

An study of relativistic energy, momentum, and particle collisions

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Final Project
PHYS 305: Computational Physics

Overview

- Background
- Objective
- Project
- Results (thus far)
- Discussion
- Further work

Background

- 1905 (“*Annus Mirabilis*”, the Miracle Year!)
- Albert Einstein—from patent office clerk to science rockstar!



Photo credit: © Bettmann/Corbis
<http://www.pbs.org/wgbh/nova/physics/einstein-the-nobody.html>

Special Relativity

- Postulate 1
 - The laws of physics remain the same in all inertial frames
- Postulate 2
 - The speed of light is invariant in all inertial frames
- What does this mean???
 - Time dilation
 - Length contraction
 - Relativistic energies and momentum!

Objective

- Demonstrate how energies and momenta in the same event depend on frame of observation

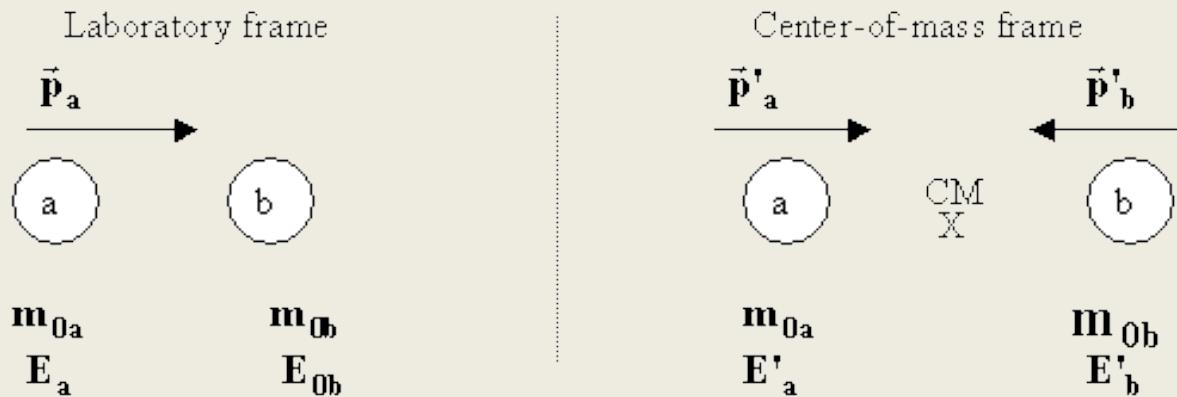


Fig. 1 Two-particle collision observed in lab and CM frames

[1]http://teachers.web.cern.ch/teachers/archiv/hst2002/bubblech/mbitu/applications_of_special_relativi.htm

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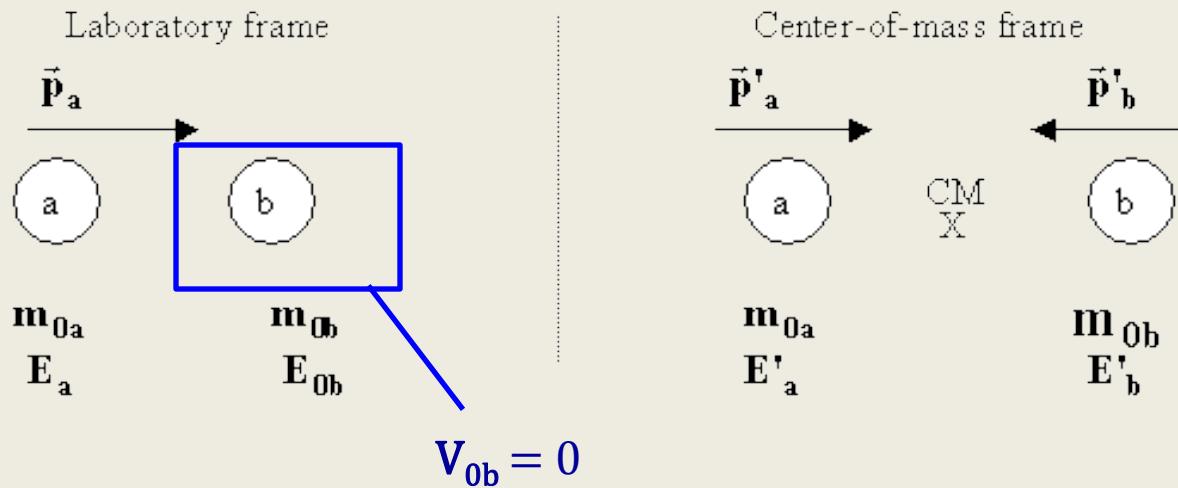
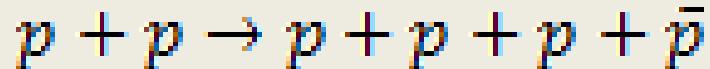


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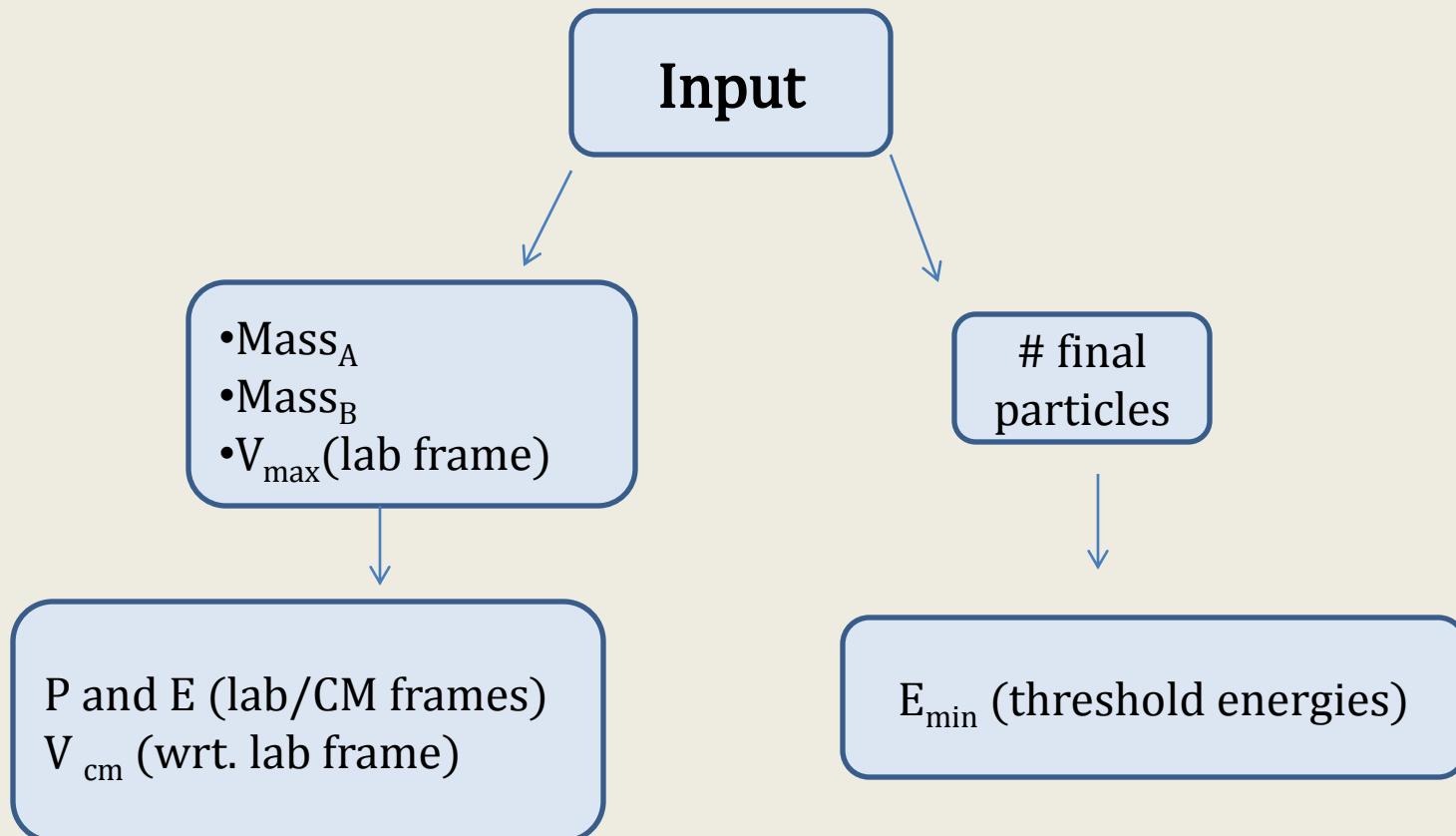
- Use previous results to display alongside the minimum threshold energies for some example collisions



- Necessary for the creation of certain particles

$$E_a^{min} = \frac{(\sum m_{final})^2 - (m_a^2 + m_b^2)}{2m_b} c^2$$

Project Approach



Calculating V_{CM}

Velocity of CM with respect to (WRT) to lab frame,

$$v_{cm} = \frac{pc^2}{E} \quad (1)$$

Total momentum and energy in lab frame,

$$\mathbf{P} = \mathbf{p}_a + \mathbf{p}_b = \mathbf{p}_a \quad (2)$$

$$E = E_a + E_b = E_a + m_{0b} c^2 \quad (3)$$

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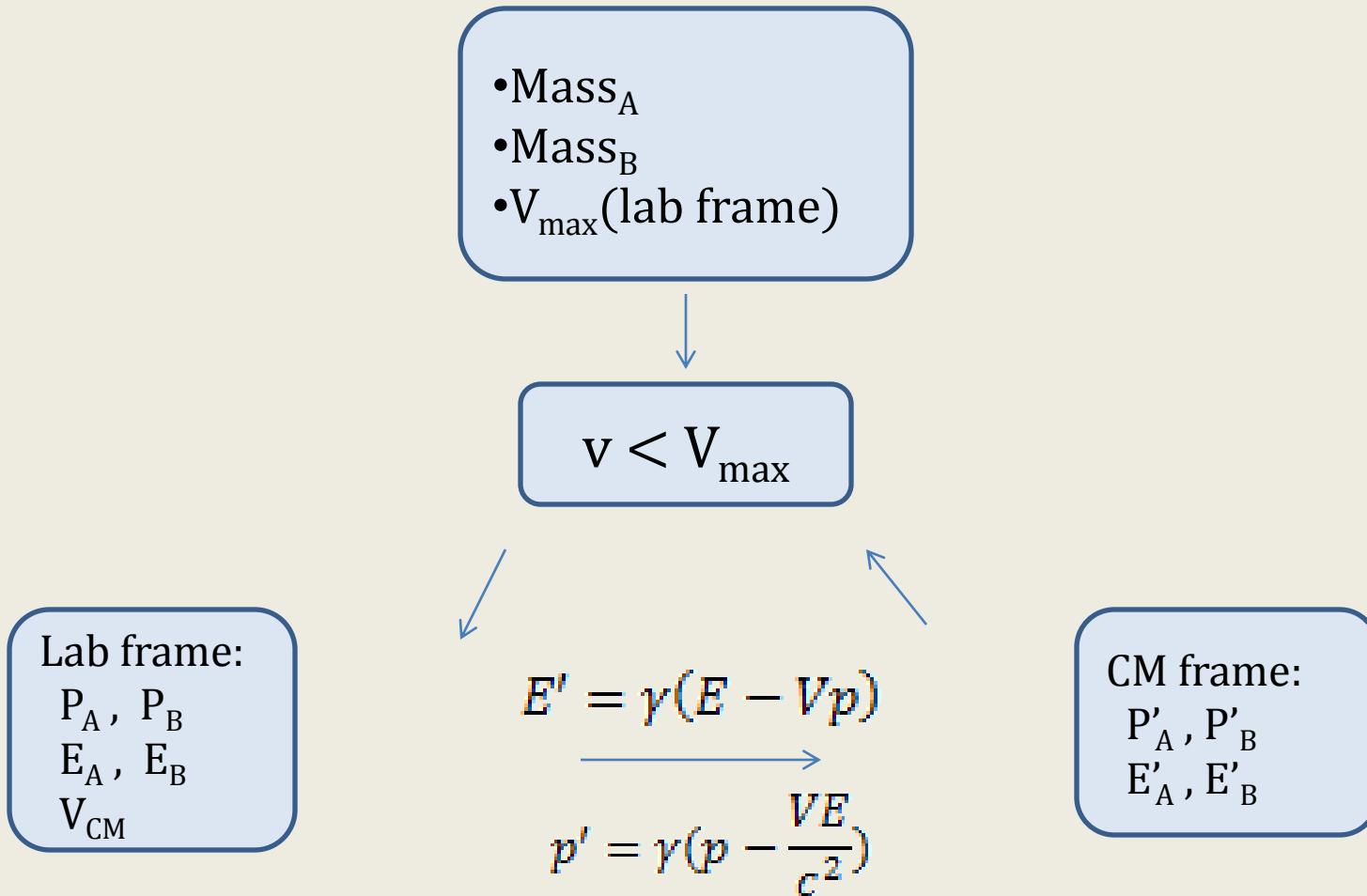
Using the energy-momentum relation,

$$E = c \cdot \sqrt{p^2 + m_0^2 c^2} \quad (4)$$

Eqn. (1) rewritten as function of total lab momentum and lab energy

$$v_{cm} = \frac{p_a c^2}{E_a + m_{0b} c^2} \quad (5)$$

Project Approach



Transforming between frames

Momentum WRT the CM frame

$$\mathbf{p}_a' = \frac{\mathbf{p}_a - \frac{v_{cm}}{c^2} E_a}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad \mathbf{p}_b' = \frac{\mathbf{p}_b - \frac{v_{cm}}{c^2} E_b}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad (6)$$

Energy WRT the CM frame

$$\mathbf{E}_a' = \frac{\mathbf{E}_a - \mathbf{p}_a v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad \mathbf{E}_b' = \frac{\mathbf{E}_b - \mathbf{p}_b v_{cm}}{\sqrt{1 - \frac{v_{cm}^2}{c^2}}}; \quad (7)$$

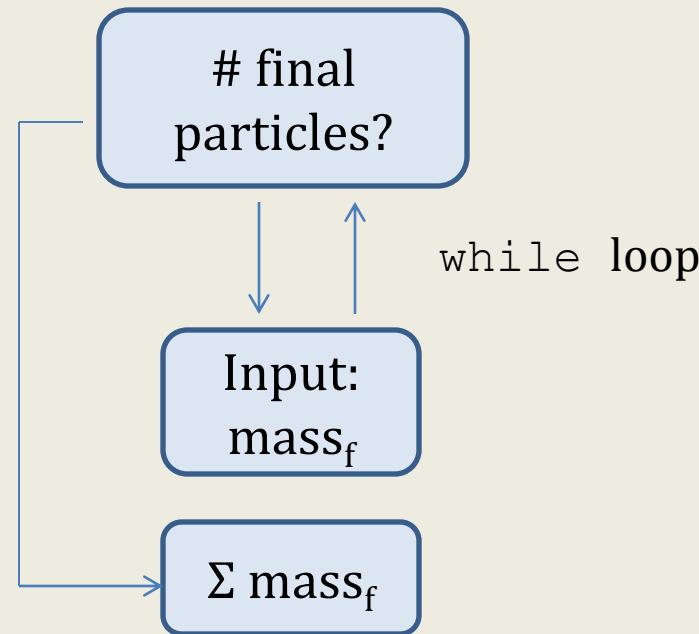
Transforming between frames

$$\begin{pmatrix} \mathbf{p}_a' \\ 0 \\ 0 \\ \frac{\mathbf{E}_a}{c} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} \mathbf{p}_a \\ 0 \\ 0 \\ \frac{\mathbf{E}_a}{c} \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} \mathbf{p}_b' \\ 0 \\ 0 \\ \frac{\mathbf{E}_b}{c} \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & -\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\beta\gamma & 0 & 0 & \gamma \end{pmatrix} \cdot \begin{pmatrix} \mathbf{p}_b \\ 0 \\ 0 \\ \frac{\mathbf{E}_b}{c} \end{pmatrix} \quad (9)$$

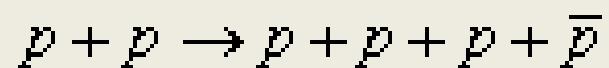
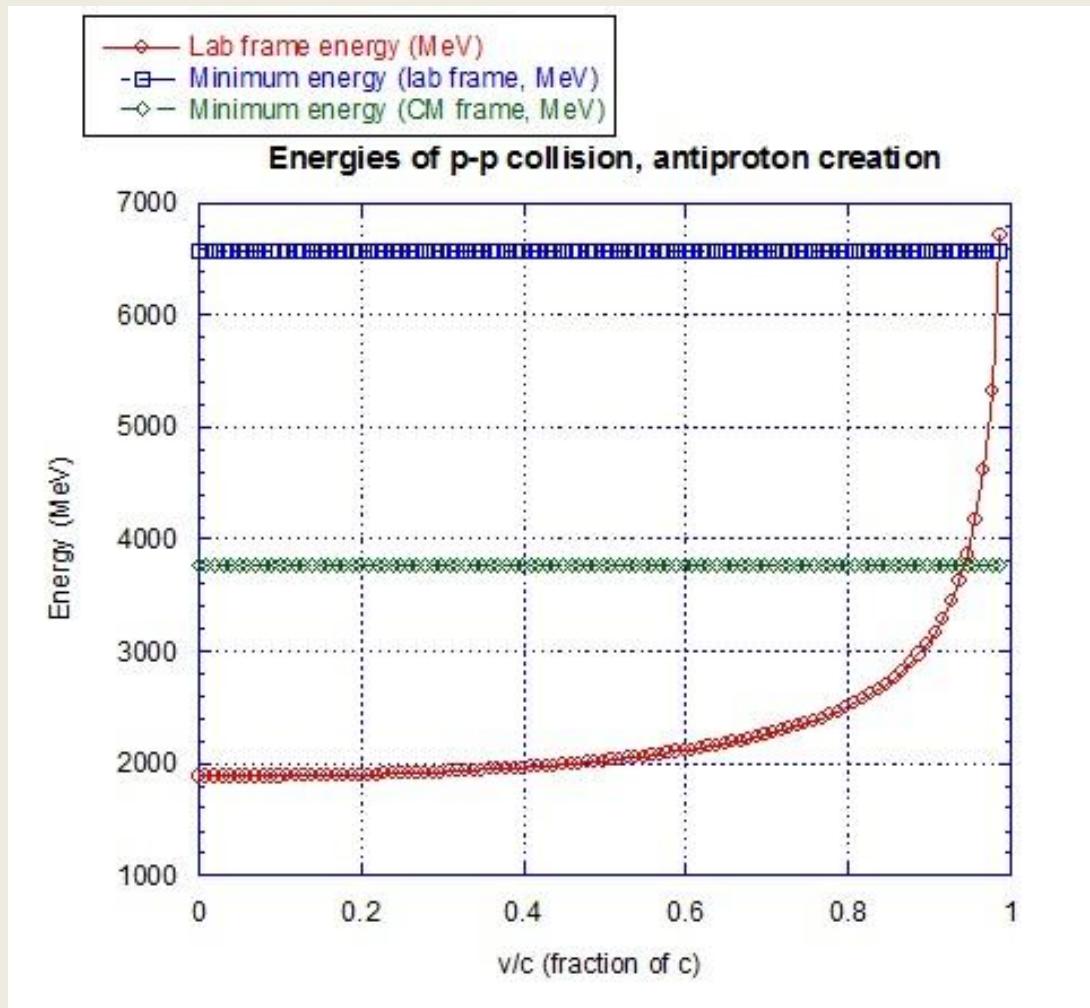
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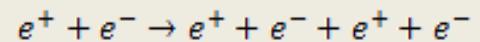
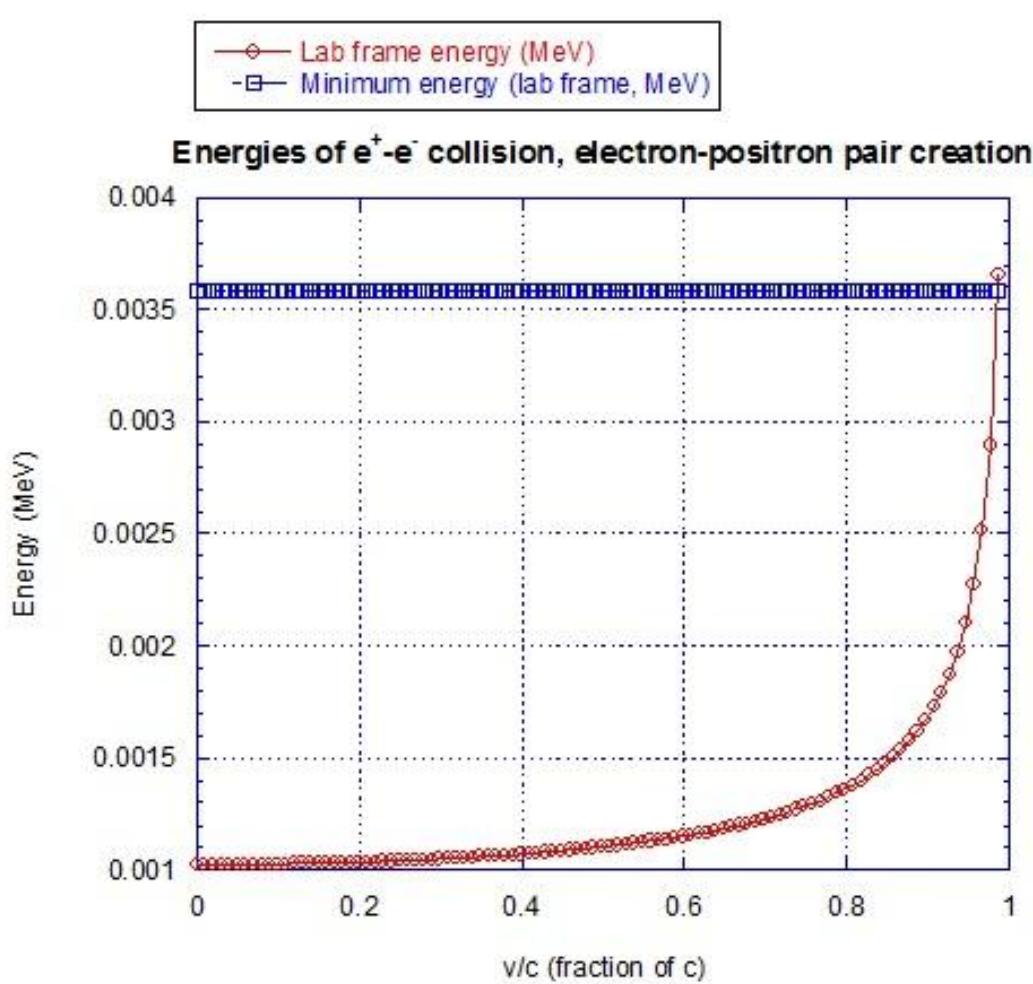
Results



$$E_{\min} \text{ lab} = 6575 \text{ MeV}$$

$$E_{\min} \text{ CM} = 3757 \text{ MeV}$$

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Discussion



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Summary

- Energies of specific events viewable in different frames
- Applications
 - particle colliders: colliding beams (CM frame) collisions with a stationary particle (lab frame)
- Further work
 - Four-vector manipulation

References

1. Bitu, R. "Applications of Special Relativity on Particle Physics."
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3. Helliwell, T. M. "Special Relativity". Harvey Mudd College. 2008.
4. Taylor, John R. "Classical Mechanics". University Science Books. 2005