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Non Perturbative

One Gluon Exchange Potential

P. González Universitat de València and IFIC (SPAIN)

Reference

P. González, V. Mathieu and V. Vento
Phys. Rev. D84, 114008 (2011).

Motivation

Recent progress in the solution of Schwinger – Dyson equations allows for a nonperturbative evaluation of the OGE potential from QCD.

What can we learn from this?

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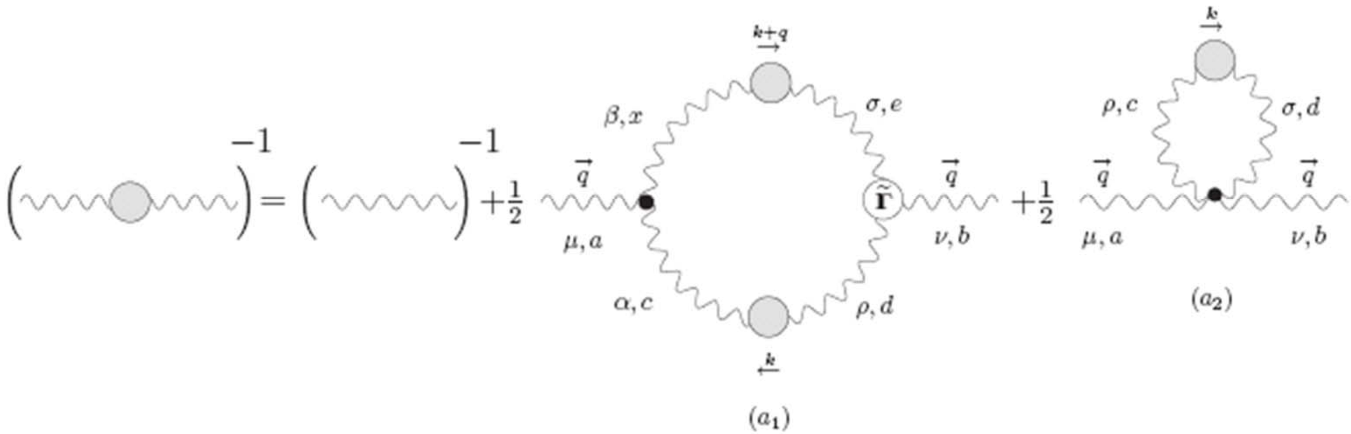
Schwinger-Dyson Equations

A QFT is completely characterized by its Green functions which are governed by SDE.

PT – BFM truncation scheme (D. Binosi, J. Papavassiliou, Phys. Rep. 479 (2009) 1-152)



$$(\Delta^{-1})^{ab}_{\mu\nu}(q) = iq^2 g_{\mu\nu}\delta^{ab} - \left[\Pi^{ab}_{\mu\nu}(q)\Big|_{a_1} + \Pi^{ab}_{\mu\nu}\Big|_{a_2}\right]$$

\+ three gluon vertex ansatz

Effective Gluon Mass

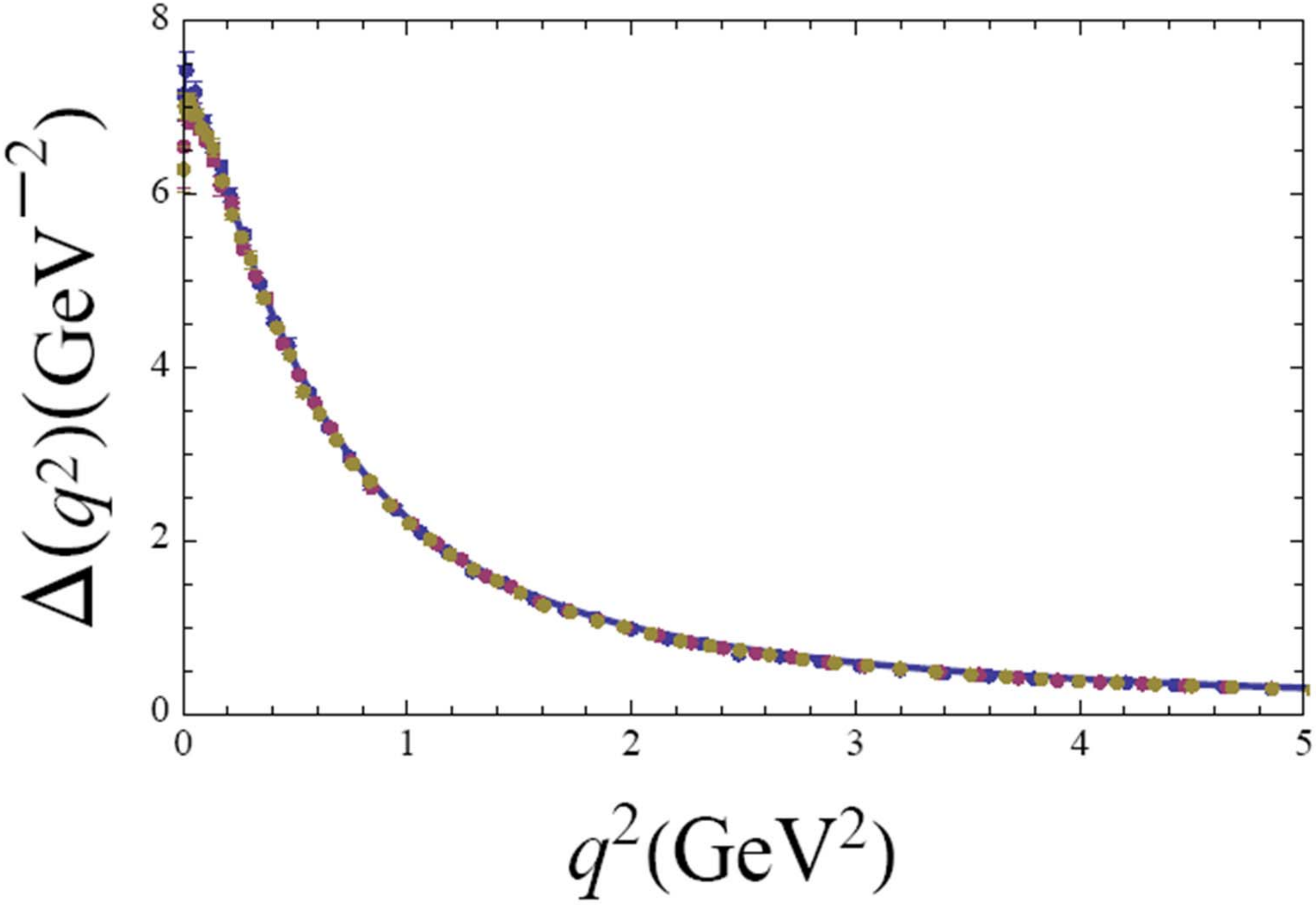
Infrared finite quenched solutions of the SDE are obtained.

These solutions may be fitted by a “massive” euclidean propagator

$$\Delta(q^2) = \frac{1}{q^2 + m^2(q^2)}$$

$$m^2(q^2) = m_0^2 \left[\ln\left(\frac{q^2 + \rho m_0^2}{\Lambda^2} \right) \Big/ \ln\left(\frac{\rho m_0^2}{\Lambda^2} \right) \right]^{-1-\delta}$$

m(0), ρ, δ : parameters fitting the lattice propagator

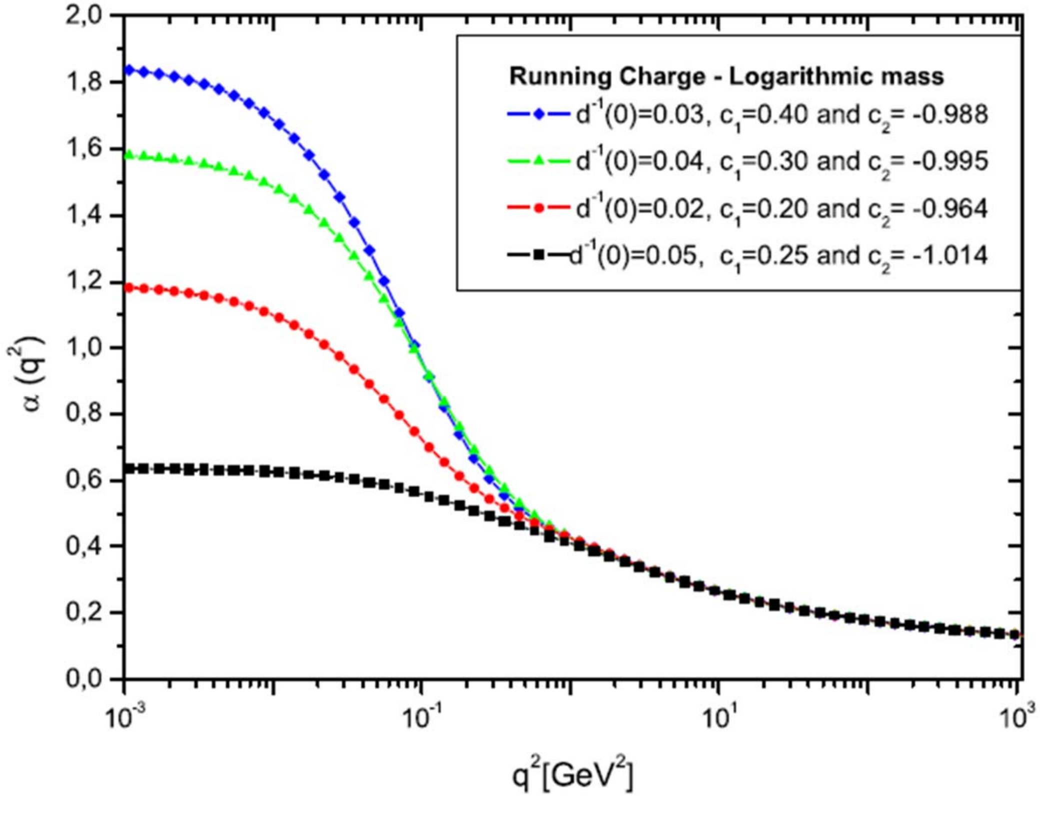


$\Delta(q^2)(\mathrm{GeV}^{-2})$
$q^2(\mathrm{GeV}^2)$
0
1
2
3
4
5
0
2
4
6
8

Strong Effective Charge

$$a(q^2) = \left[\beta_0 \ln\left(\frac{q^2 + \rho\, m^2(q^2)}{\Lambda^2}\right)\right]^{-1}$$

$$a = \frac{\alpha}{4\pi} \qquad \beta_0 = 11 - 2n_f/3$$



Quark Model Static Approach

Bethe-Salpeter Equation (Meson)

$$(p_q \partial - m_q)\Psi_{q\bar{q}}(p; P)(p_{\bar{q}}\,\partial - m_{\bar{q}}) = \frac{1}{(2\pi)^4}\int K(p,p';P)\Psi_{q\bar{q}}(p';P)d^4p'$$

Static approximation in the kernel

$$K(p,p';P) = K(\overrightarrow{q} = \overrightarrow{p} - \overrightarrow{p}') = V(\overrightarrow{q})$$

Schrödinger Equation

Fully non-relativistic approach

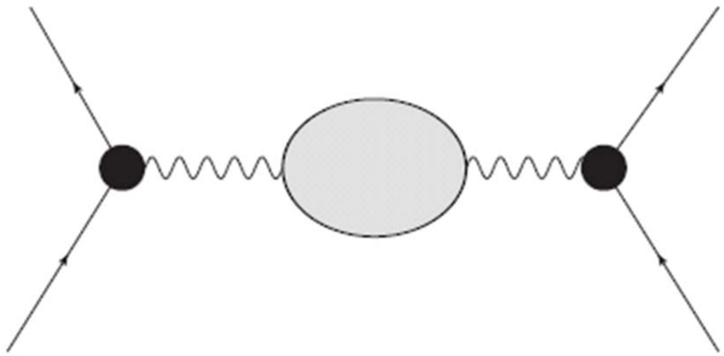
$$\left(m_q + \frac{\overrightarrow{p}_q^{\,2}}{2m_q} + m_{\overline{q}} + \frac{\overrightarrow{p}_{\overline{q}}^{\,2}}{2m_{\overline{q}}} - M \right) \phi(\overrightarrow{p}) = -\frac{1}{(2\pi)^3} \int V(\overrightarrow{q}) \phi(\overrightarrow{p}\,') d^3 p'$$

$$\phi(\overrightarrow{p}) = \int \Psi_{q\overline{q}}(p; P = 0) dp^0$$

Non Relativistic Quark Models (NRQM)

OGE Potential Model

Let us assume that the main source of dynamics is the One Gluon Exchange interaction .



$$V(\overrightarrow{q}) = -\frac{C\alpha(\overrightarrow{q}^{\,2})}{\overrightarrow{q}^{\,2}+m^2(\overrightarrow{q}^{\,2})}$$

$$V(\overrightarrow{r}) = \frac{1}{(2\pi)^3}\int d^3q V(\overrightarrow{q})e^{i\overrightarrow{q}\,\overrightarrow{r}}$$

The additive infinite self-energy contribution associated to the static sources has to be removed.

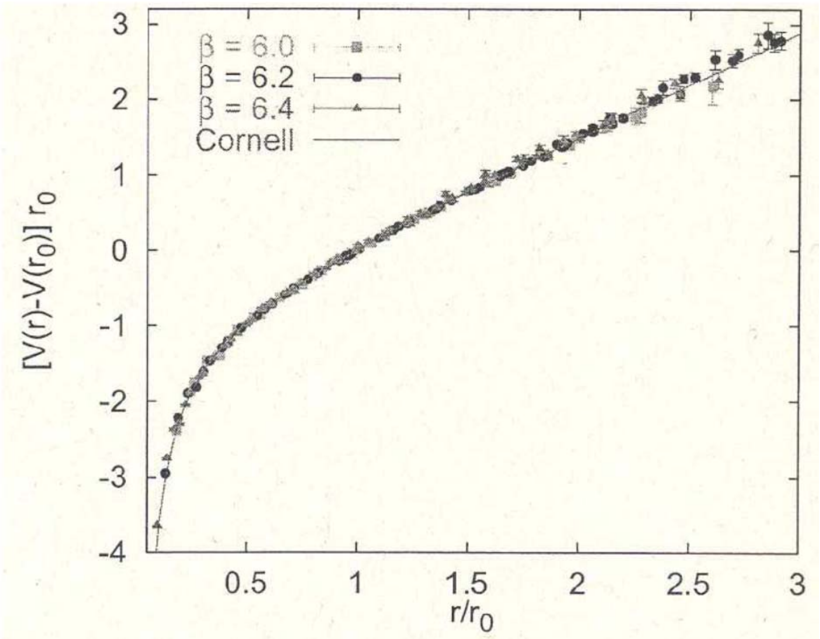
$$V(r_0) = 0 \qquad\qquad r_0 : \text{Sommer scale}$$

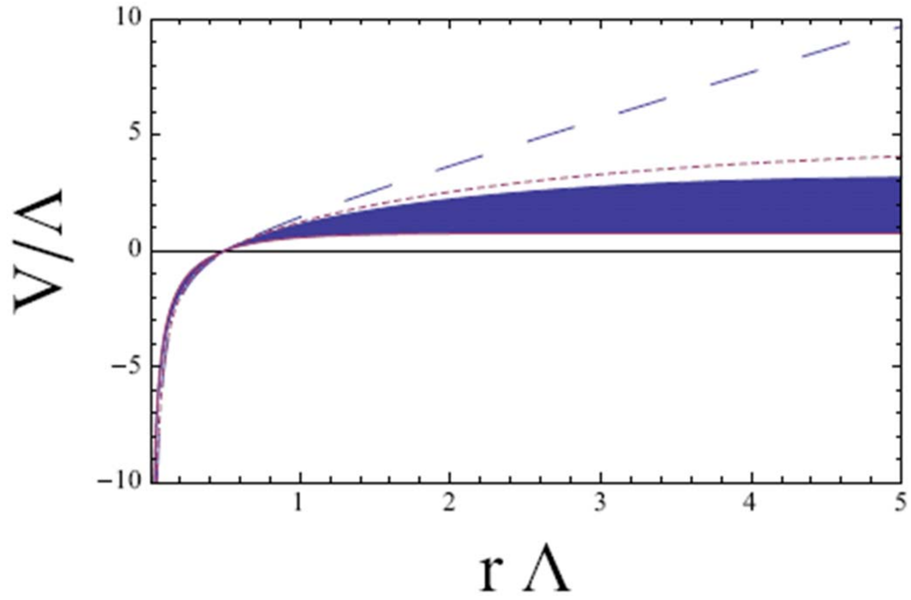
By choosing the same Sommer scale (0.35 fm) the potential fits well the quenched lattice one below 1 fm.

G. S. Bali, Phys. Rep. 343, 1 (2001)

m(0) : 360 – 480 MeV

ρ : 1 - 4

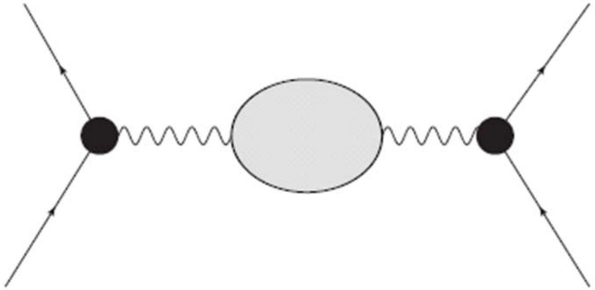




$$V(r) = -a/r + br$$

$$V(r) \sim (-\lambda/r + \mu r)\left(\frac{1-e^{-\gamma r}}{\gamma r}\right)$$

Spectral Applicability



$$q^0 \equiv (E_q)_{\text{initial}} - (E_q)_{\text{final}} = (E_{\bar{q}})_{\text{final}} - (E_{\bar{q}})_{\text{initial}} = 0$$

$$E_q = m_q\sqrt{1 + (\vec{p}^2{}_q/m_q^2)} \quad \Rightarrow \quad \frac{[(\vec{p}^2{}_q)_{\text{initial}} - (\vec{p}^2{}_q)_{\text{final}}]}{m_q^2} \ll 1$$

$$(\vec{p}_q)_{\text{initial}} = \vec{q} + (\vec{p}_q)_{\text{final}} \quad \Rightarrow \quad \frac{\vec{q}^2 + 2\vec{q}.(\vec{p}_q)_{\text{final}}}{m_q^2} \ll 1$$

$$\Downarrow$$

$$< r^2 >^{\frac{1}{2}} \simeq \frac{1}{|\overrightarrow{q}|} >> \frac{1}{m_q} \quad \Leftarrow \quad \frac{|\overrightarrow{q}|}{m_q} << 1 \qquad \frac{|\overrightarrow{p}_q|}{m_q} \not> \not> 1$$

$$m_u \sim 340 \text{ MeV}: \quad \left[< r^2 >^{1/2}\right]_{\overline{u}u} \gg 0.6 \text{ fm}$$

$$m_s \sim 500 \text{ MeV}: \quad \left[< r^2 >^{1/2}\right]_{\overline{s}s} \gg 0.4 \text{ fm}$$

$$m_c \sim 1400 \text{ MeV}: \quad \left[< r^2 >^{1/2}\right]_{\overline{c}c} \gg 0.14 \text{ fm}$$

$$m_b \sim 4800 \text{ MeV}: \quad \left[< r^2 >^{1/2}\right]_{\overline{b}b} \gg 0.04 \text{ fm}$$

Charmonium Spectrum

Cornell Potential

$$V(r) = (-a/r + br)$$

$$b \equiv \sigma \sim 0.18 \text{ GeV}^2$$
$$(\text{Regge}: \ \rho, \ \text{a}_2, \ldots)$$

$$a \sim 0.52$$

$$m_c = 1350 \text{ MeV}$$

SD OGE Potential

$\rho = 1$ $\quad\quad m(0) = 345.7$ MeV

$$m_c = 1400 \text{ MeV}$$

$$V(r) \sim (-a/r + br) \left(\frac{1 - e^{-\gamma r}}{\gamma r} \right)$$

$$\gamma = 0.38 \text{ fm}^{-1}$$

Charmonium Spectrum

$n_r L$	$M_{Cornell}$	M_{SD}	M_{PDG}
	MeV	MeV	MeV
$1s$	3069	3151	3096.916 ± 0.011
$2s$	3688	3660	3686.09 ± 0.04
$1d$	3806	3761	3772.92 ± 0.35
$3s$	4147	4004	4039 ± 1
$2d$	4228	4070	4153 ± 3
$4s$	4539	4273	4263^{+8}_{-9}
$3d$	4601	4321	$4361 \pm 9 \pm 9$
$5s$	4829	4487	4421 ± 4
$4d$	4879	4526	
$6s$	5218	4651	$4664 \pm 11 \pm 5$
$1p$	3502	3515	3525.3 ± 0.2
$2p$	3983	3886	

Caveats and Questions

Truncation scheme

+

Multigluon exchange neglected

Quenched Approach

The chosen screened Cornell potential parameterization points out the similarity with an unquenched lattice potential.

Is this similarity accidental or does string breaking imply a dilution of the confinement mechanism associated to multigluon exchanges and/or the truncation scheme ?

Physical Interpretation

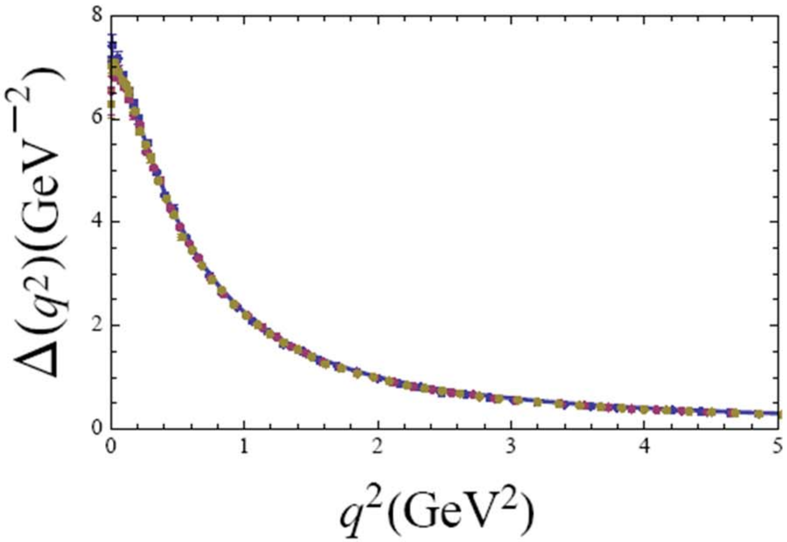
A Plausible Ad Hoc Explanation

V. Vento, FTUV-12-0501, IFIC 12-30

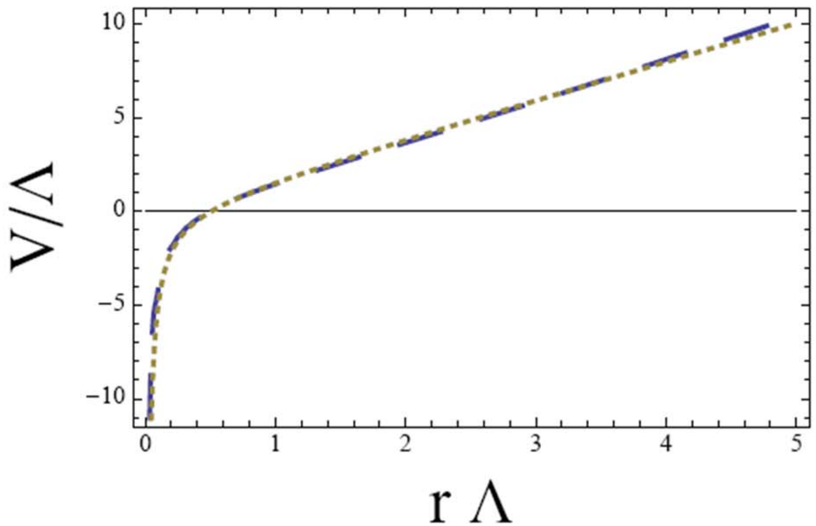
Assume a nonperturbative vertex correction to the Strong Effective Charge :

$$a_{total}(q^2) \;=\; a_{conf}(q^2) + a_{DS}(q^2)$$

$$a_{conf}(q^2) = \frac{b\Lambda^4}{q^4}$$

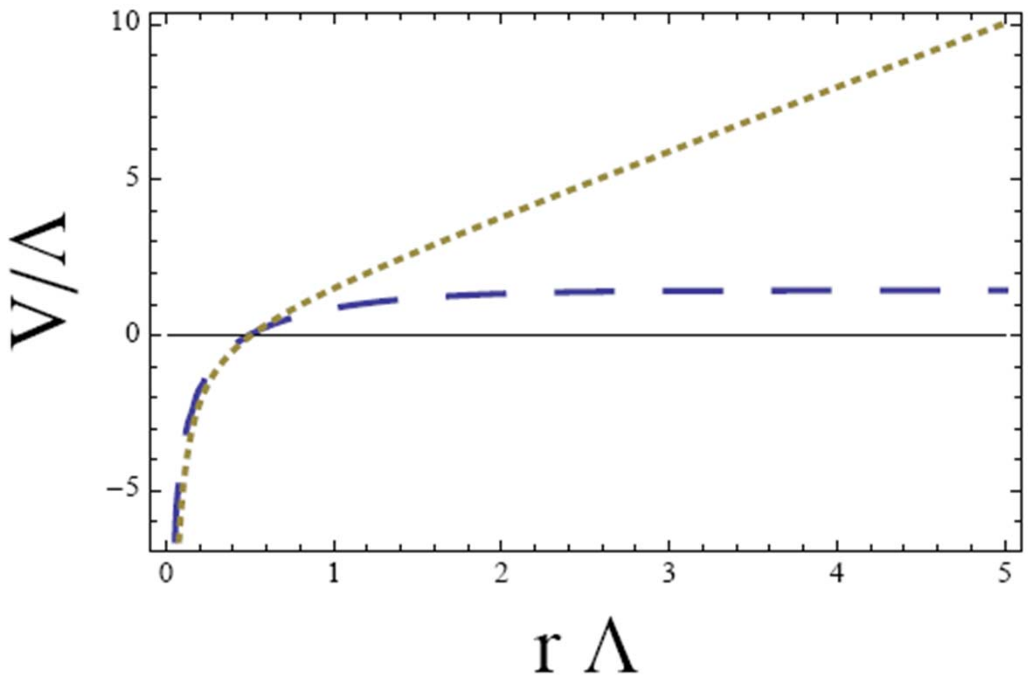


$\Delta(q^2)(\mathrm{GeV}^{-2})$
$q^2(\mathrm{GeV}^2)$



V/Λ
r Λ

Moreover if $$a_{conf}(q^2) = \frac{c\Lambda^4}{(q^2+s^2)^2}$$



If this were the solution the nonperturbative OGE Interaction Model could provide a well founded approach to heavy meson spectroscopy.

Summary

i) We have calculated the OGE static potential from an approximate solution of the quenched Schwinger-Dyson equations for the gluon propagator.

ii) The resulting OGE potential can be parametrized as a screened Cornell potential.

iii) The phenomenological application of the potential (heavy quarkonia) suggests that it might contain most of the interquark interaction dynamics.

iv) Unquenched higher order truncated Schwinger-Dyson equations solutions could provide us with a deep understanding of the meson spectrum from QCD.

THE END