Higher-order corrections to exclusive production of charmonia in NRQCD

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- Conclusions

NRQCD

• NRQCD : an effective field theory of QCD to describe the production and decay of heavy quarkonium.

$$\sigma(ij \to \mathcal{Q} + X) \sim \sum_n \hat{\sigma}_\Lambda(ij \to Q\bar{Q}(n) + X)\langle \mathcal{O}^{\mathcal{Q}}(n)\rangle_\Lambda$$

Bodwin,Braaten,Lepage,PRD51,1125(1995)

• factorization: proven in exclusive processes, but not yet in inclusive processes.

Bodwin,Tormo,Lee,PRL101(2008),PRD81(2010)

• Short distance coefficients: encode high energy effects related to the production of a heavy quark and anti-quark pair.

- expanded in terms of the strong coupling constant.

• NRQCD matrix elements: low energy effects.

- scale as the heavy-quark velocity v.

NRQCD matrix elements

• the probability to find a corresponding Fock state.

• nonperturbative, but calculable in lattice simulation in principle.

- suffers from large uncertainties.

• universal (process independent)

- holds up to corrections of v^4 in the vacuum saturation approximation.

$$\langle \text{decay ME} \rangle = \langle \text{production ME} \rangle$$

• Practically,

- color-singlet MEs: determined from exclusive electromagnetic decays of heavy quarkonium.

- color-octet MEs: fitted to the heavy-quarkonium production cross section by assuming $\langle \text{decay ME} \rangle = \langle \text{production ME} \rangle$ for the CS MEs.

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Color octet matrix elements

• inclusive production of heavy quarkonium favors the color-octet mechanism in NRQCD.

• a lot of NRQCD matrix elements are involved in inclusive production.

- 1 color-singlet, 3 color-octet matrix elements for J/ψ and several matrix elements for higher-states, ψ´ and χ_{cJ}, in the J/ψ production.
- the color-singlet matrix elements are used for inputs.

• the color-octet matrix elements are fitted to the data for the J/ψ production rate from various hadroproduction, photoproduction, two-photon scattering and electron-positron scattering.

Buthnschoen,Kniehl,1201.3862; Ma,Wang,Chao,PRL106

• the inclusive production may not be applied to test the production color-singlet matrix elements.

- have to resort to the exclusive process. (at B factories)

Higher-order corrections

1. α_s corrections

   - have been computed in a lot of processes.

2. relativistic corrections+resummation

   - have been computed in some processes.

interference

3. QED corrections

   - may be substantial for the J/ψ production. (photon fragmentation)

4. α_s^2 corrections

   - known only in $J/\psi \to e^+e^-, \eta_c \to \gamma\gamma.$

5. $\alpha_s v^2$ corrections

   - known only in $J/\psi \to e^+e^-, \eta_c \to \gamma\gamma, B_c \to l\nu.$

unknown
in the production
processes

Relativistic corrections to S-wave quarkonium

- parameterized by ratios of matrix elements of higher orders in v to the leading-order one.

$$\langle \boldsymbol{q}^{2n} \rangle_{J/\psi} = \frac{\langle J/\psi(\lambda)|\psi^\dagger(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}})^{2n}\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}{\langle J/\psi(\lambda)|\psi^\dagger\boldsymbol{\sigma}\cdot\boldsymbol{\epsilon}(\lambda)\chi|0\rangle}$$

- short-distance coefficients are determined by expanding the amplitude in terms of the relative momentum q.

$$A[J/\psi + X] = \sum_n \left[\frac{1}{n!}\left(\frac{\partial}{\partial \mathbf{q}^2}\right)^n H(\mathbf{q}^2)\right]\Bigg|_{\mathbf{q}^2=0} \left\langle \mathbf{q}^{2n}\right\rangle_{J/\psi} \left\langle O_1\right\rangle_{J/\psi}^{1/2}$$

- Gremm-Kapustin relation

Gremm,Kapustin,PLB407,323(1997)

$$\langle \boldsymbol{q}^2 \rangle \approx \epsilon_{\mathrm{B}} m_c = (M_H - 2m_c)m_c$$

ϵ_B :binding energy

- the binding energy can be obtained in the potential model.

Resummation of relativistic corrections

- generalized Gremm-Kapustin relation for S-wave quarkonium

$$\langle \boldsymbol{q}^{2n} \rangle \approx (m_c \epsilon_{\mathrm{B}})^n \approx \langle \boldsymbol{q}^2 \rangle^n.$$

Bodwin,Kang,Lee,PRD74,014014(2006)

- allows one to resum a class of relativistic corrections to all orders in v.

$$A[J/\psi + X] = \sum_n \left[\frac{1}{n!} \left(\frac{\partial}{\partial \mathbf{q}^2} \right)^n H(\mathbf{q}^2) \right] \Bigg|_{\mathbf{q}^2=0} \left\langle \mathbf{q}^{2n} \right\rangle_{J/\psi} \left\langle O_1 \right\rangle_{J/\psi}^{1/2}$$

$$= H\left(\left\langle \mathbf{q}^2 \right\rangle_{J/\psi} \right) \left\langle O_1 \right\rangle_{J/\psi}^{1/2}$$

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$$= H\left(\langle \mathbf{q}^2 \rangle_{J/\psi} \right) \langle O_1 \rangle_{J/\psi}^{1/2}$$

- Why is the resummation of relativistic corrections to all orders in v interesting?

  - confirms the convergence of expansion of relativistic corrections.

  - in a certain case, much easier to compute the resummation of relativistic corrections than v^2 corrections.

Comparison of determination of color-singlet matrix elements

- require two inputs to determine the color-singlet matrix elements.
- two ways to determine the relative order-v² matrix elements.

	BCKLY	HFC
Input experiments	$\Gamma[J/\psi \to e^+e^-]$	$\Gamma[J/\psi \to e^+e^-]$ $\Gamma[J/\psi \to \text{light hadrons}]$
Potential	Cornell	-
$\langle O_1 \rangle_{J/\psi}$	0.440	0.573
$\langle v^2 \rangle_{J/\psi}$	0.225	0.089

inputs

outputs

Bodwin,Chung,Kang,Lee,Yu,PRD77,094017(2008)

He,Fan,Chao,PRD75,074011(2007)

Comparison of determination of color-singlet matrix elements

• the main source of the small v^2 value in the HFC method is the large coefficient at order v^2 in the J/ψ→light hadrons.

$$\Gamma[J/\psi \to e^+e^-] = \frac{2e_c^2\pi\alpha^2}{3}\left((1-\frac{16\alpha_s}{3\pi})\frac{\langle 0|\mathcal{O}_1(^3S_1^\psi)|0\rangle/3}{m_c^2} - \frac{4}{3}\frac{\langle 0|\mathcal{P}_1(^3S_1^\psi)|0\rangle/3}{m_c^4}\right)$$

$$\Gamma[J/\psi \to LH] = (\frac{20\alpha_s^3}{243}(\pi^2-9))\left((1-2.55\frac{\alpha_s}{\pi})\frac{\langle 0|\mathcal{O}_1(^3S_1^\psi)|0\rangle/3}{m_c^2} - \frac{19\pi^2-132}{12\pi^2-108}\frac{\langle 0|\mathcal{P}_1(^3S_1^\psi)|0\rangle/3}{m_c^4}\right)$$

~5

He,Fan,Chao,PRD75,074011(2007)

• this implies that the BCKLY method might fail to be consistent with the decay width of J/ψ into light hadrons with v^2~0.225.

• the resummation of relativistic corrections to all orders in v might cure this problem.

$$e^+e^- \rightarrow J/\psi + \eta_c$$

- the cross section was a long standing puzzle.
  - an order of magnitude difference between theory and experiments at first.
  - later experimental values moved down while theoretical predictions moved up.

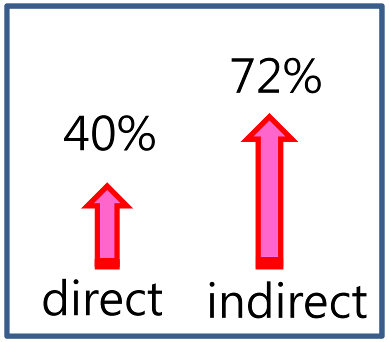
- resolved by both the QCD corrections and relativistic corrections.

- K factor from QCD corrections is 1.96.

Zhang,Gao,Chao,PRL96,092001(2006); Gong,Wang,PRD77,054028(2008)

- Relativistic corrections can come from
  - direct corrections to the short-distance process.
  - indirect corrections through the NRQCD ME.

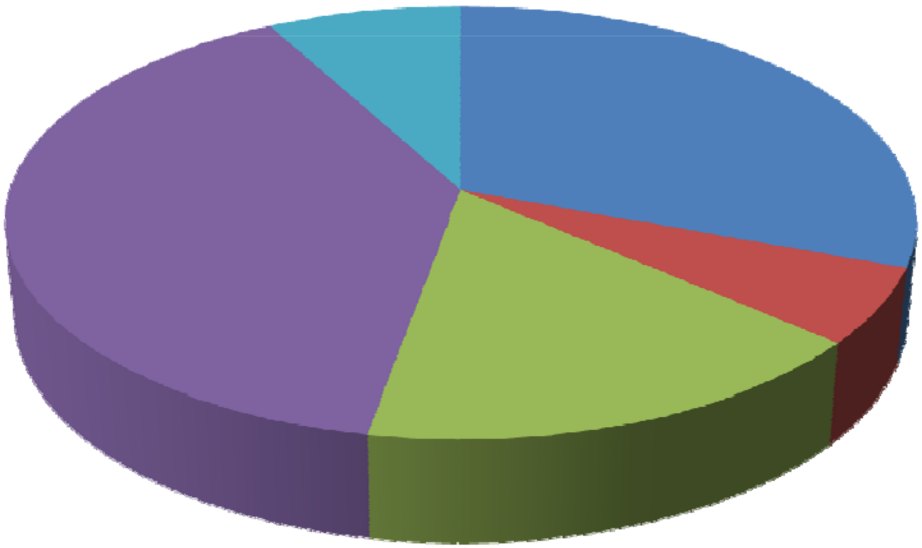
Bodwin,Lee,Yu,PRD77,094018(2008)



$$e^+e^- \to J/\psi + \eta_c$$

Belle	25.6 ± 4.4 fb
BABAR	17.6 ± 3.5 fb

Total cross section = 17.6 fb



includes indirect relativistic corrections.

LO	5.4 fb
QED	1.0 fb
Rel. Corr.	2.9 fb
QCD Corr.	6.9 fb
Interference	1.4 fb

Bodwin,Lee,Yu,PRD77,094018(2008)

$$e^+e^- \to J/\psi + \eta_c$$

• The first process to prove that the relativistic corrections are quite substantial.

• It seems that the discrepancy between theory and experiments has been resolved.

• However, there are some debatable issues.

$e^+e^- \to J/\psi + \eta_c$

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• However, there are some debatable issues.

- in experiments gathered data with at least two charged particles for η_c decay.
- how large would the cross section be if data without charged particles are included?
- NNLO corrections, increase or decrease the cross section?
- indirect $\alpha_s v^2$ corrections turned out to be small (~0.3% for J/ψ).

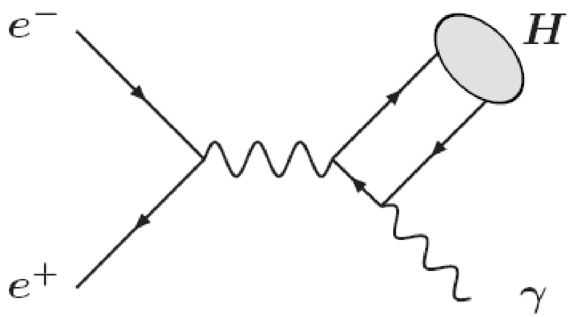
Bodwin,Chung,Lee,Yu,PRD79,014007(2009)

- direct $\alpha_s v^2$ corrections are also small?

$$e^+e^- \to \eta_c + \gamma$$

• suggested to test the color-singlet model and the convergence of relativistic corrections, especially for $\eta_c(2S)$ by Chung, Lee, and Yu.

Chung,Lee,Yu,PRD78,074022(2008)



• C=+1 quarkonium : $\eta_c, \eta_c(2S), \chi_{cJ}, \eta_b, \chi_{bJ}, X(3872)$

• may be detected by photon energy distribution.

• Now α_s corrections and v^2 corrections are available.

Sang,Chen,PRD81(2010);Li,He,Chao,PRD80(2009)

$$e^+e^- \to \eta_c + \gamma$$

Sang,Chen,PRD81(2010)

	Cross section (fb)	
Leading order	83.3	
QCD corrections	-15.3	~18%
v^2 corrections	-9.8	~12%
total	58.2	N.B. v^2~0.13.

- For v^2~0.23, v^2 corrections can reach 21%.

- the resummation of relativistic corrections to all orders in v decreases the cross section by ~17%.

PRELIMINARY

Fan,Lee,Yu, in progress

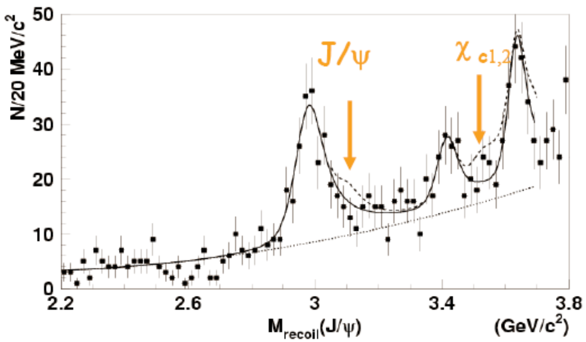
- need to combine QCD and resummed relativistic corrections.

$e^+e^- \to J/\psi + J/\psi$

- originally suggested to resolve the $e^+e^- \to J/\psi + \eta_c$ puzzle.

Bodwin,Lee,Braaten,PRL90,162001(2003)

- no evidence at Belle.



Belle,PRD70,071102(2004)

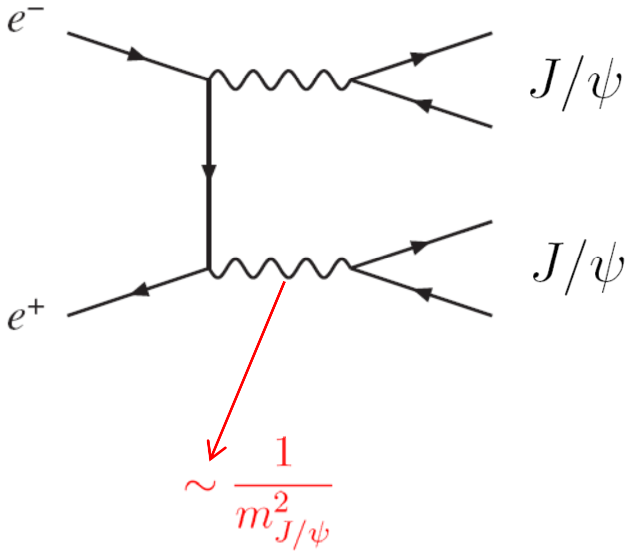
$$\sigma[e^+e^- \to J/\psi + J/\psi] \times \mathcal{B}_{>2}[J/\psi] < 9.1 \text{ fb},$$

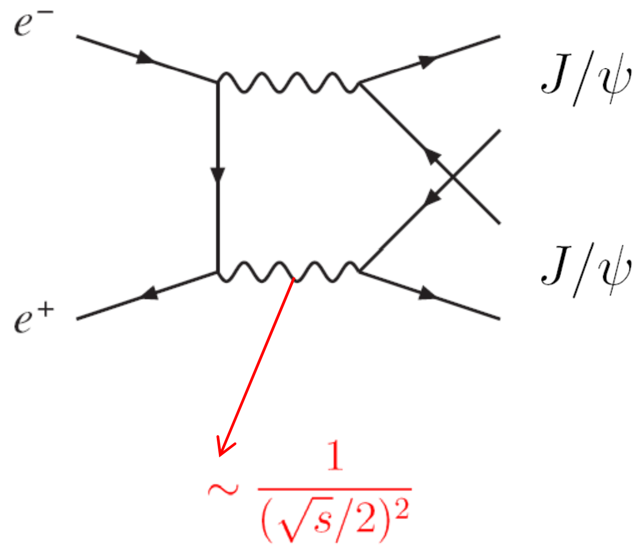
$$\sigma[e^+e^- \to J/\psi + \psi(2S)] \times \mathcal{B}_{>2}[\psi(2S)] < 5.2 \text{ fb}.$$

- disfavored by the angular distribution analysis of $e^+e^- \to J/\psi + \eta_c$ events at Belle.

$e^+e^- \to J/\psi + J/\psi$

- photon fragmentation diagram
- nonfragmentation diagram





- for the photon fragmentation diagram, the vector-meson-dominance model can be used.

Bodwin,Braaten,Lee,Yu,PRD74,074014(2006)

$$\langle J/\psi(\lambda)|J^\mu(x=0)|0\rangle = g_{J/\psi\gamma}\epsilon^\mu(\lambda)^*,$$

$$\Gamma[J/\psi \to e^+e^-] = \frac{4\pi\alpha^2 g^2_{J/\psi\gamma}}{3m^3_{J/\psi}}.$$

- for the photon nonfragmentation diagram, use NRQCD.

$$e^+e^- \to J/\psi + J/\psi$$

cross section	$J/\psi + J/\psi$	$J/\psi + \psi(2S)$	$\psi(2S) + \psi(2S)$
fragmentation	2.52 ± 0.13	1.81 ± 0.06	0.32 ± 0.02
interference	-0.98 ± 0.48	-1.09 ± 0.60	-0.30 ± 0.19
nonfragmentation	0.15 ± 0.16	0.23 ± 0.29	0.09 ± 0.14
total	1.69 ± 0.35	0.95 ± 0.36	0.11 ± 0.09

Bodwin,Braaten,Lee,Yu,PRD74,074014(2006)

- The predictions are below the upper bounds at Belle.
- Now only α_s corrections are known. Gong,Wang,PRL100,181803(2008)

m_c(GeV)	μ	$\alpha_s(\mu)$	σ_{LO}(fb)	σ_{NLO}(fb)	σ_{NLO}/σ_{LO}
1.5	m_c	0.369	7.409	-2.327	-0.314
1.5	$2m_c$	0.259	7.409	0.570	0.077
1.5	$\sqrt{s}/2$	0.211	7.409	1.836	0.248
1.4	m_c	0.386	9.137	-3.350	-0.367
1.4	$2m_c$	0.267	9.137	0.517	0.057
1.4	$\sqrt{s}/2$	0.211	9.137	2.312	0.253

$$e^+e^- \to J/\psi + J/\psi$$

- v^2 corrections have not been computed yet.

- v^2 corrections are quite complicated because the t-channel electron propagator contains the relativistic corrections.

- in this case, the resummation of relativistic corrections to all orders in v is more promising.

$$\frac{\sigma_{\mathrm{LO,v}}}{\sigma_{\mathrm{LO}}} \sim 0.42$$

PRELIMINARY

Fan,Lee,Yu, in progress

- need to combine α_s and relativistic corrections.

- Very small K factors can be reduced by using the VMD coupling for the photon fragmentation diagrams.

Conclusions

• The exclusive production of charmonia provides a unique opportunity to test a color-singlet model with relativistic corrections.

• We calculated the resummation of relativistic corrections to $e^+e^- \to \eta_c + \gamma$ and $e^+e^- \to J/\psi + J/\psi$ and find that its effects are about 17% and 52%, respectively.

• We anticipate that the cross sections for these exclusive processes will be measured at super B factories.

Thank you for your attention!