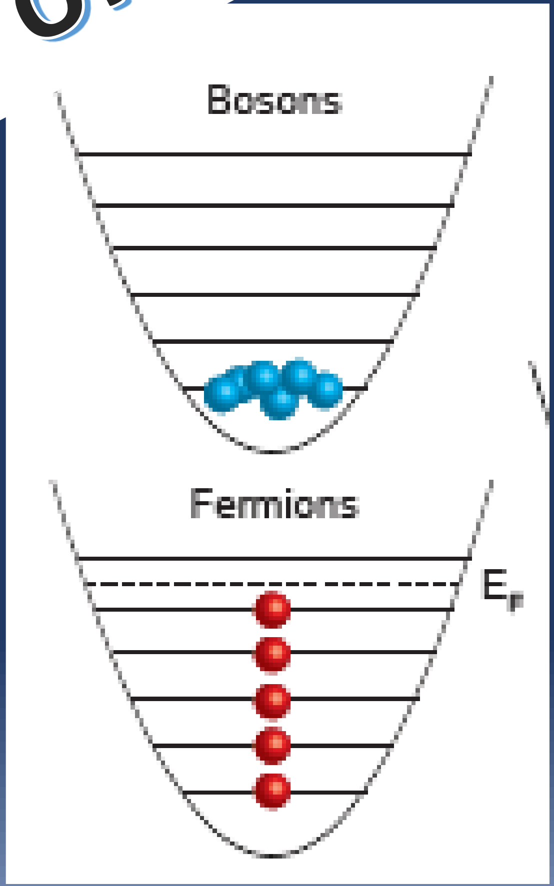
Bound not from special properties of bosons

$\sim 100\ kpc$





nate
even larger

Why Are Bosons Special?

- Bosons can occupy the same energy state

$$|n(k_1), n(k_2), \ldots\rangle = \prod_i \left\{ \frac{1}{(2\pi)^3 2\omega_{k_i}[n(k_i)+1]^{1/2}} [a^\dagger(k_i)]^{n(k_i)} \right\} |0\rangle \longleftarrow a(k)^n \neq 0$$

- In a background a decay process becomes

$$\langle 0, k_2, k_3 | \lambda\phi_3\phi_2\phi_1 | k_1, 0, 0\rangle \quad \rightarrow \quad \langle n(k_1), n(k_2)+1, n(k_3)+1 | \lambda\phi_3\phi_2\phi_1 | n(k_1)+1, n(k_2), n(k_3)\rangle$$

- Giving an enhancement

$$\langle n(k_i)|\phi_i \sim \langle n(k_i)+1| \sqrt{n(k_i)+1}$$

$$|M_0|^2 \rightarrow |M_0|^2 (1+n(k_1))(1+n(k_2))(1+n(k_3))$$

For $n(k_i) \gtrsim 1$ naively decays enhanced

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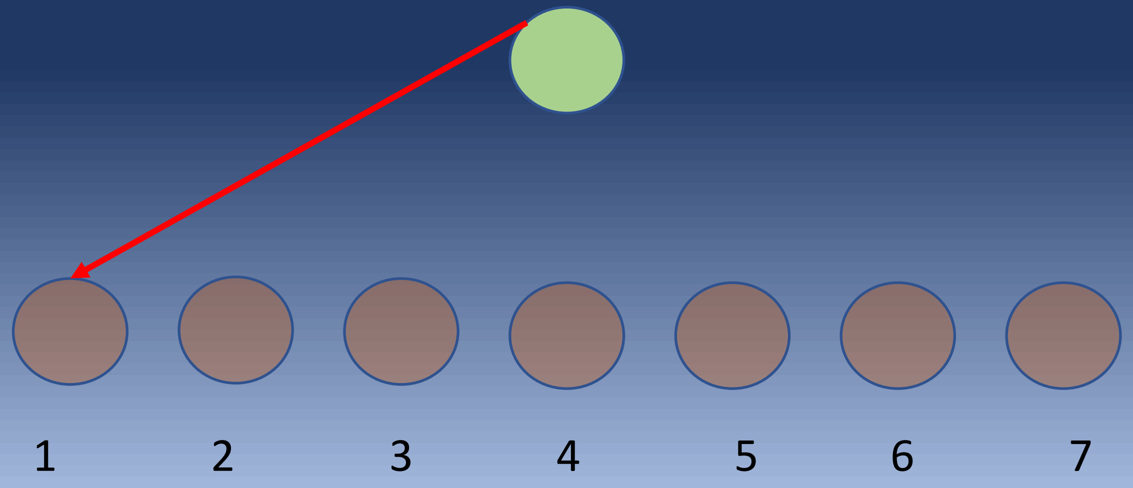
  - ❑ Where is the enhancement from?

There 1 one way



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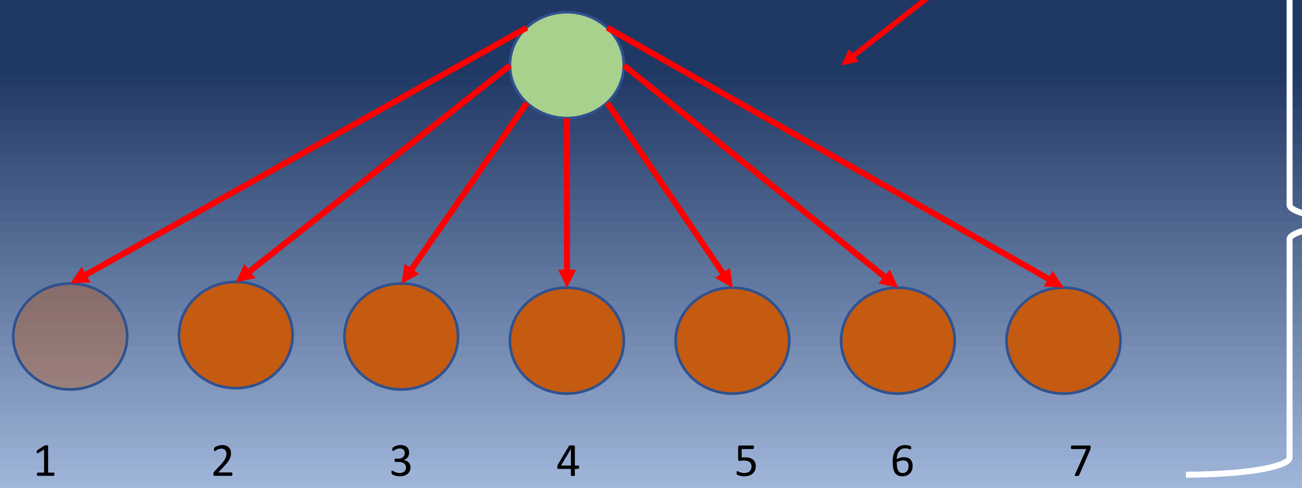
Effectively more paths for the particle to take

There are $n = 7$ ways



Bose Enhancement

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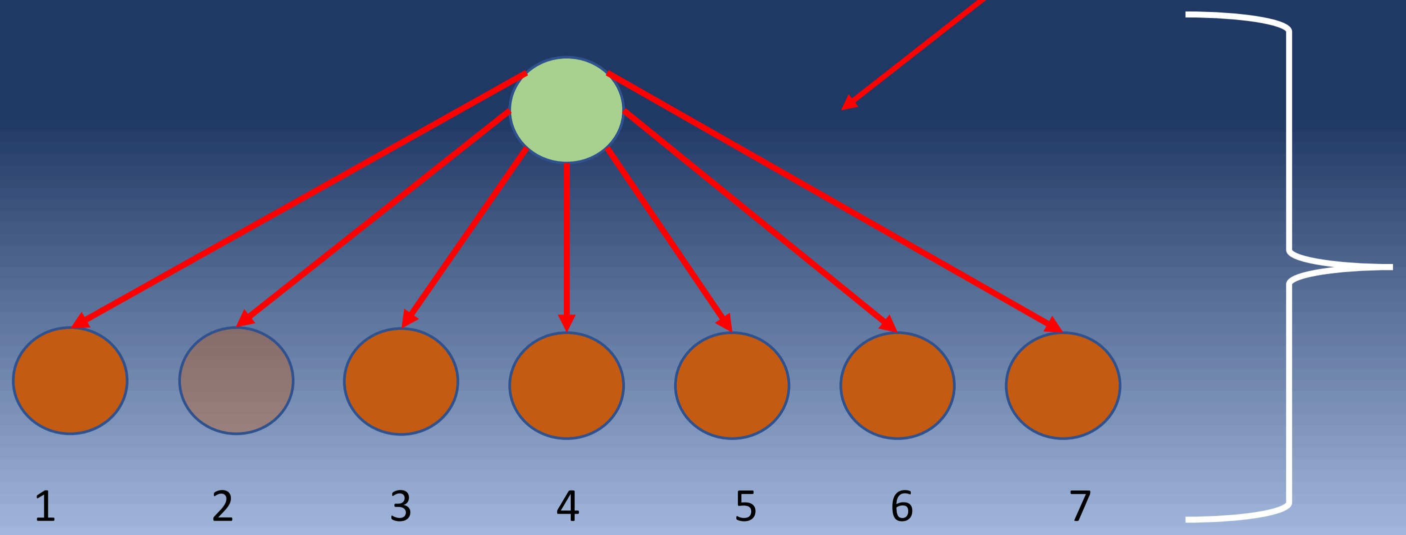

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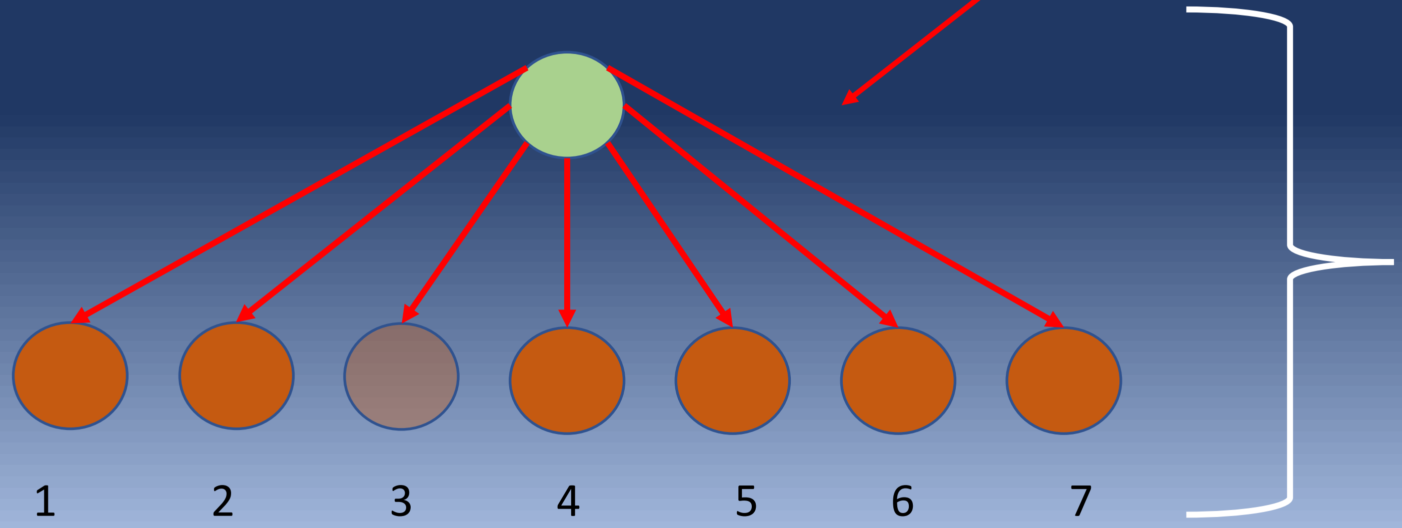

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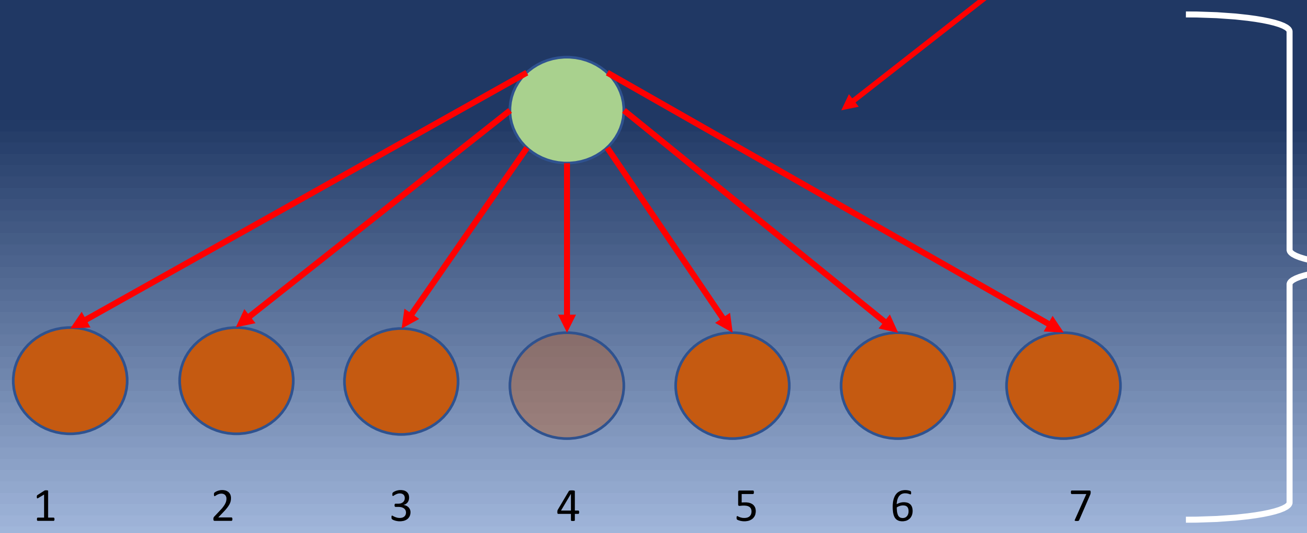

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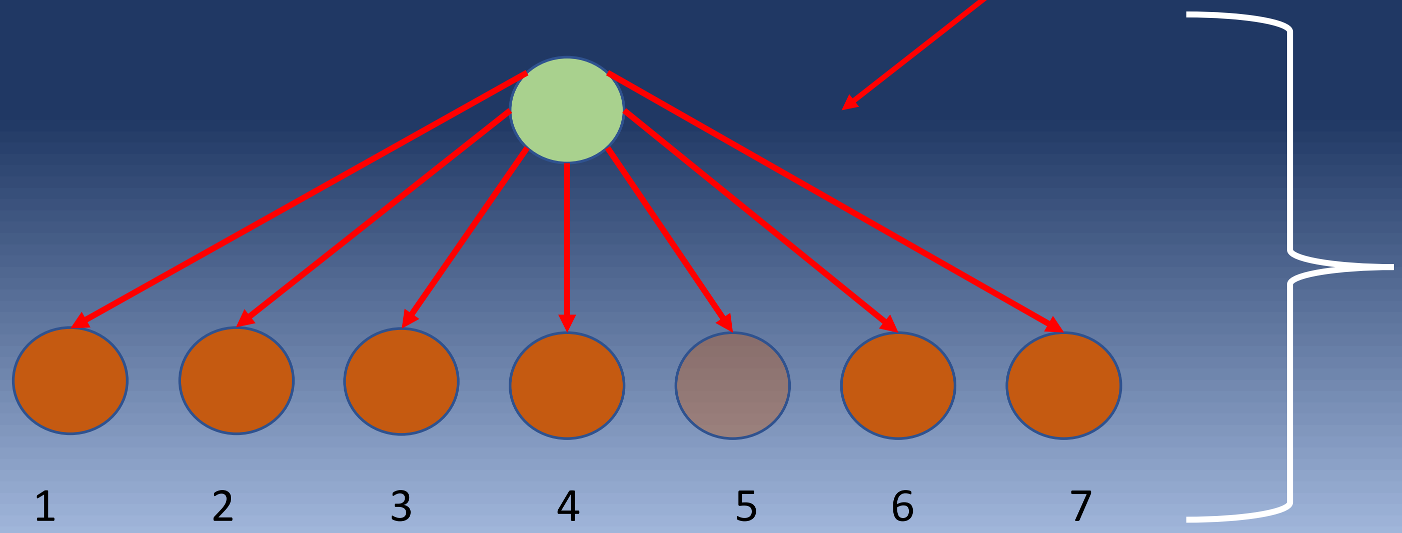

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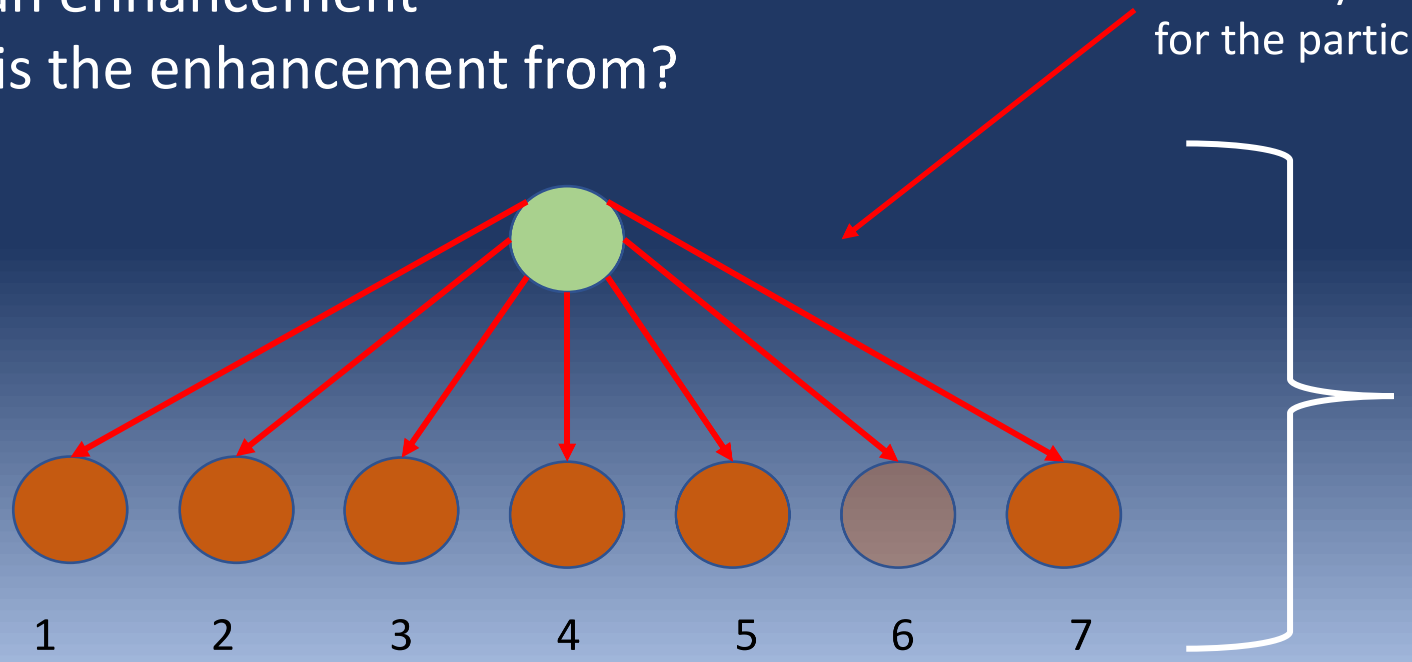

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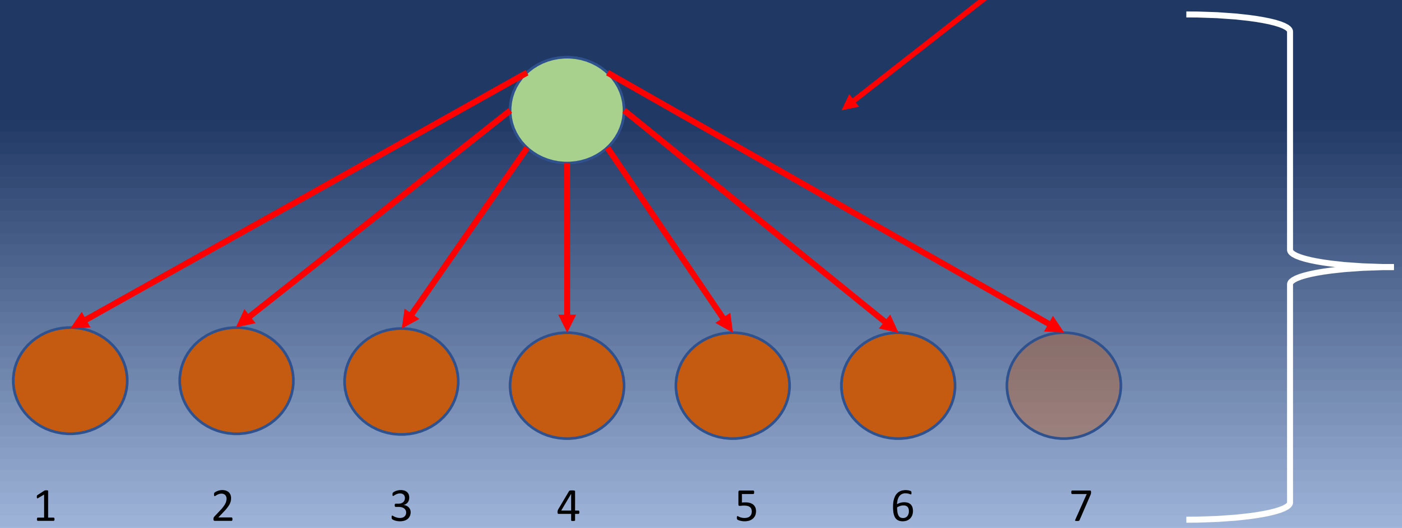

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$a(k)^n \neq 0$

- In a background a decay process becomes
- What happens to the propagator?

$$\langle 0|\phi^\dagger \phi|0\rangle \quad \rightarrow \quad \langle n|\phi^\dagger \phi|n\rangle$$

- Background provides an additional piece

$\propto \delta^3(k-k')$

Can be significantly enhanced

$\propto n\delta^3(k-k')$

No background $\langle 0|a(k)a^\dagger(k')|0\rangle = \langle 0|\left[a(k), a^\dagger(k')\right]|0\rangle$

Background $\langle n|a(k)a^\dagger(k')|n\rangle = \langle n|\left[a(k), a^\dagger(k')\right]|n\rangle + \langle n|a^\dagger(k)a(k')|n\rangle$

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$$G(k) = \frac{i}{k^2 - m^2 + i\epsilon} - 2\pi n(k)\delta(k^2 - m^2)$$

Same as time ordered thermal propagator in real time formulation.

Correction from background: Will affect loop processes

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$$|n(k_1), n(k_2), \ldots\rangle = \prod_i \left\{ \frac{1}{(2\pi)^3 2\omega_{k_i}[n(\ldots}\right.$$

$\neq 0$

- In a background a de…
- What happens t…

$|n\rangle$

…itional piece

$$G \ldots \frac{}{\ldots m^2 + i\epsilon} - 2\pi n(k)\delta(k^2 - m^2)$$

Same as time ordered thermal propagator in real time formulation.

Correction from background: Will affect loop processes

All processes potentially enhanced by background

Production of Ultralight Dark Photon Dark Matter

- Thermal production not possible
  - Dark Matter tends to be hot

$$T_{CMB} \sim 10^{-3}\ \mathrm{eV} \qquad \left.\frac{m_{DM}}{T}\right|_{\mathrm{today}} \ll 1$$

Production of Ultralight Dark Photon Dark Matter

- Thermal production out

- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field

$$A_L \sim \partial_\mu \pi$$

$$\pi(\vec{k},t) \equiv \frac{m}{k} A_L(\vec{k},t),$$

$$S_{\text{Long}} \xrightarrow{am \ll k} \int \frac{a^3 d^3k}{(2\pi)^3} dt \frac{1}{2}\left(|\partial_t \pi|^2 - \frac{k^2}{a^2}|\pi|^2\right) = \int a^3 d^3x \, dt \frac{1}{2}\left((\partial_t \pi)^2 - \frac{1}{a^2}|\vec{\nabla}\pi|^2\right)$$

  - Choose the Bunch-Davies vacuum we get

$$P_{A_L} \simeq \left(\frac{k}{m}\right)^2 P_\pi \simeq \left(\frac{kH_I}{2\pi m}\right)^2$$

Graham, Mardon, Rajendran

Production of Ultralight Dark Photon Dark Matter

- Thermal production out

- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field
  - Choose the Bunch-Davies vacuum we get
  - Power spectrum suppressed at low momentum
  - Relation between inflation scale and mass for dark matter

$$\frac{\Omega_{\rm vector}}{\Omega_{\rm cdm}} = \sqrt{\frac{m}{6\times 10^{-6}\,{\rm eV}}}\left(\frac{H_I}{10^{14}\,{\rm GeV}}\right)^2 .$$

$$m_{DM} \sim 10^{-20}\ {\rm eV} \ \rightarrow \ \frac{\Omega_{\rm vector}}{\Omega_{\rm cdm}} \sim 10^{-7}$$

However, can still place a strong constraint!!

Production of Ultralight Dark Photon Dark Matter

- ❑ Thermal production out
- ❑ Production of longitudinal modes from quantum fluctuations
- ❑ Production from inflaton induced tachyonic DP mass

Kitajima and Nakayama

$$\mathcal{L} = -\frac{f^2(\phi)}{4} F_{\mu\nu}F^{\mu\nu} - \frac{1}{2} m_A^2 A_\mu A^\mu - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \mathcal{L}_X,$$

$$V(\phi) = \frac{1}{2} m_\phi^2 \phi^2,$$

$$f(\phi) = \exp\left(-\frac{\gamma}{8}\frac{\phi^2}{M_{\rm Pl}^2}\right)$$

- ❑ Equations of motion

Tachyonic Mass

$$\ddot{\overline{A}}_i + 3H\dot{\overline{A}}_i + \left(\frac{m_A^2}{f^2} - \frac{(\alpha+4)(\alpha-2)}{4}H^2 + \frac{2-\alpha}{2}\dot{H}\right)\overline{A}_i = 0 \quad \longleftarrow \quad \alpha = \gamma < 4$$

- ❑ Have to worry about isocuvature, so need a curvaton

Production of Ultralight Dark Photon Dark Matter

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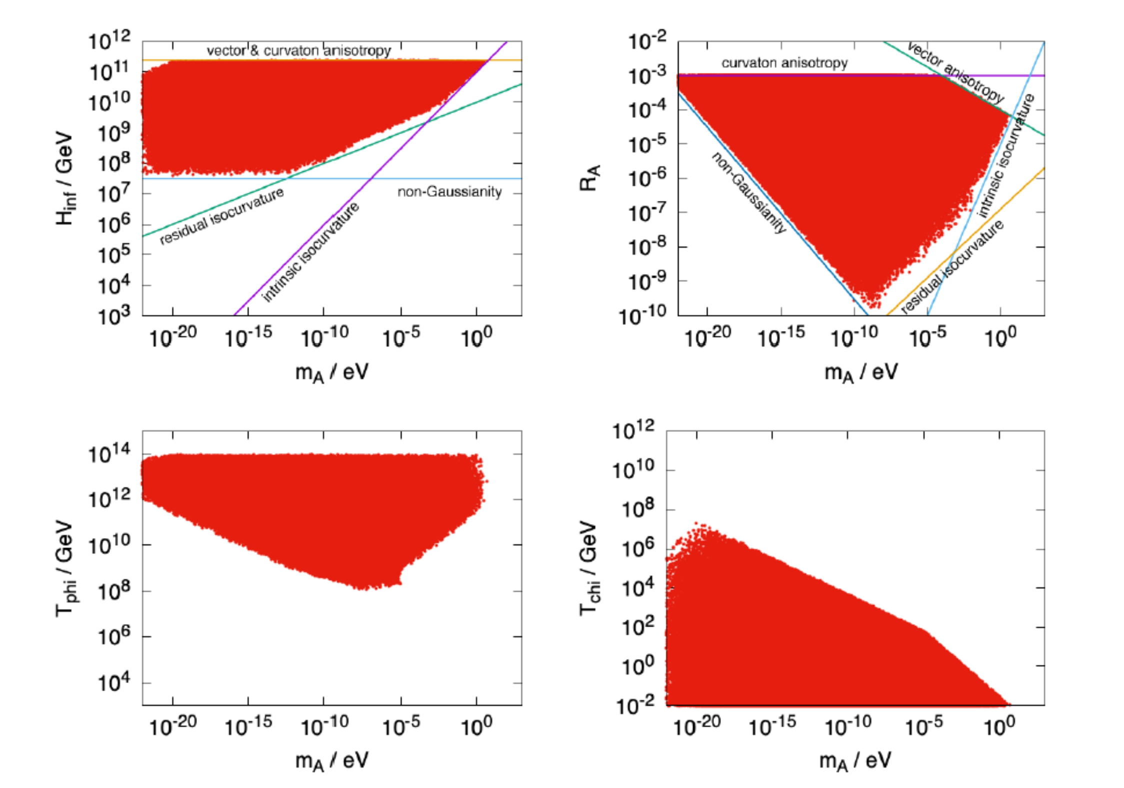
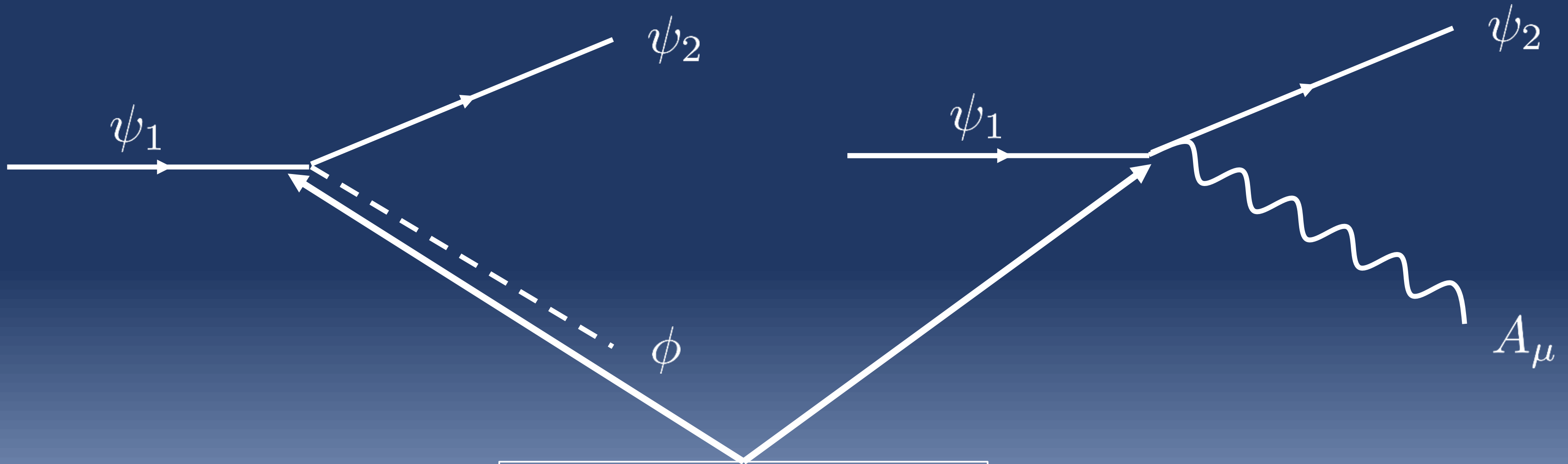


What happens for decays?

- Two body decays
  - Coupling hard to realize

$$m_{\psi_1} > m_{\psi_2}$$



Flavor Changing interactions

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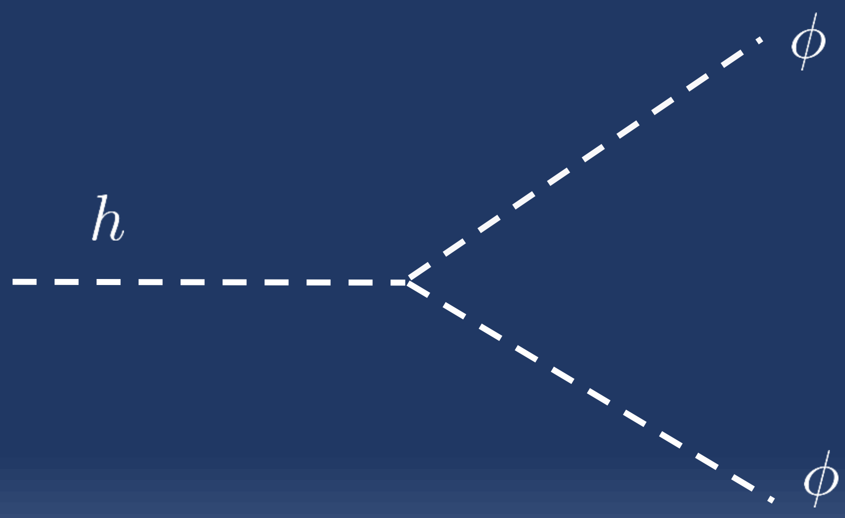
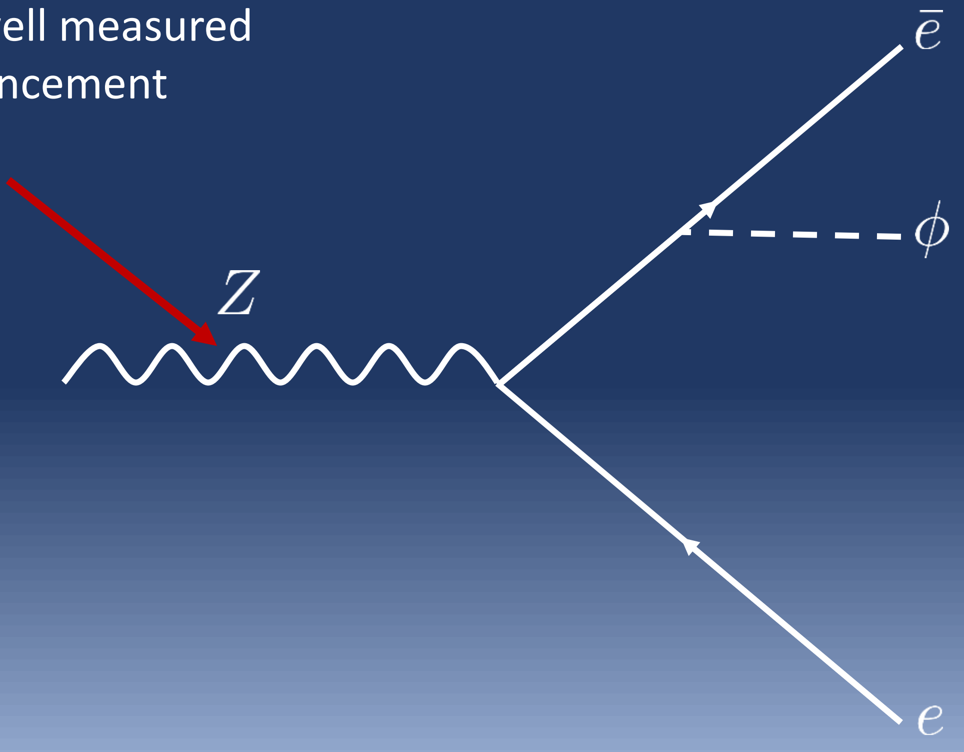


Makes it hard to keep ϕ light

What happens for decays?

- Two body decays
- Three body decays
  - Many possibilities!!

- Z decays well measured
- Small enhancement detectable



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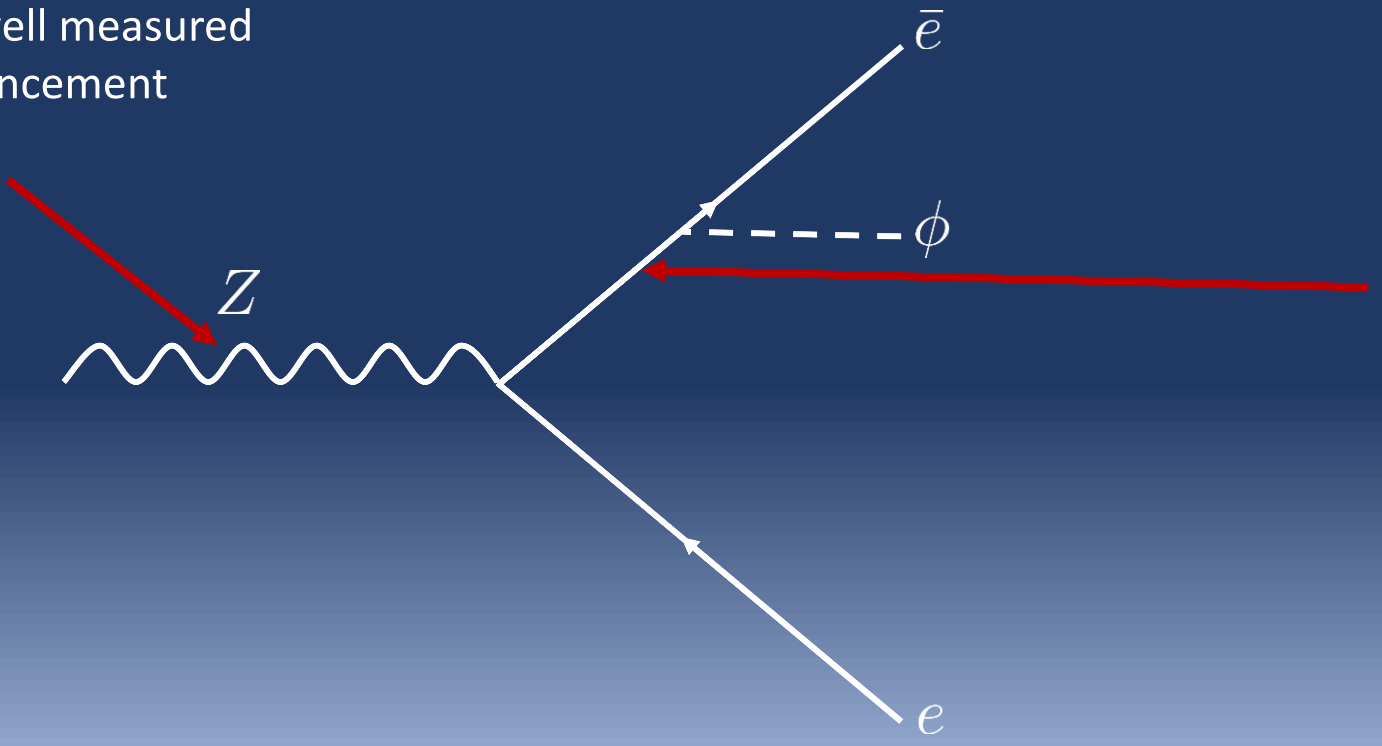

Warning

- ❑ Propagator on shell when ϕ momentum goes to zero
- ❑ IR divergences must cancel

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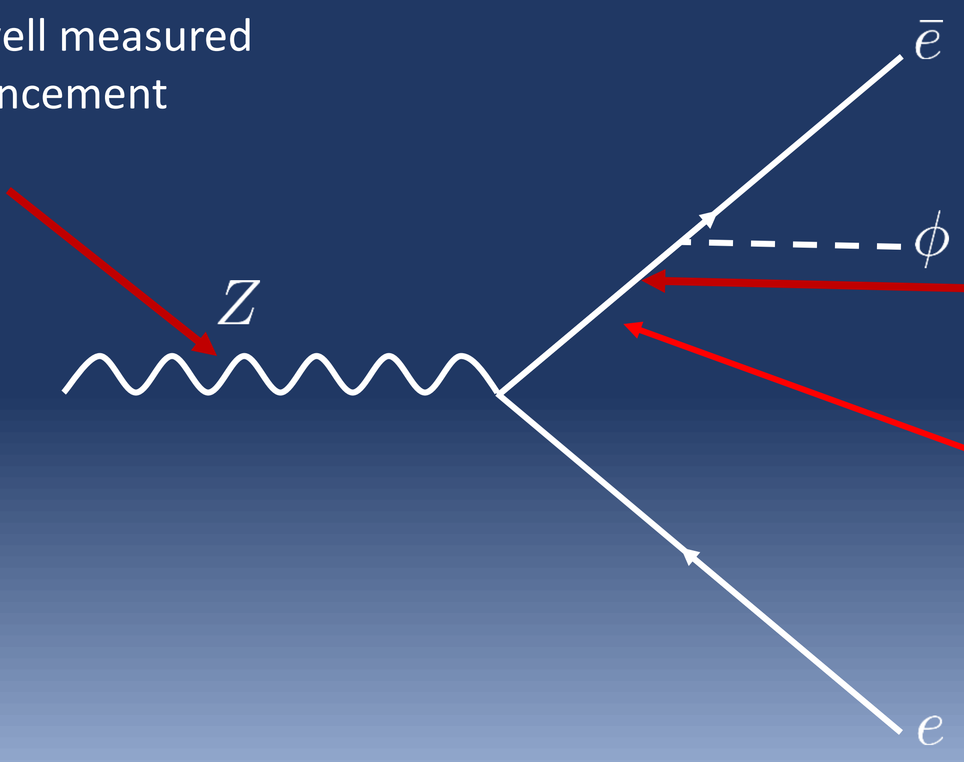

Warning

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Effectively Bremsstrahlung

What happens for decays?

- Two body decays
- Three body decays
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    - IR divergences in finite temperature
    - Have to include wavefunction renormalization

- Power Divergent IR singularity
- Shows up in decay too

$$\Sigma_\beta \supset \frac{\alpha}{4\pi^2} I_A(k)(\gamma \cdot k - m_e) \qquad I_A(k) = 8\pi \int \frac{dq}{q} n(E_q) \qquad \lim_{q\to 0} n(E_q) \sim \frac{1}{q}$$

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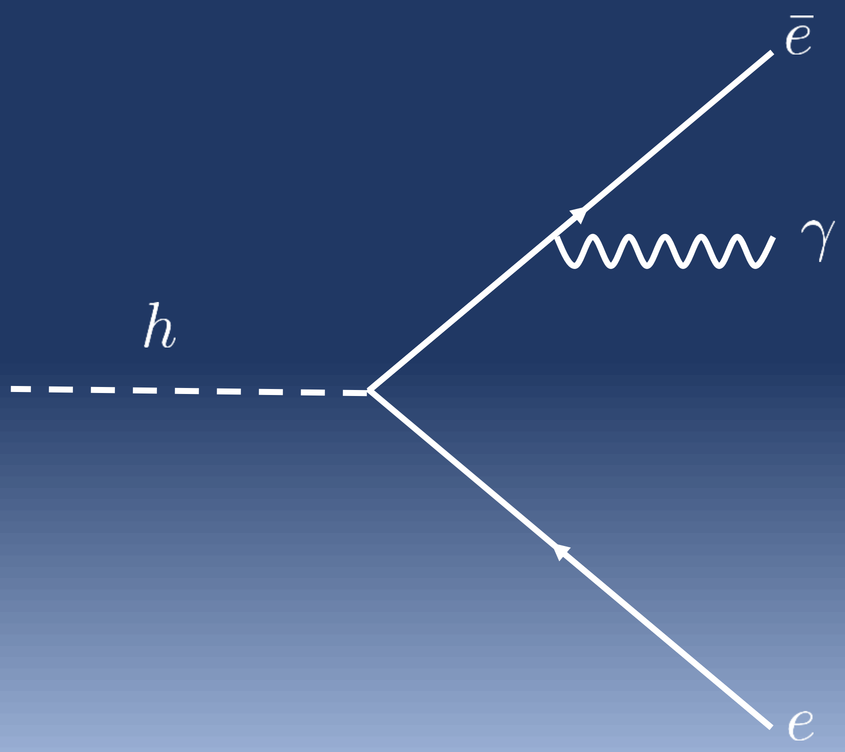
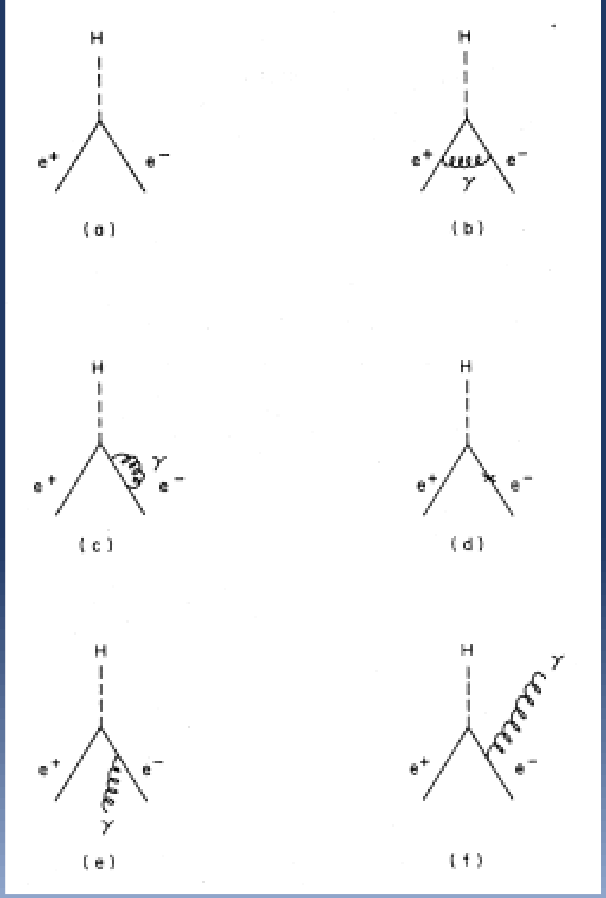
  - Example: Higgs Decay in Thermal Bath





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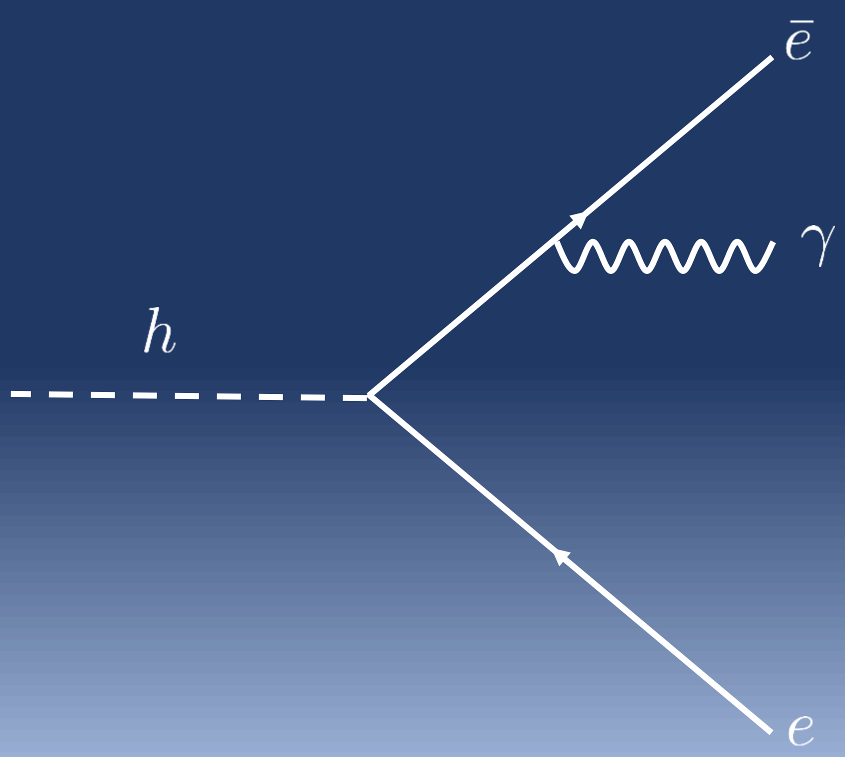
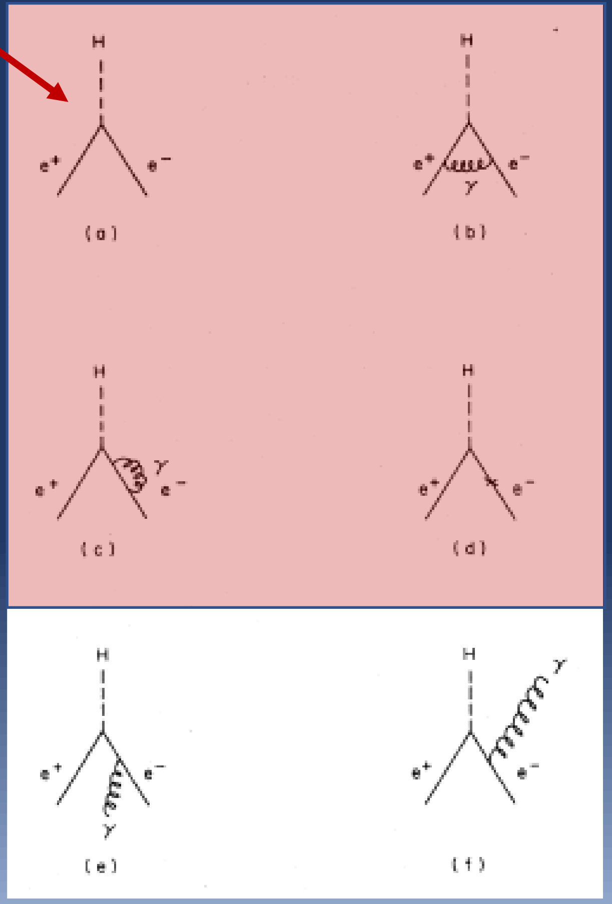


- ❑ Vertex Corrections
  - ❑ Background enhanced
- ❑ Same for $k \to 0$

Correction to this decay



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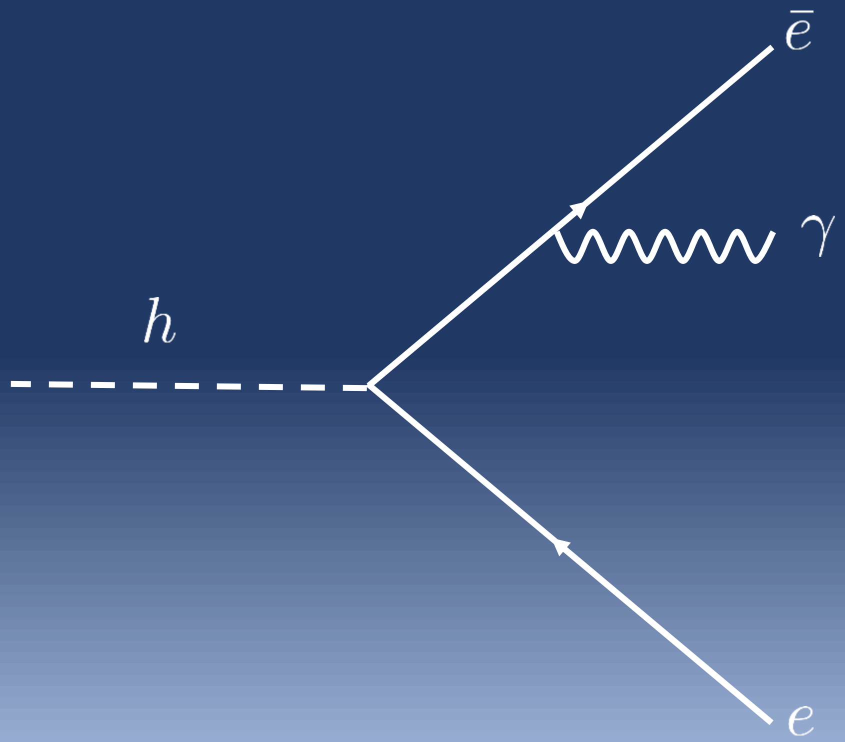
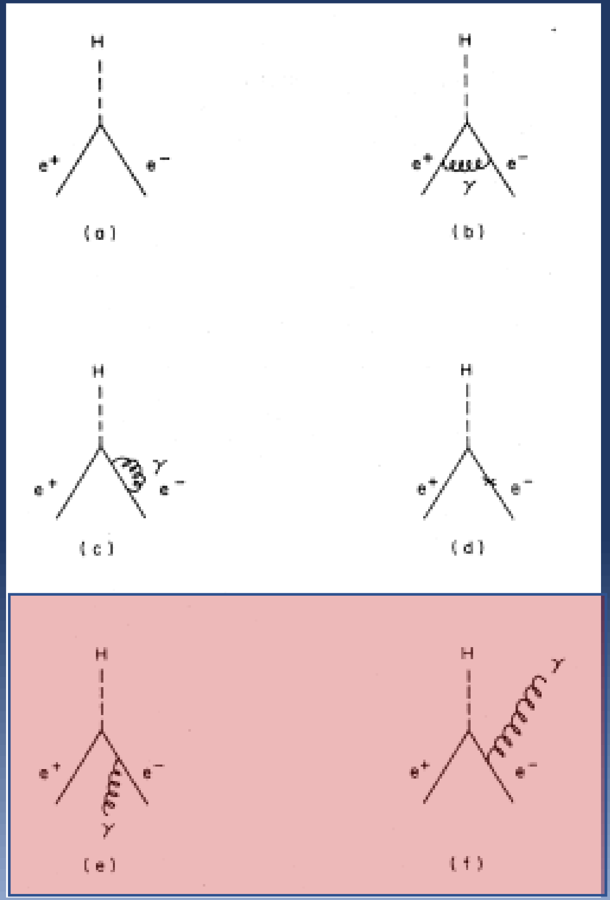
  - Example: Higgs Decay in Thermal Bath



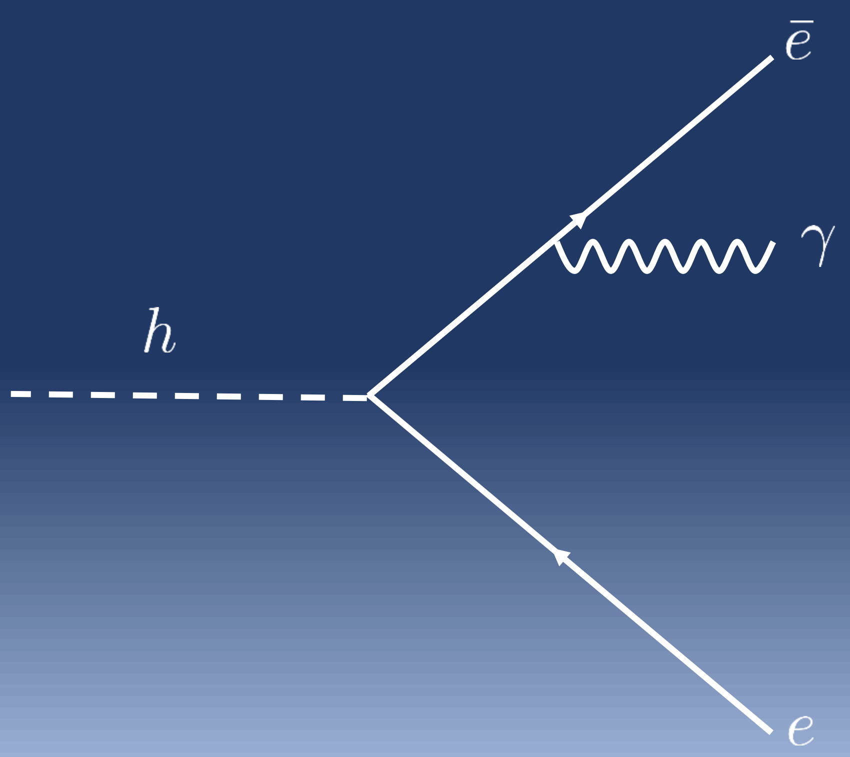
$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- Stimulated Emission/Absorption
- Background enhanced



What happens for decays?

- ❑ Two body decays
- ❑ Three body decays
  - ❑ Many possibilities!!
  - ❑ Example: Higgs Decay in Thermal Bath



$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- ❑ Enhancement completely cancels at 1-loop

Donoghue, Holstein (1983)

$$\Gamma_{T=0}(h \to e^+ e^- \gamma) = \Gamma_{T\neq 0}(h \to e^+ e^- \gamma) + \mathcal{O}(T^3)$$

Bremsstrahlung cancelations

- ❑ Cancellation of IR divergences expected

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right]$$

$$n_k(k) \sim \frac{1}{k}$$

- ❑ Finite piece goes as

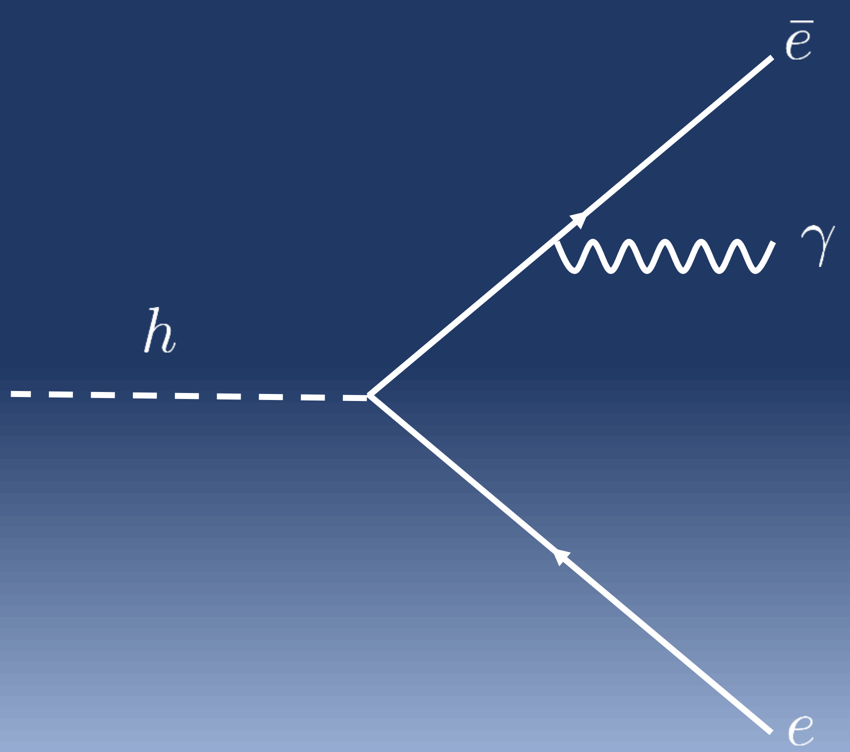
$$\int dk R_1 \left(k n_k(k)\right) \quad \sim \quad \int dk R_1 k^0$$

- ❑ What if it is a background

$$n_k(E_k) \sim \frac{\rho_{DM}}{\Delta k k^2 m_{DM}} \quad \rightarrow \quad \int dk \frac{k^2}{E_k} n_k(E_k) \sim \frac{\rho_{DM}}{m^2_{DM}}$$

What happens for decays?

- Two body decays
- Three body decays
  - Many possibilities!!
  - Example: Higgs Decay in Thermal Bath



$$|M_{SE/SA}|^2 \sim M^0_{ver} M^\alpha_{ver}$$

- Enhancement completely cancels at 1-loop

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- Cancellation of IR divergences expected

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right]$$

$\sim \mathcal{O}(T^2)$

- Not IR divergent, why does it cancel?

This vanishing appears to be accidental, but we have also calculated the radiative corrections for the decay of a pseudoscalar H (instead of scalar) and found that to be zero also.

Donoghue, Holstein (1983)

What happens to loop processes?

- What will give us the most "bang for our buck"?
  - First generation particles in loop (Denominator then $m^2_{e,d,u} m^2_{DM}$)

Enhancement comes from term like

$$\int dk \frac{k^2}{E_k} n_k(E_k) \sim \frac{\rho_{DM}}{m^2_{DM}} \quad \rightarrow \quad \frac{\rho_{DM}}{m_i^2 m^2_{DM}}$$

What happens to loop processes?

- What will give us the most “bang for our buck”?
  - First generation particles in loop (Denominator then $m_{e,d,u}^2 m_{DM}^2$)

$$\int dk \frac{k^2}{E_k} n_k(E_k) \sim \frac{\rho_{DM}}{m_{DM}^2} \quad \rightarrow \quad \frac{\rho_{DM}}{m_i^2 m_{DM}^2}$$

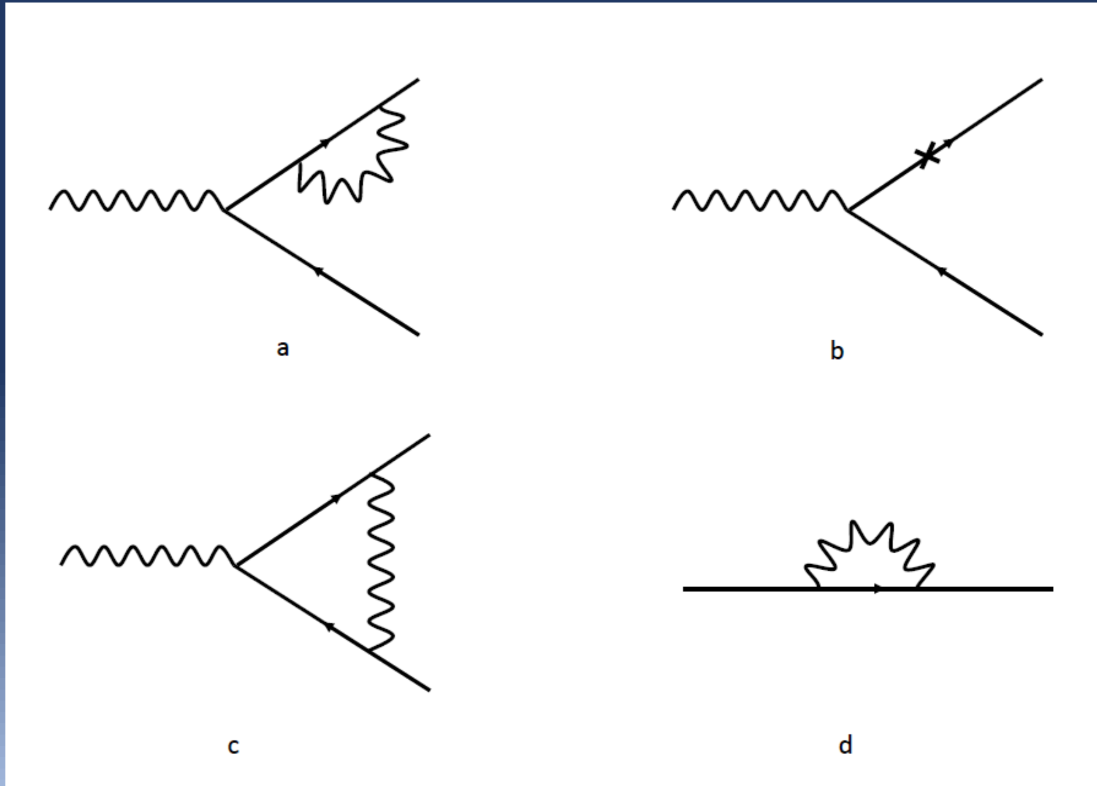
  - Precision measurement, more sensitive to everything

What happens to loop processes?

- What will give us the most “bang for our buck”?
  - First generation particles in loop (Denominator then $m^2_{e,d,u} m^2_{DM}$)
  - Precision measurement, more sensitive to everything

- $g-2$ of electron perfect

- Plan
  - Calculate $g-2$ of the electron
  - Show that charge is not renormalized in background
  - Show ward identities satisfied in background

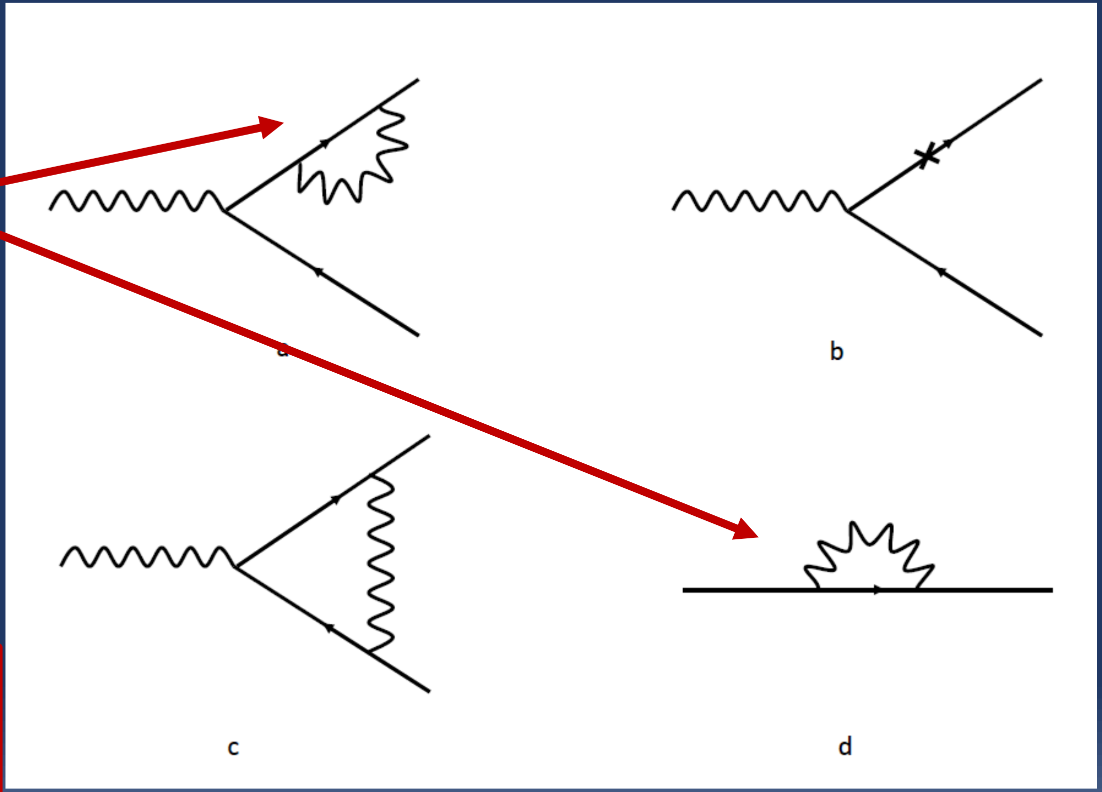


Wave Function Renormalization

- ❑ Start with wave function renormalization

$$\Sigma_n = B(k) + C(k)\,(\gamma \cdot k - m_e) + \gamma \cdot D(k)$$

- ❑ New type of contribution
  - ❑ Violates Lorentz symmetry
  - ❑ Can't be combined to C(k) or B(k)



$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{q_\mu}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$B(k) = \; 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{m_e}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$C(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{1}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

Do no specify $n(E_q)$ until end

Wave Function Renormalization

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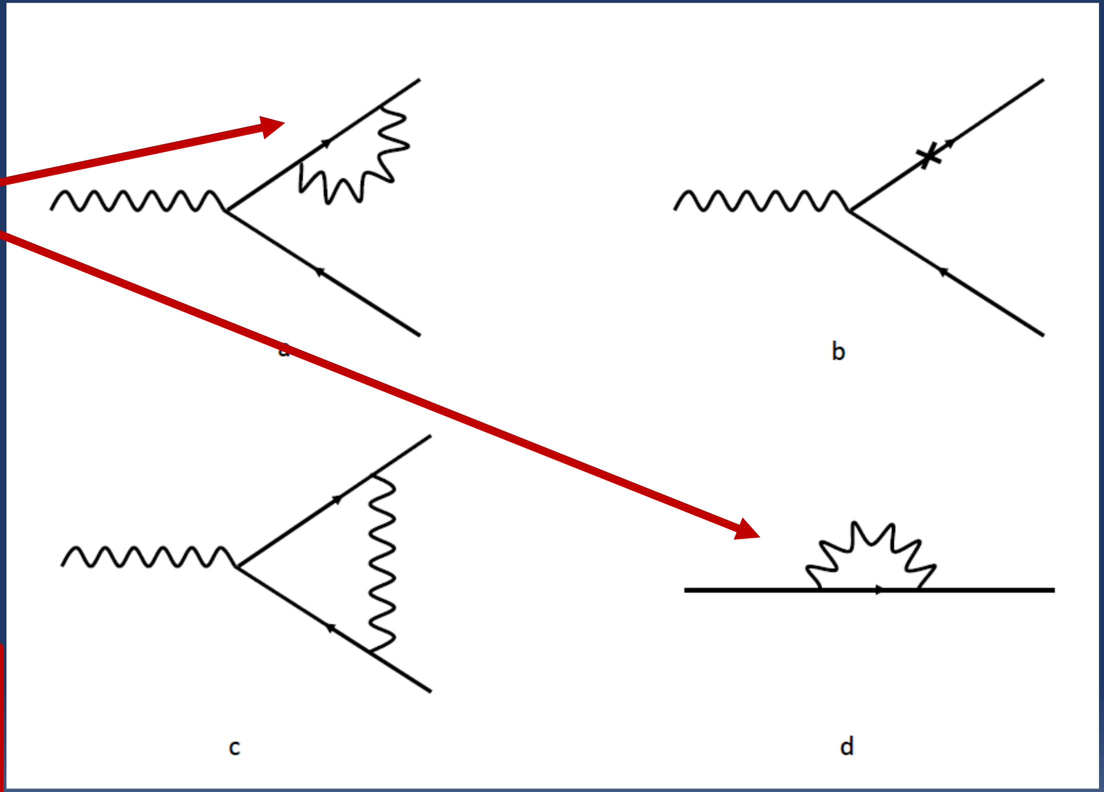
$$\Sigma_n = B(k) + C(k)\,(\gamma\cdot k - m_e) + \gamma\cdot D(k)$$



- New type of contribution
  - Violates Lorentz symmetry
  - Can’t be combined to C(k) or B(k)

Lorentz violating

$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3}\frac{q_\mu}{q^2+2q\cdot k+k^2-m_e^2}n(E_q)\delta(q^2-m_{DM})$$

$$B(k) = \;\; 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3}\frac{m_e}{q^2+2q\cdot k+k^2-m_e^2}n(E_q)\delta(q^2-m_{DM})$$

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Do no specify $n(E_q)$ until end

Gauge mixing parameter

What happens to loop processes?

- ❑ Start with wave function renormalization

$$\Sigma_n = B(k) + C(k)\,(\gamma \cdot k - m_e) + \gamma \cdot D(k)$$

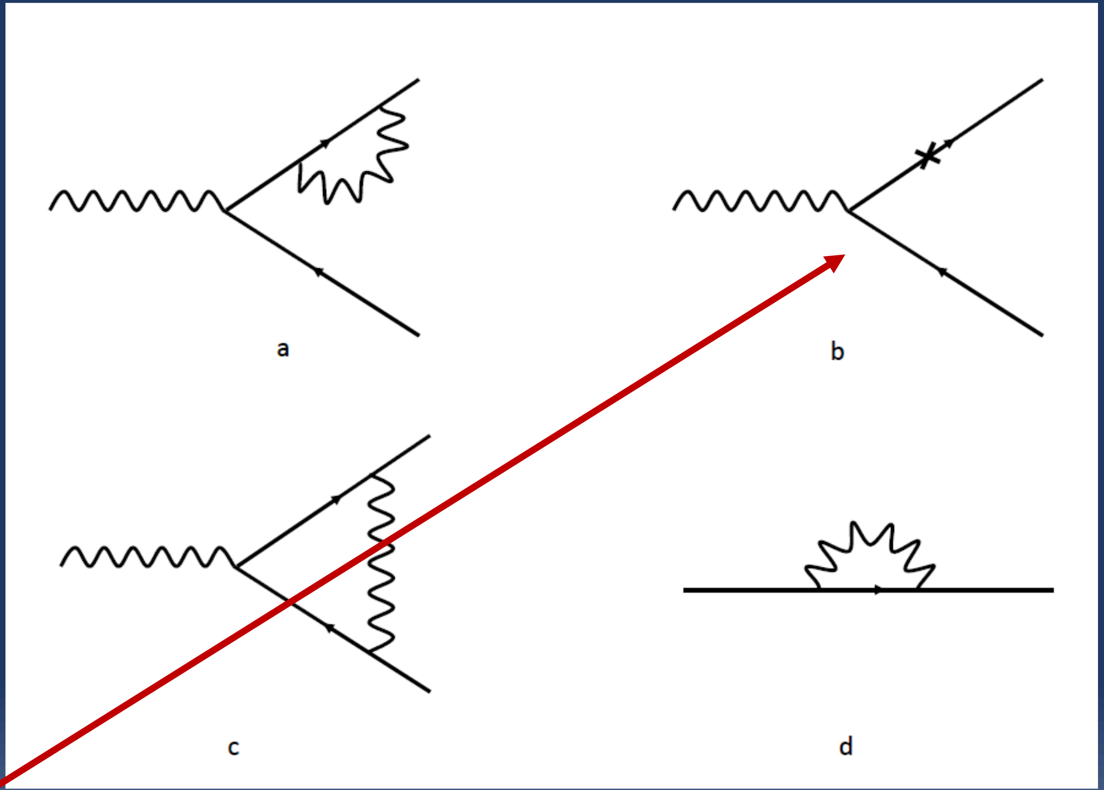
- ❑ New type of contribution
  - ❑ Violates Lorentz symmetry
  - ❑ Can't be combined to C(k) or B(k)

- ❑ Use a background dependent spinor

$$\left[\gamma \cdot k - m - \frac{\alpha}{4\pi^2}\left(\gamma \cdot D(k) + B(k)\right)\right]\psi_n = 0$$

  - ❑ So that we have "mass counterterms"

$$\text{“}\delta m\text{”} = \frac{\alpha}{4\pi^2}\left(\gamma \cdot D(k) + B(k)\right)$$



Not a real counterterm, just use it like a counterterm.
No new divergences

What happens to loop processes?

- Self energy plus counterterm contribution

$$iM_{SE+CT} = ie\bar{u}(\bar{k})\gamma_\mu\left[C(k) + \frac{1}{E_k}\left[m_e \frac{\partial B(k)}{\partial E_k}\Big|_{k^2=m_e^2} + \frac{\partial k_\mu D^\mu(k)}{\partial E_k}\Big|_{k^2=m_e^2} - D^0(k)\right]\right.$$
$$\left. + C(\bar{k}) + \frac{1}{E_{\bar{k}}}\left[m_e \frac{\partial B(\bar{k})}{\partial E_{\bar{k}}}\Big|_{\bar{k}^2=m_e^2} + \frac{\partial \bar{k}_\mu D^\mu(k)}{\partial E_{\bar{k}}}\Big|_{\bar{k}^2=m_e^2} - D^0(\bar{k})\right]\right] u(k)$$

- Derivatives arise because of definition

$$D^\mu(k) = -2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{q_\mu}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

$$B(k) = 2e^2\chi^2 \int \frac{d^4q}{(2\pi)^3} \frac{m_e}{q^2 + 2q\cdot k + k^2 - m_e^2} n(E_q)\delta(q^2 - m_{DM})$$

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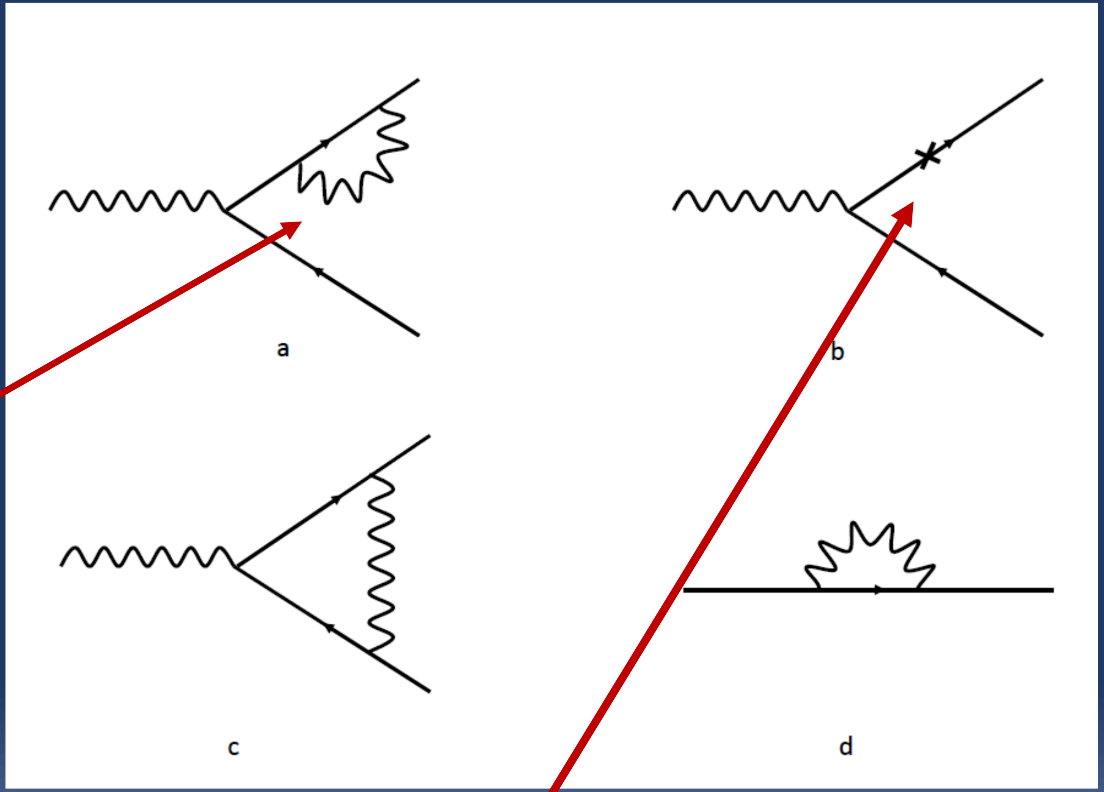


Mass counterterms on shell,
Since part of EOM

Wave Function Renormalization in a Background

- Thermally corrected propagator

$\tilde{k}$ $\tilde{m}$

$$S^{-1}(k) = \gamma\cdot k - m_R + \Sigma_n = [1+C(k)]\gamma\cdot[k + D(k)] - [1+C(k)][m_R - B(k)]$$

$$\Sigma_n = B(k) + C(k)(\gamma\cdot k - m_R) + \gamma\cdot D(k)$$

- Renormalized propagator
  - Perform k_0 integral

Wave Function Renormalization

$$i\int \frac{d^4k}{(2\pi)^4}\frac{Z_2^{-1}(\gamma\cdot\tilde{k}+\tilde{m})e^{-ik\cdot(x-y)}}{\tilde{k}^2-\tilde{m}^2+i\epsilon}$$

Residue changed if $B(k),\ k_\mu D^\mu(k)$ depend on k

- Compare to calculation of $\langle\psi\bar{\psi}\rangle$

Background dependent spinors

$$S^R(x-y) = \int\frac{d^4k}{(2\pi)^3}\left[\theta(x_0-y_0)\frac{\tilde{k}+\tilde{m}}{2\tilde{E}}e^{-ik\cdot(x-y)} - \theta(y_0-x_0)\frac{\tilde{k}-\tilde{m}}{2\tilde{E}}e^{ik\cdot(x-y)}\right]$$

$$\sum_{\text{spin}} u_n(k)\bar{u}_n(k) = \frac{\gamma\cdot\tilde{k}+\tilde{m}}{2\tilde{E}}$$

- Wave function renormalization changed by background

Standard Wave Function Renormalization

$$Z_2^{-1} = 1 + C(k) + \frac{1}{E}\frac{d}{dE}\left(mB(k) + k_\mu D^\mu(k)\right) - \frac{D^0(k)}{E}$$

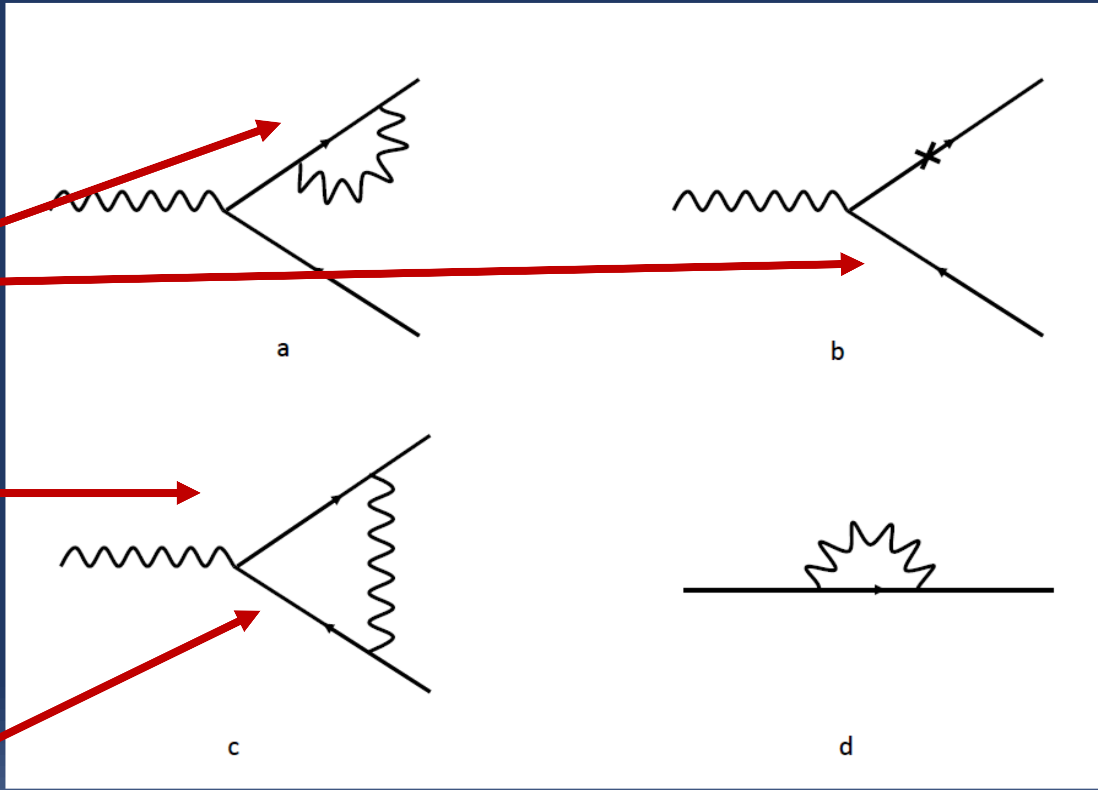
$E \to \tilde{E}$

What happens to loop processes?

- Total Vertex correction

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\left[\gamma_\mu\left[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e} + \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\right] u_n(k)$$

$$F_\mu(\Delta k) = -e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{2(\gamma \cdot q + m_e)[\gamma \cdot \Delta k, \gamma_\mu] - 4\gamma_\mu [\Delta k_\alpha q^\alpha]}{(q^2 + 2q_\nu k^\nu)^2},$$



Charge Non-Renormalization

- Charge non-renormalization
  - Apply $\Delta k = 0, \; \mu = 0$ to vertex

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\Big[\gamma_\mu\Big[1$$

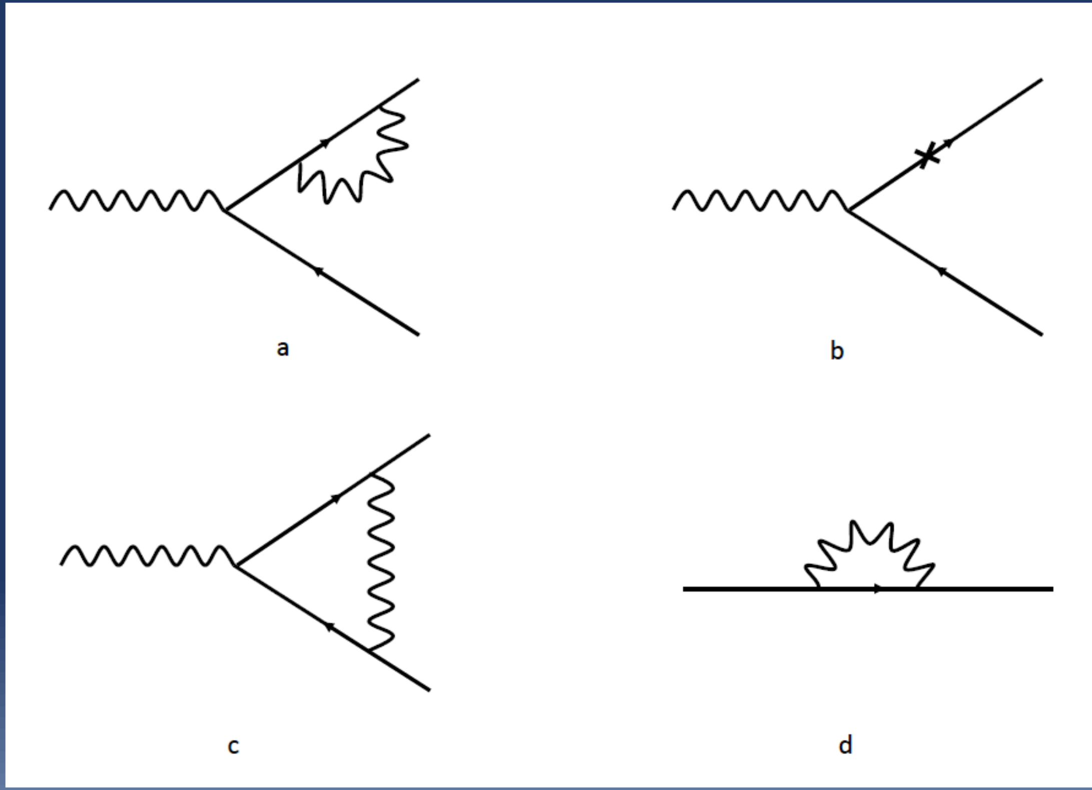
$$-\frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\Big]$$

$$+\left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e}\right.$$

$$\left. + \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\Big] u_n(k)$$

$\bar{k} \to k$ (applied to the second and third lines)

$\to 0$ (for the second through fourth lines)



Charge Non-Renormalization

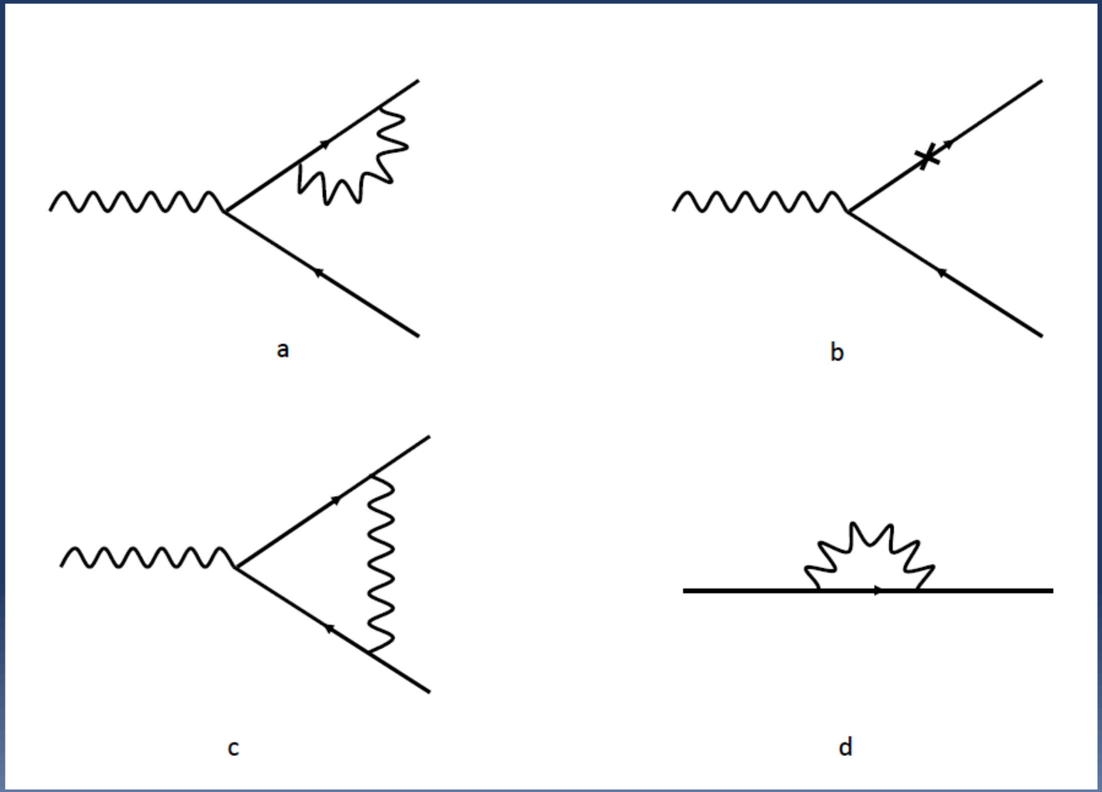
- Charge non-renormalization
  - Apply $\Delta k = 0, \;\; \mu = 0$ to vertex

$$i\, M_{TOT_0}|_{\Delta k=0} = -ie\bar{u}_n(\bar{k}) \Big[\gamma_0 \Big[1$$

$$+ \left[-\frac{m_e}{E}\gamma_0 + 1\right] \left[\frac{d}{dE}\left(B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right) + \frac{D^0(k)}{E}\right]\Big]\Big] u_n(k)$$

Cancels Due to Gordon Decomposition for $\Delta k = 0$

$$i\, M_{TOT_0}|_{\Delta k=0} = -ie$$



Ward Identities

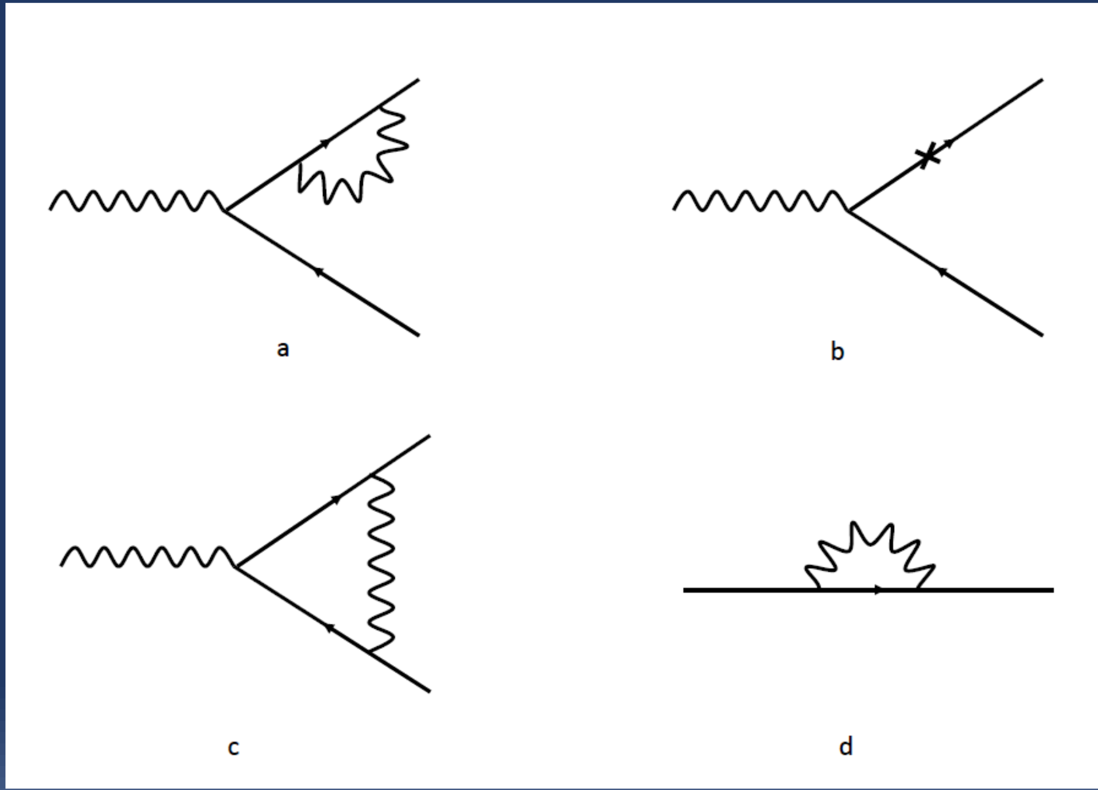
- Ward Identities apply Δk^{μ} to M_{TOT_μ}
  - Use slightly different form of M_{TOT_μ}

$$\Delta k = \bar{k} - k$$

$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\left[\gamma_\mu\left[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e} + \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\right] u_n(k)$$

Come From Gordon Decomposing

$\rightarrow \gamma \cdot \frac{dD}{dk^{\mu}}$



Ward Identities

- Ward Identities apply Δk^{μ} to $M_{TOT_{\mu}}$
  - Use slightly different form of $M_{TOT_{\mu}}$

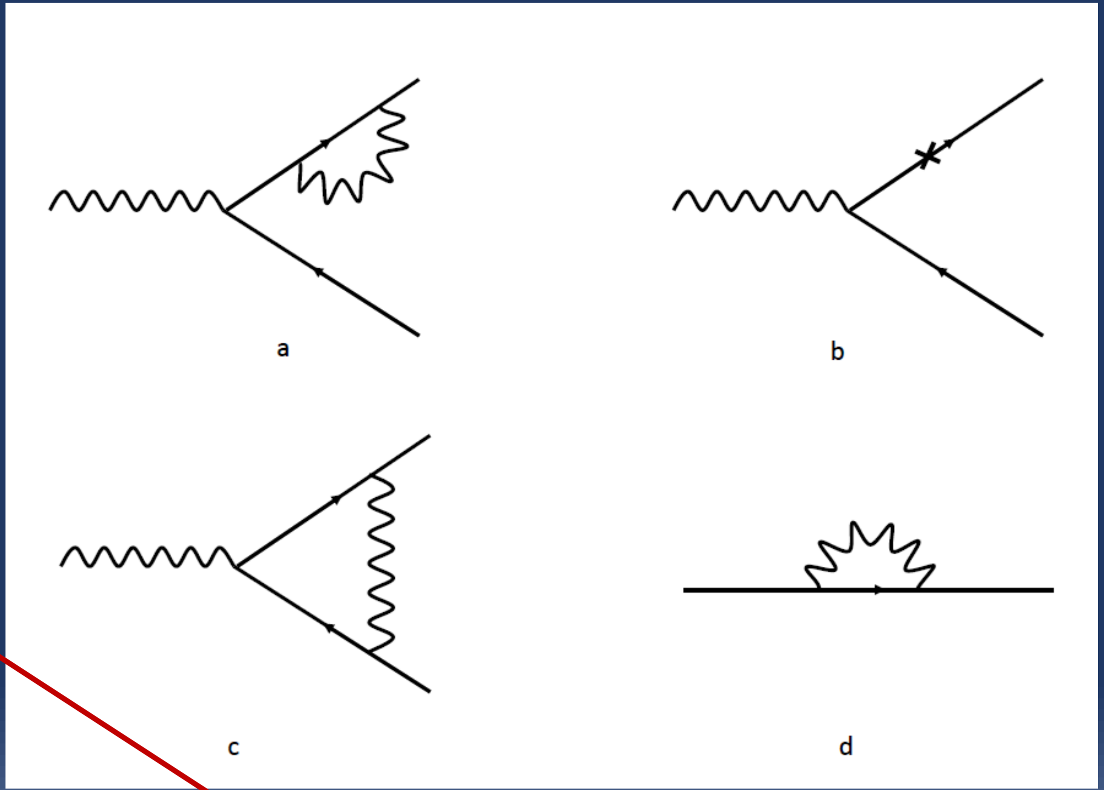
$$= B(k) - B(\bar{k}) + \gamma \cdot D(k) - \gamma \cdot D(\bar{k})$$

$$i\Delta k^{\mu} M_{TOT\mu} = -ie\bar{u}_n(\bar{k}) \left[\gamma \cdot \Delta k \left[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right]\right.$$

$$\Delta k^{\mu}\left[\frac{1}{2}\gamma \cdot \left[\frac{d}{dk^{\mu}} D(k) + \frac{d}{d\bar{k}^{\mu}} D(\bar{k})\right]\right.$$

$$\left.\left. + \frac{d}{dk^{\mu}} B(k) + \frac{d}{d\bar{k}^{\mu}} B(\bar{k})\right] + \Delta k^{\mu} F_{\mu}(\Delta k)\right] u_n(k)$$

$$= 0$$



This row of order α^2

Ward Identities

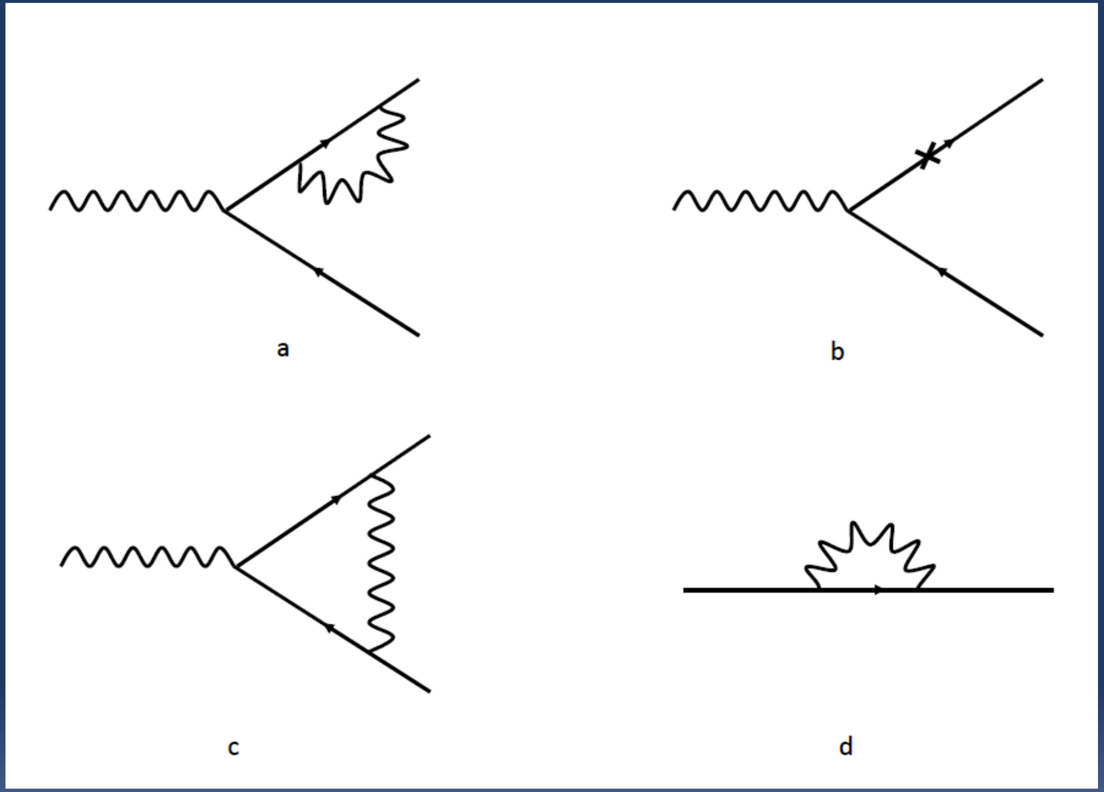
- Ward Identities apply Δk^{μ} to $M_{TOT_{\mu}}$
  - Use slightly different form of $M_{TOT_{\mu}}$
    - Generically true to order Δk^3

$$\Delta k = \bar{k} - k$$

$$\Delta k^{\mu} M_{TOT\mu} = -e\bar{u}_n(\bar{k}) \times \left[B(k) - B(\bar{k}) + \Delta k^{\mu} \left[\frac{dB(k)}{dk_{\mu}} + \frac{dB(\bar{k})}{d\bar{k}_{\mu}} \right] + \gamma_{\nu} \left[D^{\nu}(k) - D^{\nu}(\bar{k}) + \Delta k^{\mu} \left[\frac{dD^{\nu}(k)}{dk_{\mu}} + \frac{dD^{\nu}(\bar{k})}{d\bar{k}_{\mu}} \right] \right] \right] u_n(k)$$

Δk^3

Δk^3



Background Contribution to g-2

- Simplified Total Vertex correction

$$iM_{TOT_\mu} = -ie\bar{u}(\bar{k})\left[\gamma_\mu\left[1+\frac{1}{2}\left[\frac{D^0(k)}{E_k}+\frac{D^0(\bar{k})}{E_{\bar{k}}}\right.\right.\right.$$
$$\left.-R\frac{m_e}{E_k}\bar{I}_0(k)-R\frac{m_e}{E_{\bar{k}}}\bar{I}_0(\bar{k})-\frac{R}{2m_e}\Delta k_\nu\bar{I}^\nu(k)\right]$$
$$-\frac{D_\mu(k)}{2m_e}-\frac{D_\mu(\bar{k})}{2m_e}+R\left[\bar{I}_\mu(k)+\bar{I}_\mu(\bar{k})\right]$$
$$+\left[I_\mu^\nu - R\frac{k_\mu+\bar{k}_\mu}{m_e}\bar{I}^\nu(k)+2R\gamma\cdot\bar{I}(k)\delta_\mu^\nu\right.$$
$$\left.\left.+2m_e R\delta_\mu^\nu I_A(k)\right]\times\frac{[\gamma_\alpha,\gamma_\nu]_-\Delta k^\alpha}{4m_e}\right]u(k),$$

$$R=\left(\frac{m_{DM}}{m_e}\right)^2 \qquad g-2$$

$$I_A(k)=e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{2m_e}{(2q\cdot k)^2}$$
$$\sim\int dq\frac{N(E_q)}{q} \qquad N(E_q)|_{\mathrm{IR}}\sim\frac{1}{q}$$

$$\bar{I}_\mu(k)=e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{4q_\mu m_e^3}{(2q\cdot k)^3}$$
$$\sim\int dq\frac{N(E_q)}{q}$$

$$I_{\mu,\nu}=e^2\chi^2\int\frac{d^4\Pi_q}{(2\pi)^3}\frac{q_\mu q_\nu}{(q\cdot k)^2}$$
$$\sim\int dqqN(E_q)$$

$$d^4\Pi_q=d^4q\;\bar{n}(E_q)\delta(q^2-m_{DM}^2)$$

Relativistic Hamiltonian

- Magnetic Field (Approximate Penning trap)

$$A^0 = 0 \qquad \vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$$

- Momentum Integrals ($\frac{|q|^2}{E_q^2} \ll 1$)

Other components suppressed by β_{DM}^2

determines size of effect

$$R I_A(k) = \frac{\delta m_n}{2m_e^2}\left(\frac{m_e}{E_k}\right)^2 \qquad R\bar{I}_0(k) = \frac{\delta m_n}{2m_e}\left(\frac{m_e}{E_k}\right)^3 \qquad \delta m_n = \frac{e^2\chi^2}{(2\pi)^3}\frac{1}{m_e m_{DM}}\int d^3q\bar{n}(E_q)$$

- Relativistic Hamiltonian
  - Must use corrections to frequencies and compare to experiment

$$H_T' = E_\beta - \frac{e}{2E_\beta}\left[\vec{L}\cdot B + \vec{\sigma}\cdot B\right]\left[1 - 2R\frac{m_e}{E_k}\bar{I}^0(k)\right] + \frac{eR}{2E_p}\left[\frac{|k|^2}{m_e^2}\bar{I}^0(k) - 2\bar{I}^0(k) - 2I_A(k)E_p\right]\vec{\sigma}\cdot\vec{B},$$

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$$R I_A(k) = \frac{\delta m_n}{2m_e^2}\left(\frac{m_e}{E_k}\right)^2 \qquad R \bar{I}_0(k) = \frac{\delta m_n}{2m_e}\left(\frac{m_e}{E_k}\right)^3 \qquad \delta m_n = \frac{e^2\chi^2}{(2\pi)^3}\frac{1}{m_e m_{DM}}\int d^3q\bar{n}(E_q)$$

- Relativistic Hamiltonian
  - Must use corrections to frequencies and compare to experiment

$$H'_T = E_\beta - \frac{e}{2E_\beta}\left[\vec{L}\cdot B + \vec{\sigma}\cdot B\right]\left[1 - 2R\frac{m_e}{E_k}\bar{I}^0(k)\right] + \frac{eR}{2E_p}\left[\frac{|k|^2}{m_e^2}\bar{I}^0(k) - 2\bar{I}^0(k) - 2I_A(k)E_p\right]\vec{\sigma}\cdot\vec{B},$$

Spin frequency corrections

- Cyclotron Frequency also corrected
- Affects $g-2$ measurement

Experimental Constraints

- Predicted spin and cyclotron frequencies

$$\omega_c = \frac{e|B|}{2E_\beta}\left[1 - \frac{2Rm_e}{E_k}\bar{I}^0(k)\right]$$

$$\omega_{s\perp} = \omega_c\left[1 + \frac{\alpha}{2\pi}\frac{E_k}{m_e} + R\left(\left(2 - \frac{|k|^2}{m_e^2}\right)\bar{I}^0(k) + 2I_A(k)E_k\right)\right]$$

$\sim \delta m_n$

SM $\mathcal{O}(\alpha)$ prediction

- Measured quantity ratio

$$R_f = \frac{\omega_a}{\omega_c} = \frac{\omega_{s\perp} - \omega_c}{\omega_c} \simeq R_{f_0}\left[1 + \frac{\delta\omega_a}{\omega_{a_0}} - \frac{\delta\omega_c}{\omega_{c_0}}\right]$$

SM prediction

Background corrections small

Because $\omega_a \simeq \frac{\alpha}{2\pi}\frac{E_k}{m_e}\omega_c$ this dominates

Experimental Constraints

- Number density of dark matter

$$n_{DM} = \frac{1}{3}\frac{\rho_{DM}}{m_{DM}}$$

Three polarizations

- Occupation number is density per $(2\pi)^3$

$$\bar{n} = \frac{1}{3}\frac{n_{DM}}{\frac{4\pi q^2 \Delta q}{(2\pi)^3}}$$

Assumes DM velocity spread small

- Integrate occupation number

Enhanced by mass squared

$$\int d^3q\,\bar{n}(E_q) \simeq \int d^3q \frac{\rho_{DM}}{m_{DM}}\frac{(2\pi)^3}{12\pi q^2 \Delta q} \simeq \frac{(2\pi)^3 \rho_{DM}}{3 m_{DM}} \Rightarrow \delta m_n = \frac{4\pi}{3}\alpha\chi^2 \frac{\rho_{DM}}{m_e m_{DM}^2}$$

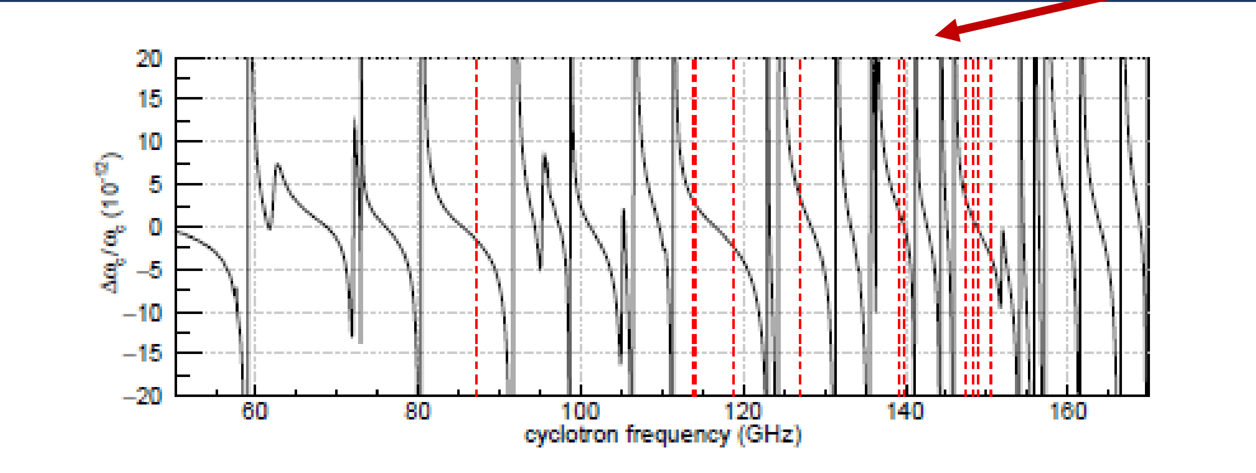
- The variation in R_f is then

$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta\omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2 \frac{\rho_{DM}}{m_{DM}^2 E_k^2}$$

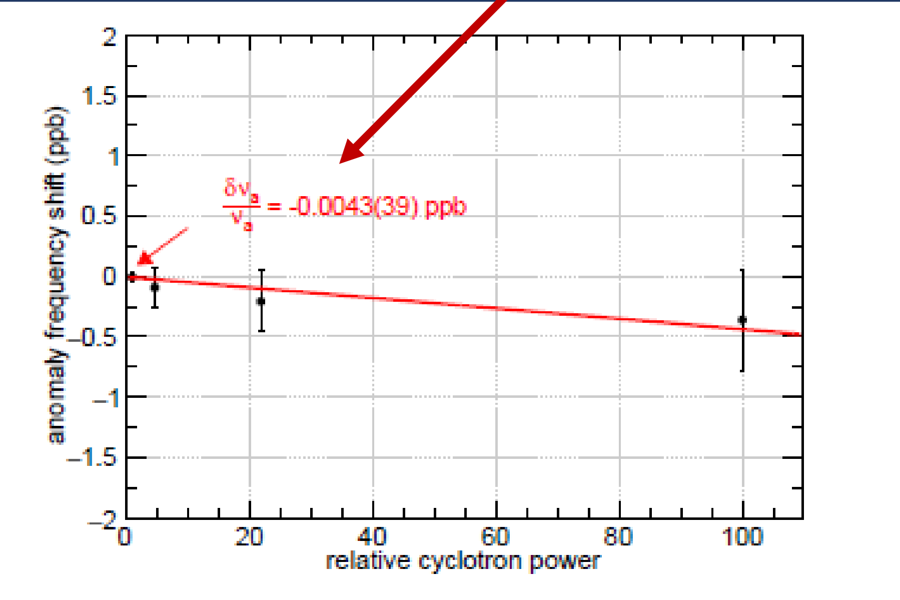
Experimental Constraints

- Experimental uncertainties

$$\frac{\Delta\omega_c}{\omega_c} \simeq \pm 2 \times 10^{-11}$$



$$\frac{\Delta\omega_a}{\omega_a} \simeq \pm 4 \times 10^{-12}$$



Fan, Xing. 2022. An Improved Measurement of the Electron Magnetic Moment. Doctoral dissertation, Harvard University Graduate School of Arts and Sciences.

Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f

Correction to Theoretical prediction of ratio

$$\frac{\delta R_f}{R_{f0}} \simeq \frac{\delta\omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2 \frac{\rho_{DM}}{m_{DM}^2 E_k^2}$$

$$\frac{\Delta R_f}{R_{f0}} \simeq -\frac{\Delta\omega_c}{\omega_{c0}} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4\times 10^{-11}$$

Dominant measurement error on ratio

Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f
  - Theory<Experiment (Measured g-2 very consistent with SM)

$$\frac{(2\pi)^2}{3}\chi^2\frac{\rho_{DM}}{m_{DM}^2 E_k^2} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4 \times 10^{-11}$$

Being Conservative

- Gives a constraint on χ for a given m_{DM}

$$\chi < 7.1 \times 10^3 \frac{m_{DM}}{eV}\left(\frac{\Omega_A}{\Omega_{cdm}}\right)^{1/2}$$

Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f

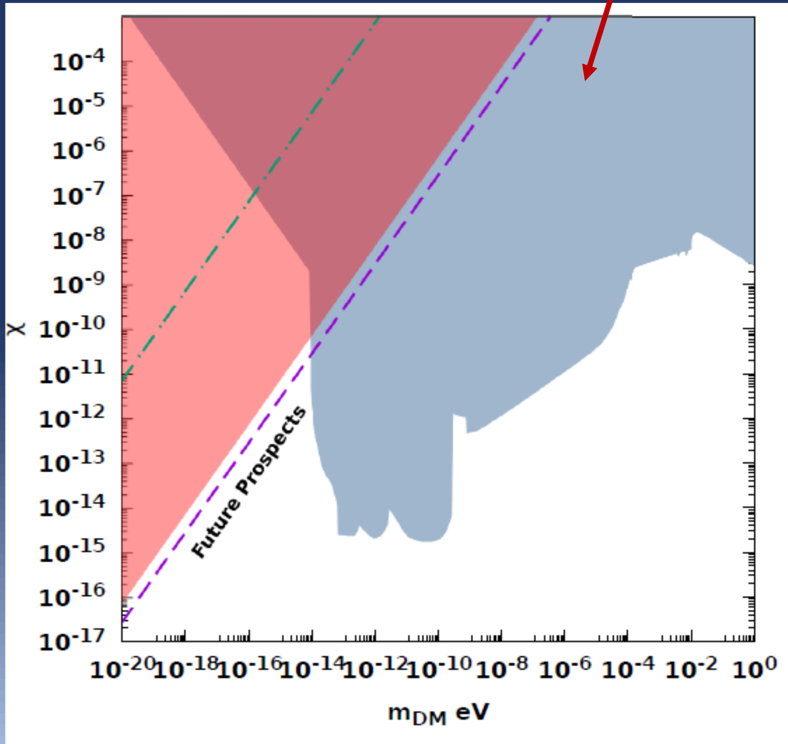
$$\frac{\Delta R_f}{R_{f_0}} \simeq \frac{\delta\omega_a}{\omega_{a_0}} \simeq \frac{(2\pi)^2}{3}\chi^2\frac{\rho_{DM}}{m_{DM}^2 E_k^2} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4\times 10^{-11}$$

Being Conservative

- Gives a constraint on χ for a given m_{DM}

- Very strong compared to previous constraints

Previous constraints from: Caputo, Millar, O'Hare and Vitagliano



Experimental Constraints

- Experimental uncertainties
- The experimental constraints on R_f

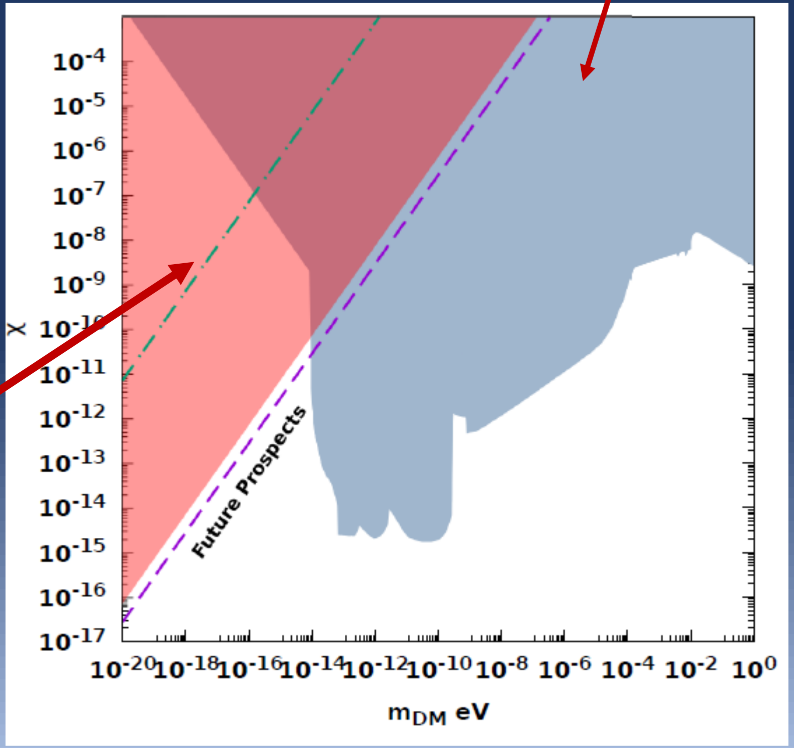
$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta\omega_a}{\omega_{a0}} \simeq \frac{(2\pi)^2}{3}\chi^2\frac{\rho_{DM}}{m_{DM}^2 E_k^2} < 2\frac{\Delta\omega_c}{\omega_c} \simeq 4\times 10^{-11}$$

Being Conservative

Previous constraints from: Caputo, Millar, O'Hare and Vitagliano

- Gives a constraint on χ for a given m_{DM}
- Very strong compared to previous constraints
- Constraints scale as ½ power of DM density

$$\frac{\Omega_A}{\Omega_{cdm}} \sim 10^{-10}$$



ALP Background Dark Matter

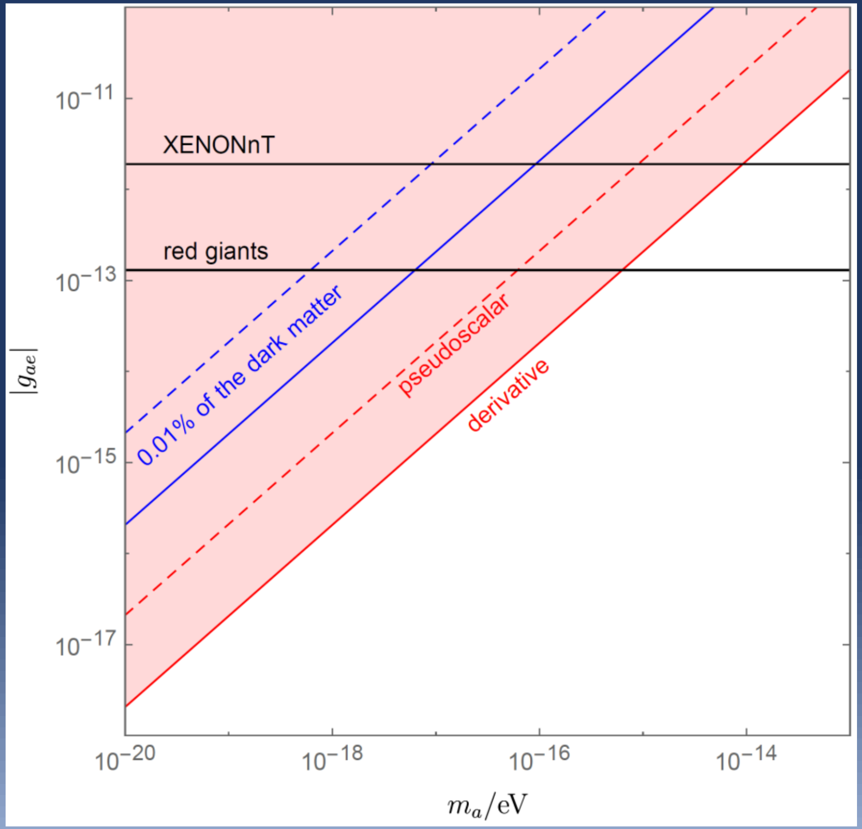
- ALP background is another motivated light background dark matter
  - Also contributes to the anomalous magnetic moment

$$\mathcal{L} \supset g_{ae} a \bar{\psi} \gamma^{\mu} \gamma_5 \psi + g_{ae} \frac{\partial_{\mu} a}{2f} \bar{\psi} \gamma^{\mu} \gamma_5 \psi$$

- Experimental constraints on its contribution

$$\frac{\Delta R_f}{R_{f0}} \simeq \frac{\delta \omega_a}{\omega_{a0}} \simeq \frac{1}{2} \frac{(2\pi)}{\alpha} \left(\frac{g_{ae} m_e}{2f} \right)^2 \frac{\vec{k}^2}{E_k m_e} \frac{\rho_{DM}}{m_{DM}^2 m_e^2} < 0.7 \times 10^{-12}$$

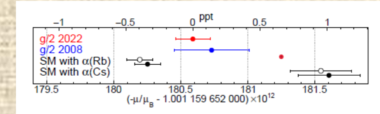
- For light ALP constraint quite strong
  - Other coupling constraints slightly weaker



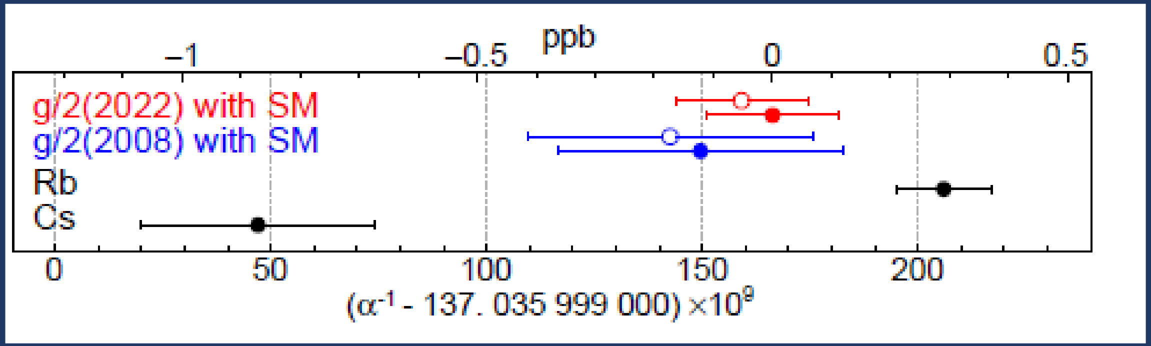
Conclusions

- Fundamental properties of dark matter can lead to constraints
  - Pauli exclusion principle prevents ultralight fermionic dark matter
  - Compton wavelength prevents super-ultralight dark matter bosons
- Production of ultralight dark matter
  - Thermal production problematic
  - Coherent condense production needed
- Ultralight dark matter is very dense throughout the universe
  - Occupation number becomes very large
  - Number of paths for processes shoots way up (Bose Enhancement)
- Bose enhancements work on loop processes
  - Anomalous magnetic contribution greatly enhanced
  - Strong constrains gauge mixing parameter for dark photon
  - Coupling of ALP to electron strongly constrained as well

Experimental Constraints



- ❑ Previous Measurements give us an average value for α



- ❑ Weighted average very close to 2022 measurement (~164)
  - ❑ Also close to theory prediction

- ❑ Allows us to call any deviation larger than experimental error a measurement

Can we treat the dark photon as a particle?

- ❑ Dark matter condensate has very long period

$$T \sim \frac{2\pi}{m_{DM}} \simeq 18 \text{ hrs} \left(\frac{10^{-20} \text{ eV}}{m_{DM}}\right)$$

- ❑ Decoherence time of condensate
  - ❑ Virilization from gravity on large object

$$\tau_{\text{dec}} \sim \frac{\Delta\beta^2}{m_{DM}}$$

- ❑ Can experiments resolve this as a particle?
  - ❑ Heisenberg uncertainty principle

$$\Delta E \Delta t > \frac{1}{2} \quad \Rightarrow \quad \Delta t > \frac{1}{2m_{DM}}$$

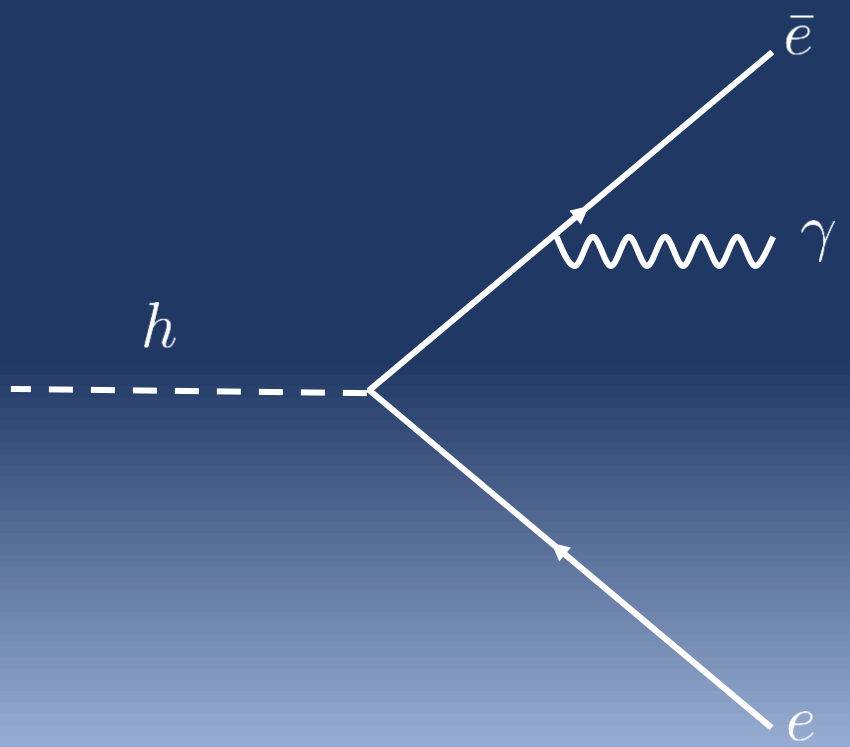
Time to resolve energy m_{DM}

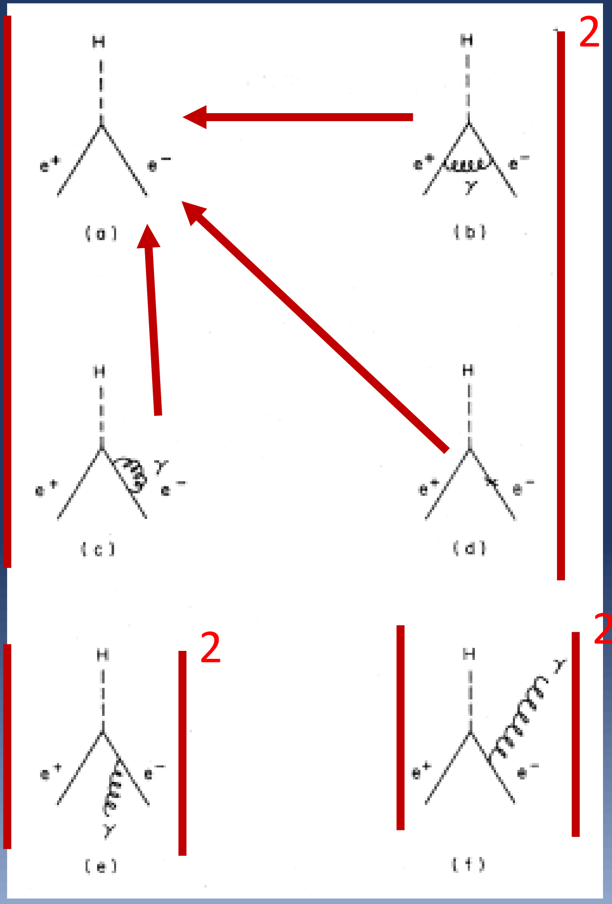
run #	time	magnetic field (T)	cyclotron frequency (GHz)
1-1	2021-12-19-14:45 – 2021-12-20-13:46	5.373	150.411
1-2	2021-12-22-12:57 – 2021-12-23-10:37		
1-3	2021-12-26-13:33 – 2021-12-27-15:31		
2-1	2021-12-29-17:43 – 2021-12-30-17:37	5.300	148.361
2-2	2021-12-31-15:15 – 2022-01-01-23:18		
2-3	2022-01-02-16:46 – 2022-01-04-11:43		
2-4	2022-01-05-12:46 – 2022-01-06-10:49		
3-1	2022-01-31-21:47 – 2022-02-02-12:01	5.269	147.498
3-2	2022-02-03-11:02 – 2022-02-04-13:58		
3-3	2022-02-04-16:13 – 2022-02-05-19:17		
3-4	2022-02-06-15:44 – 2022-02-07-16:30		
3-5	2022-02-07-17:56 – 2022-02-08-21:15		
4-1	2022-02-11-18:13 – 2022-02-14-00:14	5.326	149.091
4-2	2022-02-15-19:47 – 2022-02-17-17:15		
4-3	2022-02-19-11:38 – 2022-02-21-09:50		
5-1	2022-04-07-19:37 – 2022-04-08-19:53	4.071	113.956
5-1	2022-04-09-12:24 – 2022-04-10-21:49		
5-1	2022-04-10-21:03 – 2022-04-11-14:04		
6-1	2022-04-12-17:58 – 2022-04-13-15:10	4.245	118.822
6-1	2022-04-13-16:13 – 2022-04-14-14:32		
6-1	2022-04-14-16:58 – 2022-04-15-13:38		
7-1	2022-04-17-19:26 – 2022-04-18-22:13	4.078	114.141
7-2	2022-04-18-22:16 – 2022-04-20-10:29		
8-1	2022-06-26-11:38 – 2022-06-27-14:28	4.969	139.097
8-2	2022-06-27-15:02 – 2022-06-28-13:48		
8-3	2022-06-28-14:59 – 2022-06-29-10:19		
8-4	2022-06-29-11:33 – 2022-06-30-13:38		
9-1	2022-07-01-16:05 – 2022-07-02-10:21	5.001	139.989
9-2	2022-07-02-10:27 – 2022-07-03-11:37		
9-3	2022-07-03-12:08 – 2022-07-04-11:33		
10-1	2022-07-05-09:07 – 2022-07-06-11:10	4.537	127.007
10-2	2022-07-06-12:56 – 2022-07-07-11:57		
10-3	2022-07-07-17:10 – 2022-07-08-14:04		
11-1	2022-07-11-10:59 – 2022-07-12-10:48	3.108	87.010
11-2	2022-07-13-09:45 – 2022-07-14-11:27		
11-3	2022-07-14-11:31 – 2022-07-15-13:02		
11-4	2022-07-15-13:07 – 2022-07-16-18:38		

Table 4.1: Data sets used for the g-factor determination.

What does this mean for Decays?

- ❑ Two body decays
- ❑ Three body decays
    - ❑ Many possibilities!!
    - ❑ Higgs Decay in Thermal Bath





Ward Identities

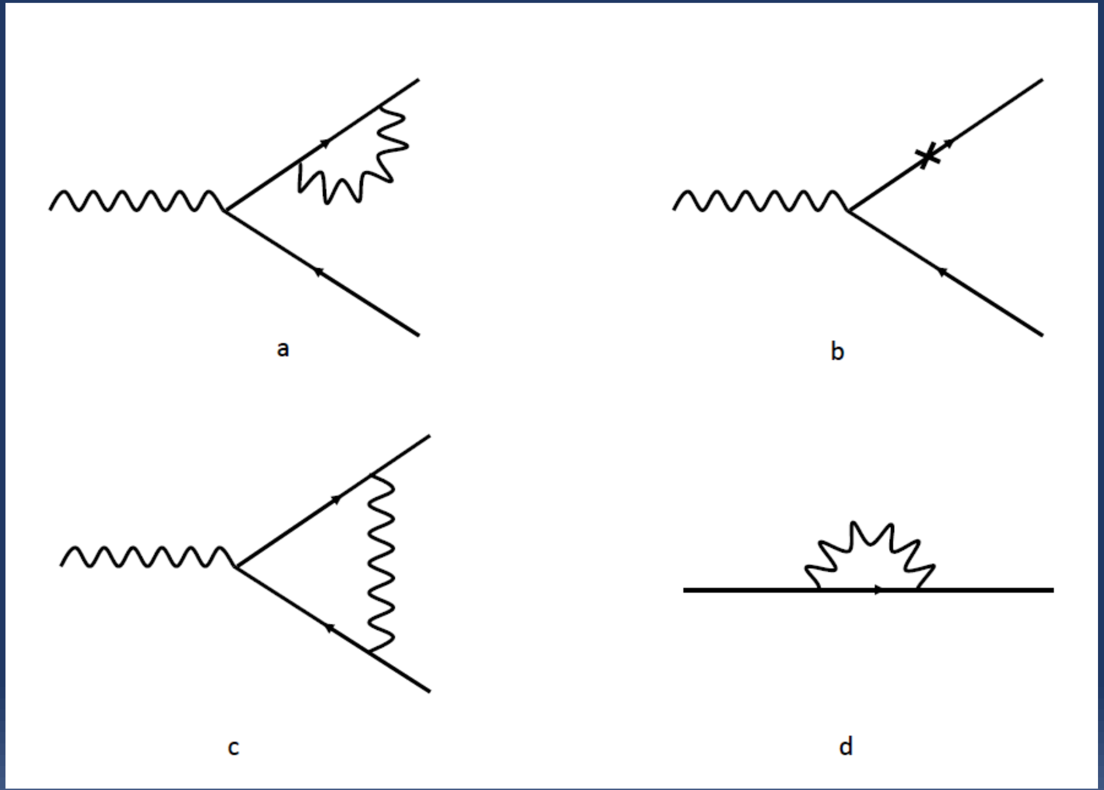
- Ward Identities

$$\Delta k^{\mu} M_{TOT\mu} = -e\bar{u}_n(\bar{k})$$
$$\times \left[B(k) - B(\bar{k}) + \Delta k^{\mu}\left[\frac{dB(k)}{dk_\mu} + \frac{dB(\bar{k})}{d\bar{k}_\mu}\right]\right.$$
$$+\frac{k_\mu + \bar{k}_\mu}{2m_e}\left[D_\mu(k) - D_\mu(\bar{k})\right] - \frac{\Delta k^{\mu}}{2m_e}\left[D_\mu(k) + D_\mu(\bar{k})\right]$$
$$+\left[\left[\frac{dD^{\nu}(k)}{dk_\mu} + \frac{dD^{\nu}(\bar{k})}{d\bar{k}_\mu}\right]\Delta k^{\mu} + D_\nu(k) - D_\nu(\bar{k})\right]$$
$$\left.\times \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{4m_e}\right] u_n(k) \ .$$

Δk^3

m^4_{DM}

Δk^3

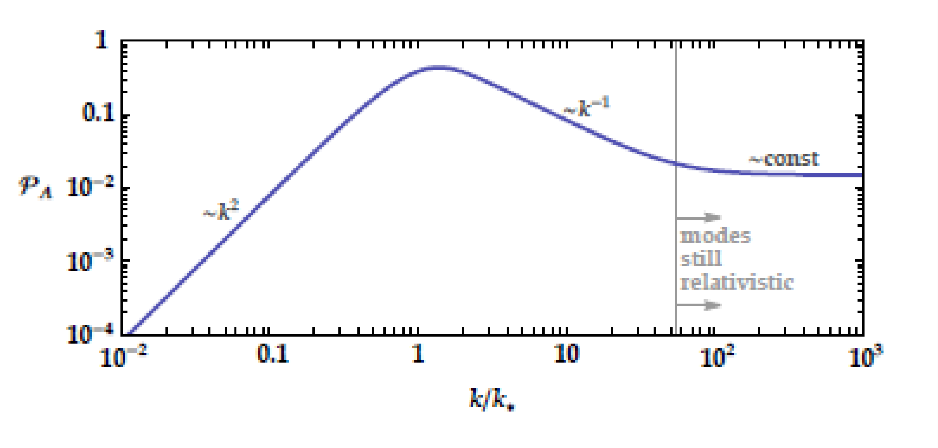


Production of Ultralight Dark Photon Dark Matter

- Thermal production out

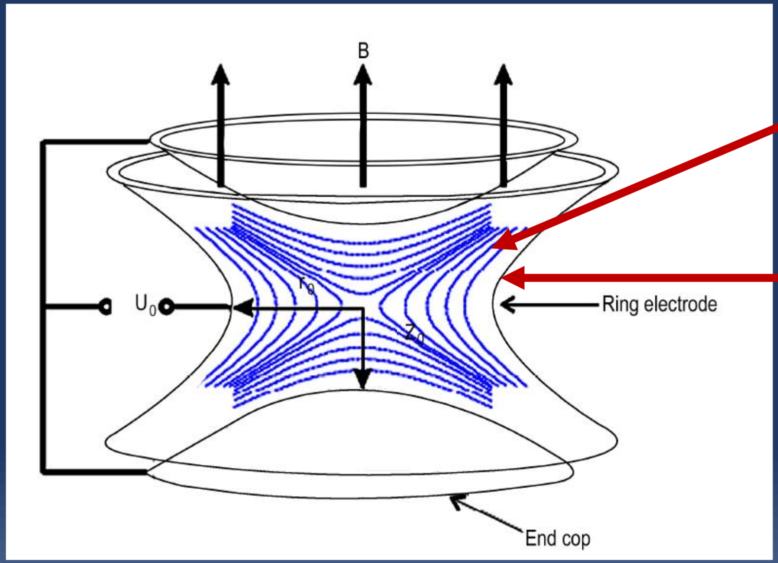
- Production of longitudinal modes from quantum fluctuations
  - longitudinal mode behaves like scalar field
  - Choose the Bunch-Davies vacuum we get
  - Power spectrum suppressed at low momentum

$$P_{A_L} \simeq \left(\frac{k}{m}\right)^2 P_\pi \simeq \left(\frac{kH_I}{2\pi m}\right)^2$$



The Penning Trap

- The Penning Trap
  - Constant magnetic Field/Quadrapole Electric Field



E field just to contain particle

Electron effectively orbits in constant B field

- Clearly not cavity since electric field non-zero inside
  - Thus, fields penetrate trap

Cavity Effects

- If the experiment were in a Cavity, this effect would cancel
  - The cavity would produce an identical background of photons
    - Except opposition spin vector

- This introduces additional enhanced propagators

$$\langle n, n'|A_\mu(x)A'_\mu(y)|n, n'\rangle \qquad \langle n, n'|A_\mu(x)A_\mu(y)|n, n'\rangle$$

$$n(E_k) = \chi n'(E_k)$$

Cavity Generated

- This then leads to a total propagator of

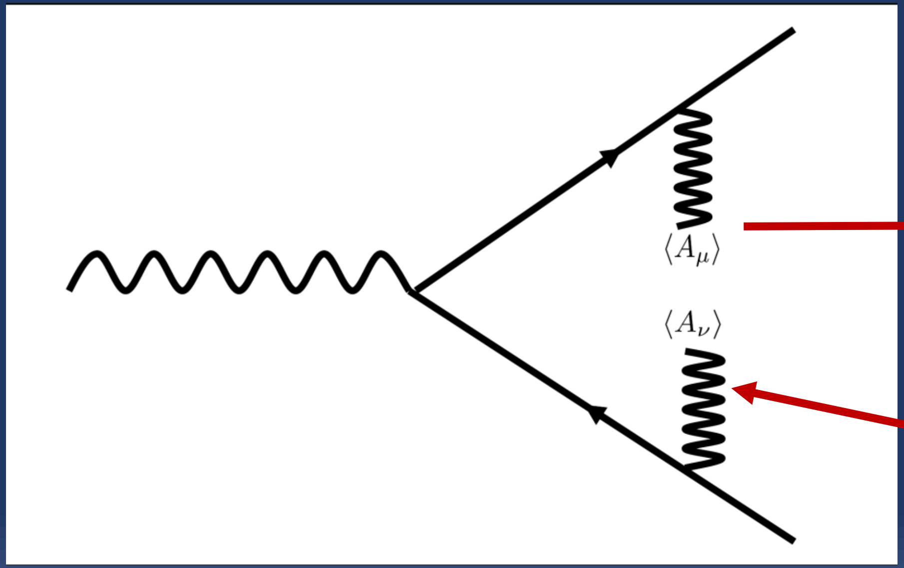
Negative: Spin sum has negative sign

$$[\chi^2\langle n, n'|A'_\mu(x)A'_\mu(y)|n, n'\rangle + 2\chi\langle n, n'|A_\mu(x)A'_\mu(y)|n, n'\rangle + \langle n, n'|A_\mu(x)A_\mu(y)|n, n'\rangle] = 0$$

- Classically this amounts to $A + \chi A' = 0$

Is there an enhancement in the classical limit

- Can do a similar cancelation with a background A'_μ



Because massive, this is not the same

Need opposite correlated momentum

- However, there is a very strong field because so light

$$\rho_{DM} \sim m_{DM}^2 A^\mu A_\mu \qquad \langle A_\mu \rangle \sim \sqrt{\frac{\rho_{DM}}{m_{DM}^2}}$$

Why is inverse scaling of dark matter mass ok?

- The integrands is expanded in k so R_i depends on k only through $n_k(E_k)$

$$\frac{d\Gamma}{dk} = n_B(k)\left[\frac{1}{k}R_{-1} + R_0 + kR_1 + O(k^2)\right]$$

- kR_1 scales as k^0
- $I_A(q), I_\mu(q)$ scale as q^{-2}
  - But $\propto R = q^2/m_e^2$
  - Effective scaling q^0
- $I_{\mu\nu}(q)$ scales as q^0
- Thus for $m_{DM} \to 0$ still well defined

$$I_A(k) = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{2m_e}{(2q\cdot k)^2}$$

$$\sim \int dq \frac{N(E_q)}{q} \qquad N(E_q)|_{\text{IR}} \sim \frac{1}{q}$$

$$\bar{I}_\mu(k) = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{4q_\mu m_e^3}{(2q\cdot k)^3}$$

$$\sim \int dq \frac{N(E_q)}{q}$$

$$I_{\mu,\nu} = e^2\chi^2 \int \frac{d^4\Pi_q}{(2\pi)^3} \frac{q_\mu q_\nu}{(q\cdot k)^2}$$

$$\sim \int dq q N(E_q)$$

$$d^4\Pi_q = d^4q\ \bar{n}(E_q)\delta(q^2 - m_{DM}^2)$$

What about no background?

- Same formulas apply to no background

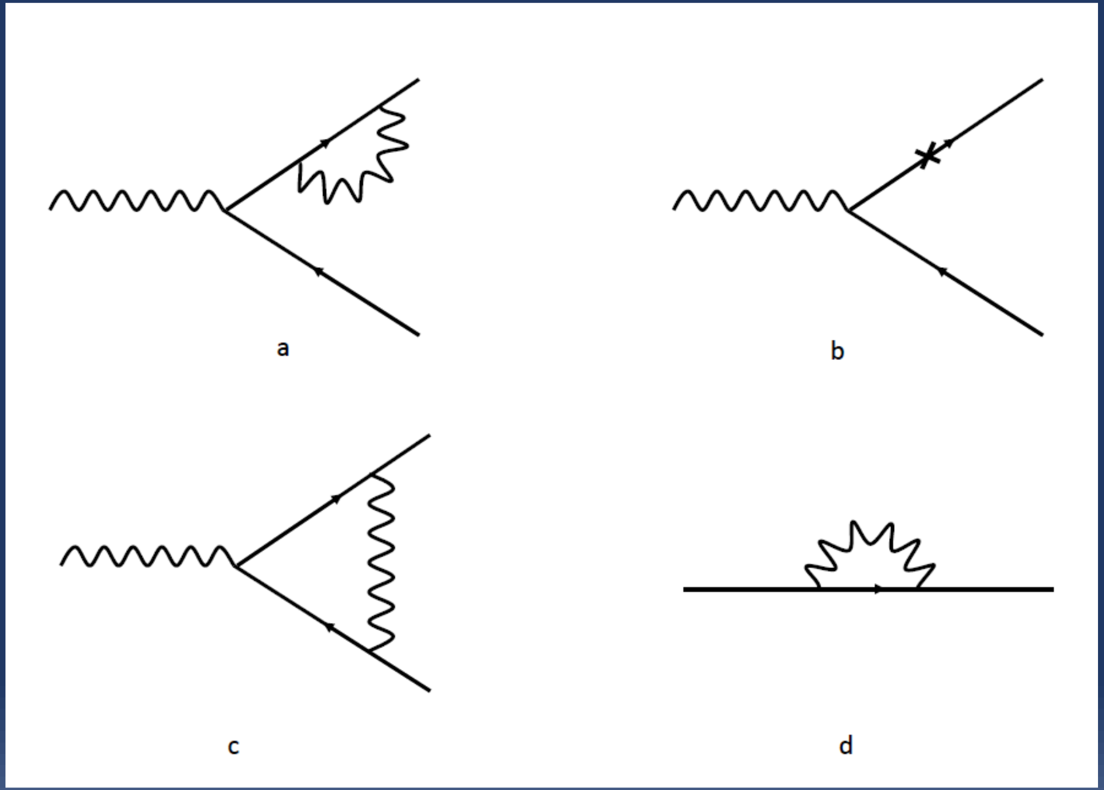
$$iM_{TOT\mu} = -ie\bar{u}_n(\bar{k})\left[\gamma_\mu\left[1 - \frac{1}{2}\frac{1}{E}\frac{d}{dE}\left(m_e B(k) + k_\nu D^\nu(k)\right) + \frac{1}{2}\frac{D^0(k)}{E} + (k \leftrightarrow \bar{k})\right] + \left[\frac{1}{2}\frac{d}{dk_\mu}\left[B(k) + \frac{k_\nu D^\nu(k)}{m_e}\right] - \frac{D^\mu(k)}{2m_e} + \frac{[\gamma_\alpha, \gamma_\nu]_- \Delta k_\alpha}{8m_e}\frac{dD^\nu(k)}{dk_\mu} + (k \leftrightarrow \bar{k})\right] + F_\mu(\Delta k)\right] u_n(k)$$

- $B(k),\ D^\mu(k),\ F(\Delta k)$ Found by

$$2\pi\delta(q^2 - m_a^2) \rightarrow \frac{i}{q^2 - m_a^2}$$

- Applied to pseudoscalar we get

$$a_e = -\left(\frac{m_e}{f_a}\right)^2 \frac{c_{ee}^2}{16\pi^2}\left[1 + 2x + x(1-x)\ln(x) - \frac{2x(x-3)\sqrt{x(x-4)}}{x-4}\ln\left(\frac{1}{2}\left[\sqrt{x} + \sqrt{x-4}\right]\right)\right]$$



Exactly what previous calculations get