

Modeling the Earths Magnetic Field Interaction with the Solar Wind

Justin R Bergonio
2 May 2012
PHYS 305

It's a Trap!

Particles in the Earth's Magnetic Field

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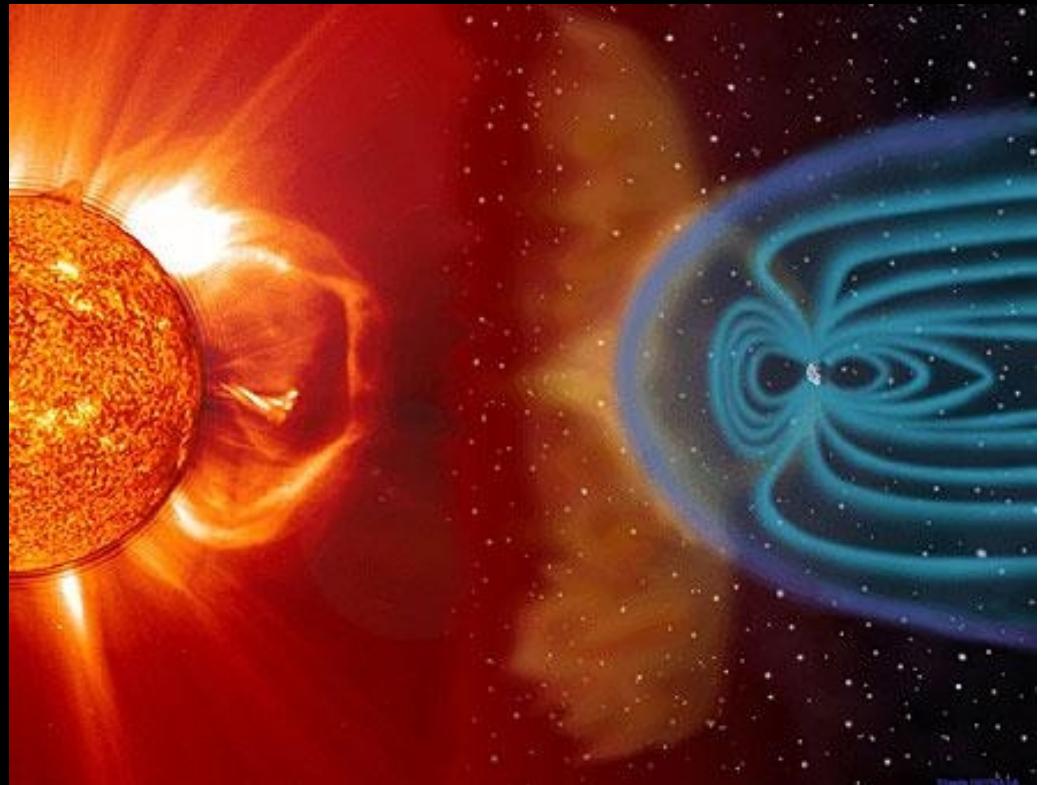
[http://shirtshovel.com/products/movies/
starwars/itsatrap-434.jpg](http://shirtshovel.com/products/movies/starwars/itsatrap-434.jpg)

Objectives

- Plot Earth's magnetic field as a simple magnetic dipole
- Track an electron's path through that field
- Observe it getting trapped (in the Van Allen Radiation Belt)

Earth's Invisible Shield

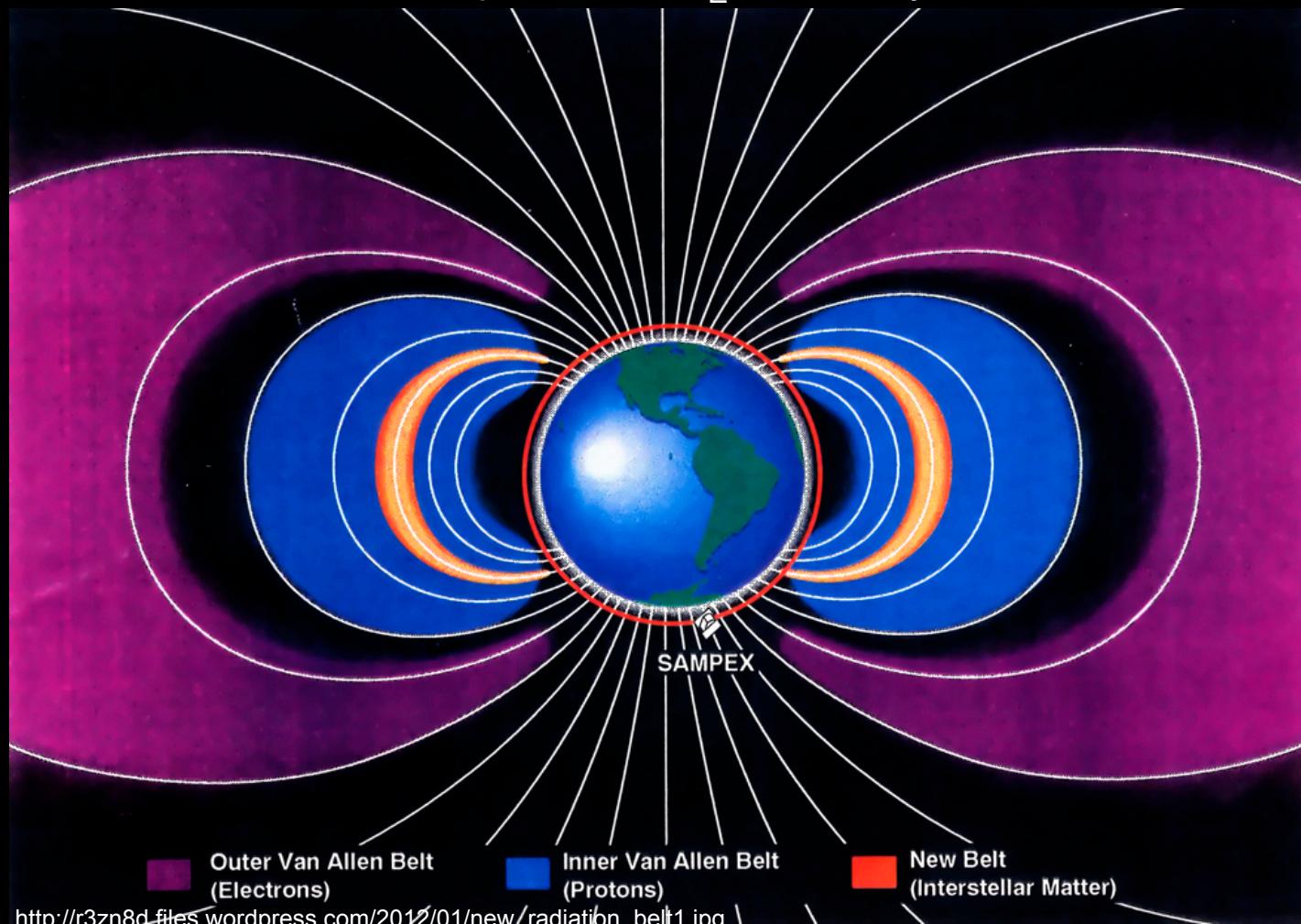
- Earth's magnetic field
 - Created by liquid Fe core
 - Approx. dipole field
 - 0.25 to 0.65 Gauss
- Solar Wind
 - Stream of charged particles
 - Ejected from Sun's upper atmosphere
 - Travels at 450 km/s



(<http://www.dailymail.co.uk/sciencetech/article-1096/discovered-Earths-magnetic-field-protects-planet-sun-rays.html>)

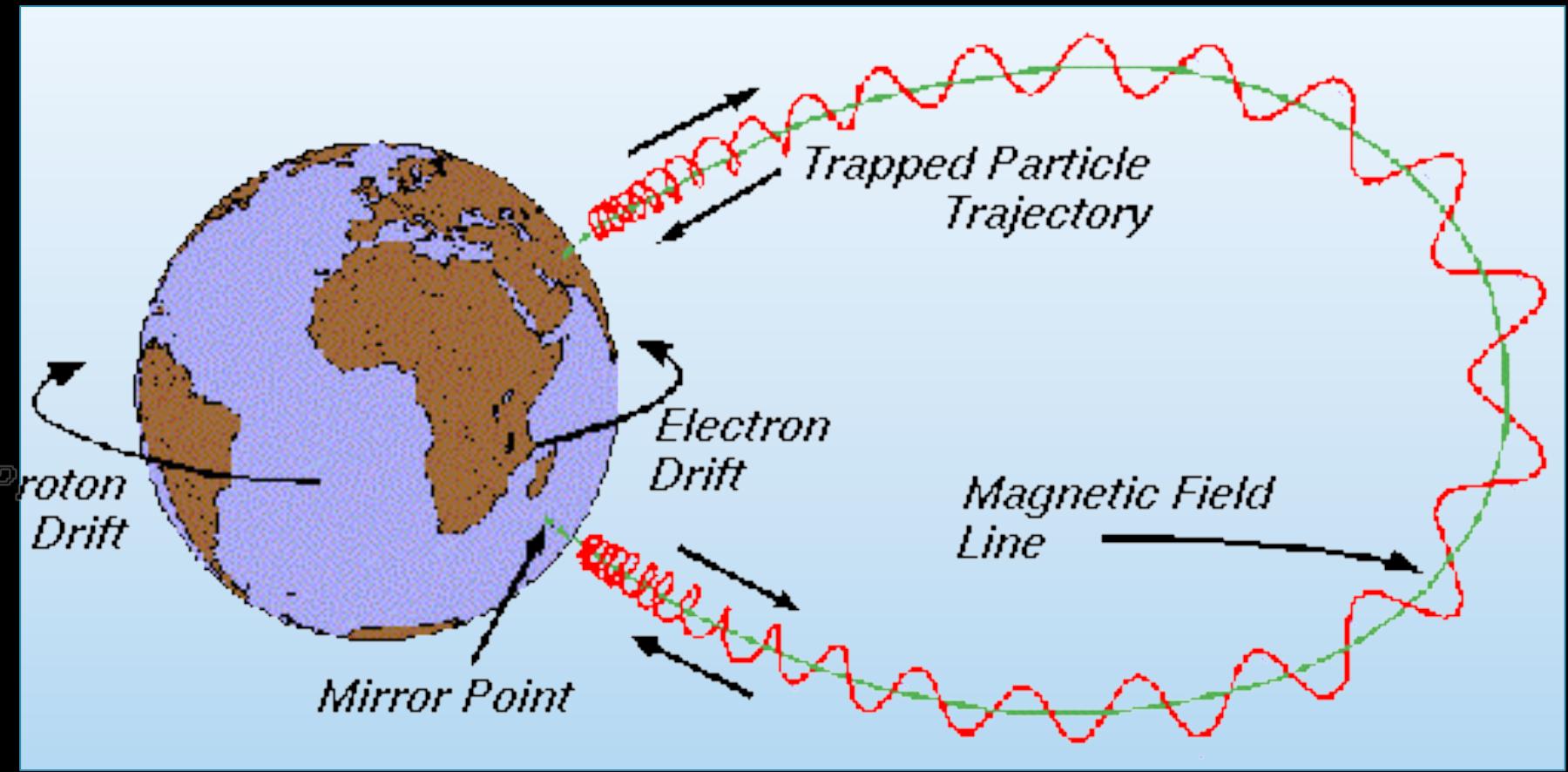
Van Allen Radiation Belts

- Discovered in early 1958 by Explorer 1 and 3
- Inner belt: protons (within $1 - 3 R_E$)
- Outer belt: electrons ($3 - 10 R_E$ - mostly between $5 - 6 R_E$)



Magnetic Mirroring

What does this look like?



<http://www.spenvis.oma.be/help/background/traprad/motion.gif>

Approach

- Create a program to generate field lines for Earth's Magnetic Field
- Use 4th Order Runge Kutta to track particle through non-uniform field

Equations for B-Field

Spherical Coordinates

$$B_r = -2B_0 \left(\frac{R_E}{r} \right) \cos \phi$$

$$B_\phi = -B_0 \left(\frac{R_E}{r} \right) \sin \phi$$

(Walt, *Introduction to Geomagnetically Trapped Radiation*)



Cartesian Coordinates

$$B_x = B_r \frac{x}{\rho} + B_\phi \frac{x}{r} \frac{z}{\rho}$$

$$B_y = B_r \frac{y}{\rho} + B_\phi \frac{y}{r} \frac{z}{\rho}$$

$$B_z = B_r \frac{z}{\rho} - B_\phi \frac{r}{\rho}$$

$$\vec{B} = B_r \hat{r} + B_\phi \hat{\phi} = B_x \hat{x} + B_y \hat{y} + B_z \hat{z}$$



$$B_0 = 3.12 \times 10^{-5} \text{ T}$$

(mean-value Earth magnetic field)

$$R_E = 6370 \text{ km}$$

(radius of Earth)

$$r = \sqrt{x^2 + y^2}$$

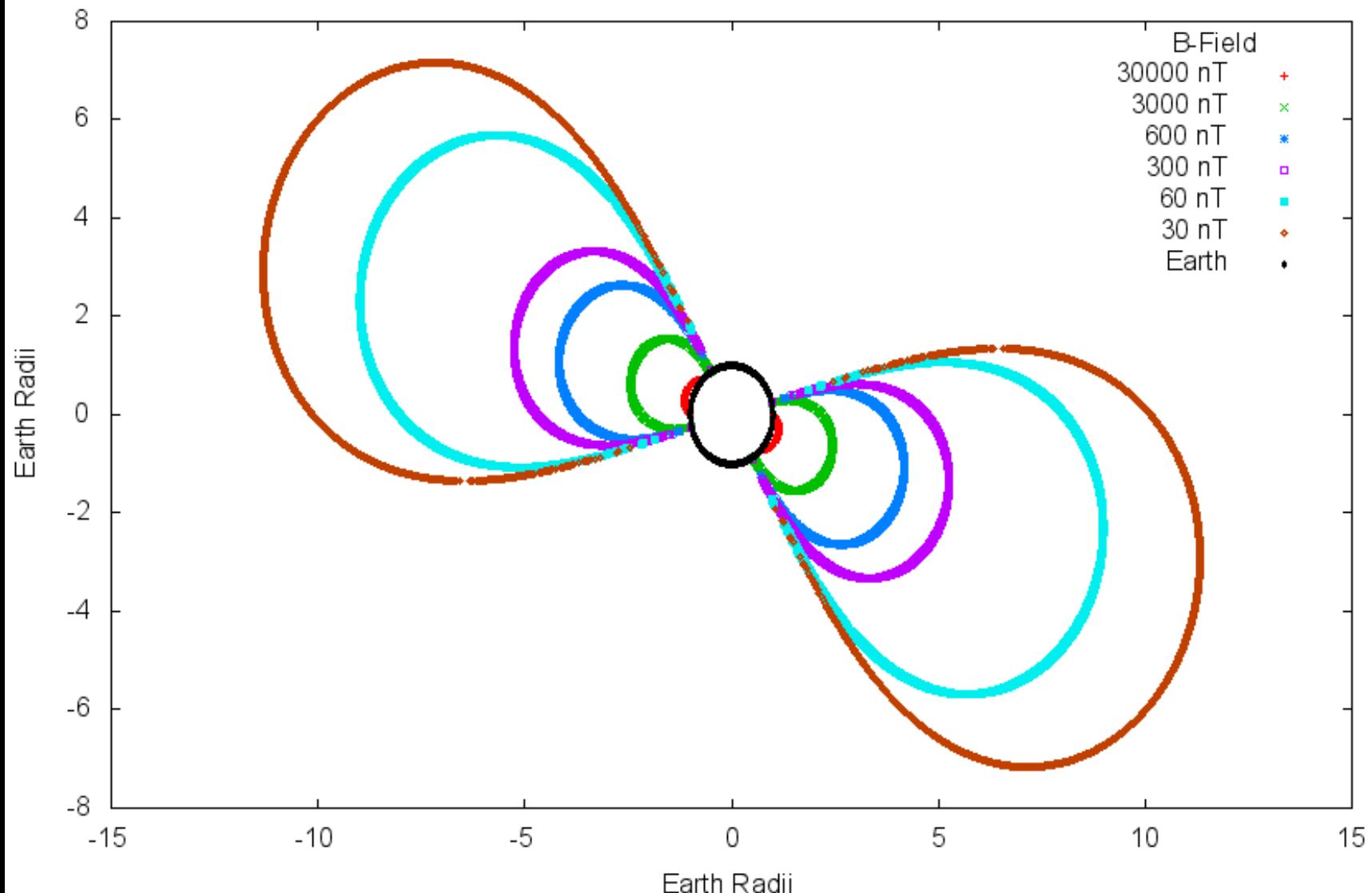
(radius in xy-plane)

$$\rho = \sqrt{r^2 + z^2}$$

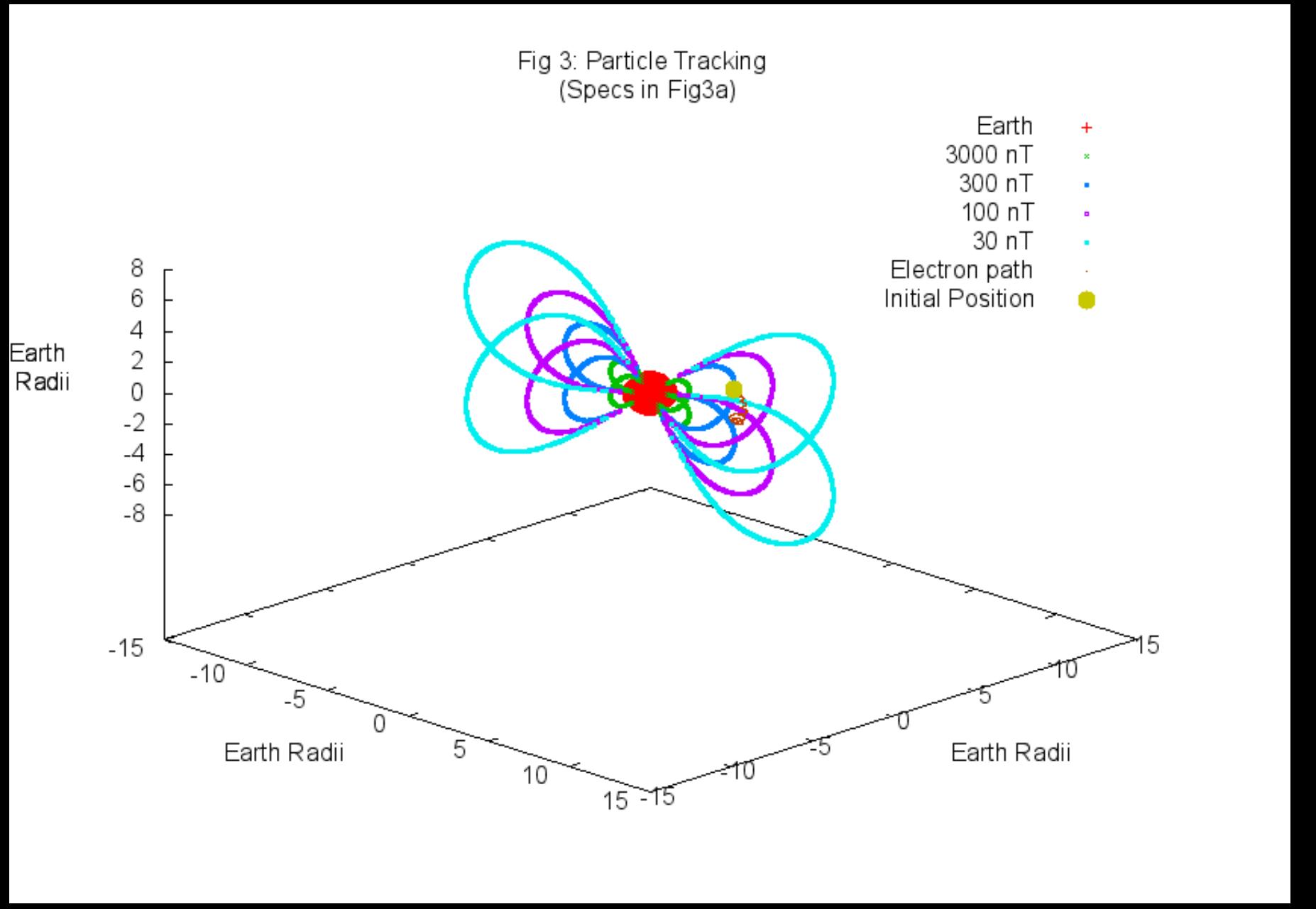
(radius in 3-dimensions)

Results

Earth Magnetic Field



Results



Equations for RK4

$x_0, y_0, z_0, v_{x_0}, v_{y_0}, v_{z_0}$ (initial conditions)

$$\vec{F} = m \frac{d\vec{v}}{dt} = q(\vec{v} \times \vec{B}) \quad (\text{Force on particle by B-field})$$

$$\frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{v} \times \vec{B}) \quad (\text{Use in RK4 to predict velocity})$$

$$\frac{d\vec{x}}{dt} = \vec{v} \quad (\text{Use in RK4 to predict position})$$

q.....charge of particle

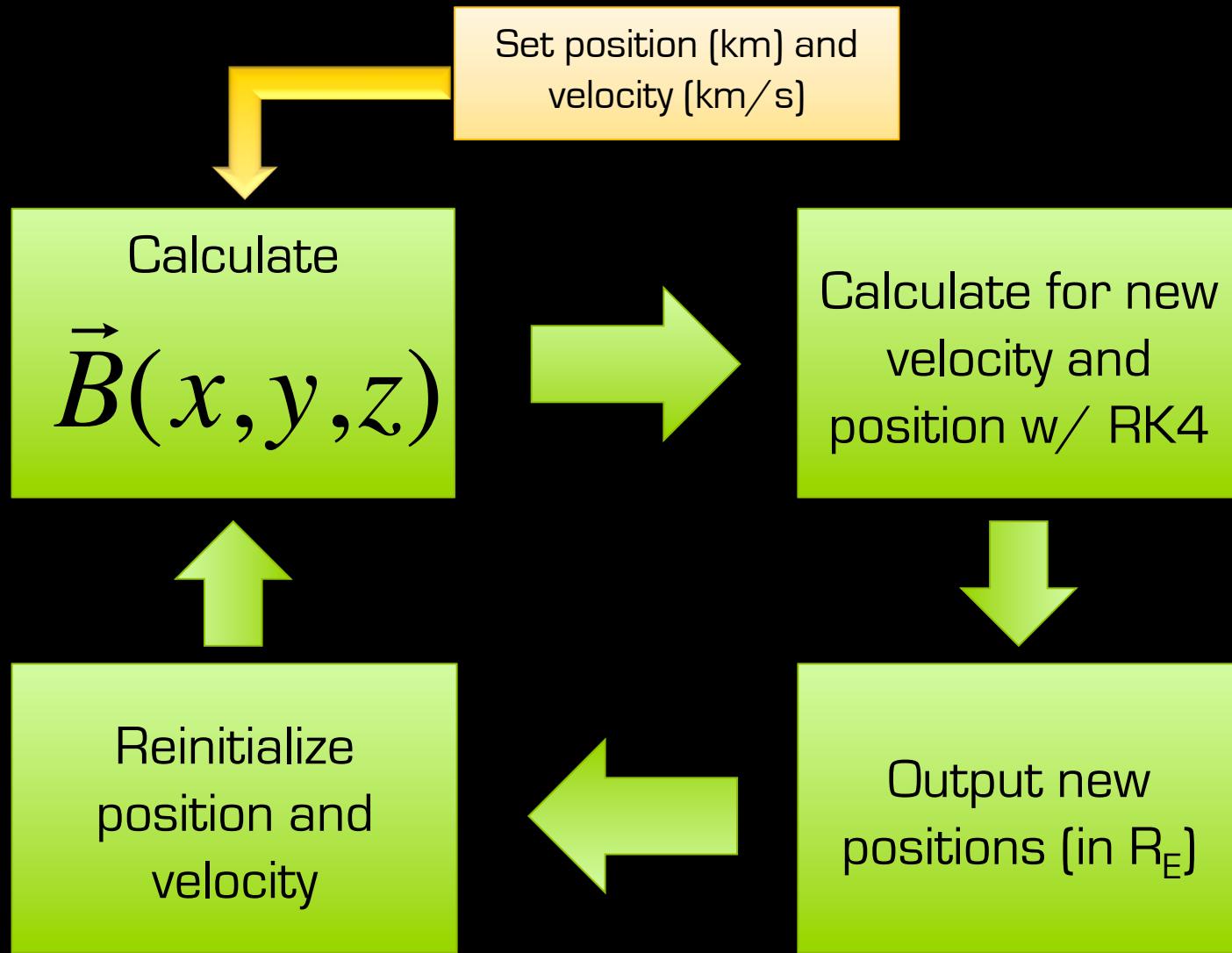
m.....particle's mass

B.....magnetic field vector

v.....velocity vector

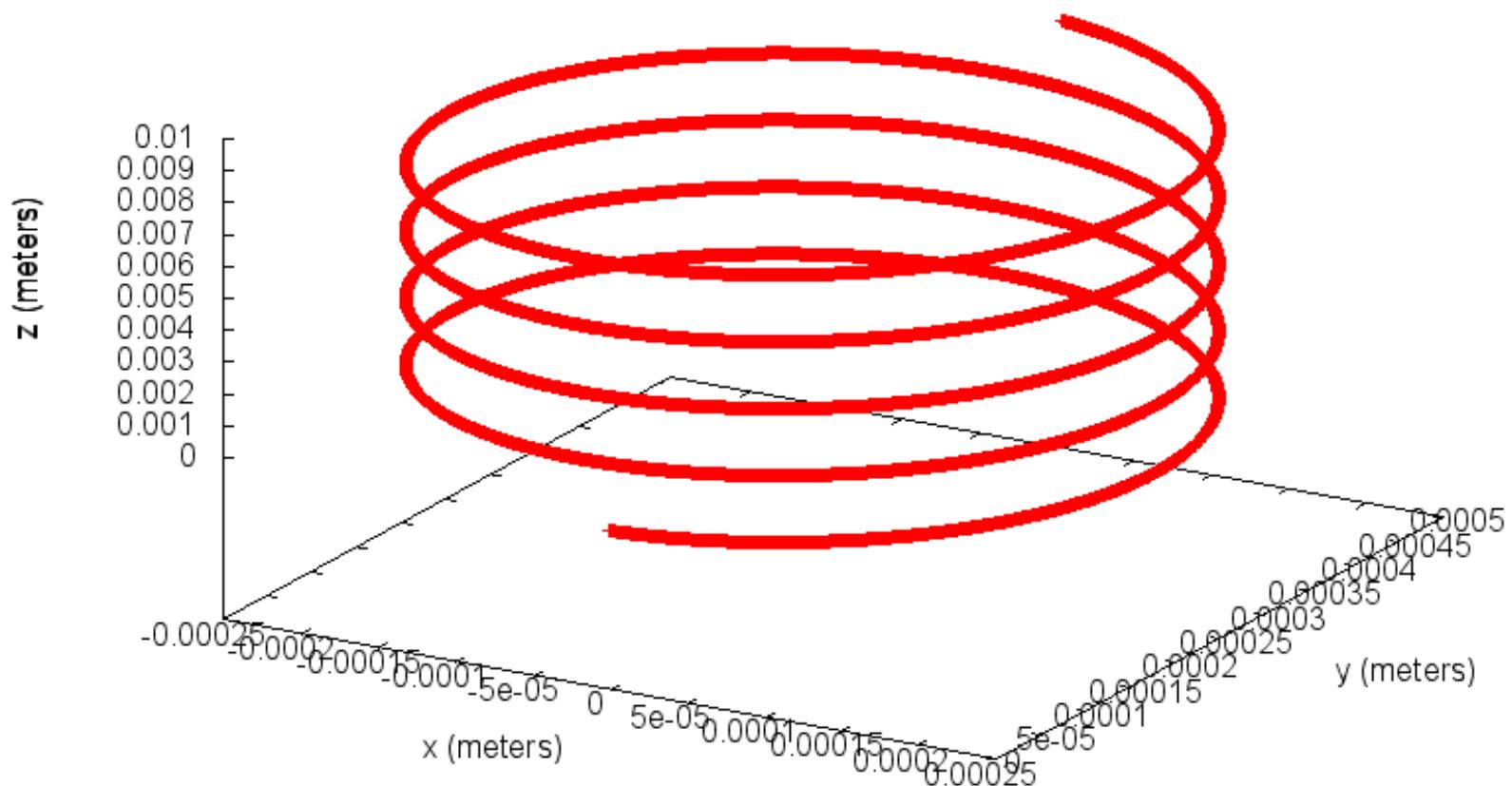
x.....position vector

Particle Tracking Algorithm

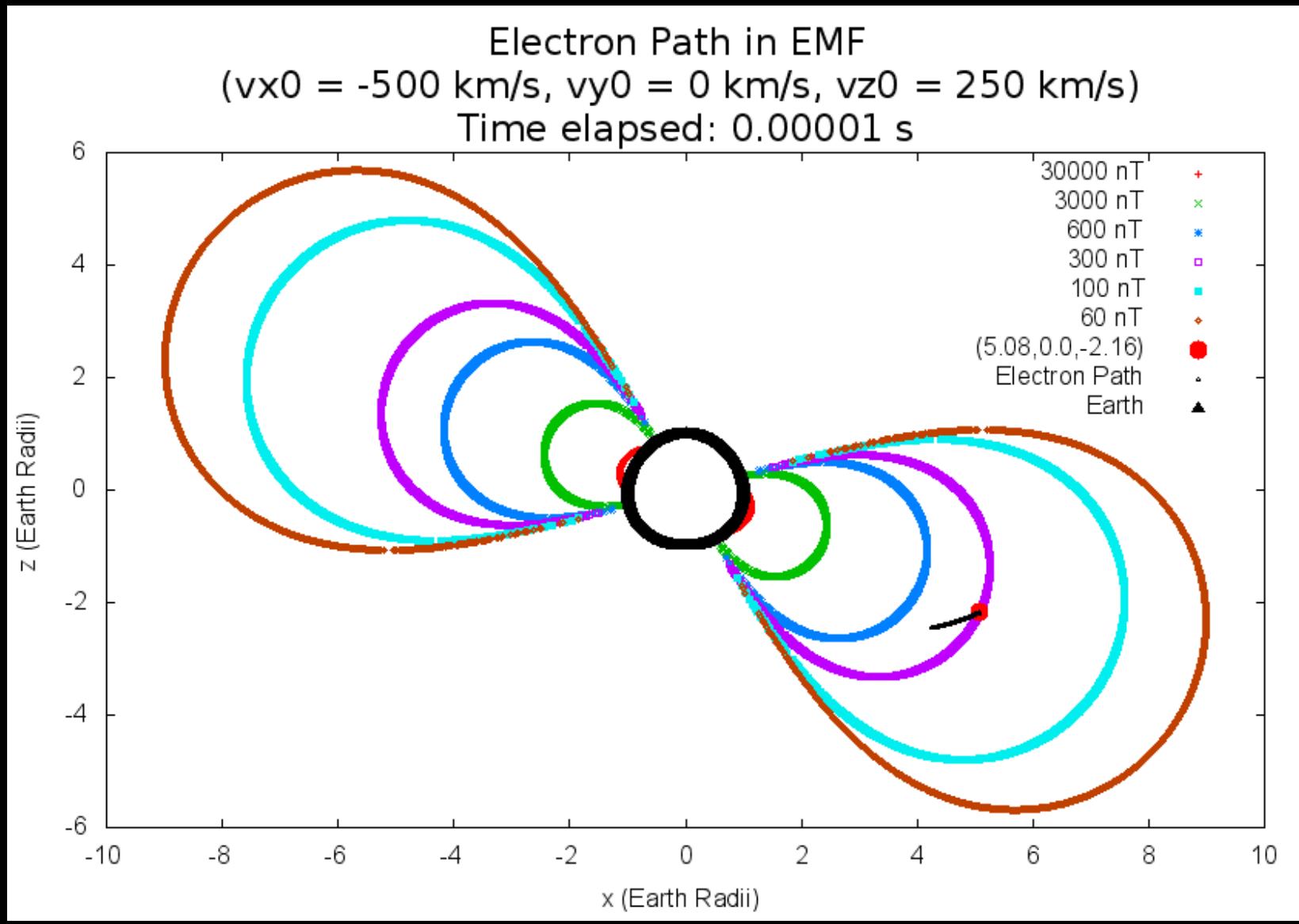


Test Accuracy of RK4

Uniform Magnetic Field
($v_x 0 = 2E7$ m/s, $v_z 0 = 2.9E7$ m/s, $B_z = 0.5$ T)

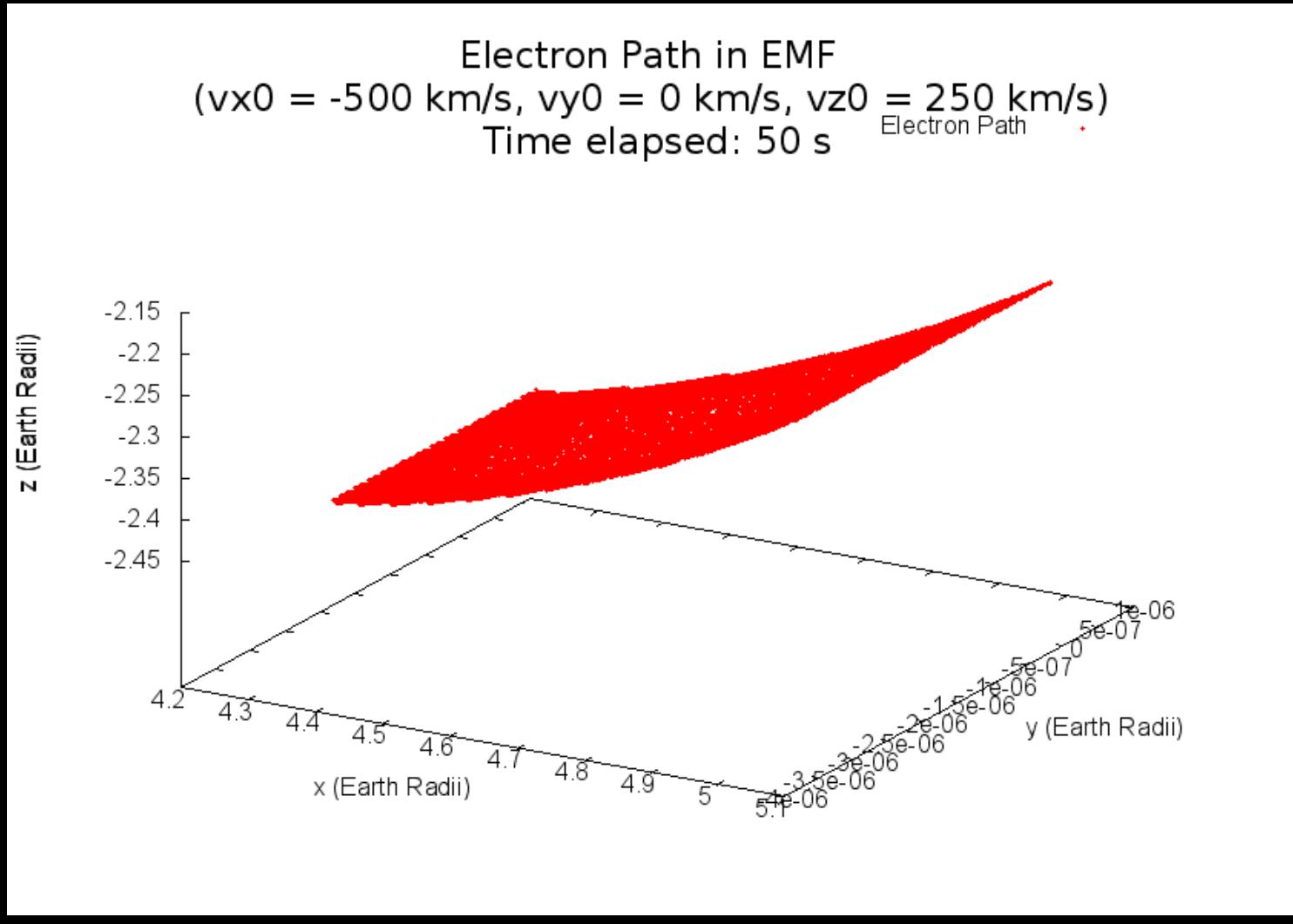


Results: Particle Tracking

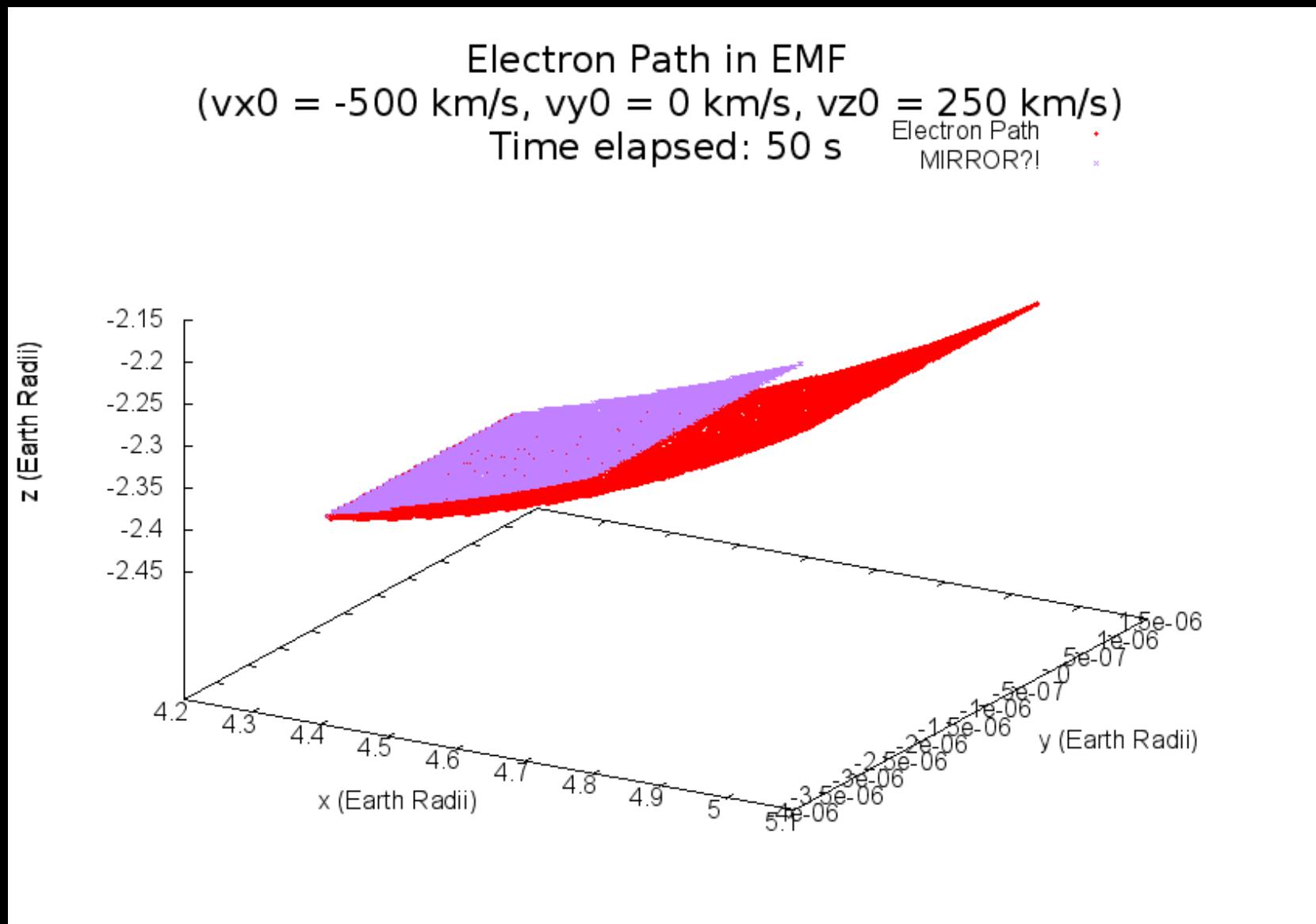


Results: Particle Trapping

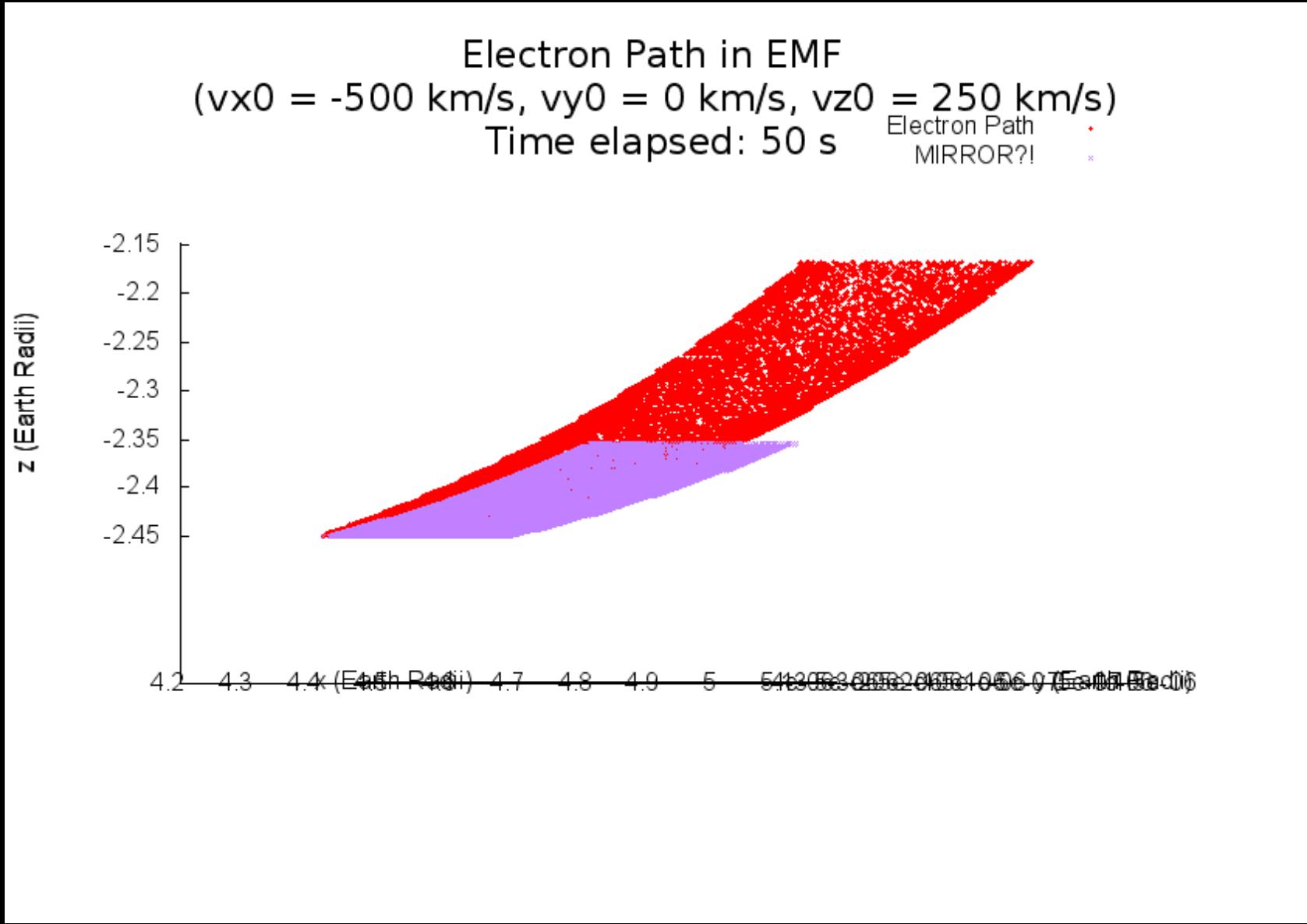
- Particles NOT getting trapped



Results: Particle Tracking



Results: Particle Tracking



Conclusion

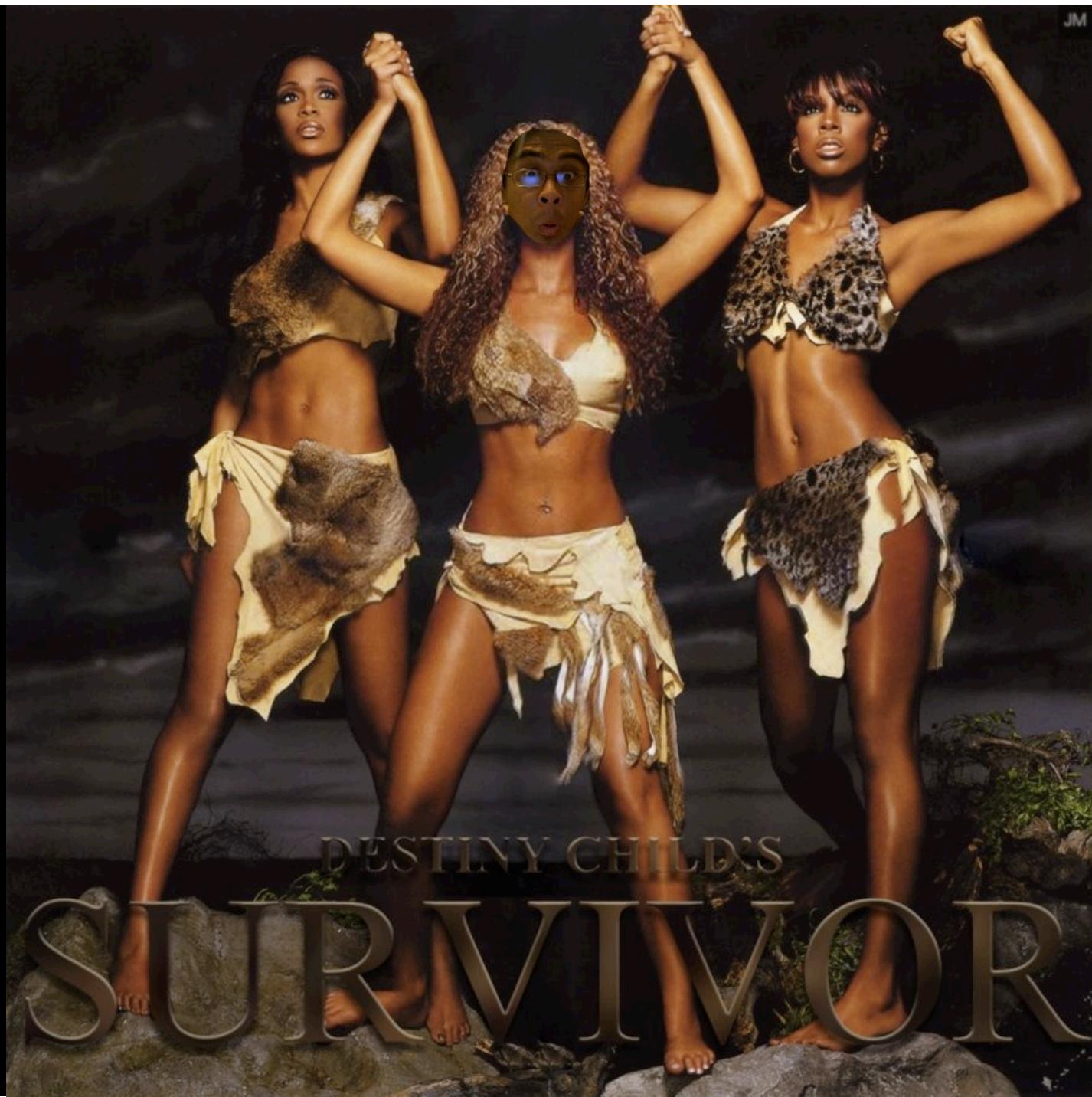
- Plot Earth's magnetic field as a simple magnetic dipole
- Track an electron's path through that field
- Observe it getting trapped (in the Van Allen Radiation Belt)
 - Need to find the right velocities/positions
 - Need to keep it out of the loss cone

Thank You's

- Tutors at Learning Emporium/Physics Library (Keita, Erin, and Pat)
- Professor Varner

•KURTIS
NISHIMURA!!!

JM



RK4: First Order Equations

$$k_{1x} = v_{x_0} \Delta t$$

$$k_{1y} = v_{y_0} \Delta t$$

$$k_{1z} = v_{z_0} \Delta t$$

$$k_{1v_x} = \frac{q}{m} \left(v_{y_0} B_z - v_{z_0} B_y \right) \Delta t$$

$$k_{1v_y} = -\frac{q}{m} \left(v_{x_0} B_z - v_{z_0} B_x \right) \Delta t$$

$$k_{1v_z} = \frac{q}{m} \left(v_{x_0} B_y - v_{y_0} B_x \right) \Delta t$$

RK4: Second Order Equations

$$k_{2x} = \left(v_{x_0} + \frac{k_{1v_x}}{2} \right) \Delta t$$

$$k_{2y} = \left(v_{y_0} + \frac{k_{1v_y}}{2} \right) \Delta t$$

$$k_{2z} = \left(v_{z_0} + \frac{k_{1v_z}}{2} \right) \Delta t$$

$$k_{2v_x} = \frac{q}{m} \left[\left(v_{y_0} + \frac{k_{1v_y}}{2} \right) B_z - \left(v_{z_0} + \frac{k_{1v_z}}{2} \right) B_y \right] \Delta t$$

$$k_{2v_y} = -\frac{q}{m} \left[\left(v_{x_0} + \frac{k_{1v_x}}{2} \right) B_z - \left(v_{z_0} + \frac{k_{1v_z}}{2} \right) B_x \right] \Delta t$$

$$k_{2v_z} = \frac{q}{m} \left[\left(v_{x_0} + \frac{k_{1v_x}}{2} \right) B_y - \left(v_{y_0} + \frac{k_{1v_y}}{2} \right) B_x \right] \Delta t$$

RK4: Third Order Equations

$$k_{3x} = \left(v_{x_0} + \frac{k_{2v_x}}{2} \right) \Delta t$$

$$k_{3y} = \left(v_{y_0} + \frac{k_{2v_y}}{2} \right) \Delta t$$

$$k_{3z} = \left(v_{z_0} + \frac{k_{2v_z}}{2} \right) \Delta t$$

$$k_{3v_x} = \frac{q}{m} \left[\left(v_{y_0} + \frac{k_{2v_y}}{2} \right) B_z - \left(v_{z_0} + \frac{k_{2v_z}}{2} \right) B_y \right] \Delta t$$

$$k_{3v_y} = -\frac{q}{m} \left[\left(v_{x_0} + \frac{k_{2v_x}}{2} \right) B_z - \left(v_{z_0} + \frac{k_{2v_z}}{2} \right) B_x \right] \Delta t$$

$$k_{3v_z} = \frac{q}{m} \left[\left(v_{x_0} + \frac{k_{3v_x}}{2} \right) B_y - \left(v_{y_0} + \frac{k_{3v_y}}{2} \right) B_x \right] \Delta t$$

RK4: Fourth Order Equations

$$k_{4x} = \left(v_{x_0} + k_{3v_x} \right) \Delta t$$

$$k_{4y} = \left(v_{y_0} + k_{3v_y} \right) \Delta t$$

$$k_{4z} = \left(v_{z_0} + k_{3v_z} \right) \Delta t$$

$$k_{4v_x} = \frac{q}{m} \left[\left(v_{y_0} + k_{3v_y} \right) B_z - \left(v_{z_0} + k_{3v_z} \right) B_y \right] \Delta t$$

$$k_{4v_y} = \frac{q}{m} \left[\left(v_{x_0} + k_{3v_x} \right) B_z - \left(v_{z_0} + k_{3v_z} \right) B_x \right] \Delta t$$

$$k_{4v_z} = \frac{q}{m} \left[\left(v_{x_0} + k_{3v_x} \right) B_y - \left(v_{y_0} + k_{3v_y} \right) B_x \right] \Delta t$$

RK4: New Values

$$x = x_0 + \frac{1}{6} \left(k_{1x} + \frac{k_{2x}}{2} + \frac{k_{3x}}{2} + k_{4x} \right)$$

← Positions!

$$y = y_0 + \frac{1}{6} \left(k_{1y} + \frac{k_{2y}}{2} + \frac{k_{3y}}{2} + k_{4y} \right)$$

$$z = z_0 + \frac{1}{6} \left(k_{1z} + \frac{k_{2z}}{2} + \frac{k_{3z}}{2} + k_{4z} \right)$$

Velocities! →

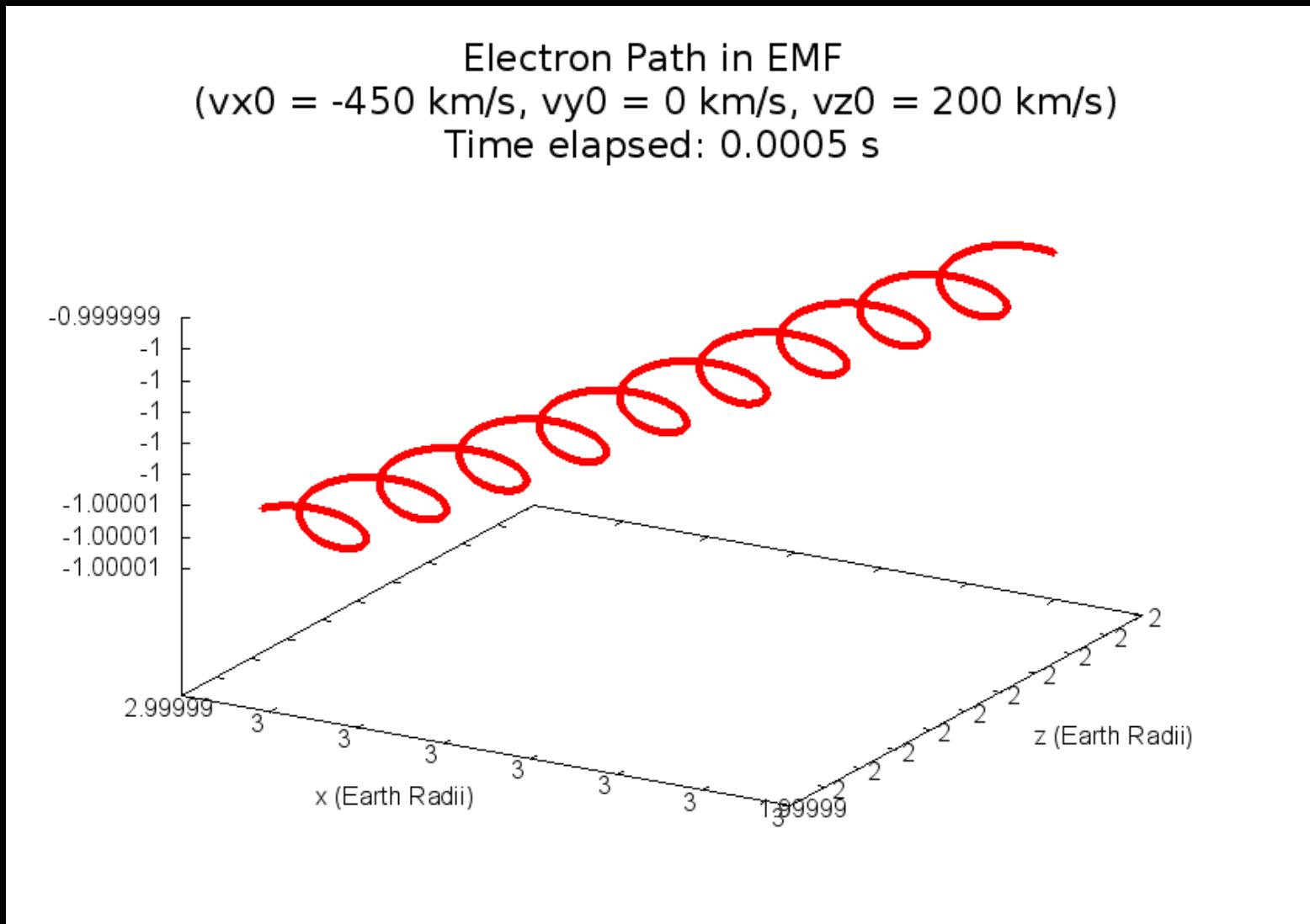
$$v_x = v_{x_0} + \frac{1}{6} \left(k_{1v_x} + \frac{k_{2v_x}}{2} + \frac{k_{3v_x}}{2} + k_{4v_x} \right)$$

$$v_y = v_{y_0} + \frac{1}{6} \left(k_{1v_y} + \frac{k_{2v_y}}{2} + \frac{k_{3v_y}}{2} + k_{4v_y} \right)$$

$$v_z = v_{z_0} + \frac{1}{6} \left(k_{1v_z} + \frac{k_{2v_z}}{2} + \frac{k_{3v_z}}{2} + k_{4v_z} \right)$$

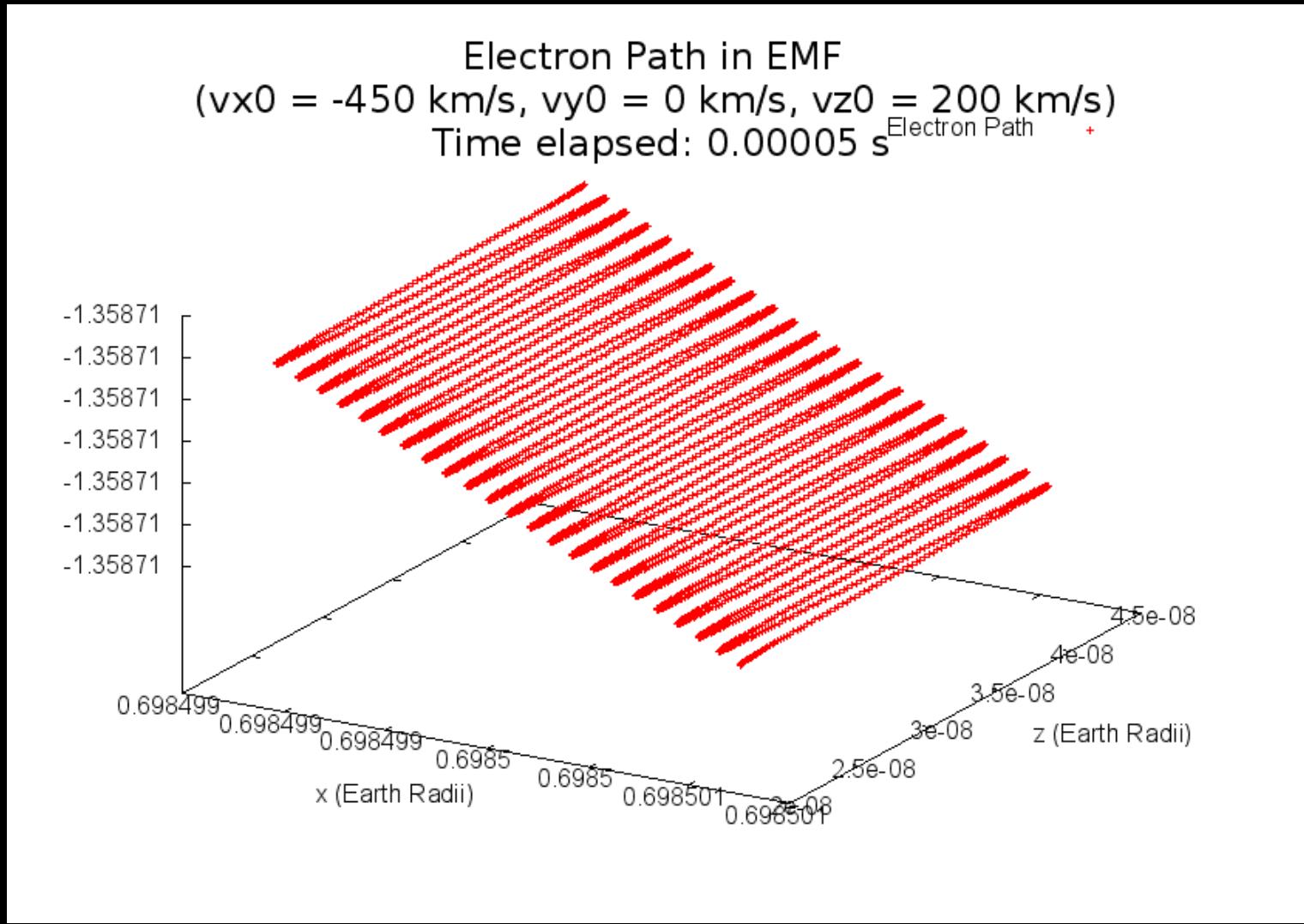
Results: Particle Tracking

- Particle approximately 3.75 Earth radii



Results: Particle Tracking

- Particle nearing Earth's south pole



Earth's Invisible Shield

