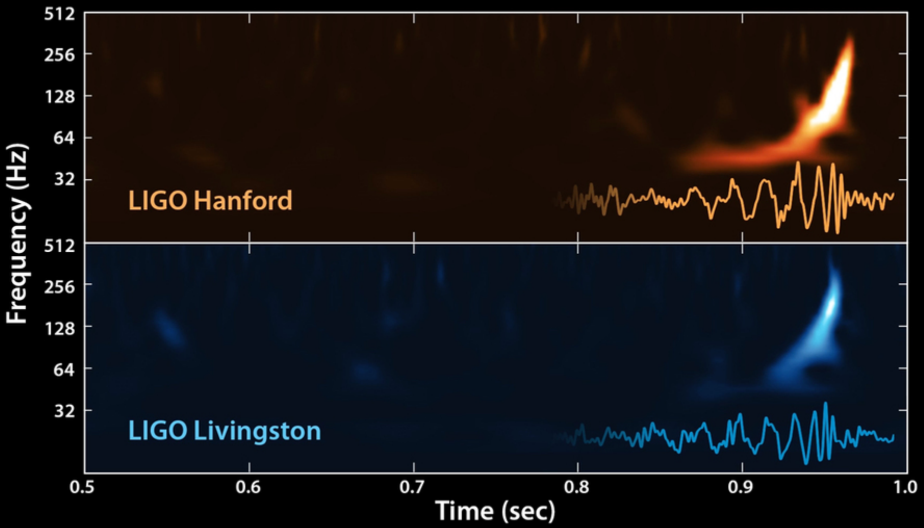
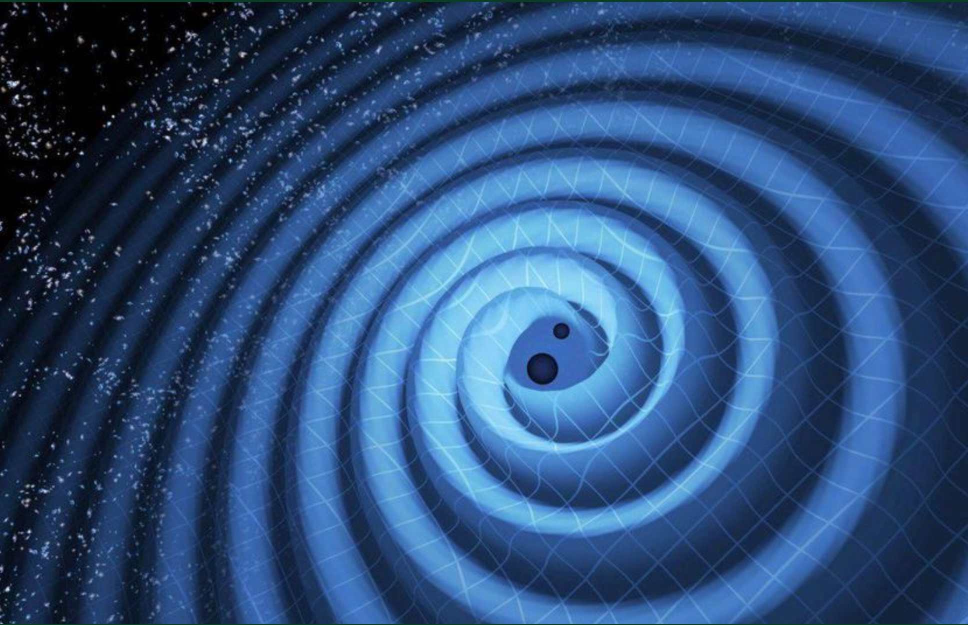
Cosmologically Coupled Compact Objects (C3Os)

A single parameter model for LVC mass-redshift distributions

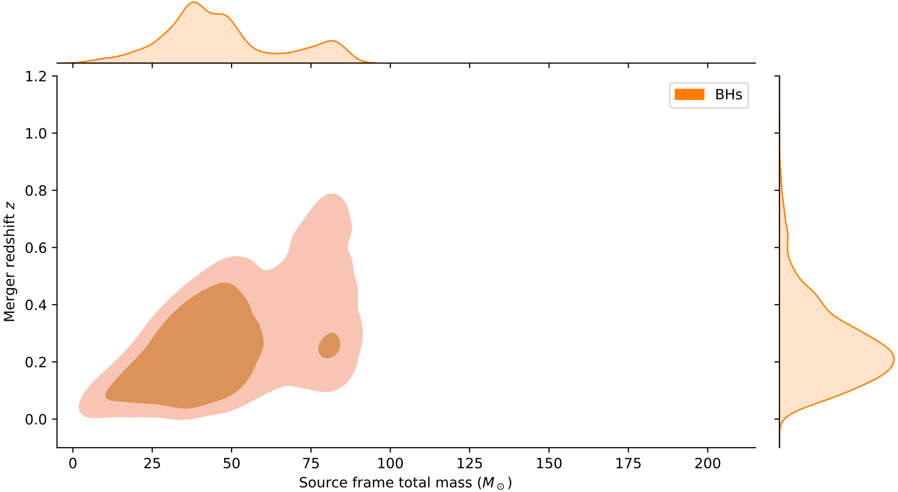
*K. Croker[1], M. Zevin[2], D. Farrah[1,3], K. Nishimura[1], G. Tarlé[4]

[1]Department of Physics and Astronomy, University of Hawai‘i at Mānoa
[2]Enrico Fermi Institute, University of Chicago
[3]Institute for Astronomy (IfA), University of Hawai‘i at Mānoa
[4]Department of Physics, University of Michigan at Ann Arbor

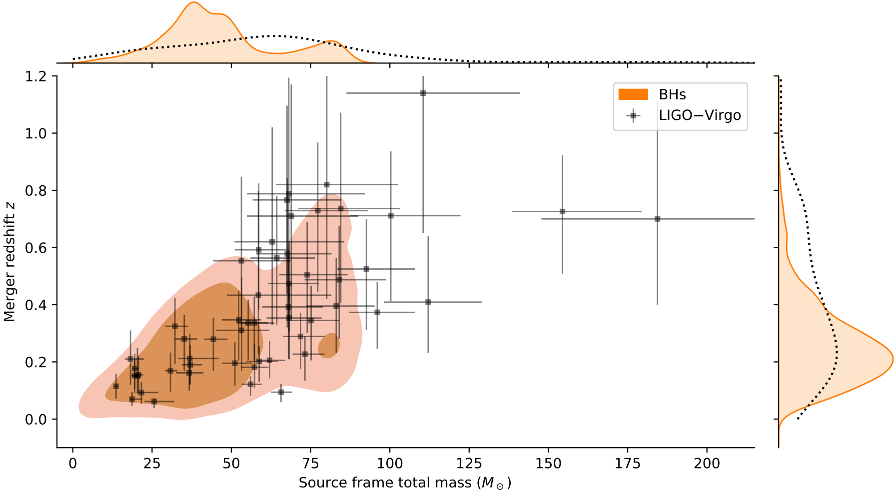

512
256
128
64
32
Frequency (Hz)
LIGO Hanford
512
256
128
64
32
LIGO Livingston
0.5
0.6
0.7
0.8
0.9
1.0
Time (sec)



Stellar population synthesis BHs



Stellar population synthesis BHs don't agree with data



Presentation outline

1. **Interpretation of BH spacetimes**
   1.1 Kerr Metric

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   3.2 Physics behind the altered distributions

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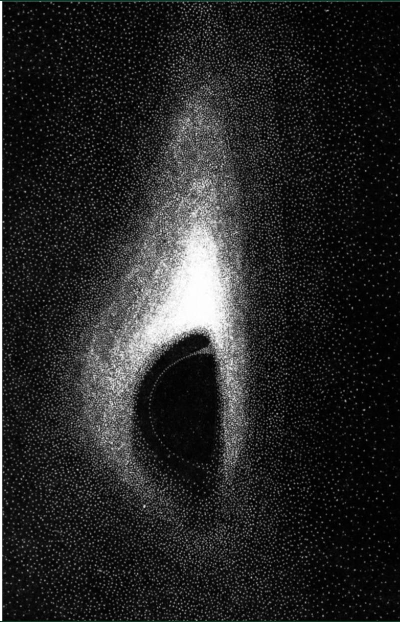
Kerr Metric

A spinning BH is modelled with the Kerr metric

$$g_{\mu\nu} := \eta_{\mu\nu} + \frac{R_s r(\mathbf{x})^3 \ell_\mu \ell_\nu}{r(\mathbf{x})^4 + A^2 z^2}$$



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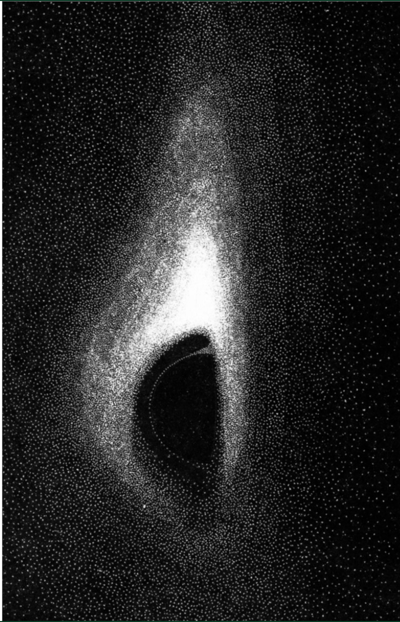

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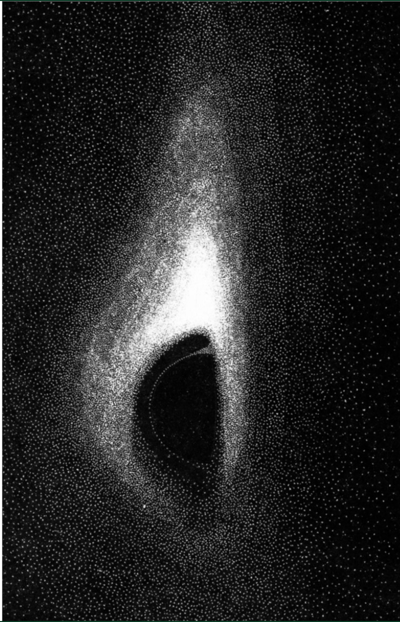

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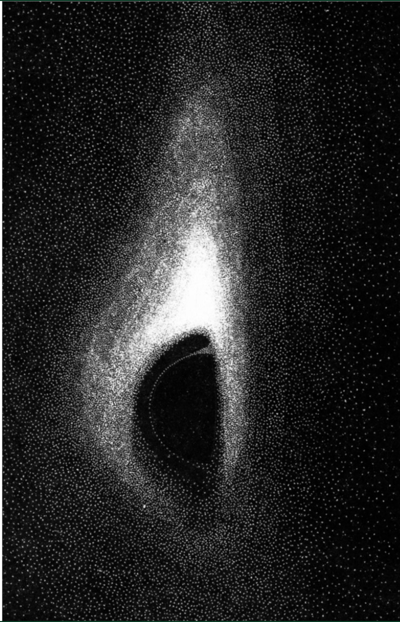

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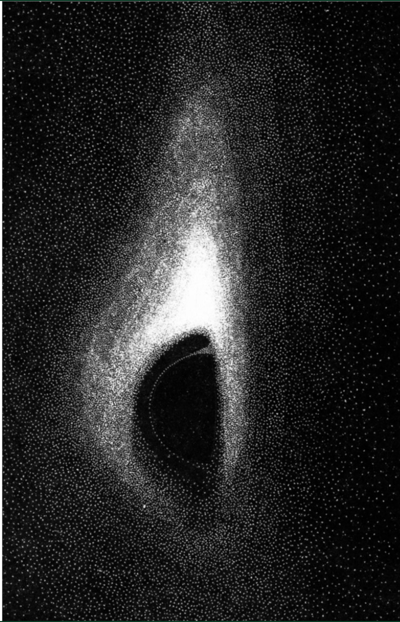
- $r(\mathbf{x})$ is a generalized radius



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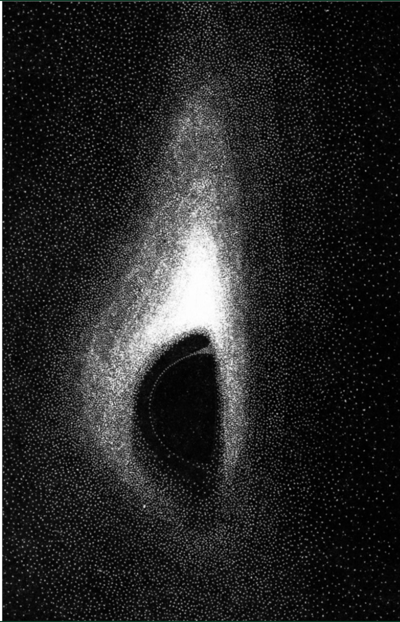
At large distances from the hole

$$\lim_{r(\mathbf{x})\to\infty} g_{\mu\nu} = \eta_{\mu\nu}$$

$\therefore$ Kerr's object exists in a static universe



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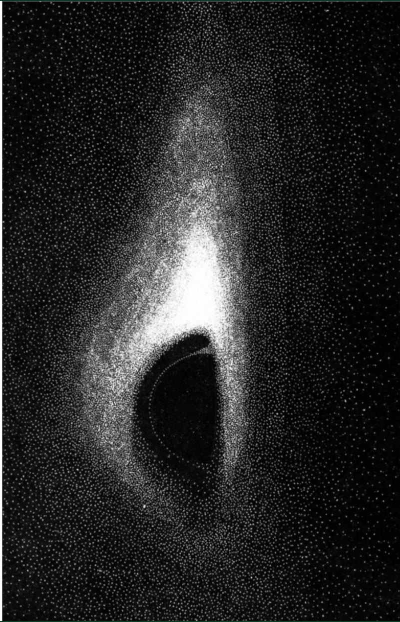
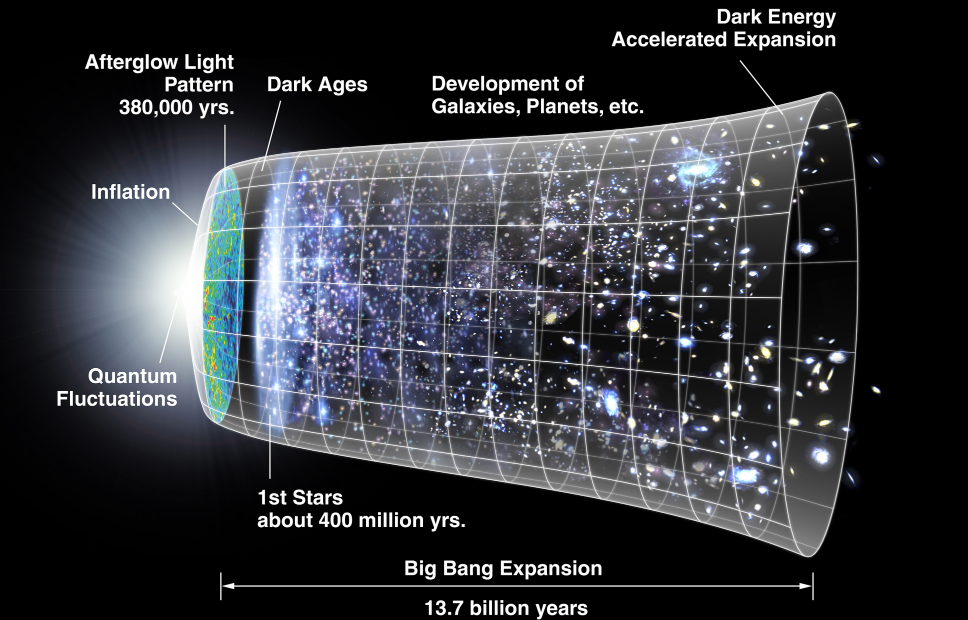
∴ Kerr's object exists in a static universe

But we don't exist there...




Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
380,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years

Robertson-Walker Metric

Size of universe measured with the Robertson-Walker (RW) metric

$$g_{\mu\nu} := a^2(\eta)\eta_{\mu\nu}$$

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Einstein's equations predict growth

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- $a(\text{now}) := 1$

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Where

- $a(\text{now}) := 1$
- $\lim a \to 0 :=$ Big Bang

Kerr within the Robertson-Walker context

Kerr's object (far away, forever)

$$g_{\mu\nu}^{\mathsf{Kerr}} \to \eta_{\mu\nu}$$

vs. the expanding universe (everywhere, forever)

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Taylor expand $a^2(\eta)$ about some η_0

$$a^2(\eta) = a^2(\eta_0) + 2a\Delta\eta \frac{\mathrm{d}a}{\mathrm{d}\eta}\bigg|_{\eta=\eta_0} + \cdots$$

and substitute ...

Kerr within the Robertson-Walker context

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vs. the expanding universe (everywhere, near η_0)

$$g_{\mu\nu}^{\mathsf{RW}} = a^2(\eta_0)\left[1 + 2Ha\Delta\eta\Big|_{\eta=\eta_0} + \cdots\right]\eta_{\mu\nu}$$

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The constant factor is just a unit redefinition ...

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vs. the expanding universe (everywhere, near η_0)

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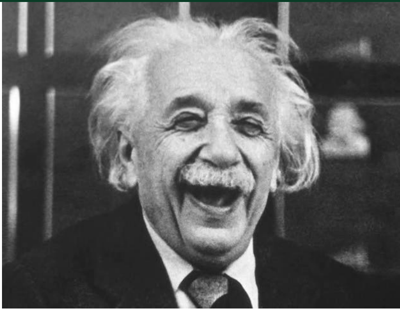
$$g_{\mu\nu}^{\mathsf{RW}} = \eta_{\mu\nu} + O\left(2\Delta\eta H a\Big|_{\eta=\eta_0}\right)$$

$\therefore$ the Kerr metric is an excellent approximation to *something else*, on timescales short compared to $1/Ha$

Astrophysical black holes

Realistic solutions to Einstein's equations are difficult. Known solutions, however, provide clues...



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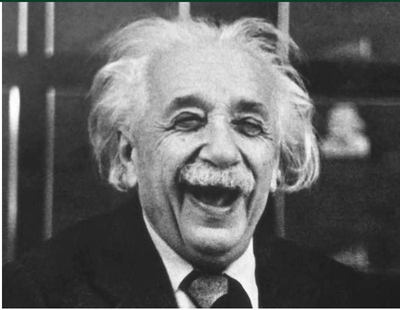

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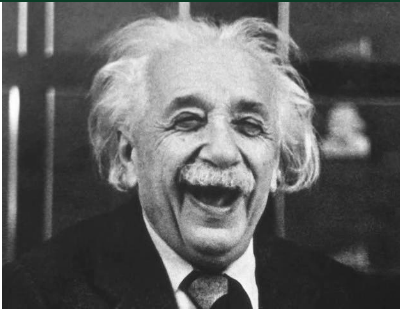
Significant: explicit counterexamples to a common misunderstanding that such effects cannot occur in GR



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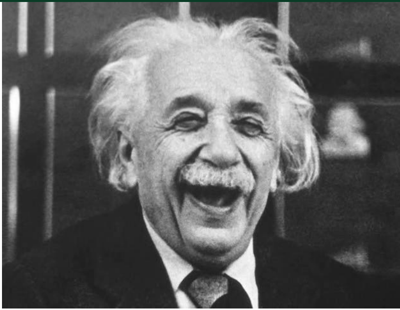
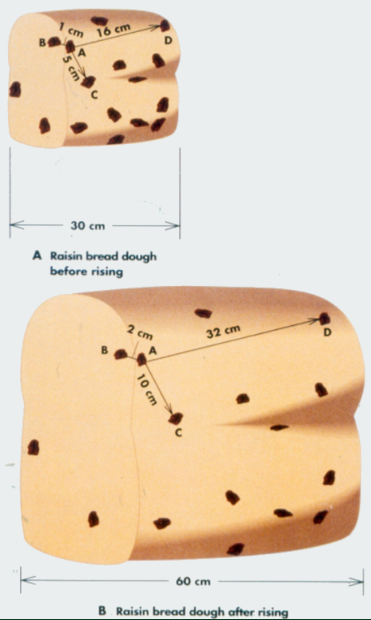
In fact, we've seen this before...



Known cosmological energy shifts

Recall that we have defined

$$a(\eta) := \text{linear scale of the universe}$$



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Recall that we have defined

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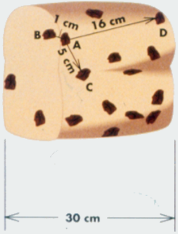
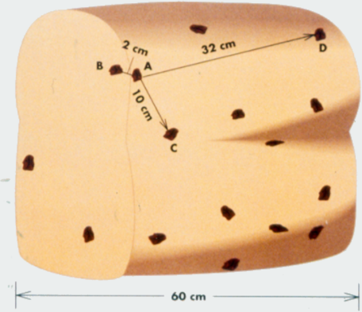
The volume of the universe increases as the cube of this linear scale

$$\mathcal{V} \propto a^3.$$



A Raisin bread dough before rising



B Raisin bread dough after rising

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Recall that we have defined

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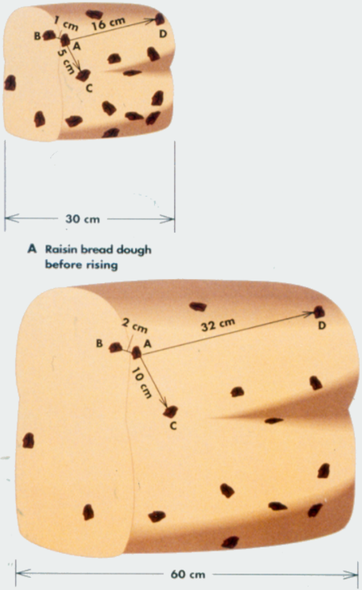
The volume of the universe increases as the cube of this linear scale

$$\mathcal{V} \propto a^3.$$

If we do not create new objects, the **number density** of objects in $\mathcal{V}$ must decrease

$$\frac{\mathrm{d}N}{\mathrm{d}\mathcal{V}} \propto \frac{1}{a^3}$$



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Known cosmological energy shifts

Conservation of stress-energy (assuming constant $\mathcal{P}/\rho$)

$$\mathrm{d}\log\rho = \mathrm{d}\log a^{-3(1+\mathcal{P}/\rho)}$$

describes how the energy density ρ changes with a.

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$\therefore E_i$ constant

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$$\boxed{\therefore E_i \propto 1/a}$$

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The effect is only visible in strongly relativistic settings...

Single parameter cosmological coupling

Hypothesis: astrophysical BHs are cosmologically coupled, like photons. Unlike photons, however, they *gain* mass-energy in time

$$m(a) := m_0 \left(\frac{a}{a_i}\right)^k \qquad a \geqslant a_i.$$

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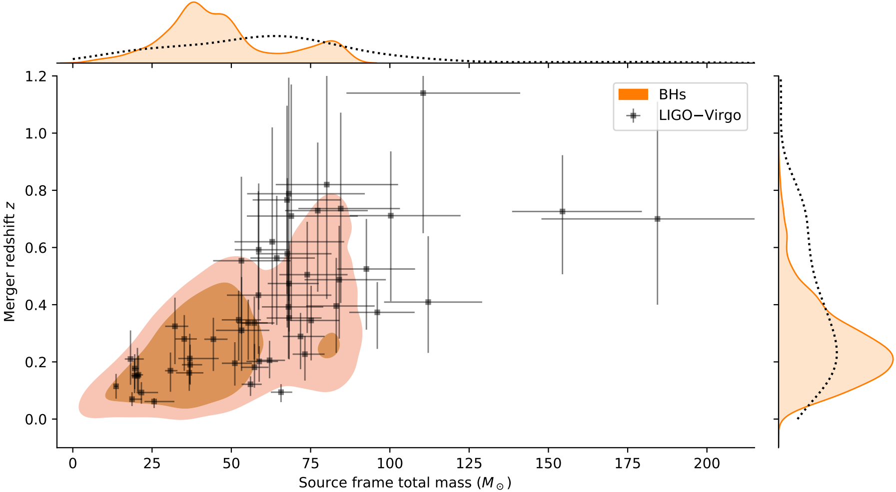
- k is the strength of the cosmological coupling

Example coupling with $k = 0.5$



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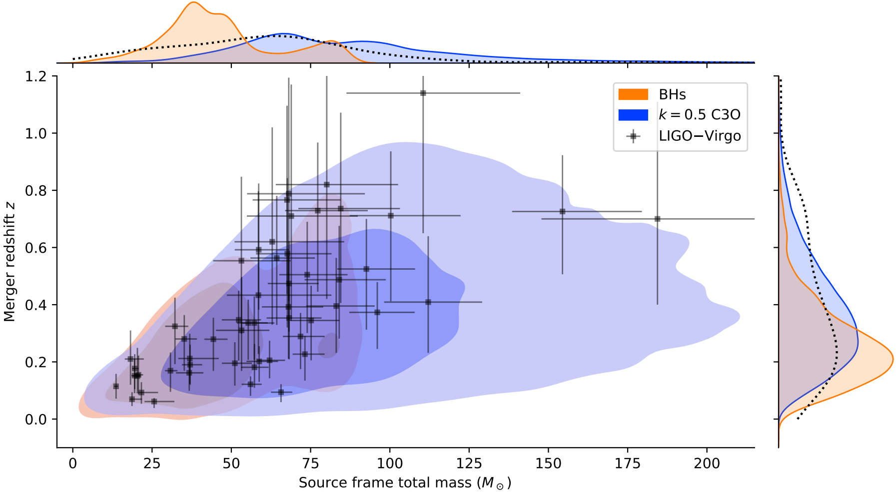

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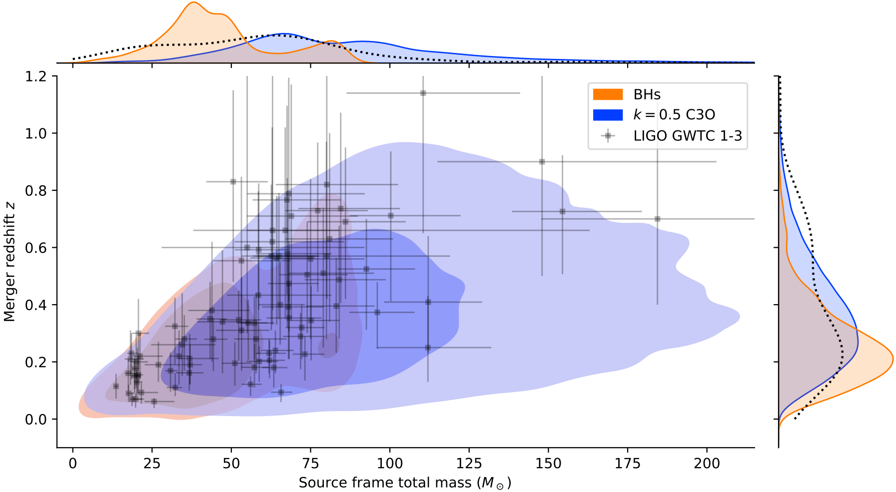
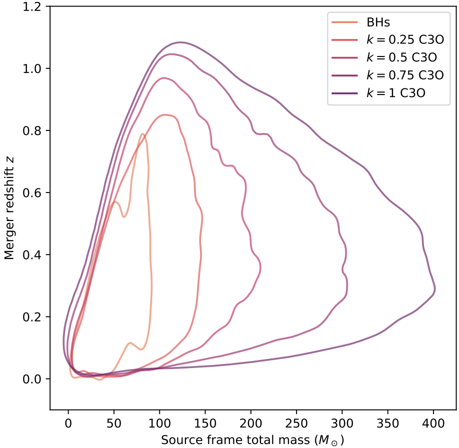


Physics behind the altered distributions

Cosmological mass growth also causes angular-momentum preserving orbital decay

- Systems with initially negligable radiative losses can now merge
- Depending on k, semi-major axes $\lesssim 10^4$ AU can now visibly merge, c.f. $\lesssim 10^{-1}$AU for BHs
- Tight binaries are culled at high redshift with low mass, and so less visible



Constraint of k with quasars

Crude upper bounds on k can be placed

- **Observation:** SMBH in quasars of $\sim 10^9 M_\odot$ at $z = 6$
- **Non-observation:** SMBH in excess of $\sim 10^{11} M_\odot$ today

$$\boxed{\pm 0.4 \text{ dex} \implies k \lesssim 3}$$

Consistent with theoretical upper bound from DEC

Conclusions

- Placing the Kerr idealization into a realistic universe allows for new dynamics in time
- Cosmological coupling in relativistic objects is theoretically and observationally well-motivated
- Cosmological coupling in astrophysical BHs eases tension between stellar population synthesis and LIGO–Virgo observations
- Coupling strength can be measured in many ways