$\bar{B} \to D^*(\to D\pi')\tau^-(\to \pi^-\nu_\tau)\nu_{\bar{\tau}}$

Presentation and Talk by Khalid Hassouna

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Significance

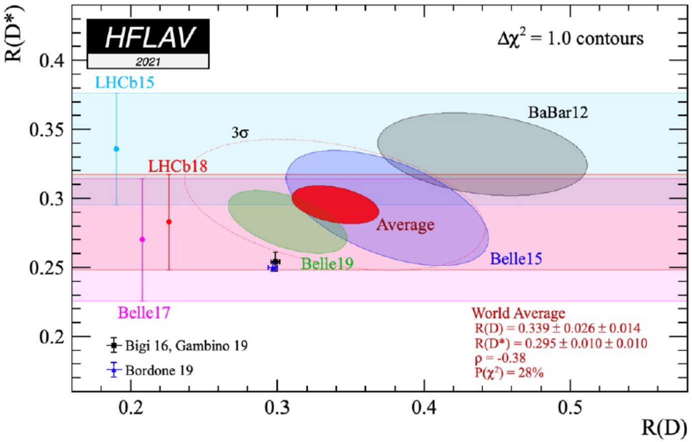
In recent years, there has been a general interest in some observables resulting from B decay. For example, $R_{D^{(*)}} \equiv \mathcal{B}\left(\bar{B} \to D^{(*)}\tau^-\bar{\nu}_\tau\right)/\mathcal{B}\left(\bar{B} \to D^{(*)}\ell^-\bar{\nu}_\ell\right)(\ell = e, \mu)$ and $R_{J/\psi} \equiv \mathcal{B}\left(B_c^+ \to J/\psi\tau^+\nu_\tau\right)/\mathcal{B}\left(B_c^+ \to J/\psi\mu^+\nu_\mu\right)$. Of course, there are reasons for this interest. For instance, these observables can serve as a test for lepton universality which is a property of the standard model that all leptons have the same coupling to all gauge bosons. In the numerator, we have the τ particle and in the denominator, we have a light lepton. Any deviation from 1 in $\frac{g_{\mu,e}}{g_\tau}$ is a violation of this universality predicted by the standard model which in turn suggests the existence of new physics. The Heavy Flavor Averaging Group (HFLAV) found a deviation from the SM of 3.1σ for the former (It is combined with R_D) and 1.7σ for the latter.

SM deviations

Observable	SM Prediction	Measurement
$R_{D^*}^{\tau/\ell}$	0.258 ± 0.005 [12]	$0.295 \pm 0.011 \pm 0.008$ [12]
$R_{D}^{\tau/\ell}$	0.299 ± 0.003 [12]	$0.340 \pm 0.027 \pm 0.013$ [12]
$R_{J/\psi}^{\tau/\mu}$	0.283 ± 0.048 [14]	$0.71 \pm 0.17 \pm 0.18$ [11]
$R_{D^*}^{\mu/e}$	~ 1.0	$1.04 \pm 0.05 \pm 0.01$ [15]

SM deviations (Image taken from `https://arxiv.org/abs/2005.03032`)

SM deviations



Results of measurements of $R(D)$ and $R(D^*)$

Significance

The measured discrepancies suggest a new physics in $b \to c\tau^-\bar{\nu}$.In this decay, many observables were measured; however, most of them are CP conserving. When the angular distribution is measured, CP-violating terms can pop out. CP-violating observables give crystal-clear evidence of new physics as it requires the interference of two amplitudes with a different weak phase which contradicts the structure of the SM that has a single amplitude. CP violation will be constructed from the asymmetry in angular distribution. Also, as will be shown later, even for CP-conserving terms, the angular distribution can yield fits that can determine the parameters of NP.

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General Structure

Consider the general decay channel: $\bar{B} \to D^*(\to D\pi)\ell^-\nu_{\bar{\ell}}$. Under the assumption of having LH neutrinos only, the decay can be parameterized by $\bar{B} \to D^* N^*(\to \ell^-\nu_{\bar{\ell}})$ such that N can be LH scalar, vector or tensor interactions encoding the new physics.

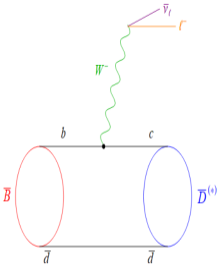
When the lepton ℓ is restricted to μ, e, there is no new physics and N=W (vector boson). This is not the case for $\ell = \tau$. For τ, all the couplings are allowed.

The τ particle is a short-lived particle. Thus, it can't be detected directly. It has to be detected through its decay particles for which the simplest hadronic decay is considered here ($\tau \to \pi\nu_\tau$).

General structure

Most measurements were done on the τ decay channel:
$\tau^- \to \ell^- \nu_\ell \nu_\tau$

The issue with this is that now we have a 3-body problem with two neutrinos. On the other hand, $\tau \to \pi \nu_\tau$ is a 2-body decay with just one missing neutrino and an easily detected hadron. The kinematics of the neutrino can be theoretically constrained from the hadronic side of the decay.



Decay Parameterization

The decay can be parameterized by 8 parameters: 5 helicity angles (having 5 final-state particles) and the 3 invariant squares of masses of the intermediate particles. These parameters are optimized.

Minimization: The following invariant mass squares are removed for their particles being on shell:

- m_{D^*}
- m_{τ^-}

Decay Parameterization

The left 6 parameters are

(i) q^2 : Invariant mass square of $\tau\bar{\nu_\tau}$ in N^* rest frame.

(ii) θ^* : Polar angle of the D-meson three-momentum in the rest frame of its parent (D^*).

(iii) θ_τ : Polar angle of the τ three-momentum in the N^* rest frame.

(iv) χ_τ : Azimuthal angle of the τ three-momentum in the N^* rest frame.

(v) θ : Polar angle of the π three-momentum in its parent τ rest frame.

(vi) χ : Azimuthal angle of the π three-momentum in its parent τ rest frame.

Problem: τ lepton is not observed directly and hence all the angles measured in the τ rest frame or defining its three-momentum are of no practical use (iii - vi). A reparametrization is needed.

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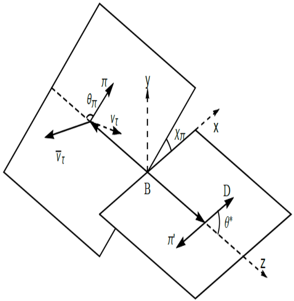
New parameters

We have 4 unmeasured angles and hence we need 4 parameters to replace them. First, we work in N^* frame as this frame is easily determined for the other side of the decay being purely hadronic). Three of the new parameters are E_π, θ_π, and χ_π. The fourth one is easily integrated over as will be shown later.



Mathematical transformation

Fermi Golden Rule for Decay:

$$d\Gamma = |\mathcal{M}|^2 \prod_{j=1}^{n_i} \left(\frac{1}{2E_j}\right) \underbrace{(2\pi)^4 \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 (2E_i)} \delta\left[\vec{P} - \sum_{i=1}^{n} \vec{p}_i\right] \delta\left[E - \sum_{i=1}^{n} E_i\right]}_{d^4 I}$$

Calculate the phase factor $(d^4 I)$ for $N^* \to \tau \bar{\nu_\tau}$ such that (ϕ_{N^*}) and $\tau \to \pi \nu_\tau$ (ϕ_τ) by substituting in the corresponding part of Fermi Golden rule.

Mathematical transformation

$$d^4 I = \underbrace{(2\pi)^4 \int \frac{d^3 p_\tau d^3 p_{\bar{\nu}_\tau}}{(2E_\tau)(2E_{\bar{\nu}_\tau})(2\pi)^6} \delta^4 \left(q - p_\tau - p_{\bar{\nu}_\tau}\right)}_{d\phi^*_N(p_\tau, p_{\bar{\nu_\tau}})}$$

$$\times \underbrace{(2\pi)^4 \int \frac{d^3 p_\pi d^3 p_{\nu_\tau}}{(2E_\pi)(2E_{\nu_\tau})(2\pi)^6} \delta^4 \left(p_\tau - p_\pi - p_{\nu_\tau}\right)}_{d\phi_\tau(p_\pi, p_{\nu_\tau})} =$$

$$\frac{1}{(4\pi)^4} \int \frac{d^3 p_\tau d^3 p_{\bar{\nu}_\tau}}{E_\tau E_{\bar{\nu}_\tau}} \delta\left(\sqrt{q^2} - E_\tau - E_{\bar{\nu}_\tau}\right) \delta^3 \left(\vec{p}_\tau + \vec{p}_{\bar{\nu}_\tau}\right)$$

$$\times \int \frac{d^3 p_\pi d^3 p_{\nu_\tau}}{E_\pi E_{\nu_\tau}} \delta\left(E_\tau - E_\pi - E_{\nu_\tau}\right) \delta^3 \left(\vec{p}_\tau - \vec{p}_\pi - \vec{p}_{\nu_\tau}\right) \tag{1}$$

All $\vec{p_x}$ and E_x are expressed in the N^* rest frame.

Mathematical transformation

Under the assumption that neutrinos are massless,

- $E_{\bar{\nu_\tau}} = |p\vec{\bar{\nu}}_\tau| = |\vec{p_\tau}|$ (In N^* frame, $\vec{p_\tau} + \vec{p_{\nu_\tau}} = 0$)
- $E_{\nu_\tau} = |\vec{p_{\nu_\tau}}| = |\vec{p_\tau} - \vec{p_\pi}|$

Perform the integrals in d^4 over ν_τ and $\bar{\nu_\tau}$,

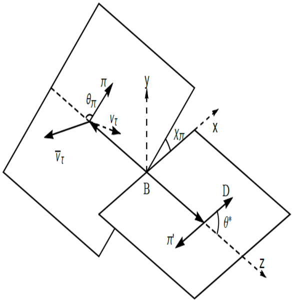
$$d^4 I = \frac{1}{(4\pi)^4} \int \frac{d^3 p_\tau}{E_\tau \, |\vec{p}_\tau|} \delta\left(\sqrt{q^2} - E_\tau - |\vec{p}_\tau|\right) \int \frac{d^3 p_\pi}{E_\pi \, |\vec{p}_\tau - \vec{p}_\pi|} \delta\left(E_\tau - E_\pi - |\vec{p}_\tau - \vec{p}_\pi|\right) \quad (2)$$

π angles

From the figure, it is possible to geometrically calculate θ_π and χ_π in N^* rest frame.

$$\cos\theta_\pi = \frac{-\vec{P}_{D^*} \cdot \vec{P}_\pi}{\left|\vec{P}_{D^*}\right| \left|\vec{P}_\pi\right|} \tag{3}$$

$$\sin\chi_\pi = \frac{[(\vec{p}_{\pi'} \times \vec{p}_D) \times (\vec{p}_{D^*} \times \vec{p}_\pi)] \cdot \vec{p}_{D^*}}{|\vec{p}_{\pi'} \times \vec{p}_D|\,|\vec{p}_{D^*} \times \vec{p}_\pi|\,|\vec{p}_{D^*}|} \tag{4}$$



Mathematical transformation

$\theta_{\tau\pi}$ can be determined theoretically, but not $\chi_{\tau\pi}$; however, the latter would be integrated over.

Using $|\vec{p_\tau} - \vec{p_\pi}| = \sqrt{|\vec{p_\tau}|^2 + |\vec{p_\pi}|^2 - 2|\vec{p_\tau}||\vec{p_\pi}|cos(\theta_{\tau\pi})}$,

$$d^4 I = \frac{1}{(4\pi)^4} \int \frac{d\,|\vec{p}_\tau|}{\sqrt{q^2}} d\cos\theta_{\tau\pi} d\chi_{\tau\pi} dE_\pi d\cos\theta_\pi d\chi_\pi$$
$$\delta\left(|\vec{p}_\tau| - \frac{q^2 - m_\tau^2}{2\sqrt{q^2}}\right) \delta\left(\cos\theta_{\tau\pi} - \frac{2E_\tau E_\pi - m_\tau^2 - m_\pi^2}{2\,|\vec{p}_\tau|\,|\vec{p}_\pi|}\right) \quad (5)$$

Mathematical transformation

$$d^4 I = \frac{1}{(4\pi)^4} \frac{1}{\sqrt{q^2}} d\chi_{\tau\pi} dE_\pi d\cos\theta_\pi d\chi_\pi \qquad (6)$$

With the following substitutions obtained from the Dirac Delta function when $|\mathcal{M}|^2$ is calculated:

- $|\vec{p}_\tau| \rightarrow \frac{q^2 - m_\tau^2}{2\sqrt{q^2}}$
- $E_\tau \rightarrow \frac{q^2 + m_\tau^2}{2\sqrt{q^2}}$
- $\cos\theta_{\tau\pi} \rightarrow \frac{2E_\tau E_\pi - m_\tau^2 - m_\pi^2}{2|\vec{p}_\tau||\vec{p}_\pi|}$

Decay Rate

$$\frac{d^5\Gamma}{dq^2 d\cos\theta^* dE_\pi d\cos\theta_\pi d\chi_\pi} = \frac{|\vec{p}_{D^*}|\,|\vec{p}_D|}{2^{15}\pi^7 m_B^2 m_{D^*}\sqrt{q^2}} \int d\chi_{\tau\pi} \frac{dp_{D^*}^2}{2\pi}\frac{dp_T^2}{2\pi}|\mathcal{M}|^2$$

Here $|\vec{p}_{D^*}| = \sqrt{\lambda\left(m_B^2; q^2, m_{D^*}^2\right)}/\left(2m_B\right)$ and $|\vec{p}_D| = \sqrt{\lambda\left(m_{D^*}^2; m_{D^2}^2, m_\pi^2\right)}/\left(2m_{D^*}\right)$, where

$$\lambda(a; b, c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca.$$

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Transferring results

A previous study was done on $\bar{B} \to D^* N^* (\to \mu^- \nu_{\bar{\mu}})$ with all couplings allowed. Citing the results, the effective Hamiltonian is given by,

$$\mathcal{H}_{eff} = \frac{G_F V_{cb}}{\sqrt{2}} \{ [(1+g_L)\,\bar{c}\gamma_\alpha (1-\gamma_5)\, b + g_R \bar{c}\gamma_\alpha (1+\gamma_5)\, b]\, \bar{\mu}\gamma^\alpha (1-\gamma_5)\, \nu_\mu$$

$$+ [g_S \bar{c} b + g_P \bar{c}\gamma_5 b]\, \bar{\mu} (1-\gamma_5)\, \nu_\mu + g_T \bar{c}\sigma^{\alpha\beta} (1-\gamma_5)\, b\bar{\mu}\sigma_{\alpha\beta} (1-\gamma_5)\, \nu_\mu$$

$h.c.$

Calculation of $|\mathcal{M}|^2$

$$|\mathcal{M}|^2 = \frac{96\pi G_F^2 \left|V_c\right|^2 m_{D^*}}{\left|\vec{p}_D\right|^3 \left(m_\tau^2 - m_\pi^2\right)^2} \frac{m_{D^*}\Gamma_{D^*}\mathcal{B}\left(D^* \to D\pi'\right)}{\left(p_{D^*}^2 - m_{D^*}^2\right)^2 + m_{D^*}^2\Gamma_{D^*}^2} \frac{m_\tau\Gamma_\tau\mathcal{B}\left(\tau \to \pi\nu_\tau\right)}{\left(p_\tau^2 - m_\tau^2\right)^2 + m_\tau^2\Gamma_\tau^2} \tag{7}$$

$$|\sum_{m=\pm,D} \mathcal{H}_{D^*}(m) \left(\mathcal{M}_{(m)}^{SP}\overline{\mathcal{L}}^{SP} + \sum_{n=t,\pm,D} g_{nn}\mathcal{M}_{(m;n)}^{VA}\mathcal{L}^{VA}(n)\right. \tag{8}$$

$$\left. + \sum_{n,p=t,\pm,0} g_{nn}g_{pp}\mathcal{M}_{(m;n,p)^T}^{T}\overline{\mathcal{L}}^{T}(n,p)\right) |^2 \tag{9}$$

In the above, the new leptonic pieces are of the form

$$\overline{\mathcal{L}}^{SP} = m_\tau u\left(\nu_\tau\right) p_\pi \left(1 - \gamma_5\right) v\left(\nu_\tau\right), \tag{10}$$

$$\overline{\mathcal{L}}^{VA}(n) = \epsilon_{VA}^{\beta}(n)\left[a\left(\nu_\tau\right) p_\pi p_\tau \gamma_\beta \left(1 - \gamma_5\right) v\left(\nu_\tau\right)\right], \tag{11}$$

$$\overline{\mathcal{L}}^{T}(n,p) = -im_\tau \epsilon_T^{\beta}(n)\epsilon_T^{\beta}(p)\left[a\left(\nu_\tau\right) p_\pi \sigma_{\beta\delta}(1 - \gamma 5)v\left(\nu_\tau\right)\right] \tag{12}$$

Decay Rate

The difference between the study of $\bar{B} \to D^* N^* (\to \mu^- \nu_{\bar{\mu}})$ and the current one is that an additional decay $(\tau \to \pi \nu_\tau)$ has to be considered whose contribution in the formula of $|\mathcal{M}|^2$ comes from the branching fraction as seen in the previous slide.

The branching fractions $\mathcal{B}\left(D^* \to D\pi'\right)$ and $\mathcal{B}\left(\tau \to \pi \nu_\tau\right)$ are as obtained in SM,

$$\begin{aligned} \mathcal{B}\left(\tau \to \pi \nu_\tau\right) &= \frac{G_F^2 \left|V_{ud}\right|^2 f_\pi^2}{16\pi m_\tau \Gamma_\tau} \left(m_\tau^2 - m_\pi^2\right)^2, \\ \mathcal{B}\left(D^* \to D\pi'\right) &= \frac{\left|\vec{p}_D\right|^3}{6\pi m_{D^*}^2 \Gamma_{D^*}} \end{aligned} \tag{13}$$

Decay Rate

Integrating over p_D^2, p_τ^2, and $\chi_{\tau\pi}^2$, we get

$$\frac{d^5\Gamma}{dq^2 dE_\pi d\cos\theta^* d\cos\theta_\pi d\chi_\pi} = \frac{3\left|V_{ob}\right|^2 G_F^2 \left|\vec{p}_{D^*}\right| \left(q^2\right)^{3/2} m_\tau^2}{2^{11}\pi^4 m_B^2 \left(m_\tau^2 - m_\pi^2\right)^2}$$
$$\times \mathcal{B}\left(D^* \to D\pi'\right) \mathcal{B}\left(\tau \to \pi\nu_\tau\right) \sum_{i,j} \Big(\mathcal{N}_i^S \left|\mathcal{A}_i\right|^2 + \mathcal{N}_{i,j}^R \operatorname{Re}\left[\mathcal{A}_i\mathcal{A}_j^*\right]$$
$$+ \mathcal{N}_{i,j}^I \operatorname{Im}\left[\mathcal{A}_i\mathcal{A}_j^*\right]\Big) \tag{14}$$

Decay Rate

The decay rate can be expressed as a product of angular and non-angular functions which is to write it as an angular distribution.

$$
\begin{aligned}
&\frac{d^5\Gamma}{dq^2 dE_\pi d\cos\theta^* d\cos\theta_\pi d\chi_\pi} = \\
&\frac{3\left|V_{cb}\right|^2 G_F^2 \left|\vec{p}_{D^*}\right| \left(q^2\right)^{3/2} m_\tau^2}{2^{11}\pi^4 m_B^2 \left(m_\tau^2 - m_\pi^2\right)^2} \mathcal{B}\left(D^* \to D\pi'\right) \mathcal{B}\left(\tau \to \pi\nu_\tau\right) \\
&\times \left[\sum_{i=1}^{9} f_i^R\left(q^2, E_\pi\right) \Omega_i^R\left(\theta^*, \theta_\pi, \chi_\pi\right) + \sum_{i=1}^{3} f_i^I\left(q^2, E_\pi\right) \Omega_i^I\left(\theta^*, \theta_\pi, \chi_\pi\right)\right]
\end{aligned}
\tag{15}
$$

Helicity Amplitudes

Coefficient $f_i^R(q^2, E_\pi)$	Angular Function $\Omega_i^R(\theta^*, \theta_\pi, \chi_\pi)$
$S_t\,\lvert\mathcal{A}_t\rvert^2 + S_{0,1}\,\lvert\mathcal{A}_0\rvert^2 + S_{SP}\,\lvert\mathcal{A}_{SP}\rvert^2 + S_{0T,1}\,\lvert\mathcal{A}_{0,T}\rvert^2$ $+R_{0T01}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*] + R_{SPt}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_t^*]$	$\cos^2\theta^*$
$S_{\perp,1}\,\lvert\mathcal{A}_\perp\rvert^2 + S_{\parallel,1}\,\lvert\mathcal{A}_\parallel\rvert^2 + S_{\perp T,1}\,\lvert\mathcal{A}_{\perp,T}\rvert^2 + S_{\parallel T,1}\,\lvert\mathcal{A}_{\parallel,T}\rvert^2$ $+R_{\parallel T\parallel,1}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*] + R_{\perp T\perp,1}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*$
$R_{SP0}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_0^*] + R_{t0}\,\mathrm{Re}[\mathcal{A}_t\mathcal{A}_0^*]$ $+R_{SP0T}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{0,T}^*] + R_{0Tt}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_t^*]$	$\cos^2\theta^*\ \cos\theta_\pi$
$S_{0,2}\,\lvert\mathcal{A}_0\rvert^2 + S_{0T,2}\,\lvert\mathcal{A}_{0,T}\rvert^2 + R_{0T0,2}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_0^*]$	$\cos^2\theta^*\ \cos 2\theta_\pi$
$R_{\perp T\parallel}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*] + R_{\perp T\parallel T}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_{\parallel,T}^*]$ $+R_{\parallel\perp}\,\mathrm{Re}[\mathcal{A}_\parallel\mathcal{A}_\perp^*] + R_{\parallel T\perp}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*\ \cos\theta_\pi$
$S_{\parallel,2}\,\lvert\mathcal{A}_\parallel\rvert^2 + S_{\perp,2}\,\lvert\mathcal{A}_\perp\rvert^2 + S_{\parallel T,2}\,\lvert\mathcal{A}_{\parallel,T}\rvert^2 + S_{\perp T,2}\,\lvert\mathcal{A}_{\perp,T}\rvert^2$ $+R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*] + R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^*\ \cos 2\theta_\pi$
$2(S_{\perp,2}\,\lvert\mathcal{A}_\perp\rvert^2 - S_{\parallel,2}\,\lvert\mathcal{A}_\parallel\rvert^2) + 2(S_{\perp T,2}\,\lvert\mathcal{A}_{\perp,T}\rvert^2 - S_{\parallel T,2}\,\lvert\mathcal{A}_{\parallel,T}\rvert^2)$ $+2(R_{\perp T\perp,2}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_\perp^*] - R_{\parallel T\parallel,2}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_\parallel^*])$	$\sin^2\theta^*\ \sin^2\theta_\pi\cos 2\chi_\pi$
$R_{\perp T0}\,\mathrm{Re}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*] + R_{SP\parallel}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_\parallel^*] + R_{t\parallel}\,\mathrm{Re}[\mathcal{A}_t\mathcal{A}_\parallel^*]$ $+R_{SP\parallel T}\,\mathrm{Re}[\mathcal{A}_{SP}\mathcal{A}_{\parallel,T}^*]\ +R_{0\perp}\,\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\perp^*] + R_{0T\perp}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*]$ $+R_{0T\perp T}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\perp,T}^*] + R_{\parallel Tt}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_t^*]$	$\sin 2\theta^*\ \sin\theta_\pi\ \cos\chi_\pi$
$R_{\parallel T0}\,\mathrm{Re}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*] + R_{0\parallel}\,\mathrm{Re}[\mathcal{A}_0\mathcal{A}_\parallel^*]$ $+R_{0T\parallel}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*] + R_{0T\parallel T}\,\mathrm{Re}[\mathcal{A}_{0,T}\mathcal{A}_{\parallel,T}^*]$	$\sin 2\theta^*\ \sin 2\theta_\pi\ \cos\chi_\pi$

Table 2

Helicity Amplitudes

Coefficient $f_i^I(q^2, E_\pi)$	Angular Function $\Omega_i^I(\theta^*, \theta_\pi, \chi_\pi)$
$I_{t\perp}\,\mathrm{Im}[\mathcal{A}_t\mathcal{A}_\perp^*] + I_{\parallel T0}\,\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_0^*] + I_{SP\perp}\,\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_\perp^*]$ $+I_{SP\perp T}\,\mathrm{Im}[\mathcal{A}_{SP}\mathcal{A}_{\perp,T}^*] + I_{0T\parallel}\,\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\parallel^*] + I_{\perp Tt}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_t^*]$	$\sin 2\theta^* \sin\theta_\pi \sin\chi_\pi$
$I_{0\perp}\,\mathrm{Im}[\mathcal{A}_0\mathcal{A}_\perp^*] + I_{0T\perp}\,\mathrm{Im}[\mathcal{A}_{0,T}\mathcal{A}_\perp^*] + I_{\perp T0}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_0^*]$	$\sin 2\theta^* \sin 2\theta_\pi \sin\chi_\pi$
$I_{\parallel\perp}\,\mathrm{Im}[\mathcal{A}_\parallel\mathcal{A}_\perp^*] + I_{\perp T\parallel}\,\mathrm{Im}[\mathcal{A}_{\perp,T}\mathcal{A}_\parallel^*] + I_{\parallel T\perp}\,\mathrm{Im}[\mathcal{A}_{\parallel,T}\mathcal{A}_\perp^*]$	$\sin^2\theta^* \sin^2\theta_\pi \sin 2\chi_\pi$

Table 3

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New Physics

The coefficients in table 2 are extracted from the angular fit to the data separated in the bins of the non-angular parameters (q^2 and E_π). Table 4 shows the dependence of the helicity amplitudes on the 5 NP parameters. Note that g_s doesn't contribute.

Helicity Amplitude	Coupling
$\mathcal{A}_0$, $\mathcal{A}_\parallel$, $\mathcal{A}_t$	$1 + g_L - g_R$
$\mathcal{A}_\perp$	$1 + g_L + g_R$
$\mathcal{A}_{SP}$	g_P
$\mathcal{A}_{0,T}$, $\mathcal{A}_{\parallel,T}$, $\mathcal{A}_{\perp,T}$	g_T

Table 4

New Physics

The couplings g_L, g_R, g_P, and g_T are in general complex. Thus, they have an independent magnitude in addition to a weak (CP-odd) phase. We can fix the phase of one of them to be 1 and end up with 7 independent parameters. It could happen that they also have NP strong (CP-even) phase which would increase the number to 11; however, it can be argued that the phase is the same as in SM. In short, we can write the products of the helicity amplitudes in tables 2 and 3 in terms of the 7 NP parameters.

New Physics

The products of helicity amplitudes in tables 2 and 3 are in the form: $|A_i^2|$, $Re[A_i A_j^*]$, and $Im[A_i A_j^*]$.

If the NP parameters have the same weak phase, $Im[A_i A_j^*] = 0$, and all the entries of table 3 vanish hence we call it CP-violating. On the other hand, table 2 contains terms in the form $Re[A_i A_j^*]$ and $|A_i^2|$ only which absorbs any phase for which it is called CP-conserving.

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CP violation

Because the strong phases are argued to be the same, the CP violation effect is tiny; however, CP violation can be obtained from the angular distribution. We get this from what we call **TP asymmetry**. TP is the triple product $\vec{p_1} \cdot (\vec{p_2} \times \vec{p_3})$ where the $\vec{p_i}$ is final-state momenta. When angular coefficients are proportional to the triple product of the three final-state momenta, a TP asymmetry can be studied as is the case for all the entries of table 3. All the entries are proportional to $\mathrm{Im}\left[A_i A_j^*\right]$

$$A_i = |A_i|\, e^{i\phi_i} e^{i\delta_i}, \quad A_j = |A_j|\, e^{i\phi_j} e^{i\delta_j} \tag{16}$$

where $\phi_{i,j}\,(\delta_{i,j})$ are the weak (strong) phases.

$$\mathrm{Im}\left[A_i A_j^*\right] = |A_i|\,|A_j|\sin\left(\phi_i - \phi_j + \delta_i - \delta_j\right) \tag{17}$$

TP can be non-zero even if the weak phases are the same if we have a non-negligible strong-phase difference. This is known as **fake TP**. We need a real CP-violating term.

CP violation

We compare the term in equation(17) with its counterpart in the CP-conjugate process. In the CP-conjugate process, the weak phases change sign, but not the strong ones. Also, all the entries in the right column of table 3 are proportional to $sin\chi_{\pi}$ and hence they pick a negative sign.

$$-\operatorname{Im}\left[\bar{A}_i\bar{A}_j^*\right] = |A_i|\,|A_j|\sin\left(\phi_i - \phi_j - \delta_i + \delta_j\right) \tag{18}$$

We just add the TP terms in the process and its CP-conjugate,.

$$\operatorname{Im}\left[A_iA_j^*\right] - \operatorname{Im}\left[\bar{A}_i\bar{A}_j^*\right] = 2\,|A_i|\,|A_j|\cos\left(\delta_i - \delta_j\right)\sin\left(\phi_i - \phi_j\right) \tag{19}$$

CP violation

Obviously, if the strong-phase difference doesn't vanish, the term in equation(17) is not a CP eigenstate. Thus, it violates CP symmetry. If the strong-phase difference is negligible, the right-hand side of the equation(17) is non-zero if the weak-phase difference is non-zero which is a sign of CP violation. In both cases, if we get non-zero entries in table 3, a CP violation is constructed which is not accommodated giving strong evidence of the new physics.

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CP conservation

The helicity amplitudes in table 2 are found in SM; however, in NP, the magnitude is different. The way to test for NP through CP conserving terms is by measuring the coefficients in $E_\pi - q^2$ (the non-angular variables) bins. A combined fit is then applied to the angular distribution to extract the magnitudes and weak phases of the NP parameters listed in table 4.

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Integrated variables categories

The decay rate for $\bar{B} \to D^*(\to D\pi')\tau^-(\to \pi^-\nu_\tau)\nu_{\bar{\tau}}$ depends on 5 independent parameters. These 5 parameters are categorized into two classes: parameters describing the lepton side (E_π, θ_π, and χ_π) and parameters describing the hadronic side (θ^* and either θ_π or χ_π). When the decay rate is integrated over all the parameters of the first category, the left decay rate is no longer dependent on the lepton-side decay because all the dependence has been integrated over. Therefore, the left decay rate and its associated s can be used to test for lepton universality. On the contrary, when the decay rate is integrated over the latter category, the observables of the decay rate depend explicitly on the τ being an intermediate state. This emerges out of the fact that the other light leptons can't decay into a pion and the left kinematic parameters are pion-dependent. The first category is studied in details in the next subsection.

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Lepton flavor universality

The starting point is to integrate E_π, θ_π, and χ_π.

$$\frac{d^2\Gamma}{dq^2 d\cos\theta^*} = \frac{3}{2}\frac{d\Gamma}{dq^2}\frac{a\left(q^2\right)+c\left(q^2\right)\cos^2\theta^*}{3a\left(q^2\right)+c\left(q^2\right)}$$
$$= \frac{3}{4}\frac{d\Gamma}{dq^2}\left[2F_L^{D*}\left(q^2\right)\cos^2\theta^* + F_T^{D*}\left(q^2\right)\sin^2\theta^*\right] \quad (20)$$

where $F_L^{D*}\left(q^2\right)$ and $F_T^{D*}\left(q^2\right) = 1 - F_L^{D*}\left(q^2\right)$ are the longitudinal and transverse polarization fractions of D^* which are obtained by the following relation,

$$F_L^{D^*} = \frac{a\left(q^2\right)+c\left(q^2\right)}{3a\left(q^2\right)+c\left(q^2\right)}, \quad F_T^{D^*} = \frac{2a\left(q^2\right)}{3a\left(q^2\right)+c\left(q^2\right)} \quad (21)$$

where $a(q^2)$ and $b(q^2)$ are given by,

Lepton flavor universality

$$a\left(q^2\right) = 2(1 + \frac{m_\tau^2}{2q^2}\Big)\left(\left|\mathcal{A}_\parallel\right|^2 + \left|\mathcal{A}_\perp\right|^2\right) + 16\left(1 + \frac{2m_\tau^2}{q^2}\right)\left(\left|\mathcal{A}_{\parallel,T}\right|^2 + \left|\mathcal{A}_{\perp,T}\right|^2\right) - \frac{24m_\tau}{\sqrt{q^2}}\left(\mathrm{Re}\left[\mathcal{A}_\parallel\mathcal{A}_{\parallel,T}^*\right] + \mathrm{Re}\left[\mathcal{A}_\perp\mathcal{A}_{\perp,T}^*\right]\right) \tag{22}$$

$$\begin{aligned} c\left(q^2\right) = {} & 2(1 + \frac{m_\tau^2}{2q^2}\Big)\left(2\left|\mathcal{A}_0\right|^2 - \left|\mathcal{A}_\parallel\right|^2 - \left|\mathcal{A}_\perp\right|^2\right) + 6\left|\frac{m_\tau}{\sqrt{q^2}}\mathcal{A}_t + \mathcal{A}_{SP}\right|^2 \\ & + 16\left(1 + \frac{2m_\tau^2}{q^2}\right)\left(2\left|\mathcal{A}_{0,T}\right|^2 - \left|\mathcal{A}_{\parallel,T}\right|^2 - \left|\mathcal{A}_{\perp,T}\right|^2\right) \\ & - \frac{24m_\tau}{\sqrt{q^2}}\left(2\,\mathrm{Re}\left[\mathcal{A}_0\mathcal{A}_{0,T}^*\right] - \mathrm{Re}\left[\mathcal{A}_\parallel\mathcal{A}_{\parallel,T}^*\right] - \mathrm{Re}\left[\mathcal{A}_\perp\mathcal{A}_{\perp,T}^*\right]\right) \end{aligned} \tag{23}$$

Lepton flavor universality

Equation(20) can be integrated further over $cos\theta^*$ to give the decay rate solely as a function of q^2,

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 \left|V_{cb}\right|^2 \left|\vec{p}_{D^*}\right| q^2}{128 m_B^2 \pi^3} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \mathcal{B}\left(D^* \to D\pi'\right) \times$$
$$\mathcal{B}\left(\tau \to \pi\nu_\tau\right) \left(a\left(q^2\right) + \frac{c\left(q^2\right)}{3}\right) \tag{24}$$

This is the formula found in the literature except for the factor $\mathcal{B}\left(\tau \to \pi\nu_\tau\right)$. A measurement of this observable, taking into account the characteristics of the τ such as its mass, can serve as a direct test for lepton universality.

Thank You for the attention