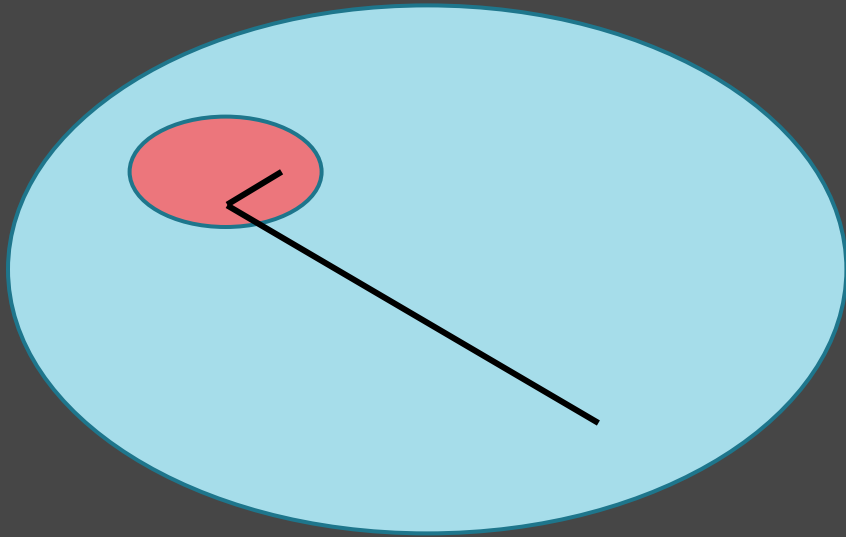


THE SPECTRAL INDEX MAY BE BLUE:
SUPERHORIZON COUPLING TO
SUBHORIZON MODES.



Joseph Bramante
University of Notre Dame

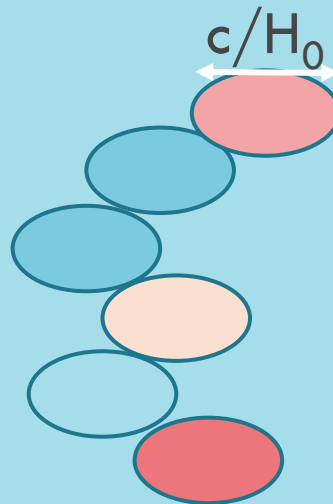
Nelson, Shandera (Penn State)
Kumar (U Hawaii)

Two Sentence Summary:

Hubble volume observables are biased by mode coupling to super-Hubble perturbations.

Result: the spectral index is shifted by super-Hubble modes coupling to non-Gaussian Hubble-scale modes.

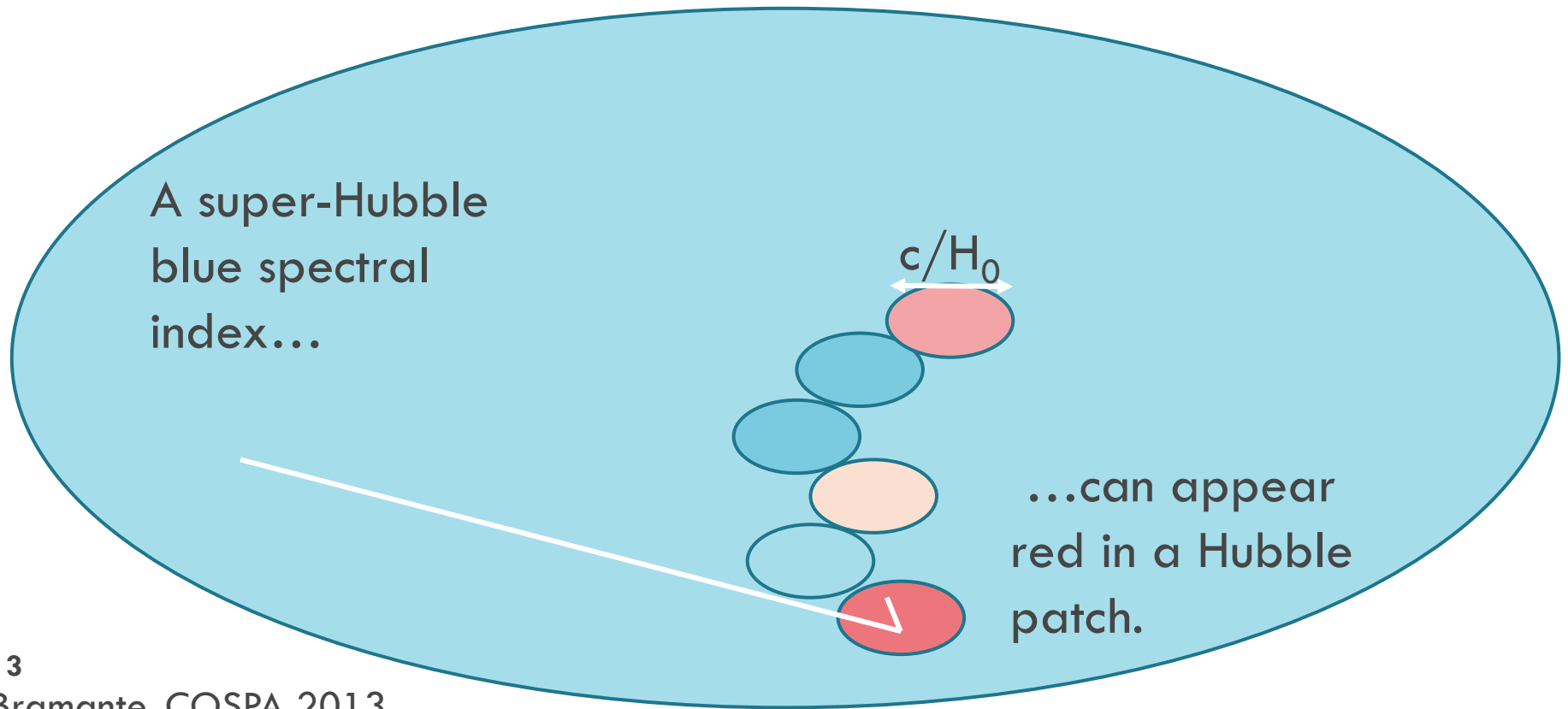
A super-Hubble
blue spectral
index...



Two Sentence Summary:

Hubble volume observables are biased by mode coupling to super-Hubble perturbations.

Result: the spectral index is shifted by super-Hubble modes coupling to non-Gaussian Hubble-scale modes.



Outline

4

- 1. Curvature Perturbations, Background vs. Foreground
- 2. Super-Hubble Mode Coupling and Running Non-Gaussianity Correct the Spectral Index
- 3. Implications for Inflaton Models and Planck
- 4. Future Work in Super Cosmic Variance

J.B., Kumar, Nelson, Shandera 1307.5083 (JCAP)

Nelson, Shandera 1212.4550 (PRL)

Nurmi, Byrnes, Tasinato 1301.3128

LoVerde, Nelson, Sandera 1303.3549 (JCAP)

LoVerde 1310.5739

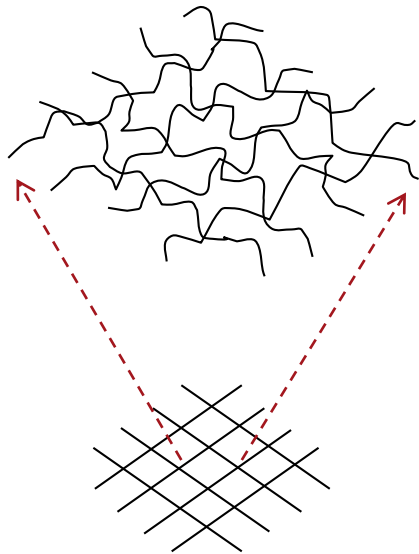
J.B., Sean Downes (In Progress)

Curvature Perturbations, Background and Foreground: How does super-Hubble structure affect our statistics?

Quantum fluctuations seed structure/scalar perturbations (ζ)

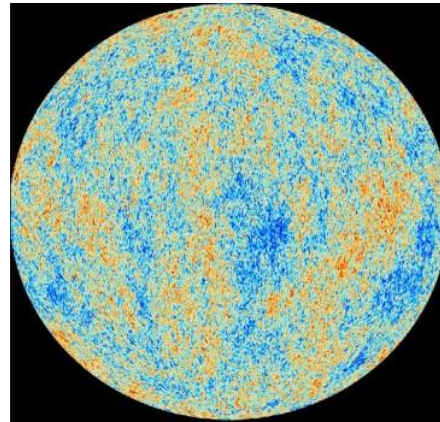
6

Inflaton Fluctuations



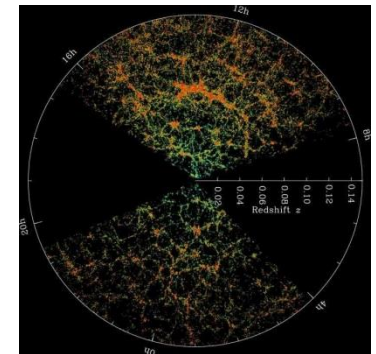
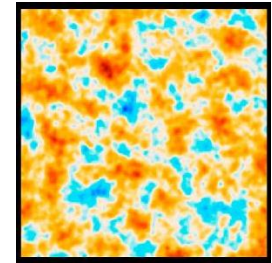
$\delta\varphi$

Primordial
Scalar Curvature
Perturbations



ζ
(curvature)

CMB, Matter,
Galactic Densities

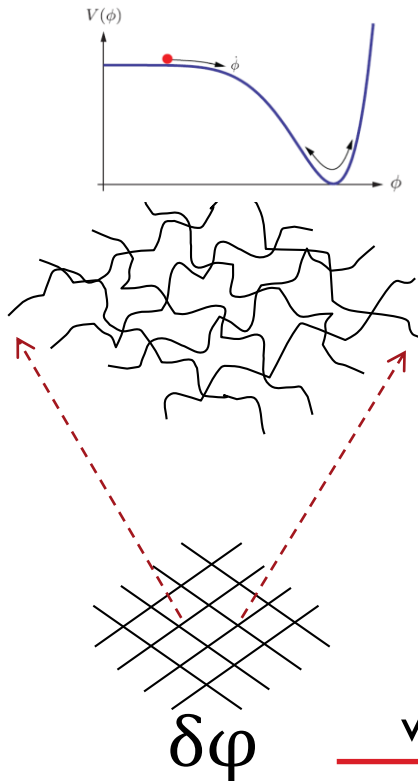


ΔT_{CMB}
 $\Delta\rho, \Delta N_{\text{galaxies}}$

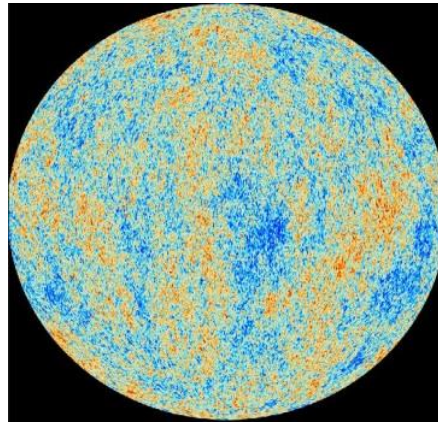
Quantum fluctuations seed structure/scalar perturbations (ζ)

7

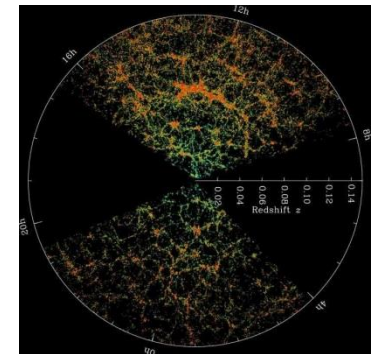
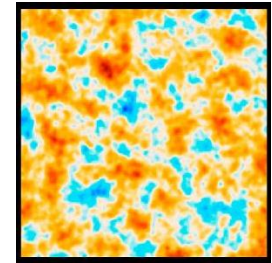
Inflaton Fluctuations



Primordial Scalar Curvature Perturbations



CMB, Matter, Galactic Densities



$\delta\phi$

vary action

ζ

(curvature)

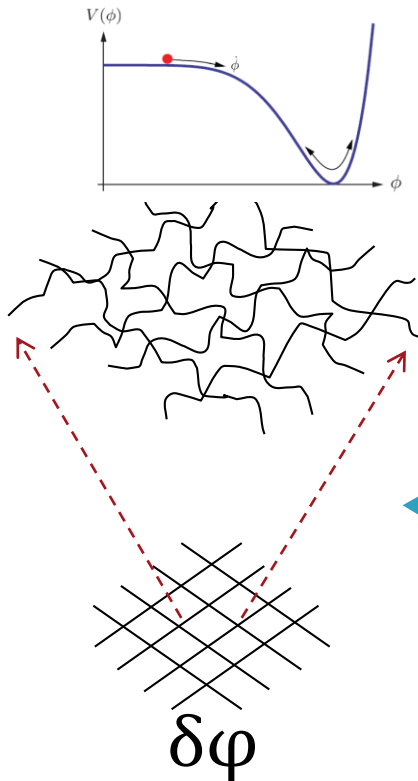
transfer functions

ΔT_{CMB}
 $\Delta\rho, \Delta N_{\text{galaxies}}$

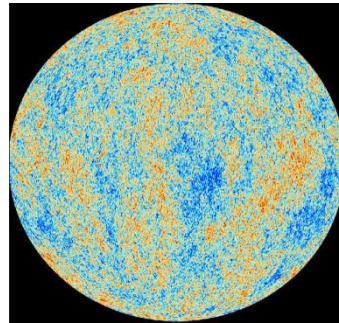
Quantum fluctuations seed structure/scalar perturbations (ζ)

8

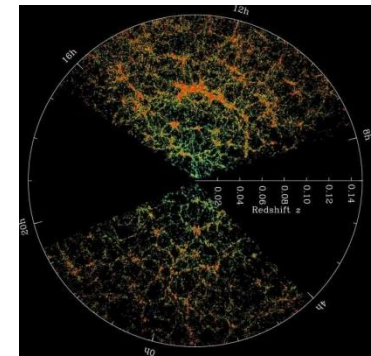
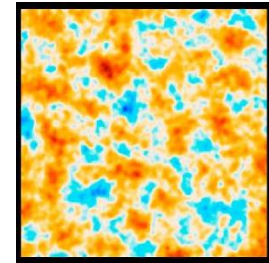
Inflaton Fluctuations



Primordial Scalar Curvature Perturbations



CMB, Matter, Galactic Densities



Use ζ to find $\delta\phi$

Use Obs. to find ζ

ζ
(curvature)

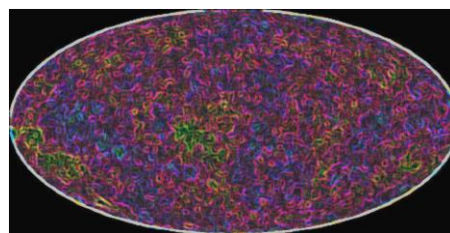
ΔT_{CMB}
 $\Delta\rho, \Delta N_{\text{galaxies}}$

If the universe inflated for >60 e-folds, separate Hubble-sized patches have different perturbation histories (ζ_L).

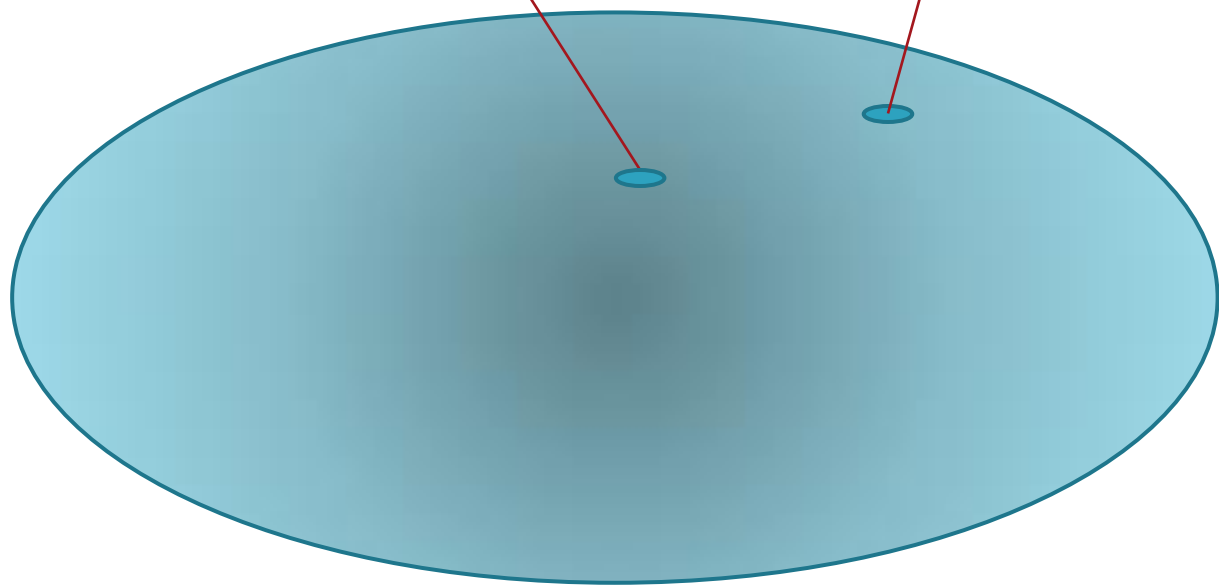
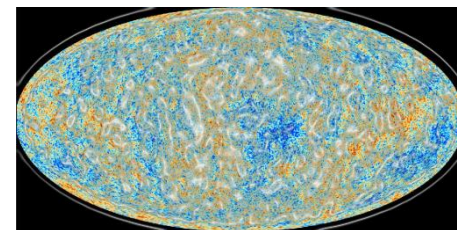
9

$$\zeta_L \equiv \langle \zeta \rangle_{\text{Hubble Volume}}$$

$$\zeta_{S1} = \zeta - \zeta_{L1}$$



$$\zeta_{S2} = \zeta - \zeta_{L2}$$



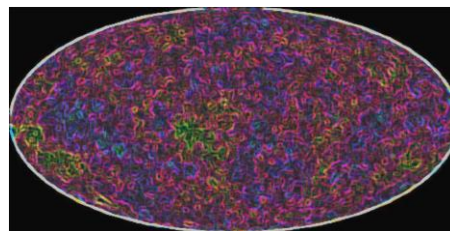
If the universe inflates for >60 e-folds, separate Hubble-sized patches will have different perturbation histories (ζ_L).

10

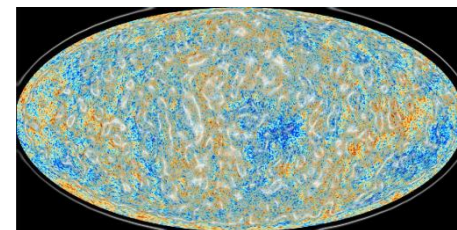
$$\zeta_L \equiv \langle \zeta \rangle_{\text{Hubble Volume}}$$

Different long
wavemode freeze-out
histories in the super-
Hubble volume lead to
different values for ζ_L ,
and different statistics in
subvolumes.

$$\zeta_{S1} = \zeta - \zeta_{L1}$$



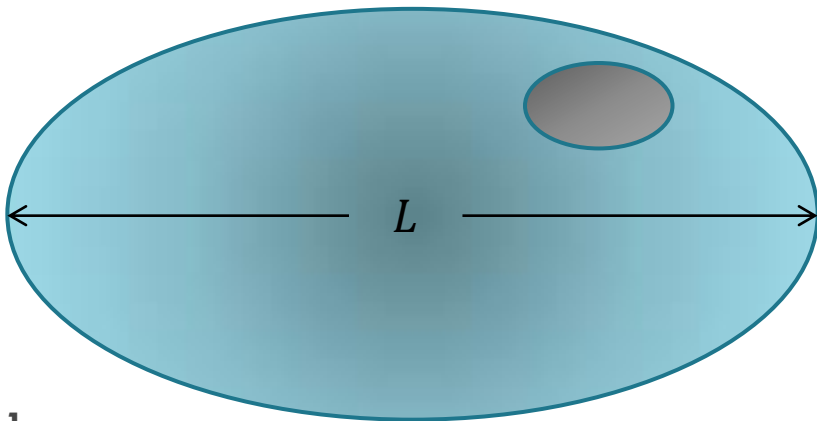
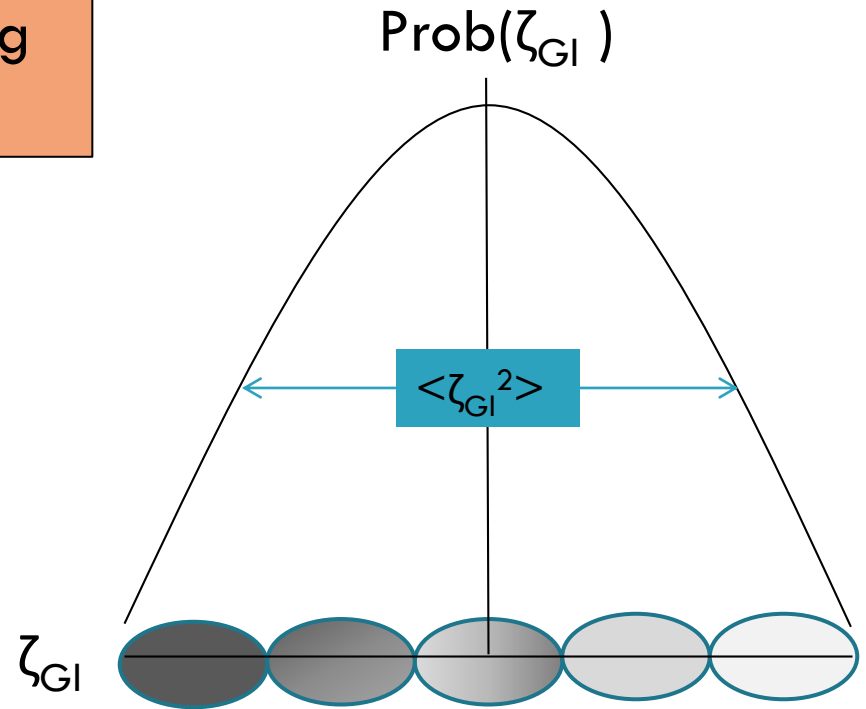
$$\zeta_{S2} = \zeta - \zeta_{L2}$$



Reheated Hubble Patches

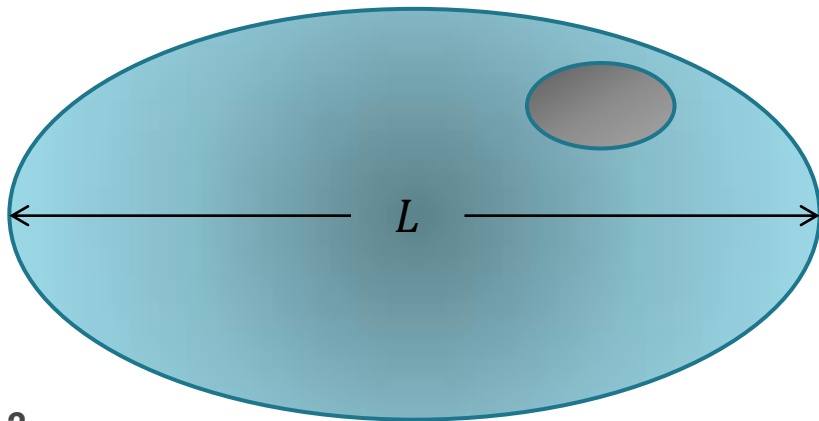
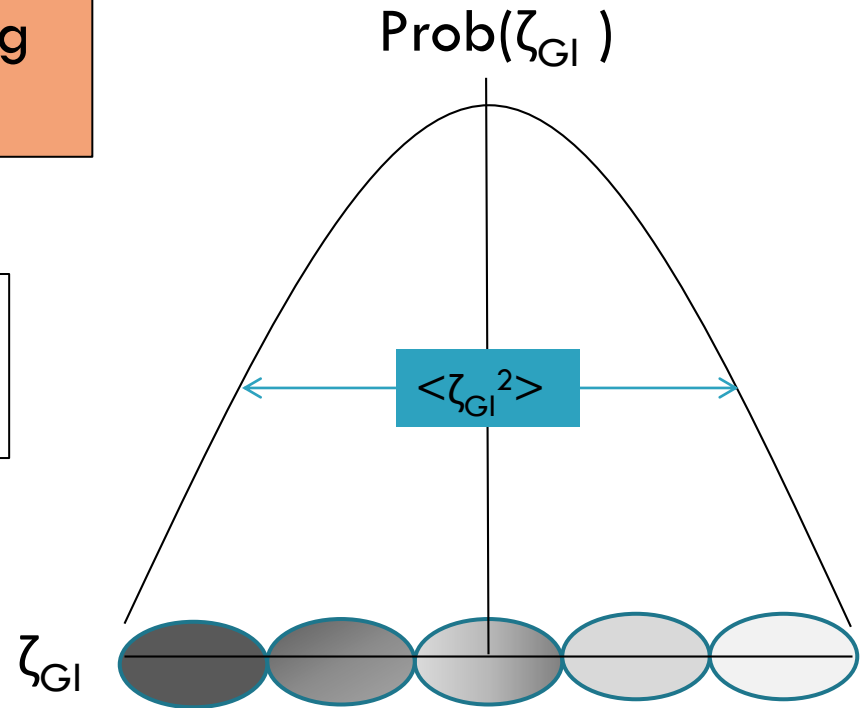
Super-Hubble Volume

What determines the mode biasing of the Hubble Volume?



What determines the mode biasing of the Hubble Volume?

$$\langle \zeta_{\text{GI}}^2 \rangle = \int_{\frac{1}{L}}^{H_0} \frac{d^3k}{(2\pi)^3} P_{\text{GI}}(k)$$

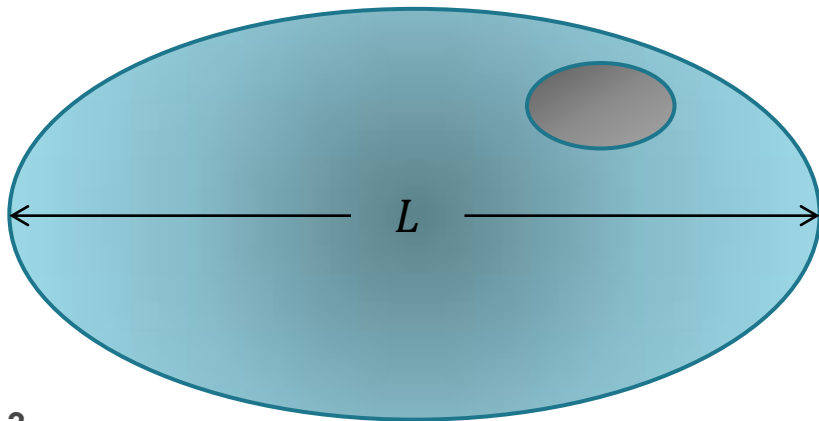
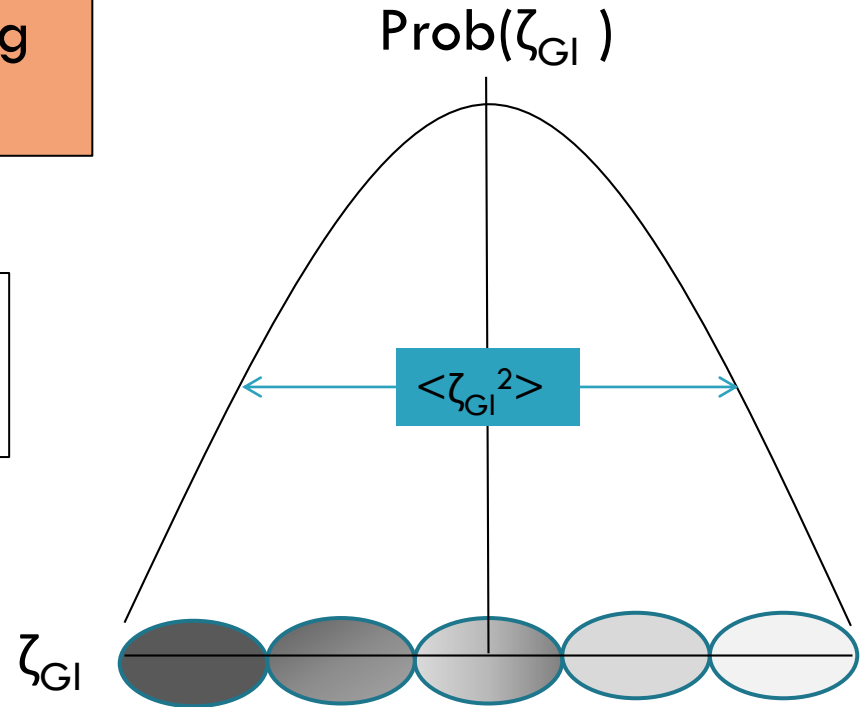


The probability of the Gaussian part of super-Hubble modes, ζ_{GI} takes different values depending on the super-Hubble power spectrum. This results in a distribution of over and under-densities in Hubble volumes.

What determines the mode biasing of the Hubble Volume?

$$\langle \zeta_{\text{GI}}^2 \rangle = \int_{\frac{1}{L}}^{H_0} \frac{d^3k}{(2\pi)^3} P_{\text{GI}}(k)$$

The super-Hubble power spectrum does – unobservable, but implied in model building.

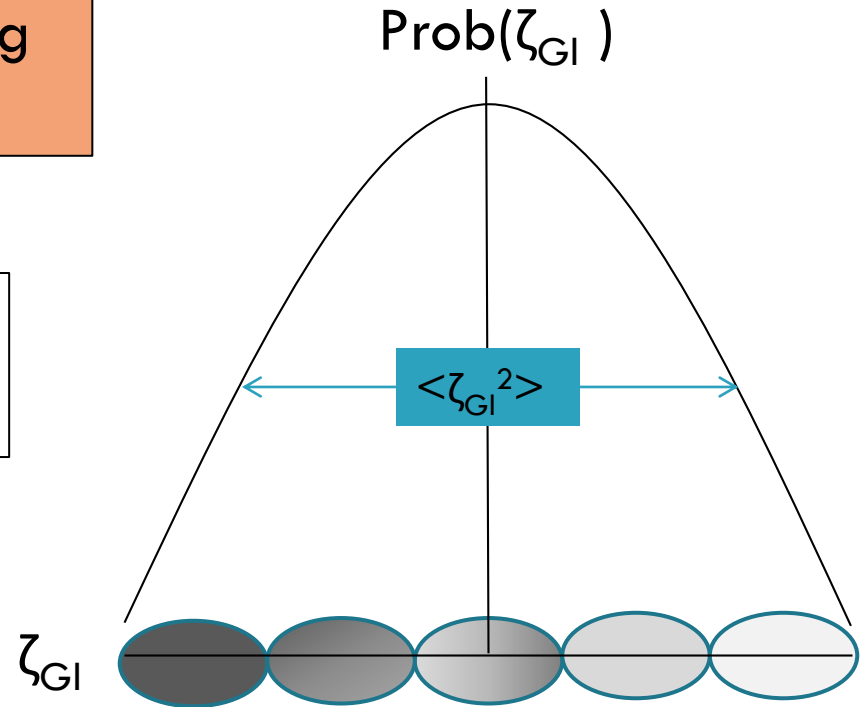
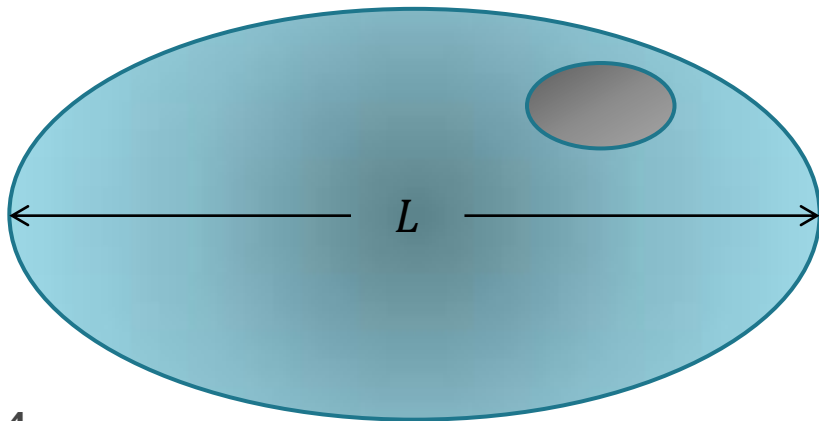


The probability of the Gaussian part of super-Hubble modes, ζ_{GI} takes different values depending on the super-Hubble power spectrum. This results in a distribution of over and under-densities in Hubble volumes.

What determines the mode biasing of the Hubble Volume?

$$\langle \zeta_{\text{GI}}^2 \rangle = \int_{\frac{1}{L}}^{H_0} \frac{d^3 k}{(2\pi)^3} P_{\text{GI}}(k)$$

We sum over superhorizon modes. This term will be constant in any Hubble volume.

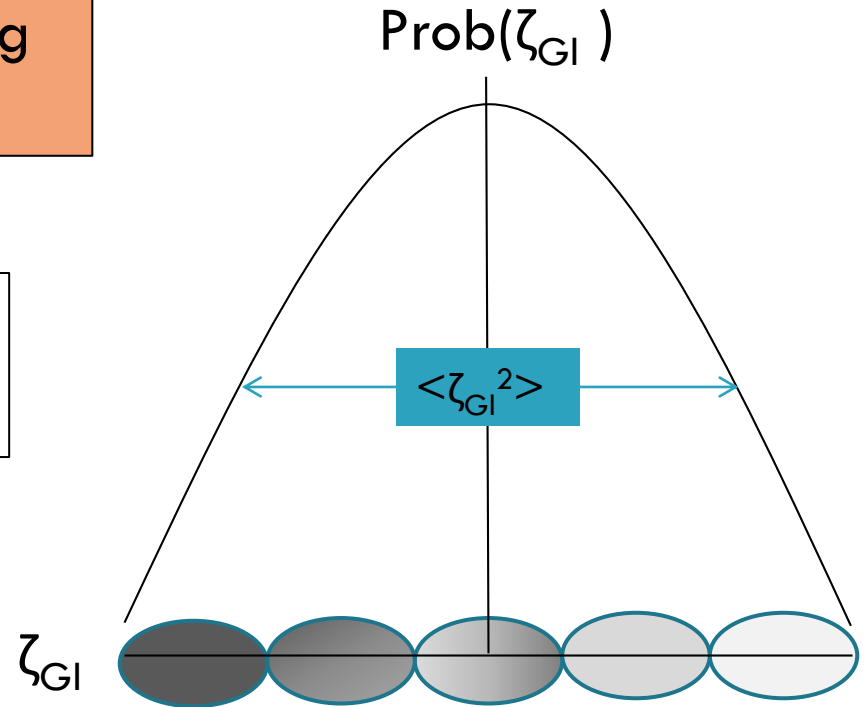
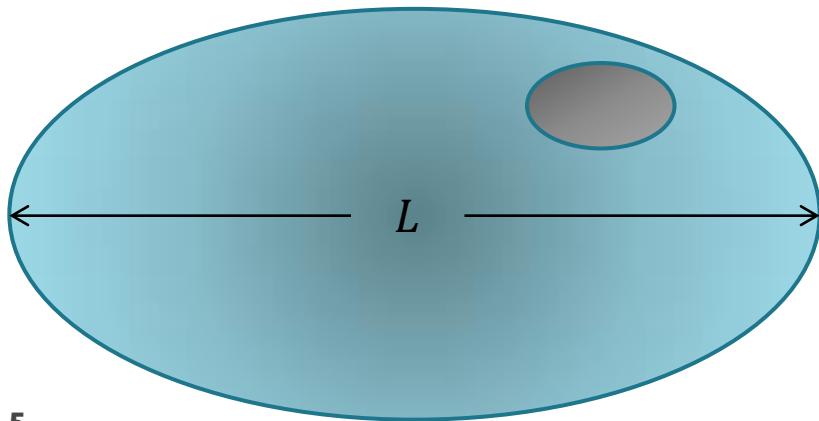


The probability of the Gaussian part of super-Hubble modes, ζ_{GI} takes different values depending on the super-Hubble power spectrum. This results in a distribution of over and under-densities in Hubble volumes.

What determines the mode biasing of the Hubble Volume?

$$\langle \zeta_{\text{GI}}^2 \rangle = \int_{\frac{1}{L}}^{H_0} \frac{d^3 k}{(2\pi)^3} P_{\text{GI}}(k)$$

$\langle \zeta_{\text{GI}}^2 \rangle = P_{\text{G}} N$ for $n_s=1$
(N is the number of superhorizon e-folds)



The probability of the Gaussian part of super-Hubble modes, ζ_{GI} takes different values depending on the super-Hubble power spectrum. This results in a distribution of over and under-densities in Hubble volumes.

Curvature Perturbations, Background vs. Foreground: Only see an effect for non-Gaussian Statistics.

16

Local ansatz expands (ζ) into Gaussian and non-G. pieces

$$\zeta = \zeta_G + \frac{3}{5}f_{\text{NL}} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

Apply a long/short wavelength split

$$\zeta_G = \zeta_{\text{Gl}} + \zeta_{\text{Gs}}$$

$$\zeta_s = \zeta_{\text{Gs}} \left(1 + \frac{6}{5}f_{\text{NL}} \zeta_{\text{Gl}} \right) + \dots$$

Hubble-scale ζ_s acquires a term dependent on Hubble-scale non-Gaussianity and super-Hubble power spectrum $\langle \zeta_{\text{Gl}}^2 \rangle$

Curvature Perturbations, Background vs. Foreground: Only see an effect for non-Gaussian Statistics.

17

Local ansatz expands (ζ) into Gaussian and non-G. pieces

$$\zeta = \zeta_G + \frac{3}{5}f_{\text{NL}} (\zeta_G^2 - \langle \zeta_G^2 \rangle)$$

Apply a long/short wavelength split

$$\zeta_G = \zeta_{\text{G1}} + \zeta_{\text{Gs}}$$

$$\zeta_s = \zeta_{\text{Gs}} \left(1 + \frac{6}{5}f_{\text{NL}} \zeta_{\text{G1}} \right) + \dots$$

constant term in Hubble volume

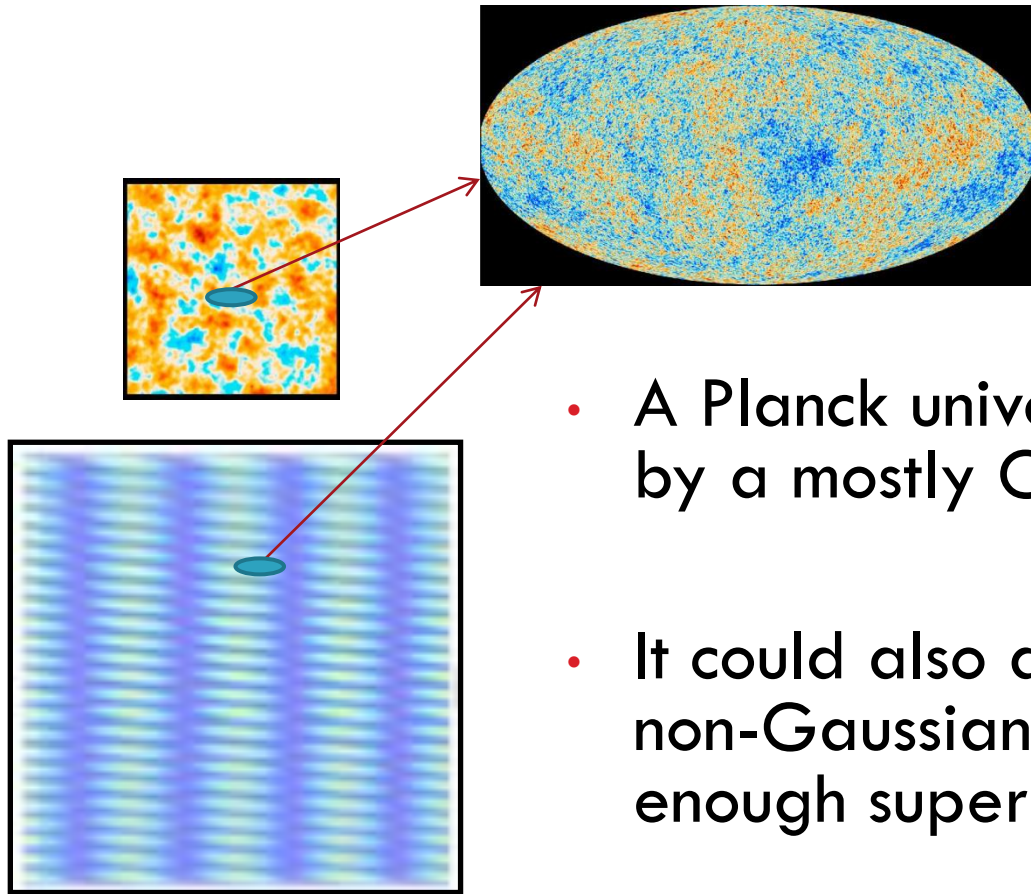
Hubble-scale ζ_s acquires a term dependent on Hubble-scale non-Gaussianity and super-Hubble power spectrum $\langle \zeta_{\text{G1}}^2 \rangle$

We see that $f_{\text{NL}}^{\text{local}}$ Hubble volumes will vary in super-Hubble backgrounds.

This will transfer to variance in the power spectrum and spectral index from Hubble volume to volume in a universe with any local non-gaussianity.

Cosmic Variance of the Power Spectrum

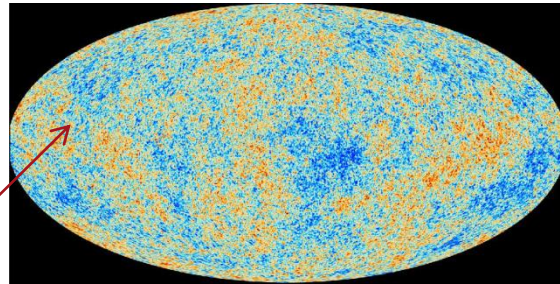
19



- A Planck universe could be sourced by a mostly Gaussian superhorizon.
- It could also arise in a very “local” non-Gaussian superhorizon with enough superhorizon e-folds.

Cosmic Variance of the Power Spectrum

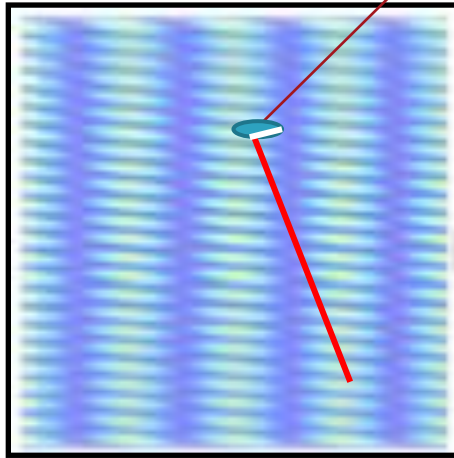
20



multifield, multiloop analysis

φ, σ scalar fields

$$\xi = P_\sigma / P_\varphi$$

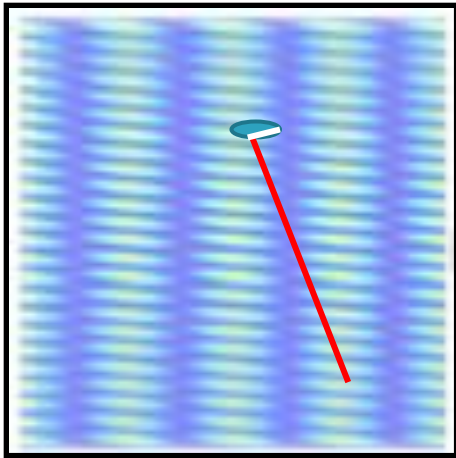


$$P_\zeta^{\text{obs}}(k) = P_\phi(k) + \left(1 + \frac{12}{5} f_{\text{NL}}(k) \underline{\sigma_{Gl}} + \frac{36}{25} f_{\text{NL}}^2(k) \underline{\sigma_{Gl}^2}\right) P_\sigma(k) + \frac{18}{25} f_{\text{NL}}^2(k) \int_{M^{-1}}^{k_{\text{max}}} \frac{d^3 p}{(2\pi)^3} P_\sigma(p) P_\sigma(|\mathbf{k} - \mathbf{p}|)$$

$$f_{\text{NL}}^{\text{CMB}} = \frac{f_{\text{NL}}(k_s) \xi_m(k_s) \xi_m(k_l) \left(1 + \frac{6}{5} f_{\text{NL}}(k_s) \underline{\sigma_{Gl}}\right) (k_s \rightarrow k_l)}{\left[1 + \frac{36}{25} f_{\text{NL}}^2(k_s) \langle \sigma_G^2(k_s) \rangle + \frac{12}{5} \xi_m(k_s) f_{\text{NL}}(k_s) \left(\underline{\sigma_{Gl}} + \frac{3}{5} f_{\text{NL}}(k_s) (\underline{\sigma_{Gl}^2} - \langle \sigma_{Gl}^2 \rangle)\right)\right] [k_s \rightarrow k_l]}$$

Cosmic Variance of the Power Spectrum

21



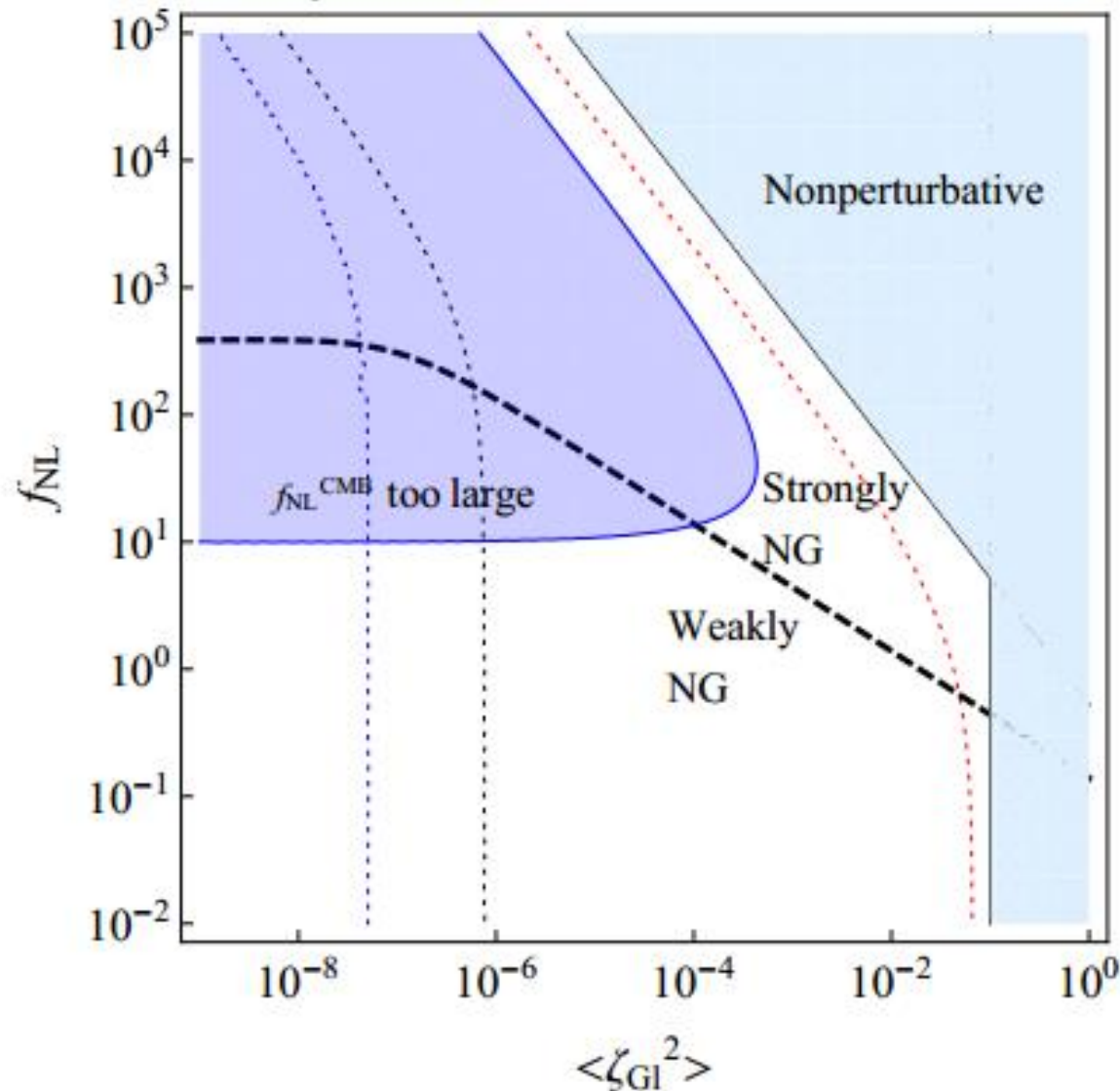
Momentum space diagrams show IR “leg” modes along with standard loop corrections.

$$P_{\zeta}^{\text{obs}}(k) = P_{\phi}(k) + \left(1 + \frac{12}{5} f_{\text{NL}}(k) \sigma_{\text{GI}} + \frac{36}{25} f_{\text{NL}}^2(k) \sigma_{\text{GI}}^2\right) P_{\sigma}(k) + \frac{18}{25} f_{\text{NL}}^2(k) \int_{M^{-1}}^{k_{\text{max}}} \frac{d^3 p}{(2\pi)^3} P_{\sigma}(p) P_{\sigma}(|\mathbf{k} - \mathbf{p}|)$$

$$P_{\zeta, \text{NG}}(k) = \begin{array}{c} \text{diagram with green line } \phi \\ \phi \end{array} + \begin{array}{c} \text{diagram with black line } \sigma \\ \sigma \end{array} + \begin{array}{c} \text{diagram with red dashed line } k \rightarrow L^{-1} \\ k \rightarrow L^{-1} \end{array}$$

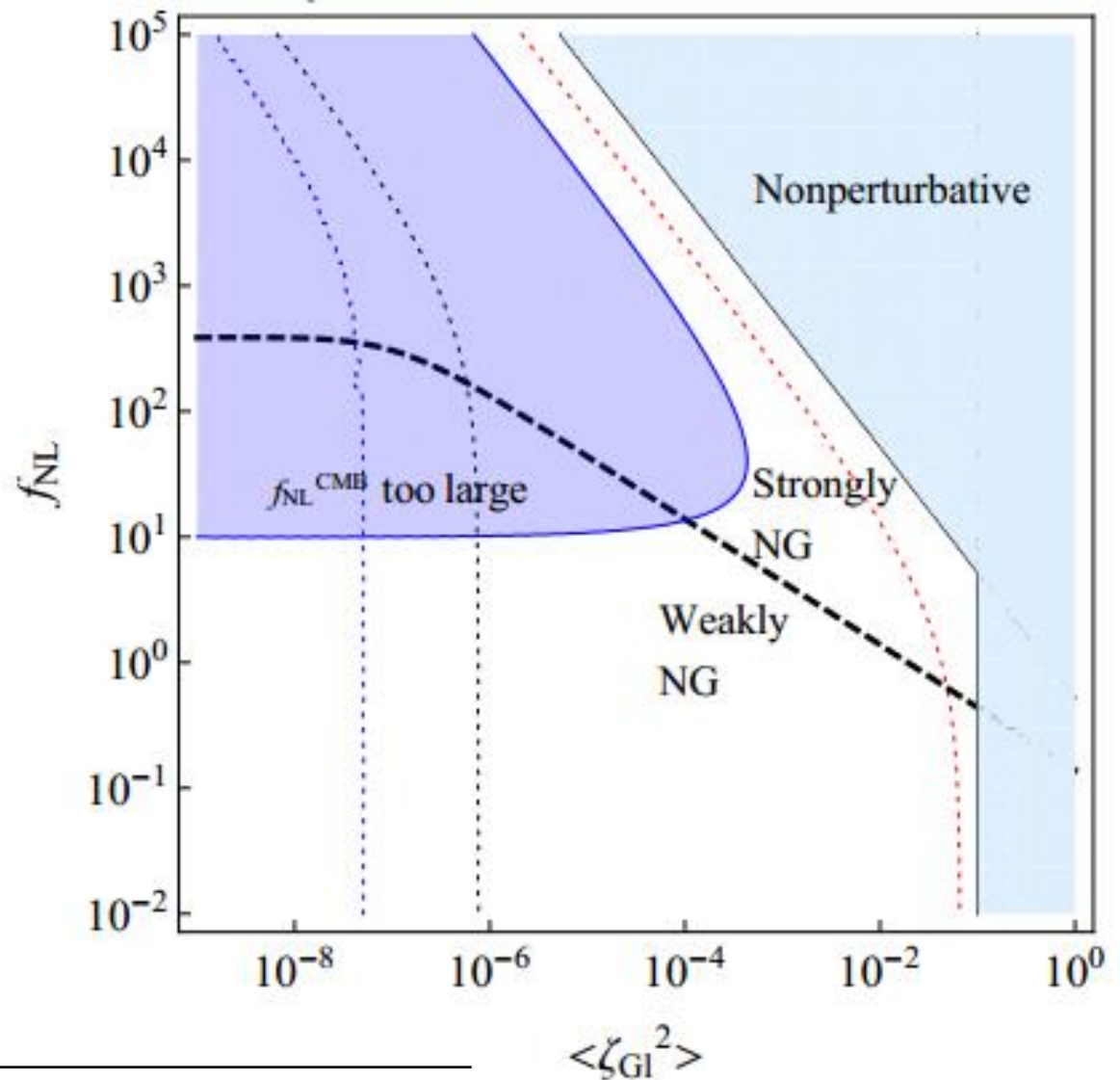
$$P_{\zeta, \text{NG}}^{\text{obs}}(k) = \begin{array}{c} \text{diagram with green line } \phi \\ \phi \end{array} + \begin{array}{c} \text{diagram with black line } \sigma \\ \sigma \end{array} + 2 \begin{array}{c} \text{diagram with red dashed line } M^{-1} \rightarrow L^{-1} \\ M^{-1} \rightarrow L^{-1} \end{array} + \begin{array}{c} \text{diagram with red dashed line } k \rightarrow M^{-1} \\ k \rightarrow M^{-1} \end{array} + \begin{array}{c} \text{diagram with red dashed line } M^{-1} \rightarrow L^{-1} \\ M^{-1} \rightarrow L^{-1} \end{array},$$

The super-Hubble power and bispectrum can differ from the Hubble power and bispectrum.



$$\langle \zeta_{\text{GI}}^2 \rangle = \int_{\frac{1}{L}}^{H_0} \frac{d^3 k}{(2\pi)^3} P_{\text{GI}}(k)$$

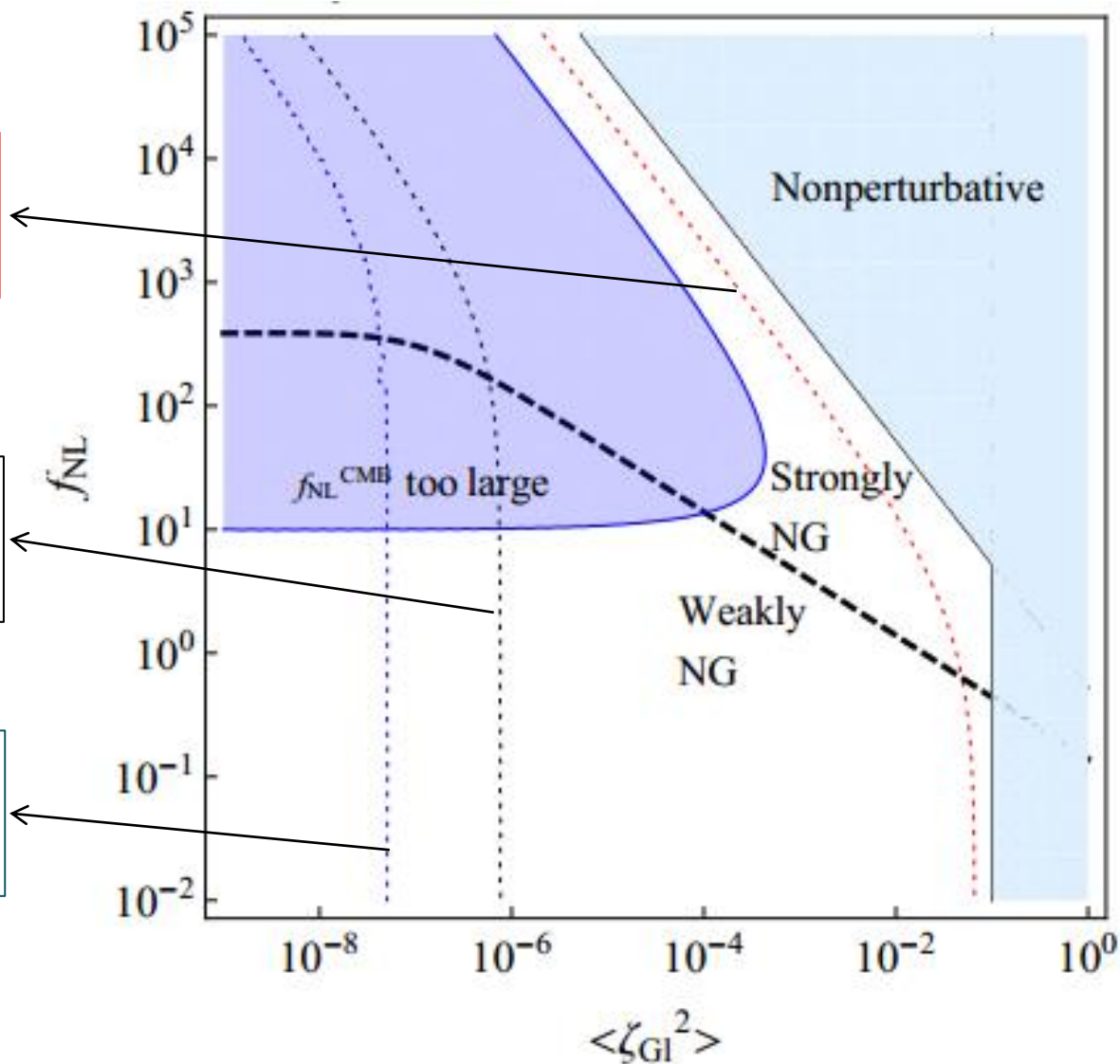
Sum over long
wavemode
curvature perturbations.



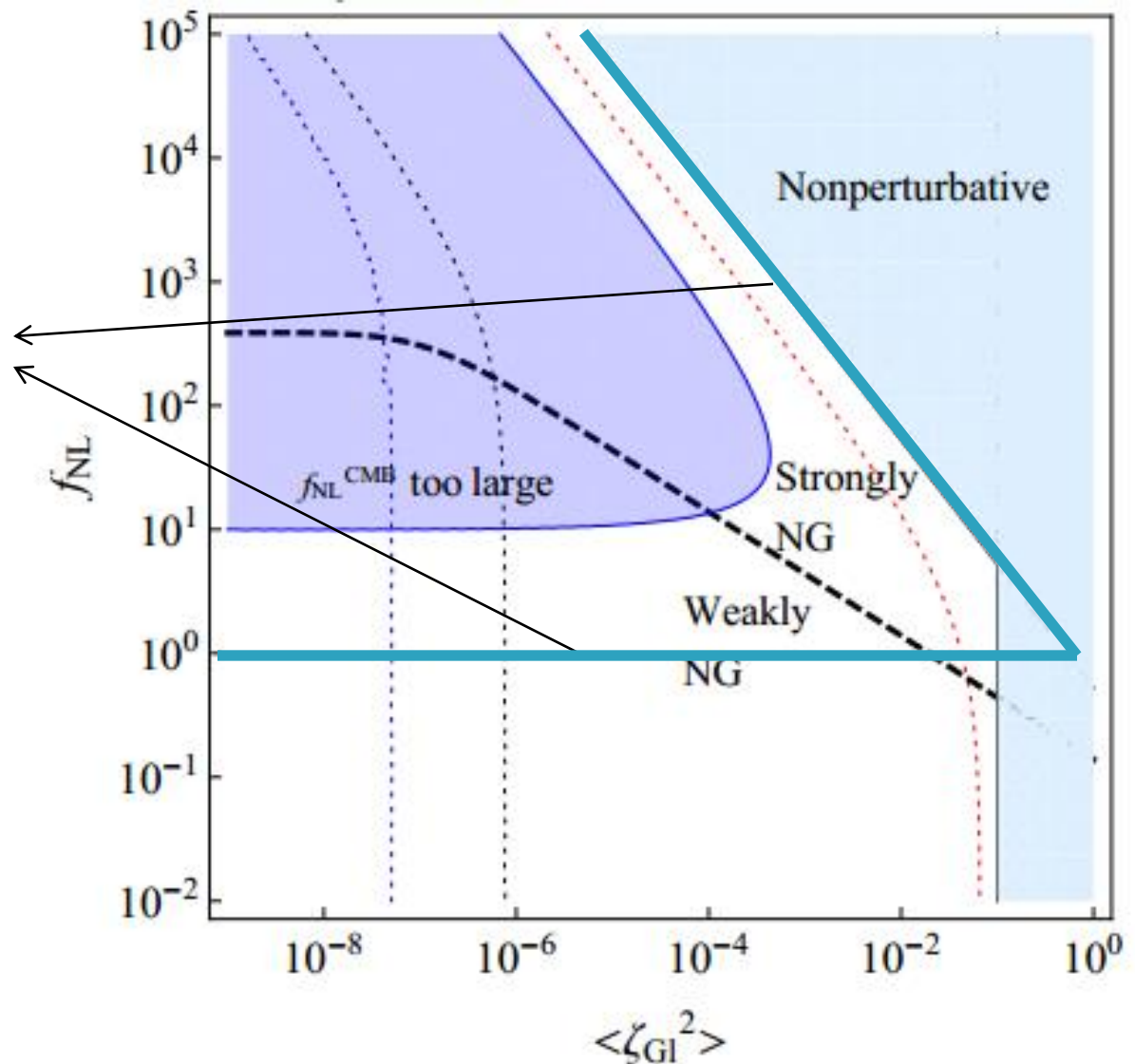
$N=350$ consistent with $n_s=0.96$

$N=350$ consistent with $n_s=1$

$N=350$ consistent with $n_s=1.02$



A bound of $f_{\text{NL}}^{\text{local}} < 1$ would rule out this parameter space. So we can rule out cosmic variance, nature permitting.



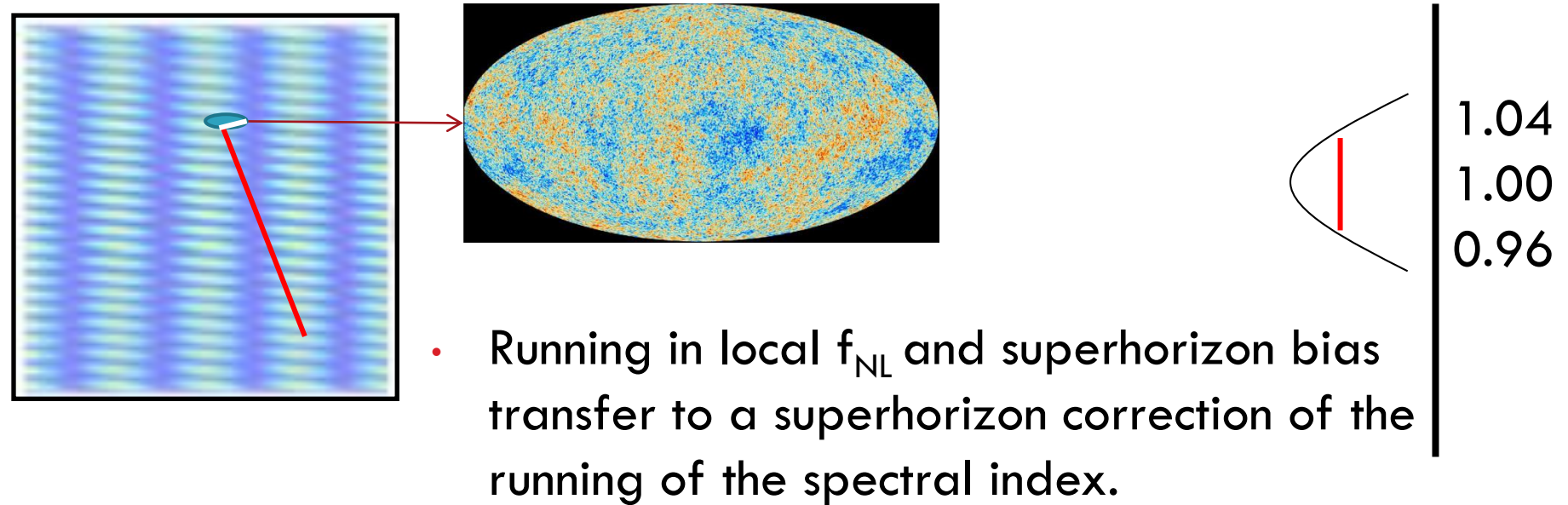
So Planck's $f_{\text{NL}}^{\text{local}} < 15$ tell us that local non-Gaussianity is very small ...

So Planck's $f_{\text{NL}}^{\text{local}} < 15$ tell us that local non-Gaussianity is very small ...

Or the universe is much larger than what we observe, which suppresses this number in the Hubble volume.

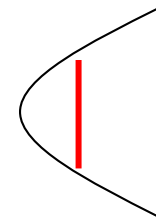
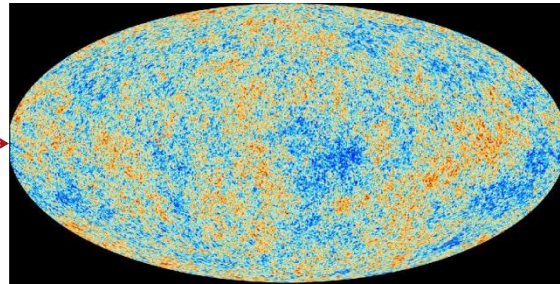
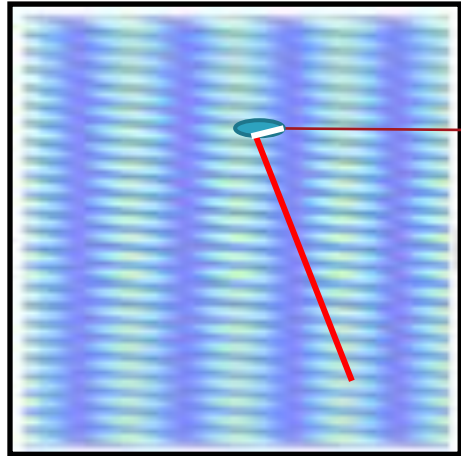
Cosmic Variance of the Spectral Index

28



Cosmic Variance of the Spectral Index

29

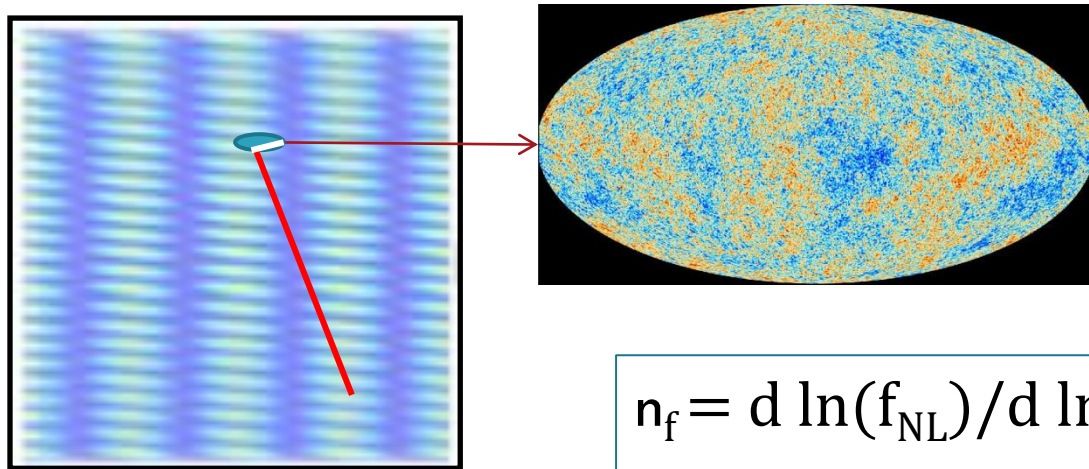


1.04
1.00
0.96

- Running in local f_{NL} and superhorizon bias transfer to a superhorizon correction of the running of the spectral index.
- When measuring power spectrum, implicitly measuring squeezed three-point function with long leg being an under/overdensity of superhorizon wavemodes...so if squeezed non-Gaussianity is running, power spectrum will be seen to run too.

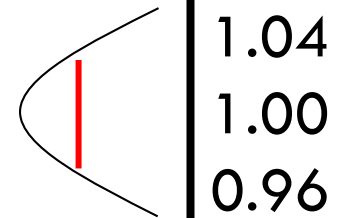
Cosmic Variance of the Spectral Index

30



$$n_f = d \ln(f_{NL}) / d \ln(k)$$

$$n_f^{(m)} = d \ln(\xi) / d \ln(k)$$



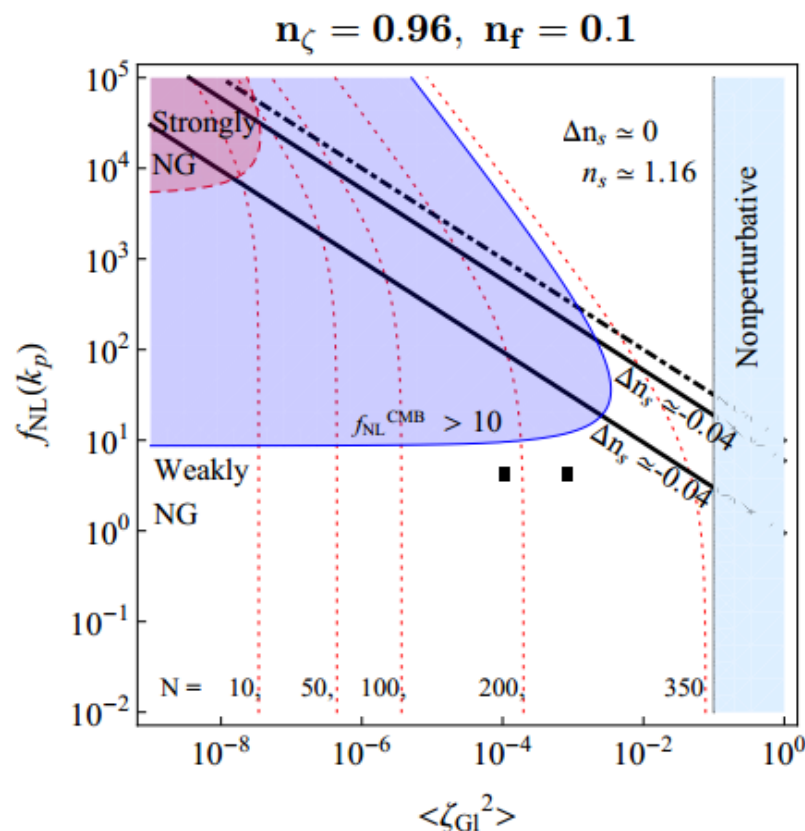
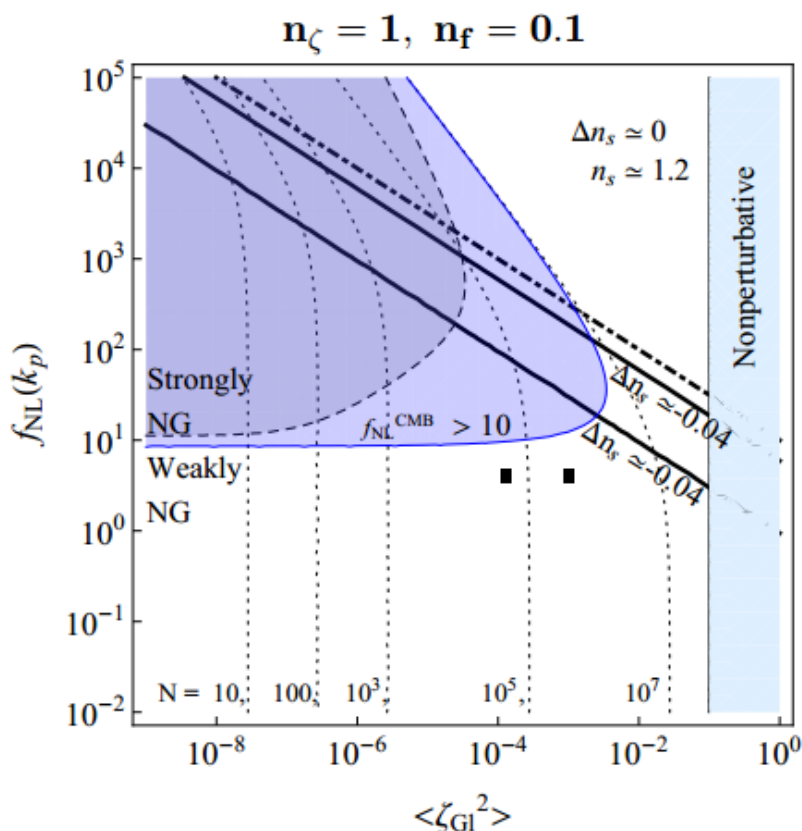
$$\Delta n_s(k) \equiv n_s^{\text{obs}} - n_s$$

$$= \frac{\frac{12}{5} \xi_m f_{NL} \left(\underline{\sigma_{GI}}(n_f^{(m)} + X_1 n_f) + \frac{3}{5} f_{NL} (\underline{\sigma_{GI}^2} - \langle \sigma_{GI}^2 \rangle) (n_f^{(m)} + X_2 n_f) \right)}{1 + \frac{36}{25} f_{NL}^2 \langle \sigma_G^2(k) \rangle + \frac{12}{5} \xi_m f_{NL} \left(\underline{\sigma_{GI}} + \frac{3}{5} f_{NL} (\underline{\sigma_{GI}^2} - \langle \sigma_{GI}^2 \rangle) \right)},$$

$$X_1 \equiv \frac{1 - \frac{36}{25} f_{NL}^2 \langle \sigma_G^2(k) \rangle}{1 + \frac{36}{25} f_{NL}^2 \langle \sigma_G^2(k) \rangle}, \quad X_2 \equiv \frac{2}{1 + \frac{36}{25} f_{NL}^2 \langle \sigma_G^2(k) \rangle}$$

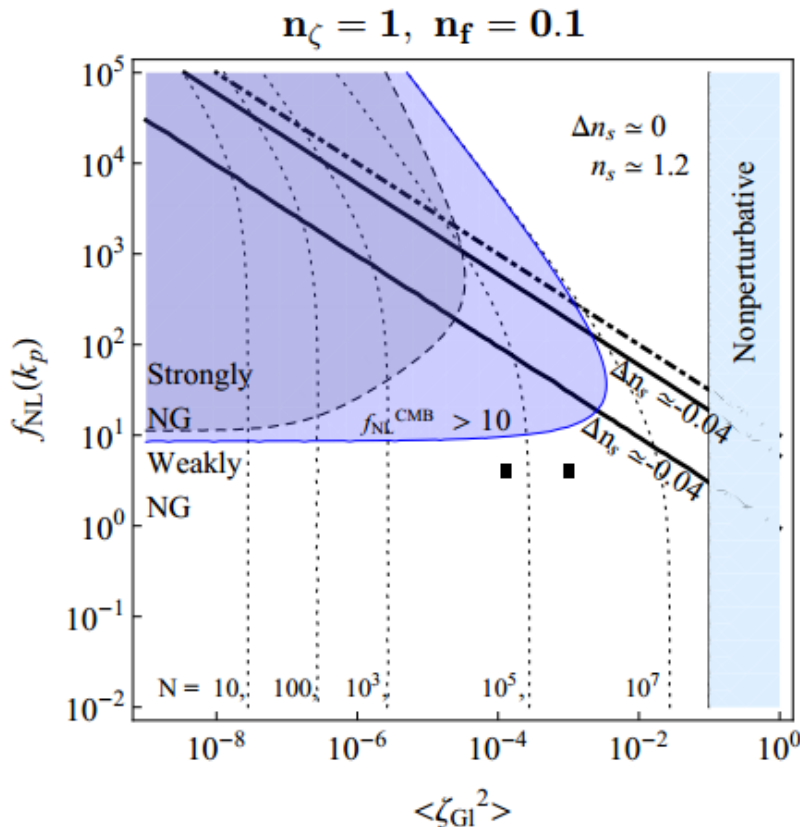
Cosmic Variance of the Spectral Index

31

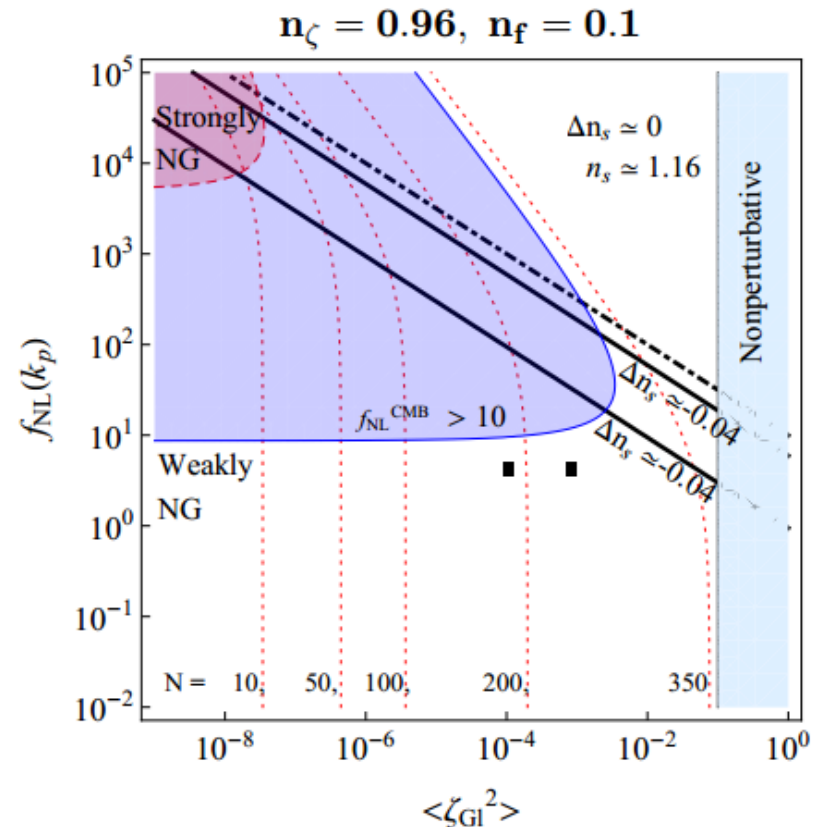


Cosmic Variance of the Spectral Index

32



$$n_f = d \ln(f_{\text{NL}}) / d \ln(k)$$

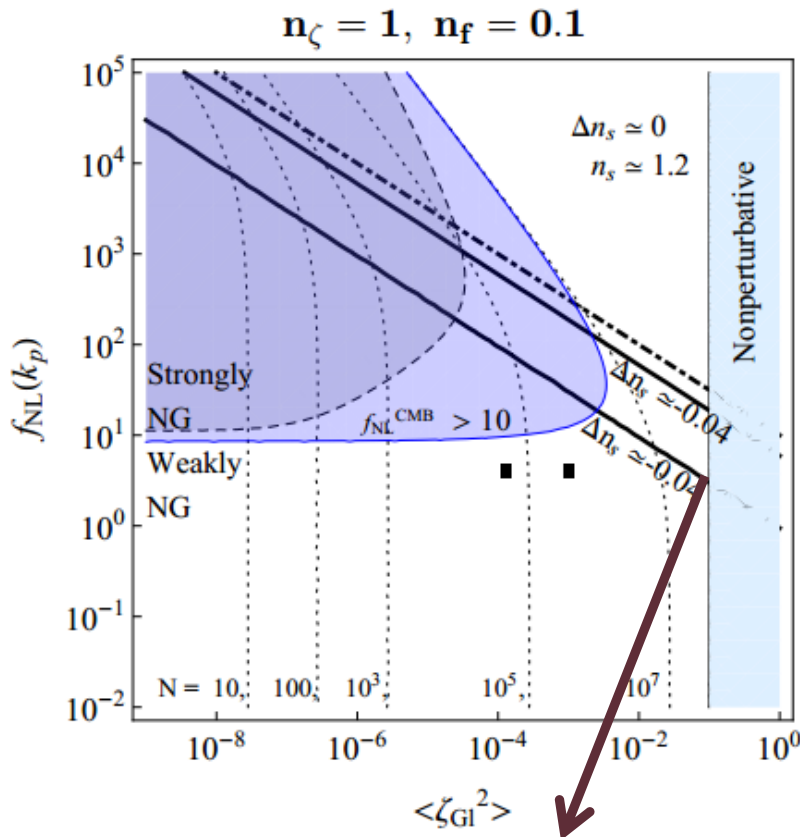


$$n_\zeta = d \ln(P_{\zeta_{\text{GI}}}) / d \ln(k)$$

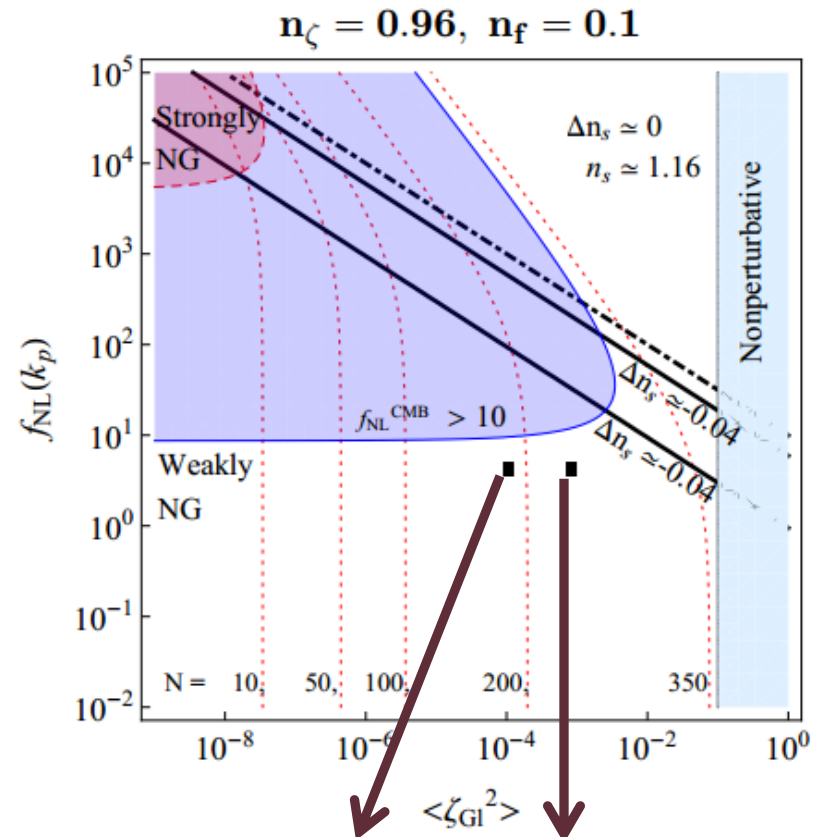
$\sim n_s$ for weakly non-Gaussian statistics.

Cosmic Variance of the Spectral Index

33



n_s could be flat for the
correction inflation model

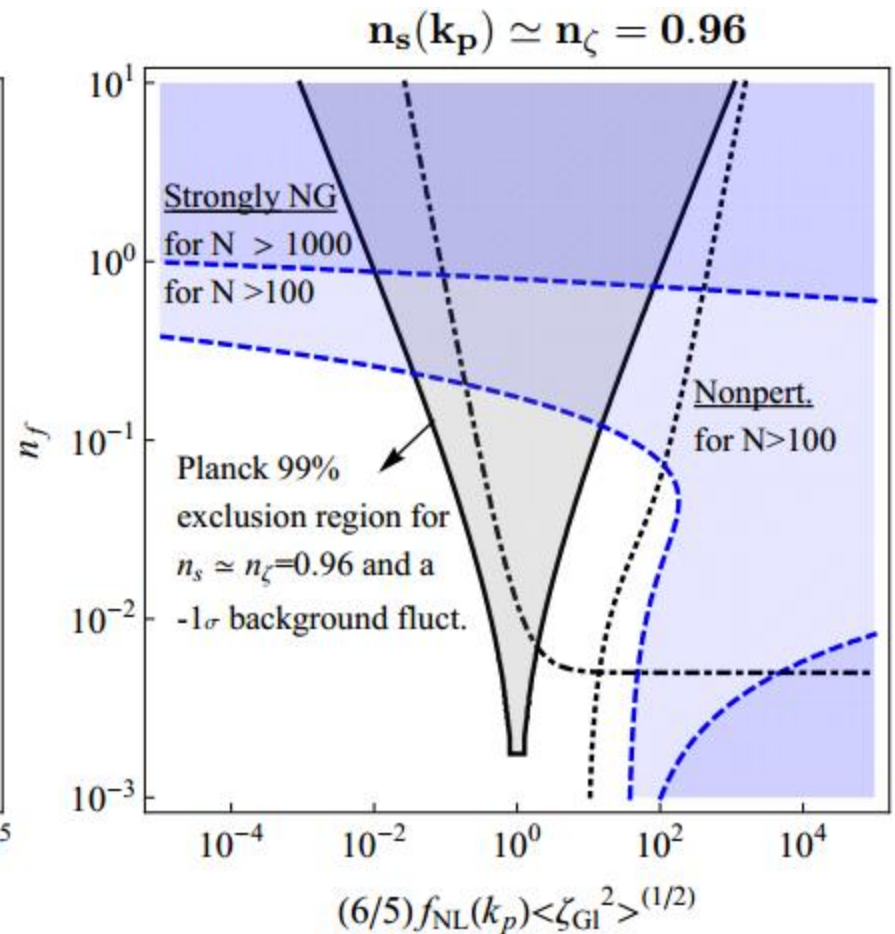
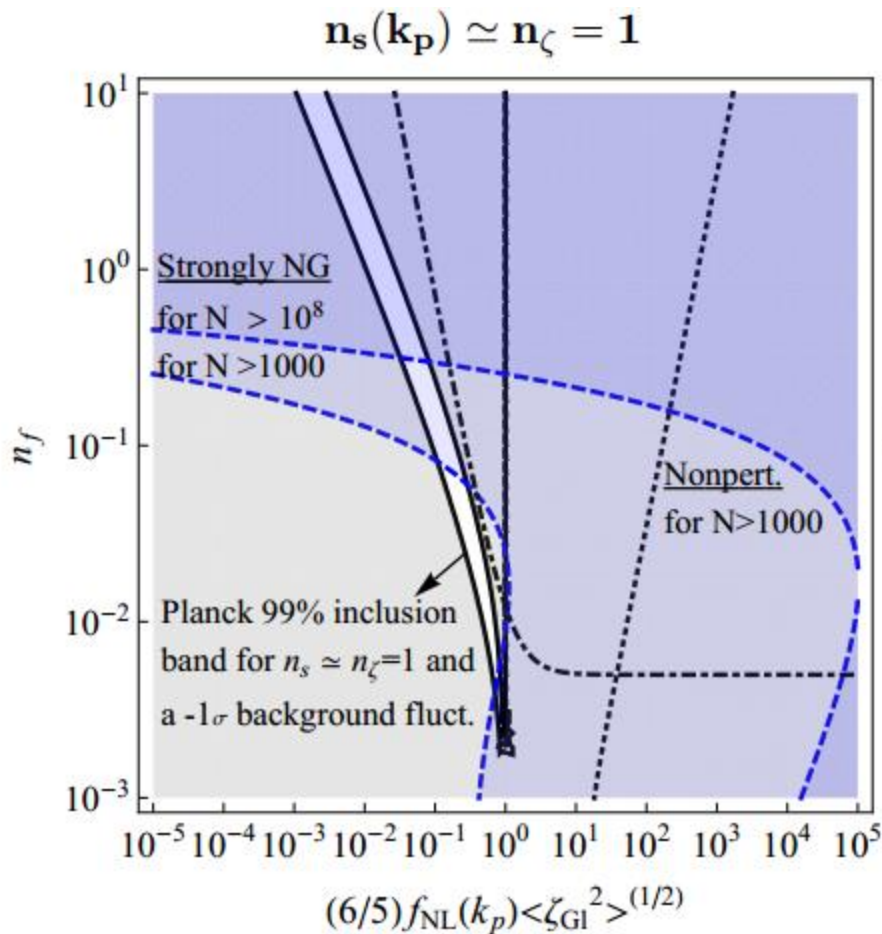


n_s could be smeared around
0.96 by cosmic variance

So either Planck tell us that the spectral index is exactly 0.9608 ± 0.0070 ...

So either Planck tell us that the spectral index is exactly 0.9608 ± 0.0070 ...

Or the universe is much larger than what we observe and there is running local non-Gaussianity...



- A perfectly flat spectral index can be reconciled with a super-Hubble volume.
- On the other hand, $n_s = 0.96$ is actually inconsistent with a large superhorizon background and significant local non-Gaussianity.

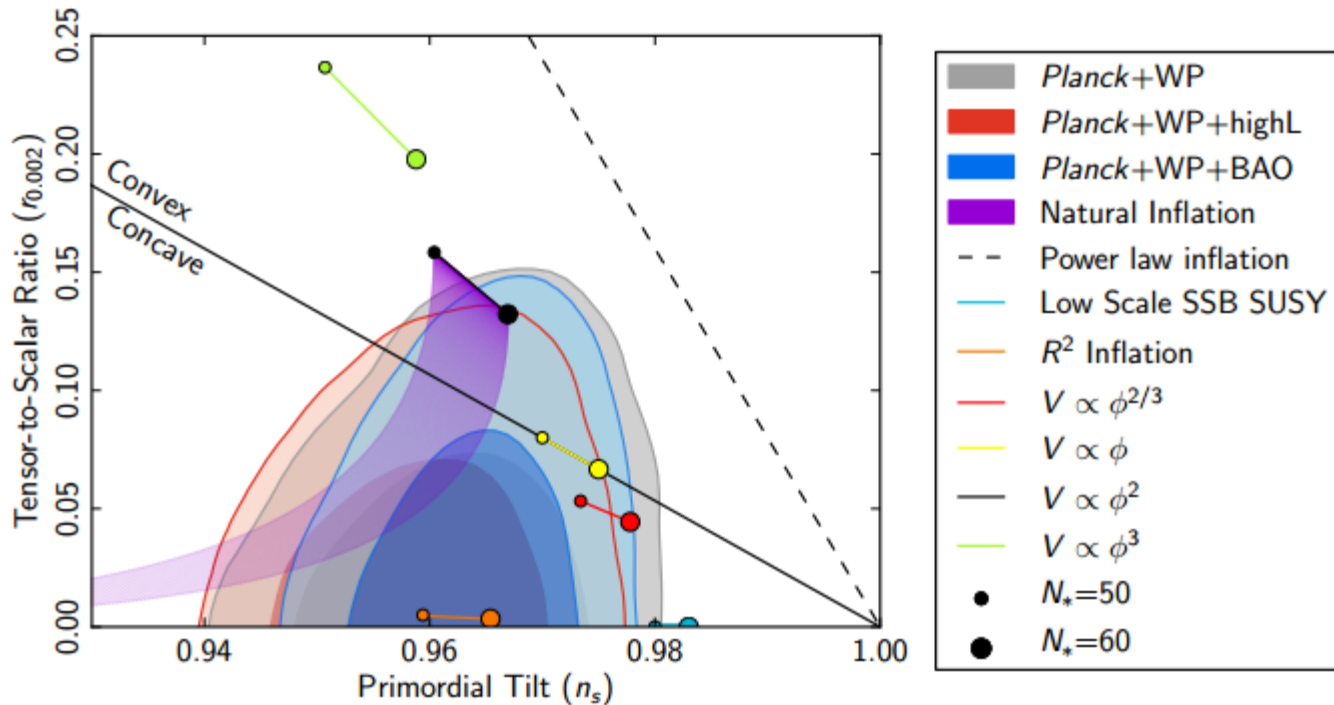
Simple Single Source Examples of Super Cosmic Variance

37

- Constant $f_{\text{NL}}^{\text{local}} = 5$. We observe $f_{\text{NL}}^{\text{local}} = 8$ if, for example, the spectral index is a constant $n_s = 0.96$ over about 200 extra e-folds of inflation and our Hubble patch sits on top of a 2-sigma under density.
- Local non-Gaussianity with constant $f_{\text{NL}}^{\text{local}} = 15$. We observe $f_{\text{NL}}^{\text{local}} = 11$ if, for example, the spectral index is a constant $n_s = 0.96$ over about 150 extra e-folds of inflation and our Hubble patch sits on top of a 2-sigma over-density.
- Scale-dependent non-Gaussianity with $f_{\text{NL}}(k_p) = -2$, $n_f = 0.04$, $n_\zeta = 0.93$, and $n_s = 0.935$. We would observe $f_{\text{NL}}(k_p) = -1$ and $n_s^{\text{obs}} = 0.956$ if our Hubble patch sits on top of a 2-sigma under density in a volume with about 190 extra e-folds.
- Scale-dependent non-Gaussianity with $f_{\text{NL}}(k_p) = 20$, $n_f = 0.03$, $n_\zeta = 0.95$, and $n_s = 1.005$. We would observe $f_{\text{NL}}(k_p) = 2.5$ and $n_s^{\text{obs}} = 0.975$ if our Hubble patch sits on top of a 0.2-sigma over density in a volume with about 280 extra e-folds.

Future Work

38



- The assumption of 60 e-folds should be re-evaluated.
- Bayesian inference methods for evaluating models should take into account super-cosmic variance.

Future Work

39

- Constrain standard inflationary models assuming superhorizon numbers of e-folds.
- Analyze impact on other CMB observables: tensors, scale of inflation...
- Use universal inflation formalism to constrain classes of inflation models in terms of e-folds.
- Re-evaluate models which may have previously been rejected for predictions of $n_s \ll 0.96$ or $0.96 \ll n_s$.

J.B., Kumar, Nelson, Shandera 1307.5083 (JCAP)

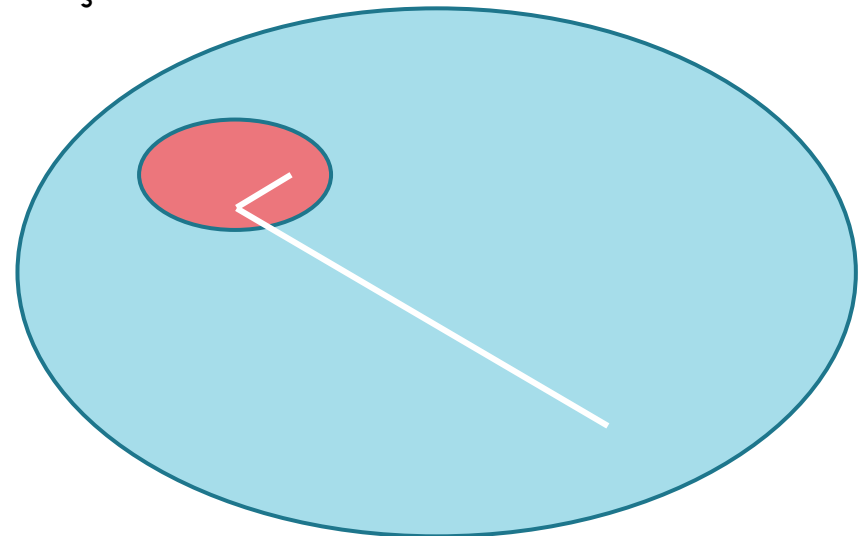
Nelson, Shandera 1212.4550 (PRL)

Nurmi, Byrnes, Tasinato 1301.3128

LoVerde, Nelson, Sandera 1303.3549 (JCAP)

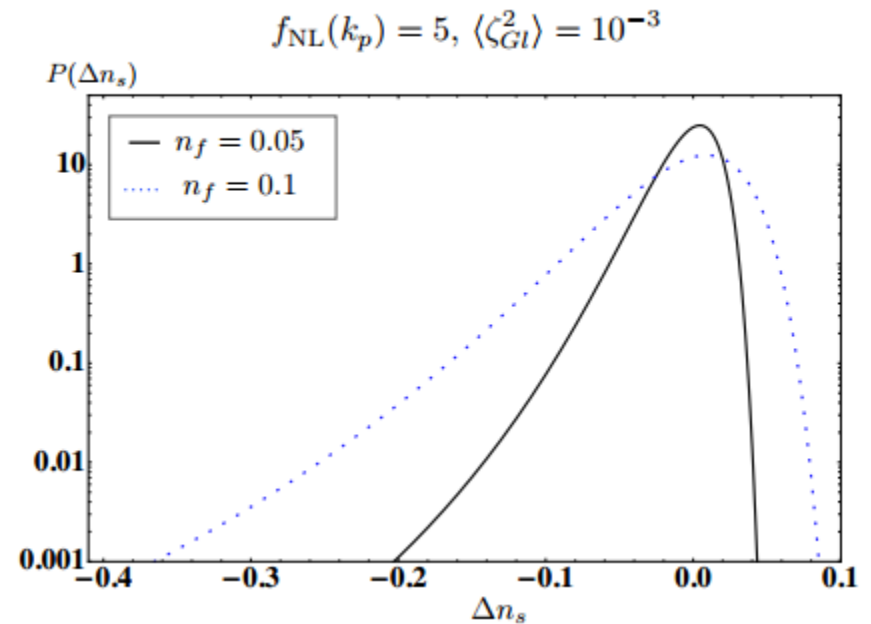
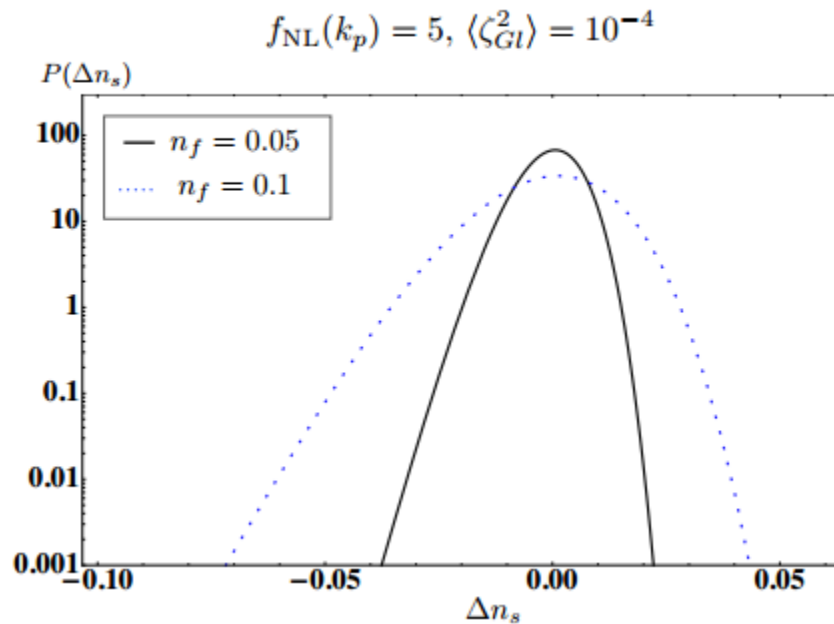
LoVerde 1310.5739

J.B., Sean Downes (In Progress)



Spectral Index Variance Plots

41



Spectral Index Constant for Large f_{nl} Bias

42

$$\begin{aligned}
 P_{\zeta, NG}(k) &= \text{diagram } \varphi + \text{diagram } \sigma + \text{diagram } k \rightarrow L^{-1} \\
 P_{\zeta, NG}^{\text{obs}}(k) &= \text{diagram } \varphi + \text{diagram } \sigma + 2 \times \text{diagram } M^{-1} \rightarrow L^{-1} \\
 &\quad + \text{diagram } k \rightarrow M^{-1} + \text{diagram } M^{-1} \rightarrow L^{-1} \text{ } M^{-1} \rightarrow L^{-1},
 \end{aligned}$$

Spectral Index Constant for Large f_{nl} Bias

43

$$P_{\zeta, NG}(k) = \text{diagram } \varphi + \text{diagram } k \rightarrow M^{-1} + \text{diagram } M^{-1} \rightarrow L^{-1}$$

$$P_{\zeta, NG}^{\text{obs}}(k) = \text{diagram } \varphi + \text{diagram } k \rightarrow M^{-1} + \text{diagram } M^{-1} \rightarrow L^{-1}$$

$$P_{\zeta, NG}^{\text{obs}}(k) \supset \text{diagram } M^{-1} \rightarrow L^{-1} \text{ and } M^{-1} \rightarrow L^{-1}$$